Perfect Symmetries

Abstract

While empirical symmetries relate situations, theoretical symmetries relate models of a theory we use to represent them. An empirical symmetry is *perfect* if and only if any two situations it relates share all intrinsic properties. Sometimes one can use a theory to explain an empirical symmetry by showing how it follows from a corresponding theoretical symmetry. The theory then reveals a perfect symmetry. I say what this involves and why it matters, beginning with a puzzle which is resolved by the subsequent analysis. I conclude by pointing to applications and implications of the ideas developed earlier in the paper.

1. Introduction

The importance of symmetry principles in physical theory is now widely acknowledged among both physicists and philosophers. Reflection on increasingly abstract symmetries has become an important heuristic in theory construction.[1] Philosophers have offered analyses of various kinds of symmetry that a theory may display, and of the relations between these.[2] Not all symmetries of a theory correspond to symmetries in nature, even when that theory succeeds in representing significant features of our world. Sometimes a theoretical symmetry is broken, as when a theory's equations have a solution that lacks their symmetry. Sometimes a theoretical symmetry associated with alternative representational devices may have no empirical consequences. But in an important class of cases one can use a theory to account for an empirical symmetry by exhibiting that symmetry as a consequence of a symmetry of the theory. Such explanations are especially satisfying and may provide convincing reasons to believe the theory that makes them possible. In such a case I shall say that the theory reveals a *perfect symmetry*.

A theory reveals a perfect symmetry when a theoretical symmetry implies a corresponding empirical symmetry. In what follows I say what this involves and why it matters. Section 2 presents a puzzle to motivate the analysis to come. Section 3 distinguishes various kinds of empirical symmetry and illustrates them with examples. Section 4 presents an analysis of one kind of theoretical symmetry and contrasts this with theoretical symmetries of other kinds. Section 5 explores different ways in which a theory can explain an empirical symmetry, focusing on the exhibition of a perfect symmetry: this section resolves section 2's puzzle. I conclude by pointing to applications and implications of the ideas developed earlier in the paper. One Appendix exhibits an important joint model, and another presents details of an argument from section 5.

2. Is Faraday in the Same Boat as Galileo?

Here are two examples of observations of a symmetry in nature.

Galileo described the first example in his *Dialogue Concerning the Two Chief World Systems*, which contains the following famous passage.

> Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though there is no doubt that when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. (Galileo [1632/1967], pp. 186-7)

The observations Galileo describes provide evidence that setting a confined system in uniform rectilinear motion has no noticeable effect on mechanical processes taking place within the system, and is in that sense a symmetry of those situations. This symmetry is closely related to the following relativity principle, often called the principle of Galilean relativity[3]:

> Situations related by a transformation from one state of uniform rectilinear motion to another are internally indistinguishable.

A uniform relative velocity transformation is a symmetry of such situations.

Galileo's ship provides a classic illustration of an empirical symmetry. His description of phenomena in its cabin in different states of motion supplies rich instances of that symmetry that are apparent even in the absence of a theory capable of accounting for them. Stimulated by these and other phenomena, Newton later found a dynamical theory of which Galilean relativity is a consequence, provided that the situations involve only mechanical processes conforming to his laws of motion, and that all masses and forces involved are independent of absolute velocity. Still later Einstein formulated a different theory (special relativity) that implies a reinterpreted extension of Galileo's relativity principle to all (non-gravitational) processes. I shall examine this implication in more detail in section 5. For now, only two things matter. First, observations like those Galileo describes in this passage provide evidence that uniform velocity boosts are empirical symmetries of such situations. Second, these observations could be, and indeed were, made prior to and independent of any theory capable of accounting for them.

Michael Faraday's description of his own observations provides the second example. In 1836, Faraday constructed a hollow cube with sides 12 feet long, covered it with good conducting materials but insulated it carefully from the ground, and electrified it to such an extent that sparks flew from its surface. An entry in his diary entry during May 1836 reads in part

> I went into this cube and lived in it, but though I used lighted candles, electrometers, and all other tests of electrical states, I could not find the least influence upon them. (Maxwell [1881], p. 53)

Here Faraday's observations bear on another symmetry in nature: charging the conducting exterior of a confined region of space has no effect on electrical phenomena in the interior of that region. These observations are often glossed by saying that by electrifying his cube Faraday had succeeded in raising it to a higher electric potential than the rest of his laboratory. That suggests the following relativity principle

Situations related by a transformation from one state of uniform electric potential to another are internally indistinguishable.

A uniform electric potential transformation is a symmetry of such situations. Taking this symmetry to heart, Maxwell developed the theory of electromagnetism that bears his name. Indeed, Maxwell's electromagnetic theory accounts for the empirical symmetry observed by Faraday in his cube. But Faraday needed no such theory to make his observations or to appreciate the importance of the empirical symmetry that grounded them.

In these passages, Faraday and Galileo describe observations of symmetries in nature. In each case, different situations are compared, and it is noted that these are indistinguishable with respect to a whole class of phenomena. The parallels are striking. In each case, the relevant symmetry was observed before the development of any theory capable of accounting for it. In each case, observation of an empirical symmetry stimulated the successful construction of a theory capable of explaining why it obtains.

The parallels appear to extend to internal features of the explanatory theories themselves. Uniform velocity boosts are among the Galilean transformations that constitute symmetries of the dynamics of Newton's theory, and also among the Lorentz transformations that constitute symmetries of the dynamics of special relativity. The addition of a constant to all electric potentials is among the local gauge transformations that constitute symmetries of Maxwell's electromagnetic theory. By a *local* gauge transformation in this context I mean a transformation in the electromagnetic 4-vector potential A_μ which takes the form $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$ for some suitably smooth function $\Lambda(\boldsymbol{x},t)$ of the space-time coordinates $(\boldsymbol{x},t)$.[4] The electric potential ϕ is the time-component of the 4-vector potential $A_\mu = (\phi, -\boldsymbol{A})$, where $\boldsymbol{A}$ is the magnetic vector potential.

Setting $\Lambda=kt$ induces the local gauge transformation $(\phi, -\mathbf{A}) \rightarrow (\phi+k, -\mathbf{A})$: so this class of local gauge transformations simply adds a constant k to the value of the electric potential everywhere.

But now a puzzle emerges. For while the Galilean and Lorentz boost symmetry of Newton's and Einstein's theories each in its own way reflects the empirical relativity of motion, local gauge symmetry is usually thought of as a purely formal feature of classical electromagnetic theory, with no empirical correlate, and indeed no empirical content. If this is correct then, as Brown and Brading ([2004], p. 657) say 'there can be no analogue of the Galileo ship experiment for local gauge transformations.' But then why doesn't Faraday's cube provide a *perfect* analogue of Galileo's ship for a class of local gauge transformations of classical electromagnetism? To solve this puzzle it will be necessary to achieve a better understanding of different ways in which the symmetries of a theory may be related to the empirical symmetries it explains. The first step is to get clearer on the nature of empirical and theoretical symmetries themselves.

