

# Reduction and Emergence in Bose-Einstein Condensates

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*Abstract*

A closer look at some proposed *Gedanken*-experiments on BECs promises to shed light on several aspects of reduction and emergence in physics. These include the relations between classical descriptions and different quantum treatments of macroscopic systems, and the emergence of new properties and even new objects as a result of spontaneous symmetry breaking.

## 1 Introduction

Not long after the first experimental production of a Bose-Einstein condensate (BEC) in a dilute gas of rubidium in 1995<sup>(1)</sup>, experiments demonstrated interference between two such condensates<sup>(2)</sup>. Interference is a wave phenomenon, and here it was naturally taken to involve a well-defined phase difference between two coherent matter waves—the BECs themselves. Experimental phenomena associated with well-defined phase-differences were already familiar from other condensed matter systems. The alternating current observed across a Josephson junction between two similar superconductors was (and is) explained by appeal to their varying phase-difference induced by a constant voltage difference across the junction. These two phenomena are now considered manifestations of quantum behavior at the macroscopic—or at least mesoscopic—level since they involve very large numbers of atomic or sub-atomic systems acting in concert, and it is the theory of quantum mechanics that has enabled us to understand and (at least in the second instance) to predict them, both qualitatively and in quantitative detail. They are among a variety of phenomena manifested by condensed matter that have been described as emergent<sup>(3),(4)</sup>, in part as a way of contrasting them with phenomena amenable to a reductive explanation in terms of dynamical laws governing the behavior of their microscopic constituents.

While some kind of contrast with reduction is almost always intended by use of the term ‘emergent’ (or its cognates), the term has been widely applied to items of many categories on diverse grounds. After briefly commenting in section 2 on philosophers’ attempts to regiment usage, I focus on a cluster of issues surrounding the emergence of a definite phase in BECs and related systems.

It is widely (though not universally) believed that the concept of broken symmetry is key to understanding not only the Josephson effect and interference of BECs but also many other phenomena involving condensed matter.

When the state of a condensate is represented by a mathematical object with  $U(1)$  symmetry, spontaneous breaking of this symmetry is associated with a definite phase—the complex argument of an order parameter such as the expectation-value of a field operator. It may be said that this phase emerges as a result of such spontaneous symmetry breaking. Analogies are often drawn between this spontaneously broken phase symmetry and the breaking of rotational symmetry as the magnetization of a Heisenberg ferromagnet or the axis of a crystal acquires a definite orientation. But the attribution of a definite value for the phase of a condensate raises a thicket of problems that challenge these analogies.

While the orientation of a crystal or a Heisenberg ferromagnet has direct operational significance, it is at most the relative phase of two or more condensates that is manifested in interference experiments: the absolute phase of a condensate is generally taken to be without physical significance. A second issue concerns measurements of the relative phase of condensates. In quantum mechanics, a measurable magnitude (an “observable”) is represented by a self-adjoint operator, and the possible results of a measurement of this observable are given by the spectrum of this operator. But there are powerful reasons for denying that observables generally have values for measurement to reveal. If the relative phase were represented by such an operator, then the appearance of a definite (relative) phase on measurement is no indication of a definite pre-existing phase in the condensate. Rather than emerging spontaneously, the definite phase would be precipitated by the measurement itself.

A number of recent papers have treated the emergence of a definite relative phase between BECs as a stochastic physical process that occurs as a result of multiple measurements of quantum observables, each on a different microscopic constituent of the BECs<sup>(5)–(11)</sup>. The measured observable is not the phase itself, so there is no need to represent this by an operator. Indeed, as section 4 explains, the emerging relative phase plays the role of a kind of “hidden variable” within a standard quantum mechanical analysis. This analysis involves no appeal to spontaneous symmetry-breaking. While some have embellished the analysis by explicit appeal to von Neumann’s controversial projection postulate (“collapse” of the wave-function on measurement), this proves unnecessary: all that is required is standard Schrödinger quantum mechanics, including the Born rule for joint probabilities. One way to look at this quantum mechanical analysis is as a reduction of the theoretical treatment of relative phase in terms of spontaneous symmetry-breaking. But this reduction would also involve elimination, in so far as it assumes there is no well-defined relative phase prior to the measurements that prompt its emergence.

A striking feature of the quantum mechanical analysis is that macroscopic values for observables also emerge in the stochastic process that produces a well-defined relative phase. These include transverse spin polarization in a region occupied by two BECs, each composed of particles with aligned spins, where the two alignments are in opposite directions. The measurements that induce this macroscopic spin polarization are themselves microscopic, and may occur in a distant region. As section 5 explains, this “nonlocal” emergence

of macroscopic values violates expectations based on a common understanding of the Copenhagen interpretation, and has been presented as a strengthening of EPR's challenge to that interpretation<sup>(12)</sup>. Section 6 considers a possible Bohrian response to this challenge and explains why this is in tension with the common view that the classical features of macroscopic objects may be derived from quantum theory. This may prompt one to question the reduction of classical to quantum physics.

For a global  $U(1)$  symmetry, Noether's first theorem implies the existence of a conserved quantity, which may in this case be identified with the number of bosons present in a condensate. Broken global  $U(1)$  symmetry then apparently implies a condensate composed of an indeterminate number of bosons. While coherent laser light has long been accepted as an example of a condensate with an indeterminate number of massless bosons, an indeterminate number of atoms in a BEC/Cooper pairs in a superconductor threatens cherished beliefs about conservation of mass, and baryon/lepton number. Section 7 addresses the question: Do we have here an emergent object—an object not composed of any definite number of its constituents?

The present paper attempts no more than a preliminary survey of a cluster of complex interrelated issues concerning reduction and emergence in Bose-Einstein condensates, each of which will repay detailed further study.

## 2 Emergence and Reduction

In physics and elsewhere, reduction and emergence are characteristically taken to label opposing views of a single relation, but lack of clarity about the nature of the relation and the identities of the relata often results in debates between “reductionists” and their opponents that generate more heat than light. One problem is that while it is typically phenomena, behavior, properties, objects, etc. that are said (or denied) to be emergent, reduction is more commonly thought of as a relation between theories, theoretical descriptions, sciences or laws (strictly, law statements). So while emergence is a relation that may or may not hold between items in the world that scientists study, reduction is a relation applicable only to products of that study. This division is not hard and fast<sup>1</sup>. But it is a division I shall respect in my usage in this paper.

In their attempts to clarify the notion of emergence, philosophers have typically begun by concentrating their efforts on the emergence of properties. No consensus has been reached, and a number of alternative analyses have been proposed<sup>(14)–(17)</sup>. Rather than take these as rival attempts to state necessary and sufficient conditions for the correct application of the term ‘emergent property’, one should view them as alternative explications of the same rough

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<sup>1</sup>In his qualified defense of reductionism, Weinberg<sup>(13)</sup> casts reduction and even reductive explanation in ontological rather than epistemological or methodological terms. He freely admits that a scientist's best strategy in understanding a phenomenon is often not to look to the fundamental laws that govern the elementary constituents of the systems involved, even while maintaining that it is those laws that “ultimately explain” it.

idea—that an emergent property is one that is somehow autonomous from more elementary underlying structures out of which it arises. Each may prove useful in marking some contrast that is important in a different application. One common application of the notion of emergence is to the mind: philosophers and cognitive scientists have debated the emergence of consciousness and other mental properties from underlying physical processes involving the brain. But here I am interested in contrasting specific physical properties (or, in one case, objects) with others as to their autonomy from or dependence on more elementary physical structures.

The phase of a condensate is the first such property, and the underlying structures are the properties and arrangement of its constituent particles. The phase of a condensate is actually a real-valued magnitude, though any qualitative (i.e. non-numerical) property may be so regarded—it’s values may be taken to be 1 (for possessed) and 0 (for not possessed). Other magnitudes of systems of condensates may also be considered emergent, including spin polarization, magnetization and electric current. We shall see that not one but several senses of emergence turn out to be usefully applied to these properties.