3. Empirical Symmetries

Abstractly, a symmetry of a structure is an automorphism–a mapping of the elements of the structure back onto themselves so as to preserve the structure. Formally, a structure $S = \langle D, R_1, R_2, \ldots, R_n \rangle$ consists of a domain D of elements and a sequence of relations R_i ($i=1,\ldots,n$) defined on D. Let $f : D \rightarrow \mathrm{D}$ be a 1-1 mapping of the domain of S onto itself. Define the transformed structure S_f by $S_f \equiv \langle D, f^*R_1, f^*R_2, \ldots, f^*R_n \rangle$, where, for each m-place relation R_i, $f^*R_i\,[f(d_1), \ldots, f(d_m)]$ if and only if $R_i\,[d_1, \ldots, d_m]$. Then f is a *symmetry of S* just in case $S_f = S$.[5]

Many different structures may be distinguished in a given object. If the structure is that of a physical object, the elements of D will generally be parts of that object, while the R_i specify properties of and relations among these parts. A physical object may have a certain size, shape, composition and pattern of colors. But abstract objects also exhibit a variety of structures. SU(2) is a group, it is non-Abelian, it is a Lie group (and so also a differentiable manifold), it is compact, it is simple, etc. We are concerned here with physical theories and the situations to which they may be applied, so we need to say what kinds of object these involve.

A physical theory specifies a set of models—mathematical structures—that may be used to represent various different situations, actual as well as merely possible, and to make claims about them. Any application of a physical theory is to a situation involving some system, actual or merely possible. Only rarely is that system the entire universe: typically, one applies a theory to some subsystem, regarded as a relatively isolated part of its world. The application proceeds by using the theory to model the situation of that subsystem in a way that abstracts from and idealizes the subsystem's own features, and also neglects or idealizes its interactions with the rest of the world.

A system may itself display a symmetric structure at a certain level of idealization. A human body has a rough bilateral symmetry, while a carefully prepared crystal more precisely displays a variety of symmetries. A single system may also display symmetric behavior: consider the motion of a pendulum, or the performance of a mirror fugue. The symmetric structure in this case is that of the situation in which the system figures rather than that of the system itself. There are even cases in which it is hard to draw a distinction between system and situation. Think of a possible world set in Newtonian space-time. A temporal inversion about an arbitrary temporal instant is a symmetry of the space-time of this world.

A symmetry involving an actual physical system and its situation is empirical. One can observe and measure the situation of the system to collect evidence that relations among its spatiotemporal parts are or would be indifferent to the action of a symmetry transformation of those parts. Such evidence may be direct (the crystal) or indirect (Newtonian space-time).

But Galileo's ship illustrates the fact that not all empirical symmetries pertain to a single situation. In that case velocity boosts are observed to be a symmetry of a class of similar but distinct situations, in each of which the ship is moving with a different velocity. Renderings of a particular tune provide another example of a symmetry of a class of situations. Renderings of the same tune all have a similar structure, even if they are in different keys and some include errors. Flawless renderings of the same tune are related by a symmetry transformation that transposes one key into another.

In such cases, a class of different situations constitutes the domain of a structure, and a symmetry of that structure maps one situation onto another. Formally, suppose that situations of a certain kind K all have a somewhat similar structure. Any situation with that structure may be transformed into another by a transformation f. If a subset D of K is closed under f, then f is a symmetry of the "larger" structure $\Sigma = < D, P_j >$, where the properties P_j define the kind K.

Galileo describes processes occurring in the cabin of a ship as having just the same dynamic structure, independent of how fast the ship moves over the sea. Different instances of each process are related by the same symmetry transformation—corresponding to a change in the uniform horizontal velocity of the ship. Here one can think of K as a class of kinematically possible motions of the objects in the cabin (i.e. those motions relative to the cabin in which every object has some continuous trajectory or other), while D contains only dynamically possible motions (i.e. just those kinematically possible motions of objects that are compatible with the forces acting on them). Renderings of the same tune (*K*) all have a somewhat similar structure, even if they are in different keys and some include errors. Flawless renderings of the same tune (*D*) are related by a symmetry transformation that transposes one key into another.

One can observe and measure situations to collect evidence that situations related by a symmetry transformation cannot be distinguished by specific procedures. Flawless renderings of a tune in different keys can be directly distinguished by someone with perfect pitch: the rest of us may need instruments. According to Galileo, measurements of purely mechanical magnitudes inside the cabin cannot distinguish between different states of uniform horizontal motion of a ship. These are two examples of the kind of empirical symmetry among situations which will be at the focus of interest here. They prompt the following abstract formulations.

As I will understand it, an empirical symmetry is a feature of a class of situations—actual as well as possible. A 1-1 mapping $\varphi : \mathcal{S} \rightarrow \mathcal{S}$ of a set of situations onto itself is an *empirical symmetry with respect to C-type measurements* if and only if no two situations related by φ can be distinguished by measurements of type *C*.

This is a contextual definition, since what it counts as an empirical symmetry depends on what measurement procedures are considered. In the case of Galileo's ship and Faraday's cube one context is particularly salient. It would be easy to observe the ship's motion over the ocean by hearing the ship's wake or viewing it through a porthole, or by consulting a GPS device in the cabin. It would have been only too easy for Faraday to observe the charge state of his cube by carelessly stepping out into his laboratory, or more safely by looking for the sparks emitted by the cube when charged. Such observations involve measurement procedures that provide (more or less) reliable information about the *relation* between the situation inside the cabin/cube and its external environment. In an idealization of the situation in which the interior is regarded as confined in such a way as to exclude any transmissions from outside, no such information is available within. Observations and measurements inside can then only provide information about the intrinsic properties of the internal situation.

I shall say that a measurement is *confined to a situation* if and only if it is a measurement of intrinsic properties of (one or more objects in) that situation.[6] Then a 1-1 mapping $\varphi : \mathcal{S} \rightarrow \mathcal{S}$ of a set of situations onto itself is a *strong empirical symmetry* if and only if no two situations related by φ can be distinguished by measurements confined to each situation. Note that the reference to measurement is not superfluous here, in so far as a situation may feature unmeasurable intrinsic properties. We shall see an example of this soon.

Spatial translations and rotations provide familiar examples of strong empirical symmetries of situations involving geometrical figures in Euclidean space. If S is any figure in Euclidean space, then a translation and/or rotation φ yields a congruent figure $\varphi(S)$. Note that situations in $\mathcal{S}$ related by a transformation φ may be in the same or different possible worlds: if φ is a strong empirical symmetry, then $\varphi(S)$ may be in the same world w as S, but only if w is itself sufficiently symmetric.