Broken symmetries associated with phase transitions in condensates have been taken to give rise to emergent phenomena by both physicists and philosophers<sup>(3),(4),(16),(18)</sup><sup>2</sup>. Weinberg<sup>(19)</sup> even defines a superconductor as “simply a material in which electromagnetic gauge invariance is spontaneously broken”. This at least suggests that it is spontaneously broken symmetry that marks properties of matter as emergent in a novel phase. If so, properties of matter in that phase that can be accounted for without appealing to broken symmetry would not count as emergent.

In one sense, emergence is a diachronic process rather than a synchronic condition. Phase transitions occur as dynamical processes, whether or not the symmetry of the prior state is physically broken during this process. So a phase of matter with striking properties may emerge dynamically even though these properties are not sufficiently autonomous from the underlying structure in the new phase to count as (synchronically) emergent.

I think there is another possible use of ‘emergent’, as applied to properties of a complex system which is, perhaps, illustrated by the emergence of a definite (relative) phase in BECs. Consider such “sensory” predicates as red, malodorous, bitter, silky or even wet or hard<sup>3</sup>. In paradigm cases, though certainly not always, these are applied to a macroscopic object on the basis of the response it elicits in a human who interacts with that object in a minimally invasive way—unfortunately, looking at a red traffic light is not an effective way to turn it green, and nor does sniffing rotten meat improve its smell. But do such predicates pick out a corresponding property of that object?

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<sup>2</sup>Though Anderson<sup>(18)</sup> doesn’t use the word ‘emergent’. It is an unfortunate linguistic accident that in the expression ‘phase transition’ the word ‘phase’ refers to states of matter themselves (e.g. superconducting), not to the complex argument of a parameter that may be used to characterize their degree of order.

<sup>3</sup>See Wilson’s<sup>(20)</sup> extended exploration of the sensory concomitants of the first and last of these terms and their bearing on the character of any corresponding property.

Many and varied answers to that question have been proposed throughout the history of philosophy and natural science. Some have defended a positive answer by claiming that a property such as the redness of an object supervenes on more fundamental properties of the microscopic constituents of that object that are not themselves red. Others have denied the existence of any property of redness, flushed with the prospect of a complete scientific explanation of our ability to perceive, classify and reliably communicate about those things we call red based only on their fundamental microphysical properties and ours. Philosophical accounts of emergence generally presuppose that emergent properties are real, even if they supervene on an underlying microphysical basis. But if one had a complete scientific explanation of our ability to perceive, classify and reliably communicate about those things we call red, that might itself be offered as an account of the emergence of redness even if there is no such property! For the account would explain the success of our common practice of calling things red and so license the continuance of that practice.

### 3 BEC Phase as Emerging from Spontaneous Symmetry Breaking?

In his seminal essay Anderson<sup>(18)</sup> takes the general theory of broken symmetry to offer an illuminating formulation of how the shift from quantitative to qualitative differentiation characteristic of emergence takes place<sup>4</sup>. In agreement with Weinberg<sup>(19)</sup> he mentions superconductivity as a spectacular example of broken symmetry, though he gives several others.

The essential idea is that in the so-called  $N \rightarrow \infty$  limit of large systems (on our own, macroscopic scale) it is not only convenient but essential to realize that matter will undergo sharp, singular “phase transitions” to states in which the microscopic symmetries, and even the microscopic equations of motion, are in a sense violated. (*op. cit.* p.395)

After the 1995 experimental production of BECs in dilute gases, Laughlin and Pines<sup>(3)</sup> were able to add “the newly discovered atomic condensates” as examples that display emergent physical phenomena regulated by higher organizing principles. Since they cite Anderson’s paper approvingly and take a principle of continuous symmetry breaking to explain (the exact character of) the Josephson effect, it is reasonable to conjecture that they would join Anderson in taking the phase transition from a normal dilute gas to a BEC as well as that from a normal metal to a superconducting state to involve spontaneous symmetry breaking.

What symmetry is taken to be broken in the transition to the condensed phase of a BEC? The transition is from a less to a more ordered state, whose order may be represented by a so-called order parameter. According to

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<sup>4</sup> “at each level of complexity entirely new properties appear” ((18), p.393).

Leggett<sup>(21)</sup> (p. 38) the order parameter characterizing a BEC (especially in the case of dilute gases including rubidium) is often taken to be a complex-valued function—the expectation value of a Bose field operator in the given quantum state.

$$\Psi(\mathbf{r}, t) = \langle \hat{\psi}(\mathbf{r}, t) \rangle \quad (1)$$

If this is written as

$$\Psi(\mathbf{r}, t) = |\Psi(\mathbf{r}, t)| e^{i\varphi(\mathbf{r}, t)} \quad (2)$$

then the phase  $\varphi(\mathbf{r}, t)$  parametrizes an element of the group  $U(1)$ . If the equations describing the field of the condensate are symmetric under global  $U(1)$  transformations, then changing the order parameter by addition of an arbitrary constant to the phase will take one solution into a distinct solution. Global  $U(1)$  symmetry will be broken by choice of one such value.

An analogy is often drawn to the broken rotation symmetry of the Heisenberg ferromagnet as the spins of all its magnetic dipoles align along some arbitrary direction in the ground state. That fits Anderson’s quoted description well, since the phase transition to one such highly ordered ground state of the ferromagnet is a good example of the kind of spontaneously broken symmetry amenable to idealized treatment as a quantum system with an infinite number of degrees of freedom<sup>5</sup>. In contrast to the case of a quantum system with a finite number of degrees of freedom, degenerate ground states of such a system cannot generally be superposed to give another state since they appear in distinct, unitarily inequivalent, representations of the fundamental commutation relations. Spontaneous breaking of the rotational symmetry of the Heisenberg ferromagnet corresponds to the adoption of one out of the many states in which the dipoles of the ferromagnet are all aligned. In two or more dimensions, this means breaking of a continuous rotational symmetry. By Goldstone’s theorem<sup>(23)</sup>, when such a continuous symmetry is broken in quantum mechanics the Hamiltonian has no energy gap<sup>6</sup>: in a quantum field theory this implies the existence of massless Goldstone bosons.

Pursuing this analogy, spontaneous breaking of the continuous  $U(1)$  phase symmetry of a BEC’s order parameter could be represented by an idealized model in which the number of constituent particles is taken to be infinite, but the density of the BEC is fixed at some low value  $\rho$  by taking the so-called thermodynamic limit  $N \rightarrow \infty$ ,  $V \rightarrow \infty$ ,  $N/V = \rho$  (a constant). Then adoption of a definite phase by a BEC would be an instance of the same kind of spontaneous symmetry breaking as adoption of a definite direction of magnetization by a Heisenberg ferromagnet. But there are problems with this analogy, as Leggett<sup>(21),(25),(26)</sup> has noted.

When rotation symmetry of a Heisenberg ferromagnet is spontaneously broken, the spins of its components are all aligned along a particular direction in space. This direction may be operationally defined in many ways having

<sup>5</sup> See, for example, Ruetsche<sup>(22)</sup>.

<sup>6</sup> As Streater<sup>(24)</sup> proved for the Heisenberg ferromagnet: this gives rise to the possibility of spin waves of arbitrarily small energy.

nothing to do with spin or magnetization: in particular, it need not be defined in relation to other Heisenberg ferromagnets, either actual or hypothetical. On the other hand, if the  $U(1)$  global phase symmetry of a BEC were to be spontaneously broken, its overall phase would become well defined only relative to some other BEC of the same kind (for example, a similarly condensed dilute gas of rubidium 87). At most, a definite phase consequent upon spontaneously broken symmetry would seem to be an emergent relational property (cf. Teller<sup>(27)</sup>) of a BEC. Moreover, difficulties in implementing multiple pairwise phase comparisons between similar BECs that have never been in contact threaten at least the operational significance even of such a relational property. Leggett<sup>(26)</sup> argues that, at least in the case of superconducting BECs, operational pairwise phase comparisons among several such BECs will fail to be transitive (though compare Leggett<sup>(28)</sup>).

A second problem arises from the need to take the thermodynamic limit to treat the emergence of relative phase in BECs as an instance of spontaneous symmetry breaking. No massive BEC system is composed of an infinite number of elementary bosons. Moreover, while the number of elementary dipoles in a macroscopic magnet will typically at least be extremely large (of the order of  $10^{23}$ ), the first dilute gas BECs contained only a few thousand atoms, and even now experimental realizations have increased this number by only a few factors of 10. If it were essential to assume that an *infinite* number of atoms is present in each of two interfering BECs to explain their observed interference (as the quote from Anderson might lead one to believe), then one may legitimately query the value of the explanation. But in fact one need not treat the emergence of relative phase here as a case of spontaneous symmetry breaking in the thermodynamic limit, as analyses by Castin and Dalibard<sup>(6)</sup> and several subsequent authors have shown.

In the context of an idealized model of two trapped condensates of the same atomic species, Castin and Dalibard<sup>(6)</sup> showed two things:

(1) No measurements performed on the condensates can allow one to distinguish between two different quantum representations of this system: By a uniform average over the unknown relative phase of two coherent states; and by a Poissonian statistical mixture of Fock states.

(2) Two different points of view on a system are available: Assuming an initial pair of coherent states with a definite relative phase, successive measurements “reveal” that pre-existing phase in an interference phenomenon; assuming each condensate is initially represented by a definite Fock state, with no well-defined relative phase, the same sequence of measurements progressively “builds up” a relative phase between the condensates as the interference phenomenon is generated.

They take the results of their analysis to show that the notion of spontaneously broken phase symmetry is not indispensable in understanding interference between two condensates. I won’t explain how they arrived at these conclusions, since the next section outlines a closely related analysis by Laloë of a similar *Gedankenexperiment* that will provide a focus for the subsequent discussion. I will merely comment that Castin and Dalibard<sup>(6)</sup> assume that

the measurements referred to in (2) are performed in a well-defined temporal sequence on individual elements of the system of condensates, and that each leaves the rest of the system in the quantum state it would be assigned if the effect of that measurement were represented by von Neumann's projection postulate.

## 4 The Appearance of Phase Without Symmetry-Breaking

In 2005 Laloë<sup>(7)</sup> began to develop an elegant framework for analyzing the emergence of phase in systems of BECs. One important application is to a system of two BECs, each composed of non-interacting bosons, and each initially represented by a Fock state corresponding to a definite number of particles. This provides a simplified and idealized model for the kind of experimental situation realized by Andrews *et. al.*<sup>(2)</sup> that first demonstrated interference between two BECs. An extension of that model is to measurements on BECs in different internal states—most simply, each in one of two different one-particle  $z$ -spin states. This enables one to consider the BECs to be initially separate systems no matter what their spatial overlap: and it naturally suggests the possibility of a variety of different kinds of measurement capable of revealing interference between them—of spin-component in any direction in the  $x$ - $y$  plane. Such measurements are considered in Mullin, Krotkov and Laloë<sup>(8)</sup>, Laloë<sup>(12)</sup>, and Laloë and Mullin<sup>(10)</sup>: here I follow Laloë's<sup>(12)</sup> presentation.

Consider a pair of noninteracting spin-polarized BECs in the normalized Fock state

$$|\Phi\rangle = \frac{1}{\sqrt{N_a!N_b!}} \hat{a}_{u_a,\alpha}^{\dagger N_a} \hat{a}_{v_b,\beta}^{\dagger N_b} |0\rangle \quad (3)$$

representing  $N_a$  particles with internal ( $z$ -spin) state  $\alpha$  and spatial state  $u_a$  and  $N_b$  particles with orthogonal internal ( $z$ -spin) state  $\beta$  and spatial state  $v_b$ , where  $|0\rangle$  is the vacuum state.

If  $\hat{\Psi}_\alpha(\mathbf{r})$  is the field operator for  $z$ -spin  $\alpha$ ,  $\hat{\Psi}_\beta(\mathbf{r})$  for  $z$ -spin  $\beta$ , and  $\dagger$  indicates the adjoint operation, then the number density operator of the BECs is

$$\hat{n}(\mathbf{r}) = \hat{\Psi}_\alpha^\dagger(\mathbf{r})\hat{\Psi}_\alpha(\mathbf{r}) + \hat{\Psi}_\beta^\dagger(\mathbf{r})\hat{\Psi}_\beta(\mathbf{r}) \quad (4)$$

and the density operator for their spin component in a direction in the  $x - y$  plane at an angle  $\varphi$  from the  $x$ -axis is

$$\hat{\sigma}_\varphi(\mathbf{r}) = e^{-i\varphi}\hat{\Psi}_\alpha^\dagger(\mathbf{r})\hat{\Psi}_\beta(\mathbf{r}) + e^{+i\varphi}\hat{\Psi}_\beta^\dagger(\mathbf{r})\hat{\Psi}_\alpha(\mathbf{r}) \quad (5)$$

Suppose that one measurement is made of the  $\varphi$ -component of particle spin in a small region of space  $\Delta r$  centered around point  $\mathbf{r}$ . The corresponding spin operator is

$$\hat{S}(\mathbf{r}, \varphi) = \int_{\Delta r} d^3\mathbf{r}' \hat{\sigma}_\varphi(\mathbf{r}') \quad (6)$$



For sufficiently small  $\Delta r$ , this has only three eigenvalues  $\eta = 0, \pm 1$  since no more than one particle would be found in  $\Delta r$ . The single-particle eigenstates for finding a particle there with  $\eta = \pm 1$  are

$$|\Delta r, \eta\rangle = |\Delta r\rangle \otimes \frac{1}{\sqrt{2}} \left[ e^{-i\varphi/2} |\alpha\rangle + e^{+i\varphi/2} |\beta\rangle \right] \quad (7)$$

where  $|\Delta r\rangle$  is a single-particle spatial state whose wave-function equals 1 inside  $\Delta r$  but 0 everywhere outside  $\Delta r$ . The corresponding  $N$ -particle projector is

$$\hat{P}_{\eta=\pm 1}(\mathbf{r}, \varphi) = \frac{1}{2} \int_{\Delta r} d^3 \mathbf{r}' [\hat{n}(\mathbf{r}') + \eta \hat{\sigma}_\varphi(\mathbf{r}')] \quad (8)$$

and the projector for finding no particle there is

$$\hat{P}_{\eta=0}(\mathbf{r}) = \left( \mathbf{1} - \int_{\Delta r} d^3 \mathbf{r}' \hat{n}(\mathbf{r}') \right) \quad (9)$$

As  $\Delta r \rightarrow 0$ , the corresponding eigenstates (for variable  $\mathbf{r}$ ) form a quasi-complete basis for the  $N$ -particle space.

Now consider a sequence of  $m$  measurements of transverse spin-components  $\varphi_j$  in very small non-overlapping regions  $\Delta r_j$ , each of volume  $\Delta$ , centered around points  $\mathbf{r}_j$  ( $1 \leq j \leq m$ ). Since the projectors for non-overlapping regions commute, the joint probability for detecting  $m$  particles with spins  $\eta_j$  in regions  $\Delta r_j$  is

$$\langle \Phi | \hat{P}_{\eta_1}(\mathbf{r}_1, \varphi_1) \times \hat{P}_{\eta_2}(\mathbf{r}_2, \varphi_2) \times \dots \times \hat{P}_{\eta_m}(\mathbf{r}_m, \varphi_m) \times | \Phi \rangle \quad (10)$$

Using (8) together with (4) and (5) this gives a product of several terms, each containing various products of field operators. Since these commute, we can push all the creation operators to the left and all the annihilation operators to the right. Expanding the field operators in terms of a basis  $|u_a, \alpha\rangle, |v_b, \beta\rangle$  of single particle states

$$\hat{\Psi}_\alpha(\mathbf{r}) = u_a(\mathbf{r}) \times \hat{a}_{u_a, \alpha} + \dots \quad ; \quad \hat{\Psi}_\beta(\mathbf{r}) = v_b(\mathbf{r}) \times \hat{a}_{v_b, \beta} + \dots \quad (11)$$

But none of the "dotted" terms will contribute to (10), since  $|\Phi\rangle$  contains no particles in states other than  $|u_a, \alpha\rangle, |v_b, \beta\rangle$ .