Uniform velocity boosts are strong empirical symmetries of a set of situations involving purely mechanical phenomena in a Newtonian world, since a Galilean boost by velocity $\boldsymbol{v}$ applied to the situation S of a mechanical system in such a world yields a situation $\varphi_v(S)$ that is indistinguishable from S with respect to all measurable intrinsic properties. The special principle of relativity guarantees that uniform velocity (Lorentz) boosts are also strong empirical symmetries of electromagnetic and all other phenomena in a special relativistic world. Even when a situation *S* actually obtains, $\varphi_v(S)$ will rarely do so. In some cases, careful laboratory manipulations may actually bring it about, but the situation $\varphi_v(S)$ will more typically obtain only in some "merely" (i.e. non-actual) possible world. Galileo's ship provides at best a rough and approximate realization of the relativity principle it is used to dramatize. A turbulence-free aircraft in level flight, and a spaceship whose rockets are not firing, supply observable situations that more closely realize strong empirical symmetries associated with uniform velocity boosts.

This is typical of the observational status of empirical symmetries. Kosso ([2000]) stated two necessary conditions for the observation of a symmetry of interest to physics. One must observe that the specified invariant property is in fact the same, before and after, and one must observe that the specified transformation has taken place. It follows that an empirical symmetry may or may not be observable. It may be too hard to create the necessary situations, or to find them realized in nature: and one may not be able to certify that one has indeed encountered the right situations. What makes a symmetry empirical is just that the necessary measurements would reveal it if they could be performed in actual situations. But as the examples of Galileo and Faraday show, one may be able to observe an empirical symmetry whether or not one has a theory that accounts for it.

Even though uniform velocity boosts are strong empirical symmetries of mechanical processes in a Newtonian world, they do effect a significant change in a mechanical system. According to Newton, a uniform velocity boost changes the absolute velocity of a system to which it is applied. But since Newton's theory itself implies that this change is not *measurable* (assuming that all masses and forces are independent of absolute velocity), no measurements on mechanical systems confined to situations related by a uniform velocity boost can distinguish between those situations.

While uniform velocity boosts are strong empirical symmetries of mechanical processes in a Newtonian world, in a special relativistic world they are not merely strong but perfect symmetries of all processes. An empirical symmetry φ is *perfect* if and only if any two situations related by φ are duplicates, where a *duplicate* of a situation is a situation that shares all its intrinsic properties. Every perfect symmetry is strong, but the converse does not hold, as is apparent from the example of uniform velocity boosts in the Newtonian world of mechanical systems: In such a world, subjecting a mechanical process to a uniform velocity boost does not produce a duplicate process since the absolute velocity of every object is now different.

4. Theoretical Symmetries

One should distinguish symmetries of a set of situations to which a theory may be applied from symmetries internal to that theory. One place to look for theoretical symmetries in a dynamical theory of physics is in its equations of motion. Since these equations pick out a class of dynamically possible models, one can alternatively focus on symmetries of this class of models. It is not necessary to endorse any version of the so-called semantic conception of scientific theories to acknowledge that many physical theories, as well as theories in other sciences, are often conveniently characterized by specifying the class of models associated with the theory. Here models are structures (typically mathematical) that may be used to represent situations. So an analysis of a theoretical symmetry as a transformation that maps models of a theory onto other models may be expected to be widely applicable. But what kind of transformation?

On the broadest conception, a theoretical symmetry would be any 1-1 function from the set of a theory's models onto itself. But while this is a symmetry of the theory in the sense that it leaves its model class invariant, it is too broad to be of much interest. As Ismael and van Fraassen ([2003]) noted, there are theoretical symmetries in this sense that transform a model of Newton's theory with one free particle into models with millions of particles interacting in complex ways. As an automorphism of the model class of a theory, an interesting theoretical symmetry should preserve more of the internal structure of the models it relates: cardinality of that structure's domain is only one very weak requirement.

Ismael and van Fraassen ([2003]) entertain another condition: that a theoretical symmetry preserve *qualitative* features of every model. They take such features to be 'quantities that can characterize a situation, distinguishable by even a gross discrimination of colour, texture, smell and so on.' (p. 376), where (as they have explained) a quality can be regarded as a quantity with range of values 1 (possessed) and 0 (not possessed). To maintain the present clear distinction between models and situations, one should rather characterize qualitative features *of a model* as those elements of the model that may serve to represent qualitative features (in their sense) of situations.[7] They distinguish this condition from a stronger condition—that a theoretical symmetry preserve *measurable* features of a model, where these generally extend beyond qualitative features in a theory-guided way. Newtonian theory, for example, connects the masses and forces its models are intended to represent to qualitative features such as positions and times in such a way as to permit the measurement of the former by observation of the latter.

In the case of space-time theories, a theoretical symmetry might be required to preserve those features of its models that serve to represent space-time structure, which gives rise to the notion of a space-time symmetry of a theory. So, for example, space-time translations and rotations are space-time symmetries of a Newtonian theory, while Galilean boosts are space-time symmetries only if that theory's models do not permit the representation of a privileged state of absolute rest.

The interesting relations among these and related conceptions of theoretical symmetry have been explored elsewhere.[8] But there is one conception of theoretical symmetry that is narrower than any of them and may at first sight seem to be of little interest. Perhaps surprisingly, this is the conception that will shed the most light on perfect symmetries. Accordingly, I will say that

> A mapping $f : \mathcal{M} \rightarrow \mathcal{M}$ of the set of models of a theory Θ onto itself is a *theoretical symmetry of* Θ if and only if the following condition obtains:
>
> For every model *m* of Θ that may be used to represent (a situation *S* in) a possible world *w*, *f*(*m*) may also be used to represent (*S* in) *w*.
>
> Two models related by a theoretical symmetry of Θ are *theoretically equivalent in* Θ.

Thus defined, the theoretical symmetries of a theory would include only the identity mapping if no two of its models could be used to represent the same situation. But we know of many a theory with a redundant set of models. Among gauge theories, such redundancy is the norm. In analytic Euclidean geometry, a spatial configuration has many algebraic models, corresponding to a choice of spatial origin, type of coordinates (rectangular Cartesian, cylindrical polar, spherical polar, etc.) and choice of coordinate axes and their orientation. Even in a coordinate-free formulation, general relativity permits the representation of a possible world by any model from within an equivalence class of diffeomorphically related models.