Each term now contains between  $\langle \Phi |$  and  $| \Phi \rangle$  a string of creation operators followed by a string of annihilation operators. If a state  $|u_a, \alpha\rangle$  or  $|v_b, \beta\rangle$  does not appear exactly the same number of times in each of these, it will not contribute to (10): if it does appear exactly the same number of times in each of these, every creation or annihilation operator will introduce a factor  $\sqrt{N_{a,b} - q}$  where  $q$  depends on the term but  $q < m$ . If  $m \ll N_a, N_b$ , these factors can be approximated by  $\sqrt{N_{a,b}}$  respectively. So now each field operator has been replaced in (10) by a factor  $\sqrt{N_{a,b}}$  multiplying a position wave-function  $u_a$  or  $v_b$  (or its complex conjugate). But we still have to take account of particle number

conservation in each sequence. This can be done by a clever trick, using the mathematical identity

$$\int_0^{2\pi} \frac{d\Lambda}{2\pi} e^{in\Lambda} = \delta_{n,0} \quad (12)$$

By multiplying each  $\hat{\Psi}_\alpha(\mathbf{r})$  (or rather  $\sqrt{N_a}u_a(\mathbf{r})$ ) by  $e^{i\Lambda}$ , and each  $\hat{\Psi}_\alpha^\dagger(\mathbf{r})$  (or rather  $\sqrt{N_a}u_a^*(\mathbf{r})$ ) by  $e^{-i\Lambda}$ , and integrating  $\Lambda$  over  $2\pi$  (and similarly for the  $b$  particles), we automatically take account of particle number conservation!

Since  $\Delta$  is very small, the spatial wave-functions are each approximately constant over each region  $\Delta r_j$ . The joint probability for detection of  $m$  particles with spins  $\eta_j$  in regions  $\Delta r_j$  ( $1 < j < m$ ), each of volume  $\Delta$ , is then

$$\frac{\Delta^m}{N_a!N_b!} \int_0^{2\pi} \frac{d\Lambda}{2\pi} \prod_{j=1}^m \left\{ \eta_j \sqrt{N_a N_b} \left( e^{i(\Lambda - \varphi_j)} u_a(\mathbf{r}_j) v_b^*(\mathbf{r}_j) + c.c. \right) \right\} \quad (13)$$

or, with  $\xi(\mathbf{r}) = \arg[u_a(\mathbf{r})/v_b(\mathbf{r})]$ ,

$$\frac{\Delta^m}{N_a!N_b!} \int_0^{2\pi} \frac{d\Lambda}{2\pi} \prod_{j=1}^m \left\{ 2\eta_j \sqrt{N_a N_b} |u_a(\mathbf{r}_j)| |v_b(\mathbf{r}_j)| \cos(\Lambda + \xi(\mathbf{r}_j) - \varphi_j) \right\} \quad (14)$$

Since we are interested in comparing relative probabilities of detection, the prefixed normalization terms can now be dropped.

First consider the case  $m = 1$ : measurement of  $\varphi$ -spin on a single particle. If  $\Lambda$  were fixed, these relative detection probabilities would be just what one would expect from a state with a definite relative phase between the two condensates, namely the (unnormalized) phase state

$$|\Lambda\rangle = \left[ N_a^{\frac{1}{2}} \hat{a}_{u_a, \alpha}^\dagger e^{-i\Lambda/2} + N_b^{\frac{1}{2}} \hat{a}_{v_b, \beta}^\dagger e^{i\Lambda/2} \right]^{N_a + N_b} |0\rangle \quad (15)$$

with one-particle pure spin-state density matrix  $W$  proportional to

$$\begin{pmatrix} N_a |u_a(\mathbf{r}_1)|^2 & \sqrt{N_a N_b} e^{-i\Lambda} u_a^*(\mathbf{r}_1) v_b(\mathbf{r}_1) \\ \sqrt{N_a N_b} e^{i\Lambda} u_a(\mathbf{r}_1) v_b^*(\mathbf{r}_1) & N_b |v_b(\mathbf{r}_1)|^2 \end{pmatrix} \quad (16)$$

Moreover, the expectation value of  $z$ -spin in state  $W$  is proportional to  $N_a |u_a(\mathbf{r}_1)|^2 - N_b |v_b(\mathbf{r}_1)|^2$ , also just what one would expect if  $|\Lambda\rangle$  represented the definite, pre-existing relative phase between the condensates. But the uniform integral over  $\Lambda$  "washes out" the appearance of any definite phase relation between the two condensates, so the overall probability distribution for measurement of  $\varphi$ -spin on a single particle corresponds to no interference.

Now consider the case  $m = 2$ : joint measurement of  $\varphi_1$ -spin and  $\varphi_2$ -spin with results  $\eta_1, \eta_2$  respectively on two particles. The  $\Lambda$ -probability distribution for result  $\eta_2$  conditional on outcome  $\eta_1$  is now weighted by a factor that depends

both on the angle  $\varphi_1$  of the measurement on particle 1 and on its outcome and location  $(\eta_1, \mathbf{r}_1)$  and is proportional to

$$N_a |u_a(\mathbf{r}_1)|^2 + N_b |v_b(\mathbf{r}_1)|^2 + 2\eta_1 \sqrt{N_a N_b} |u_a(\mathbf{r}_1)| |v_b(\mathbf{r}_1)| \cos(\Lambda + \xi(\mathbf{r}_1) - \varphi_1) \quad (17)$$

This may well already give rise to a slight correlation between the results  $\eta_1, \eta_2$ : if  $\eta_1$  is +1 and  $\varphi_1$  and  $\varphi_2$  are close, then  $\eta_2$  is more likely than not also to equal +1. But as one considers additional transverse spin measurements, strong correlations become apparent. The probability distribution for the transverse spin of the  $(m + 1)$ st particle *conditional on outcomes  $\eta_j$  for the other  $m$  measurements* becomes strongly peaked as  $m$  increases. Laloë<sup>(12)</sup> comments

When more and more spin measurements are obtained, the  $\Lambda$ -distribution becomes narrower and narrower....Standard quantum mechanics considers that  $\Lambda$  has no physical existence at the beginning of the series of measurements, and that its determination is just the result of a series of random perturbations of the system introduced by the measurements. Nevertheless (14) shows that all observations are totally compatible with the idea of a pre-existing value of  $\Lambda$  which is perfectly well defined but unknown, remains constant, and is only revealed (instead of created) by the measurements. (p. 43)

It is tempting to think of the emergence of a definite phase here as a stochastic, dynamical process in which each subsequent transverse spin-measurement (with increasing probability) renders the relative phase of the two condensates more definite. But the parameter  $\Lambda$  enters the above quantum mechanical analysis only as a convenient mathematical device for calculating conditional probabilities, and not (as in (15)) as a way of characterizing the state of the condensates themselves. Moreover, the analysis nowhere appealed to the evolution of the state of the condensates, whether unitary (in accordance with the Schrödinger equation) or non-unitary (in accordance with von Neumann's projection postulate). Though if one were instead to assume a temporal sequence of projective  $\varphi_j$ -spin measurements, then (neglecting Schrödinger evolution) the state of the remaining condensate would progressively come to approximate a state of definite phase.

## 5 A Strengthened EPR argument?

Laloë<sup>(12)</sup> goes on to consider alternative spatial wave-functions  $u_a, v_b$  for the two condensates. He takes one such configuration to justify this claim in the abstract to his paper.