The redundancy of representational devices is both a resource and a danger for the theorist. It permits one to choose whichever of a set of theoretically equivalent models offers the most convenient representation of a given situation, and so simplifies its treatment within the theory. But it may also mislead one into believing that a mere choice of representational device has empirical significance. Einstein himself was so misled while struggling to formulate his general theory of relativity.[9] For a while he was convinced by the so-called "hole" argument that the theory could not be generally covariant, precisely because he had not yet come to appreciate that diffeomorphically related models may be taken to represent the same physical situation.

This conviction was not the result of a simple mistake. The case of analytic geometry is atypical. It is often hard to say when distinct models of a theory simply represent the same situation, and when they may represent distinct situations. It is especially hard when it is the theory itself that provides our only initial access to those features of situations it represents by newly introduced structures—hard, but not impossible. As the next section will show, there are cases when a theoretical symmetry of a theory itself implies an empirical symmetry of situations it models. In such cases the empirical symmetry is perfect. While models related by a theoretical symmetry may always be used to represent the very same situation, here they may be used alternatively to represent distinct but intrinsically indistinguishable situations.

5. Explaining Empirical Symmetries

Even when a theory explains an empirical symmetry, the explanation need not appeal to a theoretical symmetry of that theory. Newton's explanation of the principle of Galilean relativity provides an example.

Assuming that no masses or forces depend on absolute velocities, Newton's laws of motion entail that uniform velocity boosts are strong empirical symmetries of a set of situations involving purely mechanical phenomena in a Newtonian world. One can use Newton's theory to explain the principle of Galilean relativity by noting that we live in an approximately Newtonian world in which Galilean relativity (restricted to mechanical phenomena) is a consequence of this strong empirical symmetry of uniform velocity boosts. But a non-zero uniform velocity boost is not a theoretical symmetry of Newton's theory. Two models related by such a transformation *cannot* be used to represent the same situation, since each would represent the absolute velocity of every object in that situation differently.

Uniform velocity boosts are dynamical but not theoretical symmetries of Newton's theory. Nor are they space-time symmetries of the Newtonian space-time structure on which Newton based his theory. Earman ([1989]) argued that any theory whose dynamical symmetries exceeded its space-time symmetries was not well-formulated, and that it posited excess spatiotemporal structure. In this case, the excess structure is provided by Newton's enduring absolute space, the trajectories of whose points define a privileged but unobservable state of absolute rest. By eliminating from the models of Newton's theory anything capable of representing this superfluous theoretical structure, one arrives at an empirically equivalent theory, set in what Geroch ([1978]) calls Galilean space-time, of which uniform velocity boosts *are* theoretical symmetries. In this revised version, Newtonian theory explains the symmetries associated with Galilean relativity by appeal to these theoretical symmetries. It thereby exhibits uniform velocity boosts as not merely strong but *perfect* symmetries of mechanical systems in a Newtonian world.

Einstein ([1905]) extended the set of situations among which uniform velocity boosts are empirical symmetries to situations involving non-mechanical phenomena, and specifically electromagnetic phenomena. His reasoning in that paper is interesting. It begins as follows 'It is known that Maxwell's electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena.' The paper goes on to defend a new understanding of Maxwell's theory whose application to moving bodies leads to no such asymmetries. Of course, this new understanding involves the radical changes in how we conceive of space and time for which Einstein is famous. But from the present perspective what is striking is the way in which Einstein was able successfully to fulfill his desire to provide an alternative theoretical explanation of the empirical symmetries of electrodynamic phenomena *which eliminated the theoretical asymmetries* in the existing account. Within the new space-time structure, Maxwell's theory explained the strong empirical symmetries of electrodynamic phenomena by deriving them from a theoretical symmetry of the theory. It thereby exhibited these as perfect symmetries.

These two cases exhibit a very similar structure. We begin with an empirical symmetry. This is then explained as a consequence of a theory. The explanation entails that the empirical symmetry is strong, but not perfect. It does not appeal to a theoretical symmetry of the theory itself. But the theory is perceived as in some measure defective, since its models mark a distinction between situations—a distinction which that theory itself implies is neither observable nor measurable by us. Indeed, it is the same distinction in each case—the distinction between one uniform absolute velocity and another. After the theory has been reformulated, the defect is remedied. A structure is removed from the models that had been intended to represent the problematic, unobservable feature of these situations. This introduces a new theoretical symmetry into the theory, which serves as the basis for a new kind of explanation of the empirical symmetry that reveals this to be not merely strong but perfect.[10]

In both these cases, a theory comes to reveal a perfect symmetry by implying that distinct but symmetrically related situations are in fact duplicates of one another—they share all the same intrinsic properties. This may seem surprising. As analyzed in section 3, a theoretical symmetry concerns only a single situation. How can it imply that two distinct situations are duplicates? Moreover, the puzzle presented in section 2 suggests that there may be theories whose theoretical symmetries carry no such implication. To remove the surprise and solve the puzzle we need to exhibit the mechanism by which a theoretical symmetry of a theory may be transferred to an empirical symmetry of situations its models represent. It turns out that this mechanism will work only if the theory has certain special features.

How exactly does the theory of special relativity imply that Lorentz boosts are perfect symmetries of situations represented by its models? Here is a sketch of the argument: further details are spelled out in Appendix A. Consider a localized situation *S* represented by model *m* of the special theory of relativity. *S* is localized because it occupies a compact region of space-time. Because Lorentz boosts are theoretical symmetries of the theory, *S* is equally represented by a Lorentz-boosted model $m'=\Lambda_{-\mathbf{v}}(m)$: one can think of m' as representing *S* from the perspective of a frame F' moving at velocity $\mathbf{v}$ with respect to a frame *F* from whose perspective *S* is represented by *m*. Now special relativity itself implies that *m* also represents a distinct situation $S'=\Lambda_{\mathbf{v}}(S)$ in exactly the same way that m' represents *S*. Since *m* and m' represent *S* as having exactly the same intrinsic properties, it follows that *m* represents *S* and S' as having exactly the same intrinsic properties. Hence special relativity implies that Lorentz boosts are perfect empirical symmetries of all localized situations to which the theory applies.

But doesn't an exactly parallel argument show that a theoretical gauge symmetry of classical electromagnetic theory (addition of a constant to the value of the electric potential at each point inside the cube) implies a corresponding perfect empirical gauge symmetry of the situation inside Faraday's cube? It does not, since there is a crucial disanalogy between this case and the case of Lorentz boosts in special relativity. Special relativity itself implies that situation S' differs from situation S precisely by application of a Lorentz boost, thereby justifying the equation $S'=\Lambda_{\boldsymbol{v}}(S)$. It implies that these situations differ in just this way, because every joint model in the theory of the combined situation $S\oplus S'$ represents S, S' as related by a Lorentz boost. But classical electromagnetic theory has no corresponding implication.