We study in this article how the EPR argument can be transposed to this case, and show that the argument becomes stronger, mostly because the measured systems themselves are now macroscopic. (p.35)

He makes the simplifying assumption that  $u_a, v_b$  have the same phase at each point  $\mathbf{r}$  (though their amplitudes may differ) so  $\xi = 0$ . He then considers an arrangement in which  $u_a, v_b$  overlap only in two distant regions  $A, B$  and  $m$  successful transverse spin-measurements are considered in non-overlapping small regions of  $A$ . The foregoing analysis shows that, for  $m \sim 100$ , the conditional probabilities for the outcomes of additional transverse spin-measurements *in small non-overlapping small regions of  $B$*  will differ little from corresponding unconditional probabilities for a phase state with some definite  $\Lambda$  (whose value depends on the outcomes of the  $m$  measurements *in  $A$* ). In particular, if  $B$  contains a macroscopic number of particles there will be some angle  $\varphi_\Lambda$  such that each of, say,  $10^{23}$  successful individual measurements of  $\varphi_\Lambda$ -spin on particles in  $B$  is almost certain to give outcome  $\eta_\Lambda = +1$ , conditional on the outcomes of the  $m$  measurements in  $A$ . Laloë<sup>(12)</sup> comments

Here we have a curious case where it is the measured system itself that spontaneously creates a pointer made of a macroscopic number of parallel spins. Moreover, for condensates that are extended in space, ...this process can create instantaneously parallel pointers in remote regions of space, a situation obviously reminiscent of the EPR argument in its spin version given by Bohm. (p.37)

Recall Bohm's spin version of the EPR *Gedankenexperiment*, featuring two spin  $\frac{1}{2}$  particles in the spin singlet state

$$|\psi_s\rangle = \frac{1}{\sqrt{2}} [|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle] \quad (18)$$

where  $\uparrow$  ( $\downarrow$ ) labels a  $z$ -spin eigenvector with positive (negative) eigenvalue and the order of the arrows in each component of the superposition corresponds to that of the particles' state spaces. Transposed to this situation, the intended conclusion of the EPR argument is that quantum mechanical description is incomplete since the state  $|\psi_s\rangle$  does not describe certain "elements of reality" associated with each of these two particles: for each direction, one such "element of reality" corresponds to the (eigen)value of spin-component in that direction which a well-conducted measurement of that spin component would reveal, were one to be carried out.

After removing the excess erudition of which Einstein complained right after its publication, the original EPR argument went like this<sup>7</sup>. Einstein, Podolsky and Rosen<sup>(29)</sup> assumed the following sufficient criterion for reality

If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this quantity. (p.777)

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<sup>7</sup>In a letter to Schrödinger of June 19th, 1935 Einstein said that the main point was, so to speak, overwhelmed by erudition ("die Hauptsache ist, sozusagen, durch Gelehrsamkeit verschüttet").

Suppose one were to get outcome  $\eta$  in a measurement of the (arbitrary)  $\varphi$ -component of spin on particle  $a$  of a Bohm-EPR pair in spin state  $|\psi_s\rangle$  at a time when their spatial state corresponds to  $a, b$  being far apart (with negligible probability of finding  $a$  outside  $A$  or  $b$  outside  $B$ , where  $A, B$  are widely separated spatial regions). Assuming this is a projective measurement, the resulting state is

$$|\psi'\rangle = |\varphi_\eta\varphi_{-\eta}\rangle \quad (19)$$

Applying the Born rule to  $|\psi'\rangle$  one could predict with probability unity that a (well-conducted) measurement of the value of the  $\varphi$ -component of spin on particle  $b$  would yield outcome  $-\eta$ . EPR further take state  $|\psi'\rangle$  to describe  $b$  as certainly (with probability unity) *having* value  $-\eta$  for its  $\varphi$ -component of spin even if no measurement is performed on  $b$ . Assuming (locality) that such a hypothetical measurement on  $a$  alone would not disturb  $b$ , they infer that in the hypothetical situation in which (only)  $a$  is measured with result  $\eta$ , the  $\varphi$ -component of spin of  $b$  would be  $-\eta$ , prior to and independent of the measurement on  $a$ : similarly, in the hypothetical situation in which (only)  $a$  is measured with result  $-\eta$ , the  $\varphi$ -component of spin of  $b$  would be  $\eta$ , prior to and independent of the measurement on  $a$ . Hence in any hypothetical situation in which (only)  $a$  is measured,  $b$  would have had a definite (though as yet unknown)  $\varphi$ -component of spin, prior to and independent of the measurement on  $a$ . It follows that  $b$  always *has* a definite (though unknown)  $\varphi$ -component of spin in the spin singlet state, irrespective of what measurement (if any) one contemplates performing on  $a$  or  $b$ : by symmetric reasoning, so too does  $a$ . Together with EPR's necessary condition for completeness ("every element of the physical reality must have a counterpart in the physical theory") this establishes the incompleteness of quantum mechanical description.

Lalö<sup>(12)</sup> takes his BEC *Gedankenexperiment* to strengthen the EPR argument "mostly because the measured systems themselves are now macroscopic". In evaluating this claim later, it will be helpful to bear in mind a more straightforward extension of the Bohm-EPR scenario to the macroscopic scale, even though the resulting *Gedankenexperiment* is so far beyond the bounds of practicality as to challenge credulity (cf. Schrödinger's own reference to his infamous cat scenario as "ridiculous")<sup>8</sup>. So consider a pair of spatially separated macroscopic systems  $a, b$ , composed of  $N \sim 10^{23}$  spin  $\frac{1}{2}$  particles each, in an entangled spin state  $|\psi_{Mac}\rangle$  of total  $z$ -component of angular momentum zero

$$|\psi_{Mac}\rangle = \frac{1}{\sqrt{2}} [|(N \uparrow)(N \downarrow)\rangle - |(N \downarrow)(N \uparrow)\rangle] \quad (20)$$

Applied to this scenario, the EPR reasoning would lead one to conclude that in  $|\psi_{Mac}\rangle$  each of  $a, b$  has a definite macroscopic  $z$ -spin that quantum mechanics fails to describe. To reach this conclusion, the argument would consider a hypothetical measurement of the  $z$ -spin on  $a$  (a macroscopic object) and its outcome

<sup>8</sup>Schrödinger called *Gedankenexperimenten* like that of his eponymous cat "burleske Fälle". By contrast the Bohm-EPR *Gedankenexperiment* famously leant itself to implementation as an *actual* experiment with profound results for quantum nonlocality.

(a macroscopic value for the  $z$ -spin on  $a$ ) to conclude—*independent of any measurements*—that  $b$  has a definite macroscopic value of  $z$ -spin in state  $|\psi_{Mac}\rangle$ . Note that this argument need *not* involve consideration of measurements of any other (incompatible) spin-component on  $a$ .

However elegant the argument, EPR's conclusion is now generally taken to be mistaken, primarily because of Bell's work and the subsequent experimental violation of his eponymous inequalities. Now if EPR's argument is valid but not sound, which of their assumptions are false? Even after Bohr's prompt refutation and extensive more recent discussions of quantum nonlocality, I believe there is still no consensus on exactly how to answer that question. But I think many would follow Gisin<sup>(30)</sup> in pinning the blame on EPR's locality assumptions, taking the failure of quantum mechanics to satisfy all these assumptions to show why some of its (verified) predictions violate Bell inequalities derived from them. More specifically, the condition Shimony<sup>(31)</sup> called Outcome Independence fails for quantum mechanics, as illustrated by the fact that the quantum mechanical probability for outcome  $\eta$  of a measurement of  $\varphi$ -spin on  $a$  in the Bohm EPR scenario conditional on a measurement of  $\varphi'$ -spin on  $b$  depends on the outcome  $\eta'$  of the latter measurement (though it does not depend on *which*  $\varphi'$ -spin-component (if any) is measured on  $b$  if the outcome of any such measurement is ignored). This failure of Outcome Independence does not facilitate signalling between spacelike separated locations, and a variety of proofs have been offered that quantum mechanical nonlocality is innocuous because it does not permit such superluminal signalling.