Classical electromagnetic theory does have joint models of a combined situation $C_0\oplus C_+$ involving the interiors C_0, C_+ of uncharged and charged Faraday's cubes (respectively) that represent the electric potentials inside the cubes by functions differing by a constant k. But it also has joint models of that *same* situation $C_0\oplus C_+$ that represent the electric potentials inside the cubes by identical functions.[11] So while an argument parallel to that in the case of Lorentz boosts in special relativity does establish the *existence* of a situation C_+ distinct from C_0, it does not prove that C_+ is related to C_0 by an empirical gauge symmetry. Local gauge symmetry is merely a theoretical symmetry of classical electromagnetism; and local symmetries of the form $(\phi, -\boldsymbol{A}) \rightarrow (\phi + k, -\boldsymbol{A})$ imply no corresponding empirical gauge symmetry, perfect or not.

The relation between C_+ and C_0 is worth considering further, because these situations are related by an empirical symmetry that does *not* correspond to any empirical gauge symmetry of classical electromagnetism. Note first that C_+ is related to C_0 by a space-time translation—certainly a perfect symmetry of classical electromagnetism, but by no means an empirical gauge symmetry. But the empirical symmetry relating C_+ and C_0 may appear richer and more interesting than a mere space-time translation.

The charge accumulated on the surface of the charged cube will give rise to an electric field outside the cube(s). Every joint model in classical electromagnetic theory of a situation $C_0 \oplus C_+$ incorporating C_+ and C_0 must represent this electric field. The line integral L of this electric field along a curve joining any two points, one in the interior of the charged cube, the other in the interior of the uncharged cube, is an invariant of all situations $C_0 \oplus C_+$ incorporating both C_+ and C_0.[12] So classical electromagnetic theory implies that the transformation $C_0 \rightarrow C_+$ is an empirical symmetry that seems to be more than just a space-time translation, and indeed this is a perfect empirical symmetry.

But while the value of L represents a relation between C_0 and C_+, this relation obtains only by virtue of their relations to the electromagnetic situation outside the cube(s). In David Lewis's ([1986, p. 62]) terminology, it is neither an internal nor an external relation between C_0 and C_+. I follow custom in calling a relation (e.g. *having the same owner*) whose obtaining depends on relations between its relata and some distinct object (e.g. the owner) an *extrinsic* relation. The line integral of the electric field joining points in the interior of the cube(s) represents an extrinsic relation between C_0 and C_+. To suppose one can change the electromagnetic condition inside Faraday's cube by charging its exterior is just as mistaken as to think one can move a car from New York to Los Angeles merely by selling it.

Despite appearances, according to classical electromagnetism the only empirical symmetry relating C_+ and C_0 is a space-time translation. This is not an empirical correlate of the theoretical local gauge transformation of adding a constant (L) to the value of the electric potential at each point inside the cube. Moreover, a constant difference in electrostatic potential is not an invariant of the models of $C_0 \oplus C_+$ within classical electromagnetism. Contrast this with the case of Lorentz boosts in special relativity, where every model agrees on the size of the velocity difference between boosted and un-boosted situations.[13] In classical electromagnetic theory, the perfect empirical symmetry between C_+ and C_0 does not reflect a theoretical gauge symmetry.

Another way to see this is to note that this empirical symmetry is also a consequence of a formulation of classical electromagnetism whose models include no gauge potentials, but only electric and magnetic fields. While these suffice to exhibit the extrinsic difference between C_+ and C_0 represented by the line integral of the electric field between them, the models of this formulation admit no gauge transformations of which this is an invariant magnitude.

It is natural to describe the state of Faraday's cube when charged by saying that it has been raised to an electric potential with respect to the ground. But this is not something that we observe—all we observe are differences in electric field outside the cube when charged and uncharged. And it is not entailed by classical electromagnetic theory, since that theory has models that represent this electric field as arising not from a difference in electric potential, but from variations in magnetic vector potential (see Appendix B).

It is interesting to note that the empirical symmetry between C_+ and C_0 *does* reflect a theoretical symmetry of a *different* theory. Classical electrostatics with potentials can model C_0, C_+ and $C_0 \oplus C_+$. But in classical electrostatics the only models of $C_0 \oplus C_+$ are those that represent two distinct cubes at rest with respect to one another. Moreover, every such model represents C_0, C_+ as at different electrostatic potentials ϕ_0, ϕ_+ respectively. In this way, classical electrostatics represents a relation between C_+ and C_0 that is *external* rather than extrinsic. This theory not only implies a perfect *electrostatic* empirical symmetry between C_+ and C_0: it also implies that this uniquely corresponds to the theoretical symmetry of adding a constant to the value of the electric potential at each point in a model of a localized situation. In classical electrostatics, but not the full theory of classical electromagnetism, this empirical symmetry follows from a corresponding theoretical symmetry that might well be regarded as a gauge symmetry.

In a world with static electricity but no magnetism, the empirical success of electrostatics might have justified belief in the empirical adequacy of a theory in which the only joint models of the combined situation $C_0 \oplus C_+$ represented an electric potential difference between the cubes of the same constant value $\Delta\phi$. In such a world, the limited theoretical "gauge" symmetry $\phi_0 \rightarrow \phi_+ = \phi_0 + \Delta\phi$ would have implied a corresponding empirical gauge symmetry. We would have had indirect empirical evidence that differences in electric potential are real, as are transformations from one state of electric potential to a state of lower or higher potential—just as, in our world, we have (both direct and) indirect evidence that differences in uniform velocity are real, as are transformations from one uniform velocity to another.

6. Conclusion

I conclude by drawing some morals from this investigation of perfect symmetries. The first moral is that most, if not all, empirical symmetries relate localized rather than global situations.

Sometimes a symmetry masquerades as empirical when it is a merely theoretical symmetry of a theory's global models. Gauge theories provide examples of this phenomenon, as I have discussed elsewhere.[14] General covariance provides another important example.

In discussions of the general covariance of a space-time theory like the general theory of relativity, it is important to distinguish covariance of a theory's equations under smooth coordinate transformations from diffeomorphic equivalence among models of the theory. This distinction is commonly glossed by calling a coordinate transformation *passive* while a manifold diffeomorphism is *active*. The terminology suggests that while performing a coordinate transformation in a generally covariant theory merely leads to an alternative representation of a given situation, performing a diffeomorphism on a model results in a model of a physically distinct situation. Taking this suggestion seriously leads to the notorious "hole argument".[15]

Diffeomorphisms are theoretical symmetries of a generally covariant theory. But a symmetry of global models of such a theory (i.e. models that serve to represent all of space-time) cannot imply a corresponding empirical symmetry in accordance with the argument illustrated in the previous section and detailed in Appendix A. Lacking such an argument, there seems no good reason to believe that the theoretical symmetry under diffeomorphisms of a generally covariant theory reflects any corresponding empirical symmetry.[16] Indeed, careful general relativity texts now point out that diffeomorphically related models merely offer alternative representations of the same global situation.

Contrast this with another instance of a symmetry related to general relativity, this time involving *localized* situations. On the road to general relativity, Einstein focused on a particular empirical symmetry of gravitational phenomena that followed from the equality of (passive) gravitational and inertial mass. Measurements of mechanical phenomena cannot distinguish a situation involving a uniform gravitational field from a similar situation involving a uniform acceleration. Consequently,

'The gravitational field has only a relative existence [...] Because for an observer freely falling from the roof of a house—at least in his immediate surroundings—there exists no gravitational field.'

Einstein called this 'the happiest thought of my life'! His general theory of relativity finally enabled him to account for the perfect empirical symmetry among situations in which we take a system to be subject to no gravitational field. An appropriate diffeomorphism applied to a localized model of a freely-falling system will produce a model of a distinct situation involving a similar freely-falling system elsewhere in a common space-time.

The second moral is that even though empirical symmetries concern nature rather than our representations of it, theories are very important tools in revealing perfect empirical symmetries.

One reason is that theoretical models often introduce the structures that provide our best way of representing situations related by a perfect symmetry. Many symmetries of Euclidean geometry may be represented without introducing mathematical structures such as real numbers to serve as measures of angles and lengths. But it is not so easy to represent empirical Lorentz boost symmetry or flavor SU(3) symmetry (say) without appeal to a fair amount of the mathematical structure that accompanies theoretical models of those symmetries.

A second reason is deeper. Real life situations almost never display perfect empirical symmetries. The world is a complicated and messy place, and actual situations are rarely duplicated in all their details. In theory construction physicists abstract from and idealize real life situations in order to focus on what they consider their essential features. Debs and Redhead ([2007]) mark this process by interposing what they call a conceptual model between a situation in unvarnished nature and a mathematical model that a theory uses to represent it. While a brilliantly original thinker like Galileo may have been able to arrive at a good conceptual model without the aid of a theory, I believe it is increasingly common for physicists to view the world through the lens of their theories. From this perspective, the mathematical model may precede the conceptual model in their thinking, in which case the features of an actual situation will naturally come to be represented in terms of the theoretical models applied to it.

For these reasons, when a theory reveals an empirical symmetry as perfect, it may not simply recharacterize a previously acknowledged empirical symmetry. Rather, the theory gives us a new way of conceptualizing the situations it relates, along with a set of mathematical models to represent them. If there is an isomorphism between model and (idealized) situation, this is then something that is read into the situation by reconceptualizing its structure, rather than discovered on the basis of an independent, pre-existing description of that situation.

This has implications for the notion of intrinsic properties. Recall that situations related by a perfect empirical symmetry are duplicates—each shares all intrinsic properties of the other. When a theory exhibits a perfect empirical symmetry corresponding to a theoretical symmetry, it thereby displays or at least circumscribes what these intrinsic properties are. It is through the development of physical theories that we learn more about the fundamental intrinsic properties of the world.

In this way we get a better grasp not only on the list of intrinsic properties, but also on what it is for a property to be intrinsic. Here is a way in which physical science merges with metaphysics. A property of an object is intrinsic just in case every duplicate object has that property: a relation among parts of an object is intrinsic just in case it holds among the corresponding parts in every duplicate of that object. Duplicate situations share all intrinsic properties. This is true just in case there is a 1-1 correspondence between duplicate objects in the two situations that induces an isomorphism between their intrinsic relations. Duplicate situations are those that are related by perfect empirical symmetries. The progress of physical theorizing gives us richer and more widely applicable models with which we are able successfully to represent situations. Physics claims a perfect empirical symmetry when it is implied by a theoretical symmetry of such models.

According to Lewis ([1986], p. 60) 'What physics has undertaken, whether or not ours is a world where the undertaking will succeed, is an inventory of the *sparse* properties of this-worldly things.' The sparse properties are supposedly those that 'carve at the joints': he later calls such properties *natural*, and even relies on the notion of a natural property in his attempts to analyze the distinction between intrinsic and extrinsic properties. But we do not need to assume our world comes "pre-packaged" in order to appreciate the significant improvements in and enrichments of our categorizations made possible by advances in our physical theories.

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[1] By extending isospin symmetry from a global to a local gauge symmetry, Yang and Mills ([1954]) produced the first of the gauge theories that now bear their name. These theories are now fundamental to the Standard Model of elementary particles. Flavor SU(3) symmetry was key to the origin of the quark model. BRST symmetry, and symmetries associated with the

renormalization group, are now basic theoretical tools of high energy and condensed matter physics.

[2] Earman ([1989]) gave a classic analysis of the relation between dynamical and space-time symmetries of a theory. A number of essays in (Brading and Castellani (*eds*), [2003]) distinguish different kinds of symmetry that may be associated with a theory.

[3] Galileo may have taken his observations rather to provide support for a principle of the relativity of uniform circular motion about the earth's center. Galileo's ship could not have been moving at constant uniform velocity even if its motion across the ocean were perfectly smooth. Horizontal motion follows the curvature of the earth; the earth rotates about its axis and orbits the sun; etc. But to detect the ship's slight resulting deviation from inertial motion observations much more precise than those described by Galileo would be needed.

[4] In a different theoretical context where electromagnetism is coupled to quantum mechanical matter of charge e represented by a wave-function or quantized field ψ, a local gauge transformation also acts on ψ as follows: $\psi \rightarrow \exp(-ie\Lambda)\psi$. A *global* gauge transformation corresponding to $\Lambda = constant$ then induces a constant phase shift in ψ : but such a global gauge transformation induces only the trivial identity transformation on A_μ. Throughout the paper I apply the term 'local' only to gauge transformations in this customary way to forestall misunderstandings, despite misgivings about the aptness of that terminology (cf. Earman [2002]).

[5] f satisfies the condition that $S_f = S$ just in case f is an automorphism of S: *i.e.* a bijective map from D onto itself such that $\forall a,b \in D(aR_i b \text{ iff } f(a)R_i f(b))$ $(i=1,...,n)$. (Thanks to Fred Muller—first for questioning this equivalence, and then for proving that it holds.)

[6] Intuitively, a property of an object is intrinsic if and only if the object's having that property does not depend on the existence or (intrinsic!) properties of any object distinct from it. I shall have more to say about the notion of an intrinsic property, which has proven notoriously resistant to deeper philosophical analysis, in the conclusion to this paper (section 6).

[7] Ismael and van Fraassen think of a theory as specifying sets of possible worlds (the physically possible as a subset of the metaphysically possible), rather than models (regarded as mathematical structures), which serves to blur this distinction.