But, as Maudlin<sup>(32)</sup> pointed out, there is still a problem reconciling quantum mechanical nonlocality with relativity. Recall that, according to the EPR argument, a measurement on  $a$  projects  $|\psi_s\rangle$  onto the state  $|\psi'\rangle$  in which the  $\varphi$ -spin of  $b$  is definite. EPR took this to be a straightforward application of quantum mechanics itself, unlike their reality criterion and locality assumptions which were motivated by more general physical considerations. If quantum mechanical description is complete, in conflict with EPR's conclusion, then the  $\varphi$ -spin of  $b$  was *not* definite prior to the measurement on  $a$ . But if the  $a, b$  measurements are spacelike separated events, then they have no invariant temporal order, and any attempt to specify the spacetime location at which the  $\varphi$ -spin of  $b$  becomes definite must appeal to structure not provided by a relativistic space-time and hard to reconcile with it.

Despite this problem, Laloë<sup>(12)</sup> takes the analysis of his BEC *Gedankenexperiment* to predict that spin-component measurements on a few microscopic particles in  $A$  will *immediately* create a spontaneous transverse polarization of a macroscopic assembly of spins in  $B$ .

what standard quantum mechanics describes here is not something that propagates along the state and has a physical mechanism... it is just 'something with no time duration' that is a mere consequence of the postulate of quantum measurement (wave packet reduction).  
(p.45)

In fact the analysis he has given does not even establish the claim that, following these measurements in  $A$ , a *single measurement* of the *total*  $\varphi$ -spin in  $B$  would (almost) certainly yield the predicted, definite macroscopic outcome. For that analysis concerns only multiple (successful) *microscopic* measurements of  $\varphi$ -spin on individual particles in specific tiny regions of  $B$ . But it is true that successfully measuring the  $\varphi$ -spin of each of a macroscopic number of particles within  $B$  and adding the results would be *one* way of measuring (a significant portion of) the total  $\varphi$ -spin in  $B$ . Moreover, one can show that the expectation value of total  $\varphi$ -spin in  $B$  will be macroscopic after even a *single* microscopic transverse spin measurement in  $A$ . So it would be very surprising if an extension of Laloë's<sup>(12)</sup> analysis did *not* establish this claim.

How does the EPR argument apply to Laloë's<sup>(12)</sup> BEC *Gedankenexperiment*, in which the  $a, b$  condensates overlap only in remote regions  $A, B$ , a few  $m \sim 100$  successful transverse spin-component measurements are performed in  $A$ , and a macroscopic number of particles is present in  $B$ ? Here is what he says

We have a situation that is similar to the usual EPR situation: measurements performed in  $A$  can determine the direction of spins in both regions  $A$  and  $B$ . If we rephrase the EPR argument to adapt it to this case, we just have to replace the words 'before the measurement in  $A$ ' by 'before the series of measurements in  $A$ ', but all the rest of the reasoning remains exactly the same: since the elements of reality in  $B$  cannot appear under the effect of what is done at an arbitrary distance in region  $A$ , these elements of reality must exist even before the measurements performed in  $A$ . Since the double Fock state (3) of quantum mechanics does not contain any information on the direction of spins in  $B$ , this theory is incomplete.  
(p.46)

There is one clear disanalogy between the Bohm-EPR scenario and Laloë's<sup>(12)</sup> BEC *Gedankenexperiment*. Even if an individual spin-component measurement is projective, the sequence of measurements performed in  $A$  does not collapse the state (3) into an eigenstate of total  $\varphi$ -spin in  $B$ : at most it produces a state of the BECs for which a measurement of total  $\varphi$ -spin in  $B$  is *very likely* to give a particular result. Hence the EPR reality criterion cannot be applied as stated, since it specifies probability *unity*. This disanalogy does not appear for the macroscopic Bohm-EPR state  $|\psi_{Mac}\rangle$ , which is in this respect a better macroscopic generalization of the original Bohm-EPR state  $|\psi_s\rangle$ . Does this disanalogy matter? I think it does.

Since they are arguing that quantum mechanical description is incomplete, EPR need to have in mind a clear rival view of what it would be for it to be complete. Quantum mechanics represents the (pure) state of systems by a wavefunction or state vector: how could such a mathematical object be considered to offer a complete description of a system's properties? A natural answer is that given by the so-called eigenvalue-eigenstate link: observable  $O$  represented by self-adjoint operator  $\hat{O}$  has value  $o_i$  on a system if and only if the state of that

system can be represented by pure state  $|\omega_i\rangle$  where  $\hat{O}|\omega_i\rangle = o_i|\omega_i\rangle$ . Indeed, EPR apply this link in both directions in section 1 of their paper. To adapt EPR's reasoning to Laloë's<sup>(12)</sup> BEC *Gedankenexperiment* one would have to modify it to avoid relying on the eigenvalue-eigenstate link.

Einstein's own preferred variant on EPR<sup>9</sup> does *not* rely on the eigenvalue-eigenstate link. Instead it directly argues for incompleteness of description by the wave-function. As applied to the Bohm-EPR scenario Einstein<sup>(34)</sup> would reason that while a measurement of  $z$ -spin on  $a$  would collapse  $b$ 's state onto an eigenstate of  $z$ -spin, a measurement of  $x$ -spin on  $a$  would collapse the state of  $b$  onto an eigenstate of  $x$ -spin. By locality ("Grundsatz II" of Einstein<sup>(34)</sup>), neither measurement could influence the real state of  $b$ , which would therefore be the same no matter what measurement (if any) were performed on  $a$ . But there is no way to understand both an  $x$ -spin eigenstate and a  $z$ -spin eigenstate as offering a complete description of the *same* real state of  $b$ , since these eigenstates imply radically different statistical predictions for the results of measurements on  $b$ .

Einstein's preferred mode of reasoning cannot be applied directly to Laloë's<sup>(12)</sup> BEC *Gedankenexperiment*. For even if the  $m$  transverse spin measurements on particles in  $A$  are projective, they do not project the quantum state onto a pure state that has the form of a tensor product, one factor of which has support confined to  $B$  and so could be taken to describe just the contents of  $B$ .

Both the reasoning of the EPR argument and that of Einstein's preferred variant *may*, however, be readily applied to the macroscopic generalization of Bohm-EPR represented by the state  $|\psi_{Mac}\rangle$ , provided only that one takes the conclusion to be the incompleteness of the description offered by  $|\psi_{Mac}\rangle$  of the real state of  $b$  in that scenario. On the other hand, Laloë's<sup>(12)</sup> BEC *Gedankenexperiment* has the distinct advantage of not being totally beyond the bounds of practicality. As he says, progress in experimental studies of dilute gas BECs may bring us within reach of producing systems of condensates for whose quantum mechanical modeling the double Fock state provides a reasonable idealization, and whose temporal evolution does not render an analysis in terms of multiple simultaneous measurements wholly irrelevant. Just as the Bohm-EPR scenario is no longer merely a *Gedankenexperiment*, we may be on the verge of realizing variants of Laloë's<sup>(12)</sup> BEC *Gedankenexperiment* as real experiments.

## 6 What Bohr would (or should?) have said

Laloë<sup>(12)</sup> presents his *Gedankenexperiment* as a challenge to Bohr's<sup>(36)</sup> refutation of the EPR argument in these words:

What is new here is that the EPR elements of reality in  $B$  correspond to a system that is macroscopic. One can no longer invoke its microscopic character to deprive the system contained in  $B$  of any physical reality! The system can even be at our scale, correspond to

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<sup>9</sup>For which, see Einstein ((33), pp. 340-2; (34), pp. 320-24; (35), pp. 82-87).



a macroscopic magnetization that can be directly observable with a hand compass; is it then still possible to state that it has no intrinsic physical reality? When the EPR argument is transposed to the macroscopic world, it is clear that Bohr's refutation does not apply in the form written in his article; it has to be at least modified in some way. (pp. 46-7)

On the contrary, I venture that Bohr's reasoning in his refutation of EPR applies equally well to Laloë's<sup>(12)</sup> *Gedankenexperiment*. I say "venture" rather than "claim" since any analysis based on an interpretation of just what Bohr meant in his refutation must remain tentative. What follows may, with some justification, be considered an attempt to put words into Bohr's mouth that he would never have let pass his lips!