[8] See (Earman [1989]; Belot [2001]; Ismael and van Fraassen [2003]; Roberts [2008]).

[9] See the papers by Norton and Stachel in (Howard, D. and Stachel, J. eds. [1989]).

[10] There are also important differences between the cases. Elimination of a structure representing absolute space from the models of Newton's theory did not necessitate any further radical revisions—in the structure of time, or in dynamics (though dropping absolute velocity did require a corresponding relativization of magnitudes such as kinetic energy): elimination of a structure representing absolute space from the models of Maxwell's theory entailed radical revisions in the structure of time, and in Newtonian dynamics. By postulating the special principle of relativity, Einstein immediately generalized the empirical symmetries of uniform velocity boosts not just to electrodynamic phenomena but to *all* physical phenomena; even after its revision, Newton's theory only implied that uniform velocity boosts are empirical symmetries of mechanical phenomena within the scope of the theory.

[11] In such a joint model, electric fields external to the cubes derive instead from a time-varying (curl-free) magnetic vector potential. The existence of such models is a simple consequence of the theoretical local gauge symmetry of classical electromagnetism (see Appendix B).

[12] Thanks to David Wallace for stressing this point. If C_+ and C_0 coexist throughout an interval of time, this will be a curve in their common space: If $C_0 \oplus C_+$ is a situation in which a single uncharged cube is charged, so that C_+ is its equilibrium internal situation at a later time than C_0, then it will be a time-like curve.

[13] One can alter the size of their relative 3-velocity by changing frames, but this merely changes how their 4-vector relative velocity is projected onto different hyperplanes of simultaneity. The magnitude of the difference in 4-velocities is Lorentz invariant.

[14] In my ([2007]). See in particular the discussion of "large" gauge transformations and the θ-vacuum of non-Abelian gauge field theories in chapter 6.

[15] For which, see (Earman and Norton ([1987]) and the literature it spawned: and (Howard, D. and Stachel, J. eds. [1989]) for details of Einstein's original "hole argument".

[16] Some may suppose that space-time substantivalism provides such a reason. I disagree, for reasons outlined in my ([1995]).

Appendix A

Let $\alpha, \beta \in \mathcal{M}(\Theta)$ be structures with domains D_α, D_β respectively.

Since $\alpha \in \mathcal{M}(\Theta)$, there is a possible situation a and a bijective representing function r_a such that α represents a *via* r_a ($r_a : D_a \to D_\alpha$, where D_a is the domain of objects in a).

Suppose that $\varphi : \mathcal{M}(\Theta) \to \mathcal{M}(\Theta)$ is a theoretical symmetry of Θ, with $\beta = \varphi(\alpha)$. Then β also represents a *via* $r'_a = \varphi \circ r_a$ ($r'_a : D_a \to D_\beta$). Moreover, φ defines a bijective map $\bar{\varphi} : D_\alpha \to D_\beta$ between the domains of α, β.

Now make the following

Assumption: h_α, h_β define embeddings of α, β as substructures α', β' of a structure $\alpha \oplus \beta \in \mathcal{M}(\Theta)$ with domain $D_{\alpha\oplus\beta}$ such that $D_{\alpha'} = h_\alpha(D_\alpha), D_{\beta'} = h_\beta(D_\beta)$ and $D_{\alpha'}, D_{\beta'} \subset D_{\alpha\oplus\beta}$ with $D_{\alpha'} \cap D_{\beta'} = \emptyset$.

Then α' represents a in $\alpha \oplus \beta$ *via* the bijective function $h_\alpha \circ r_a : D_a \to D_{\alpha'}$. Since $\alpha \oplus \beta \in \mathcal{M}(\Theta)$, this representation of a may be extended to a representation of a situation $a \oplus b$ of which a is a sub-situation, *via* a bijective function $r : D_{a\oplus b} \to D_{\alpha\oplus\beta}$ such that $r_{\restriction D_a} = h_\alpha \circ r_a$. Then α' represents a and β' represents a sub-situation b (*via* r), where D_b is the domain of objects in b, and $D_a \cap D_b = \emptyset$, since r is bijective.

Now consider the function $r_b \equiv h_\beta^{-1} \circ \left(r_{\restriction D_b} \right)$: β represents b *via* r_b. Moreover $r_b \neq r'_a$, since they have non-overlapping domains. So β not only represents situation a *via* r'_a, β also represents a *distinct* situation b *via* r_b. a and b are distinct because $\alpha \oplus \beta$ models a situation $a \oplus b$ incorporating both a and b as sub-situations with non-overlapping domains.

Now specialize to a case in which we are considering a theory Θ that models the behavior of physical systems located in a fixed, flat space-time. As model of such a space-time we have a structure $< M, A_i >$, where M is a 4-dimensional differentiable manifold representing space-time, and the A_i are fixed geometric object fields representing the flat space-time background.

For a special relativistic theory, the A_i include the Minkowski metric tensor η determining the light-cone and spatio-temporal metric structure, and the unique compatible symmetric affine connection D that makes this a flat affine space, representing the inertial structure.

For a Newtonian theory, the A_i include objects representing an absolute temporal as well as spatial metric, a flat affine connection compatible with these defining the inertial structure, and possibly also a vector field defining a timelike congruence of inertial lines representing the trajectories of points of absolute space.

A global model of Θ extends this model of space-time to a model $< M, A_i, O_j >$ of dynamical processes in space-time by the addition of geometric object fields O_j intended to represent such processes. For example, source-free electromagnetism includes the Maxwell-Faraday tensor F_{ab}, intended to model the behavior of electromagnetic fields.

But Θ will also be assumed to possess localized models of the form $< U, A_{i\upharpoonright_U}, O_j >$, where U is a 4-dimensional submanifold of M, the $A_{i\upharpoonright_U}$ are restrictions of the A_i to U, and the O_j are geometric object fields on U intended to model the behavior of systems in a restricted region of space-time modeled by U.

Now consider two models $\alpha, \beta \in \mathcal{M}(\Theta)$ of Θ: $\alpha =< U_\alpha, A_{i\upharpoonright_{U_\alpha}}, O_j^\alpha >$, $\beta =< U_\beta, A_{i\upharpoonright_{U_\beta}}, O_j^\beta >$ related by a theoretical symmetry φ that defines a diffeomorphism $\bar{\varphi} : U_\alpha \to U_\beta$ representing a space-time translation: $A_{i\upharpoonright_{U_\beta}}|_{\bar{\varphi}(x)} = A_{i\upharpoonright_{U_\alpha}}|_x$.

α represents a situation a *via* representing function $r_a : R_a \to U_\alpha$, where R_a is the region of space-time occupied by a. β also represents a *via* $r'_a = \varphi \circ r_a$.

Assumption: There exists a structure $\alpha \oplus \beta \in \mathcal{M}(\Theta)$ of the form $< U, A_{i\upharpoonright_U}, O_j >$ such that α, β are each sub-structures of $\alpha \oplus \beta$ with $U_\alpha \cap U_\beta = \emptyset$:

where $U_\alpha, U_\beta \subset U$, $A_{i\restriction U_\alpha} = (A_{i\restriction U})_{\restriction U_\alpha}$, $A_{i\restriction U_b} = (A_{i\restriction U})_{\restriction U_b}$, $O_j^\alpha = O_{j\restriction U_\alpha}, O_j^\beta = O_{j\restriction U_\beta}$.

N.B. The Assumption may hold when φ itself simply represents a space-time translation: in that case, $O_j^\beta = \bar{\varphi}^* O_j^\alpha$, where $\bar{\varphi}^*$ is the drag along of $\bar{\varphi}$: $\left(\bar{\varphi}^* O_j^\alpha\right)|_{\bar{\varphi}(x)} = O_j^\alpha|_x$. We are chiefly interested in cases in which the transformation φ represents something other than a simple space-time translation.

In $\alpha \oplus \beta$, α represents a *via* the bijective function $r_a : R_a \to U_\alpha$. Since $\alpha \oplus \beta \in \mathcal{M}(\Theta)$, this representation of a may be extended to a representation of a situation $a \oplus b$ of which a is a sub-situation, *via* a bijective function $r : R_{a\oplus b} \to U$ such that $r_{\restriction R_a} = r_a$. Then α represents a and β represents a sub-situation b, where R_b is the space-time region occupied by b, and $R_a \cap R_b = \emptyset$, since r is bijective.

Now β represents b *via* $r_{\restriction R_b}$. Moreover $r_b \neq r'_a$, since they have non-overlapping domains. So β not only represents a *via* r'_a, β also represents a *distinct* situation b: a and b are distinct because $\alpha \oplus \beta$ models situation $a \oplus b$ incorporating both a and b as sub-situations occupying disjoint space-time regions.

So when the Assumption holds, the theoretical symmetry φ implies a perfect empirical symmetry f between situations a and b. For a single model of Θ (namely β) may be used to represent either situation.

But what kind of symmetry is f? In particular, is it some new dynamical symmetry that is uniquely represented by φ?

Suppose that $\alpha \oplus \beta$ represents $a \oplus b$ *via* r, and that φ does not simply represent a space-time translation.

There may be a distinct model γ that also represents $a \oplus b$ *via* r_γ: indeed, any non-trivial theoretical symmetry of Θ applied to $\alpha \oplus \beta$ will produce such a model. If some model $\gamma \in \mathcal{M}(\Theta)$ representing $a \oplus b$ represents a and b as related just by a space-time translation, then φ does not represent some new dynamical symmetry: it relates distinct situations that Θ can represent (by γ) as differing simply by a space-time translation.

Suppose on the other hand that every structure γ related to $\alpha \oplus \beta$ by a theoretical symmetry of Θ represents a as differing from b in some way that does *not* correspond to a mere space-time translation. Suppose further that there is some new dynamical relation $\mathcal{R}_\varphi$ that all such models γ represent a as bearing to b, and that this depends uniquely on φ (as the notation $\mathcal{R}_\varphi$ indicates). $\mathcal{R}_\varphi$ is a new dynamical relation since it is represented in $\alpha \oplus \beta$ by a difference between the O_j^β and the $\bar{\varphi}^* O_j^\alpha$ that would have represented the space-time translates of the O_j^α if φ had simply represented a space-time translation of a to b. Then the theoretical symmetry φ of Θ implies a corresponding new dynamical symmetry f. f is a perfect empirical symmetry of situations modeled by Θ, and any two f-related situations are related by $\mathcal{R}_\varphi$.

Appendix B

We wish to exhibit a joint model $C_0 \oplus C_+$ of the charged and uncharged cubes within classical electromagnetism that represents the electric potentials inside both cubes by the same function.

Suppose the charged and uncharged cubes are very big, and are put side by side, with a small gap between them. In the static situation there will be no magnetic fields, and we can ignore the exterior of the two cubes except for the small gap between them, where there will be a uniform electric field normal to their adjoining faces. To further simplify, assume the electric field is zero everywhere *inside* the cubes. The problem then has only one spatial dimension of interest: put the origin of the x-axis on the face of the uncharged cube, with the adjacent face of the charged cube at $x = a$.

In one gauge, the electromagnetic potential takes the form

$$\begin{aligned} \varphi(x) &= 0 \quad (x < 0), \text{ inside the uncharged cube} \\ &= \varphi_0 \left(\frac{x}{a}\right) \qquad (0 < x < a), \text{ between the cubes} \\ &= \varphi_0 \quad (x > a), \text{ inside the charged cube} \\ \mathbf{A}(x) &= 0 \end{aligned} \tag{1}$$

Now perform a gauge transformation of the form

$$\begin{aligned} \varphi &\rightarrow \varphi + \partial\Lambda/\partial t \\ \mathbf{A} &\rightarrow \mathbf{A} - \nabla\Lambda \end{aligned} \tag{2}$$

with

$$\begin{aligned}
\Lambda &= f(x)t \\
\nabla \times \nabla\Lambda &= 0 \qquad\qquad \text{(trivially)} \\
f(x) &= 0 \qquad\qquad (x < 0) \qquad (3) \\
&= -\left(\frac{x}{a}\right)\varphi_0 \quad (0 < x < a) \\
&= -\varphi_0 \qquad\qquad (x > a)
\end{aligned}$$

The transformed potentials are

$$\begin{aligned}
\varphi &= 0 \qquad x \neq 0, a \\
A_y &= A_z = 0 \qquad (4) \\
A_x &= 0 \qquad (x < 0) \qquad (5) \\
&= \frac{\varphi_0}{a}t \quad (0 < x < a) \\
&= 0 \qquad (x > a)
\end{aligned}$$

The fields are given by

$$\begin{aligned}
\mathbf{E} &= -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t} \qquad (6) \\
\mathbf{B} &= \nabla \times \mathbf{A}
\end{aligned}$$

which gives

$$\begin{aligned}
\mathbf{E} &= 0 \qquad\qquad (x < 0) \\
&= -\frac{1}{a}\varphi_0 \qquad (0 < x < a) \qquad\qquad (7) \\
&= 0 \qquad\qquad (x > a) \\
\mathbf{B} &= 0
\end{aligned}$$

both for the original and for the transformed potentials, with discontinuities at $x = 0, a$ where charge has accumulated. In this new gauge, the electric potential inside both cubes is given by the same function, namely $\varphi = 0$.

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