In his refutation, Bohr<sup>(36)</sup> charged their reality criterion with fatal ambiguity. The key passage is notoriously obscure, so I quote it at length.

Of course there is in a case like that considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system*. Since these conditions constitute an inherent element of the description of any phenomenon to which the term 'physical reality' can properly be attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete. (p.700)

Note that Bohr here appears to *deny* that the flaw in the argument is the falsity of EPR's locality assumptions, while pointing to a different, and perhaps deeper, problem with their assumptions about physical reality. The problem is deeper in so far as the falsity of *these* assumptions would undermine the *applicability* of notions of locality that rest on them. Note also that in the quoted passage Bohr does not mention any division between microscopic and macroscopic systems. Why, then would Laloë suppose that his refutation is based on the denial of any physical reality for microscopic systems that would not apply equally to *macroscopic* systems?

The key phrase is surely that which Bohr himself stresses, namely "*the very conditions which define the possible types of predictions regarding the future behavior of the system*". What does Bohr think those conditions are? I believe a close reading of the rest of his reply to EPR shows that what he has in mind here are the experimental conditions set up by an experimenter who wishes to perform the measurement in question. Moreover, this reading receives support from others of Bohr's writings. Bohr would insist that any consideration even of a hypothetical measurement must be based on some specification of the experimental conditions in order to be well-grounded enough to play a role in an

argument like that of EPR. His idea is that since any ascription of physical reality to a quantity is meaningful only in a well-defined experimental context, the element of reality EPR argue for cannot be detached from the (hypothetical) experimental context in which it is initially inferred to play an independent role in the argument, including its conclusion.

Bohr maintained that the experimental conditions must be specified in ordinary language, suitably enriched with the vocabulary of classical physics. In his words (Bohr<sup>(37)</sup> )

it is decisive to recognize that *however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms.* The argument is simply that by the word 'experiment' we refer to a situation where we can tell others what we have done and what we have learned and that, therefore, the account of the experimental arrangement and of the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics. (p.209)

The main point here is the distinction between the *objects* under investigation and the *measuring instruments* which serve to define, in classical terms, the conditions under which the phenomena appear. (pp. 221-2)

It has often been assumed that the distinction to which Bohr refers here is one based on *size*: that the apparatus is macroscopic, and so accurately treatable by classical physics, while the object under investigation is microscopic, and so must be treated quantum mechanically. That this assumption is false becomes apparent when one reads the discussion in Bohr<sup>(37)</sup> of his debates with Einstein, in which he applies quantum mechanics to macroscopic objects without even feeling the need to comment on the fact.

The distinction Bohr has in mind is a *pragmatic* one: in order to apply quantum mechanics to a system at all, he believes, one must describe the whole experimental arrangement surrounding that system *classically*. That is true whether the system under investigation is microscopic or macroscopic. But any object that one (perforce) described classically when it figured in the experimental arrangement for investigating some other system may *itself* be made the system under investigation in the context of a *different* experimental arrangement, in which case it would be legitimate to apply quantum mechanics to it in that context, and even necessary if classical physics proved inadequate to predict results of observing it in that context.

Bohr's response to EPR did not rest on the denial of physical reality to isolated microscopic systems. Instead, it rested on the denial of context-independent attributions of properties (or rather physical quantities) to systems of *any* size. He took this denial to be required by the transition to quantum physics from classical physics. As he put it,

The necessity of discriminating in each experimental arrangement between those parts of the physical system considered which are to be treated as measuring instruments and those which constitute the objects under investigation may indeed be said to form a *principal distinction between classical and quantum-mechanical descriptions of physical phenomena*. ((36), p.701)

On my reading, this passage makes clear just how radical Bohr's view of quantum mechanical description was. In his view, with quantum mechanics, all ascriptions of physical reality to properties of systems become contextual: taken out of context, they lack significance. This denial of significance rests on a pragmatist rather than a verificationist view of meaning<sup>10</sup>. To supply the context needed to render meaningful the ascription of a property to a system that one has decided to treat as "an object under investigation" when applying quantum mechanics, one must describe *other* surrounding systems classically. This does not mean that those other systems *are* classical rather than quantum mechanical. Still less does it mean that there is a special class of systems ("the macroscopic systems") which must be described classically. But it does mean that there can be no purely quantum mechanical description of the world, or even of any part of the world to which one contemplates applying quantum mechanics. Laloë's<sup>(12)</sup> BEC *Gedankenexperiment* helps to bring out this radical character of Bohr's view even though it does not challenge it. On Bohr's view, once one has decided to apply quantum mechanics to the system of BECs in this *Gedankenexperiment*, even the ascription of a macroscopic magnetization to part of that system in a region lacks significance, absent classically described conditions external to the system. It may be hard to accept, but it is no refutation of this view, that bringing up a hand compass renders that ascription not only meaningful but true.

This response to Laloë's<sup>(12)</sup> BEC *Gedankenexperiment* has interesting implications for the claim that classical physics (or parts of it, including classical mechanics) is reducible to quantum physics (including quantum mechanics). If one endorses the response, then one has strong grounds for denying such claims of reducibility. According to classical physics, the behavior of a hand compass near region *B* containing enough of the BEC system following 100 or so measurements of transverse spin on particles in region *A* would warrant ascribing a macroscopic magnetization to (the contents of) that region. Any reduction of classical physics to quantum physics here would involve showing that this is true<sup>11</sup>. But if one endorses (what I take to be) Bohr's response, this is *not* true, or even significant, outside of an appropriate context. So the most one could

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<sup>10</sup>Here an analogy may be helpful. Consider the following message carved on a tree-trunk deep in a forest: "I'll meet you here tomorrow". This message is significant only in a context which specifies to whom 'I' and 'you' refer and on what day the message was carved. Absent such a context, the message is useless and so lacks pragmatic significance even though its semantic role in English is perfectly clear. Supplying the context renders the message verifiable.

<sup>11</sup>One might qualify this with 'approximately', but the qualitative nature of the ascription renders this irrelevant.

expect is a *contextual* reduction of classical to quantum physics. But even this would elude one in so far as at some stage the assumed *context* could not be described within a significant application of quantum physics<sup>12</sup>.

## 7 Emergent Properties and Emergent Objects

Laloë summarizes the essence of his (12) as follows

in some quantum situations where macroscopic systems populate Fock states with well defined populations, the EPR argument becomes significantly stronger than in the historical example with two microscopic particles. The argument speaks eloquently in favour of a pre-existing relative phase of the two states ... but certainly not in favour of the orthodox point of view where the phase appears during the measurements. (p.51)

In spite of the objections I have lodged against his argument, Laloë here expresses an important insight that should not be lost if we wish to understand the emergence of relative phase between BECs. The EPR argument was directed against a popular version of the Copenhagen interpretation that takes the quantum state to describe intrinsic properties of a system it represents, and measurement to project the quantum state onto a new one that describes the system's new intrinsic properties. If one tries to understand the emergence of relative phase between BECs initially in a double Fock state as a stochastic dynamical process mediated by successive projective measurements on individual particles in the condensate, then, as Laloë goes on to say, surprising non-local effects appear in the macroscopic world (which, I might add, are extremely difficult to reconcile with relativistic spacetime structure, even though they do not admit superluminal signalling).

Must one who rejects this popular version of the Copenhagen interpretation conclude that the relative phase between BECs was definite already prior to measurements on its constituent particles, which simply progressively reveal that pre-existing phase  $\Lambda$ ? Drawing this conclusion on the basis of EPR-type reasoning, one would take  $\Lambda$  to be an *additional* variable characterizing the BECs in quantum state (3), initially hidden but gradually revealed by transverse spin measurements. But further investigations by Laloë and Mullin<sup>(10)</sup> effectively block this route. They derive (Bell-Clauser-Horne-Shimony-Holt)-type inequalities for carefully chosen observables of particles in quantum state (3) on the assumption of a pre-existing relative phase between the condensates,

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<sup>12</sup>This train of thought may be what Landau and Lifshitz<sup>(38)</sup> had in mind when they said

"quantum mechanics occupies a very unusual place within physical theories: it contains classical mechanics as a limiting case, yet at the same time it requires this limiting case for its own formulation." p.3

and show that quantum mechanics predicts their violation in that state.<sup>13</sup> So, just as in the Bohm-EPR case, the intended conclusion of EPR-type reasoning here proves to be incompatible with quantum mechanics itself.

There is a different way to use a pre-existing relative phase  $\Lambda$  to account for the interference exhibited by a system of two similar condensates as a result of transverse spin measurements. It is to *deny* that their initial quantum state is correctly represented by (3), and to claim that it is rather a phase state (15). The analysis of section 4 shows that these two quantum states lead to identical interference patterns for the phenomena considered there. Of course, by taking this line one is evading rather than solving the problem of understanding how a relative phase emerges in the double Fock state (3). Such evasion could be justified by an argument as to why any natural preparation procedure for a system of condensates of the type we have been considering would give rise to the phase state (15) instead. But if one recalls that the whole discussion of interference between similar BECs was provoked by experiments like those of Andrews *et. al.*<sup>(2)</sup>, the prospects of developing such an argument seem bleak.

Leggett<sup>(21)</sup>, for example, says this

The authors start with a trap which is split into two by a laser-induced barrier so high that the single-atom tunnelling time between the two wells is greater than the age of the universe. They then condense clouds of <sup>87</sup>Rb atoms independently in the two wells and allow them to come to thermal equilibrium. At this point there seems no doubt that the correct quantum mechanical wave-function of the system is, schematically, [of the form of a double Fock state] (p.138)

He goes on to show that the time-evolution of each component to bring them into overlap after removal of the laser barrier will not make this double Fock state approach a phase state. Laloë<sup>(12)</sup> argues that environmental decoherence can favor phase states over double Fock states, but dismisses this as a reason to reject his analysis in terms of double Fock states. Since coupling with the environment tends to produce an improper *mixture* of phase states, if there is any interference in a system like that analyzed, this cannot be accounted for by appeal to a *pure* phase state of the BEC system.

It is interesting to contrast the case of a system of dilute gas BECs in a double Fock state (3) with other systems involving a pair of condensates that exhibit interference phenomena accounted for by appeal to a relative phase between them. When a pair of conductors separated by a thin metal oxide junction is cooled to become superconducting, a current flows across the junction even in the absence of an applied voltage. This DC Josephson effect may be explained quantum mechanically by appeal to a well-defined phase difference  $\phi$  across the junction in the wave-function representing the state of the system: the DC

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<sup>13</sup>It is, however, noteworthy that experimental violation would be extraordinarily difficult to arrange because it would be essential to perform measurements on *all the particles* in the system of condensates.

current is proportional to  $\sin \phi$ . Leggett and Sols<sup>(25)</sup> write the wave-function as follows,

$$\Phi \sim \left( |a| e^{i\phi/2} \psi_L + |b| e^{-i\phi/2} \psi_R \right)^N \quad (21)$$

where the system consists of  $N$  "bosons" (Cooper pairs) and  $\psi_L$  ( $\psi_R$ ) is the Schrödinger amplitude for a boson to be on the left (right) of the junction. Note the analogy with the phase state (15)<sup>14</sup>. If positing a phase state like (21) is indeed the only way to explain the Josephson effect, then just as in the case of the dilute gas BECs, one should ask how a relative phase emerges. One possible answer is that there is *always* some relative phase difference between any pair of similar superconductors (even those prepared independently and arbitrarily far away from each other), and its (random) value emerges as a result of spontaneous breaking of the  $U(1)$  symmetry. Leggett and Sols<sup>(25)</sup> reject this answer, and Leggett<sup>(26)</sup> advances an interesting argument for doing so.

To set the context for this argument, note that the state (21) may be expanded in a basis of double Fock states  $|N_a, N_b\rangle$  as

$$\Phi \sim \sum_{M=-N/2}^{+N/2} |C_M| e^{iM\phi} |N_a, N_b\rangle \quad (22)$$

where  $(N_a + N_b) = N$  and

$$|N_a, N_b\rangle \sim \hat{a}_{\psi_L}^{\dagger N_a} \hat{a}_{\psi_R}^{\dagger N_b} |0\rangle \quad (23)$$

It follows that in state (21) the difference between the number of bosons in the left-hand condensate and the number in the right-hand condensate is indeterminate, even though together they contain exactly  $N$  bosons. This may be acceptable in the typical situation in which one takes (21) to represent the state of a pair of similar condensates, prepared together and spatially separated only by a thin junction. But it is harder to stomach if the left and right hand condensates have been separately prepared in different continents!

Leggett<sup>(26)</sup> rejects this *outré* suggestion, and presents a thought experiment as a reason for doing so.

The "experiment" simply consists in weighing them at separate times ... that can be arbitrarily far separated, so as to determine the number difference  $[N_a - N_b]$  at these times, without ever making Josephson contact between them. (p.459)

If (21) correctly represents their total state, then there is no reason to expect the results to agree: indeed, one would expect them to differ by an amount of the order of  $N^{\frac{1}{2}}$ . If, on the other hand, the correct representation is a double Fock state (or mixture of these), then the results would be expected to agree (within the margin of error of the experiment). Leggett<sup>(26)</sup> concludes

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<sup>14</sup>Leggett and Sols (1991) actually apply their analysis to a generic Josephson effect in a Bose superfluid, of which a superconductor is one example, another being superfluid Helium.

I can see no reason whatever to doubt that it is this latter conclusion which would be found experimentally, so that in *this* (operationally defined) sense, the statement that "two superfluids which have never seen one another before nevertheless have a definite relative phase" is, I believe, false. (*ibid.*)

But suppose we take spontaneous symmetry breaking absolutely seriously here and consider what we should say if the results of Leggett's thought experiment were to confound his firm expectations. In that case, I submit, we should have evidence for more than just the emergence of relative phase in BECs consequent upon spontaneous symmetry breaking: we should have reason to accept the spontaneous emergence of composite *objects*—the BECs themselves.

Here we have at least a conceptual possibility not (to my knowledge) contemplated by philosophers interested in emergence. When philosophers have considered the possibility of emergent objects, they have had in mind a case in which an object composed of a perfectly determinate set of microscopic parts possesses an emergent *property* (however that notion is analyzed).<sup>15</sup> But what we are presently contemplating is a case in which each of two objects, composed of nothing but microscopic parts of a certain kind, contains no definite *number* of these objects, although together the *pair* is composed of a definite number of these constituent parts.

At first this may seem analogous to more familiar cases: consider a cat's tail and the rest of its body, Siamese twins, two colliding galaxies, or the stratosphere and troposphere. But in such cases the parts of the total system are spatially contiguous and the indeterminateness of composition of each where they join is naturally attributed to the vagueness of the language we use to describe them. If two similar BECs, independently prepared on different continents, had a definite relative phase, then each BEC would be an emergent object in a much stronger sense. The indeterminateness of composition could not be localized to any spatially intermediate region and would be distributed equally among all their component bosons. Perhaps we have here a new candidate for the metaphysician's disputed category of vague objects?

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<sup>15</sup>See, for example, section 1 of Bedau<sup>(15)</sup>.

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