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Sir Michael Anthony Eardley Dummett, 1925–2011

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Sir Michael Anthony Eardley Dummett died on 27 December 2011, at the age of 86, after a long illness. He spent his entire philosophical career at Oxford University, first as a Fellow at All Souls College and then at New College, as Wykeham Professor of Logic, from 1979 until his retirement in 1992. Sir Michael, who was knighted in 1999 for 'Services to Philosophy and to Racial Justice', was also a Fellow of the British Academy.[1]

Dummett's philosophical interests were primarily focused on the philosophy of language, logic, and mathematics, on metaphysics, and on the philosophy of Gottlob Frege, though he also did important work on causation, on the philosophy of time, on the history of analytic philosophy more generally, and on modal logic. I will focus here on Dummett's contributions to philosophy of mathematics, and not just because this journal is devoted to that subject. In many ways, the philosophy of mathematics was the single area of philosophy most important to Dummett, and his work there drove many of his other concerns. Indeed, even after he had otherwise stopped actively working on philosophy, Dummett continued to attend Oxford's Philosophy of Mathematics Seminar. He only stopped doing so in 2010, when his deteriorating health made it impossible.

Dummett's most important contributions to philosophy of mathematics may be divided into three broad categories: (i) his engagement with and canonization of Frege; (ii) his interpretation and defense of mathematical intuitionism; and (iii) his invocation and development of the notion of an 'indefinitely extensible' concept. There is more as well, most notably the criticism of 'strict finitism' in his seminal paper 'Wang's Paradox' [Dummett, 1978g], which is primarily concerned with vagueness, and his review of Wittgenstein's *Remarks on the Foundations of Mathematics* [Dummett, 1978h], which remains worth reading. But only so much can be discussed here.

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[1] For further details about Dummett's life, see Daniel Isaacson's obituary notice, at http://www.philosophy.ox.ac.uk/news__events/older_news/in_memoriam_michael_dummett_1925-2011.

Dummett on Frege

It would be difficult to over-state the significance of Dummett's work on Frege. Today, Frege is widely regarded as one of the most important philosophers in the history of the subject, but it was not always so. Frege's work was, of course, well-known to such luminaries as Russell, Wittgenstein, Carnap, and Quine, but until the 1950s Frege's influence on the field as a whole was usually indirect, in large part because it was only then that English translations of much of Frege's work started to become available. Even once they did, however, Frege's influence was limited, and he was generally regarded as a somewhat minor figure whose contributions to philosophy were confined to tidbits like the puzzle about Hesperus and Phosphorus.

Everything changed, however, in 1973, with the publication of Dummett's *Frege: Philosophy of Language* [1973]. *FPL* not only established Dummett himself as a major figure, but established Frege as a philosopher of genius, and when one reads the literature on Frege from the 1950s and 1960s nowadays, one cannot but be struck by its comparative naïvete.[2] As Dummett himself noted, Frege's interests were limited in scope compared with those of the philosophers who had preceded him into the canon. Even Russell had an interest in epistemology that Frege did not really share. But Dummett showed in his book that Frege's views constitute a powerful and coherent whole with implications throughout the philosophy of language and logic and, therefore, well beyond, given the central importance of logic, and of general questions about the nature of representational content, throughout philosophy. Moreover, Dummett showed by example that Frege's views are worthy of sustained attention, thus both inviting and initiating the explosion of interest in Frege's work that followed.

Dummett intended to publish a companion volume, *Frege: Philosophy of Mathematics* [1991a], shortly afterward, but *FPM* would not appear until 1991. The reason was largely the sudden need for Dummett to engage the burgeoning literature on Frege for which *FPL* was responsible and to defend his interpretation of Frege, which was derided in some quarters as ahistorical. There is some truth to the charge. Dummett does not approach Frege in *FPL* the way a scholar would, but largely treats him as a contemporary. And *FPL* is, in many ways, not really a book about Frege, but a book about what Frege has to teach us about the philosophy of language. Still, Dummett's knowledge of Frege's writings was so comprehensive and so deep that, far more often than not, he had Frege right. I have often had the experience myself of struggling, sometimes for months, with some

[2] There are, of course, shining exceptions to this general principle, including still-famous papers by Peter Geach and Charles Parsons.

aspect of Frege's views, finally to arrive at a resolution, only then to stumble across a passage in Dummett's writings that proves he had been there before me.

Because of the long delay in publication, *FPM* did not have the kind of influence that *FPL* did, since, by then, the study of Frege's philosophy of mathematics had become a growth industry, the proximal cause being Crispin Wright's *Frege's Conception of Numbers as Objects* [1983]. As Wright acknowledges, however, his book owes a large debt to Dummett's work. The conception of reference to abstract objects that is developed in the early chapters of *FCNO* is a direct descendent of one Dummett had defended in an early paper [1978a] but abandoned in *FPL* [1973, Ch. 14]. But it is not so much in the details that Dummett's influence is felt, but in the very fact that Wright would bother paying such careful attention to Frege's work. Without Dummett's example, I am not sure any of us would ever have done so.

Dummett on Intuitionism

Mathematical intuitionism originates in the work of the Dutch mathematician L.E.J. Brouwer. Logically speaking, what is distinctive of Brouwer's approach is his rejection of the law of excluded middle and, relatedly, certain quantificational inferences, such as from $\neg\forall x(Fx)$ to $\exists x(\neg Fx)$. Intuitionism is thus a form of constructivism, and intuitionistic mathematics diverges from classical mathematics, especially in the case of real analysis. What justifies the approach, however, in Brouwer's writings, is a broadly Kantian conception of mathematics as a psychological activity and of mathematical objects as a sort of mental construction, which in turn supports the intuitionist's characteristic rejection of 'completed' infinities in favor of 'potential' infinities.

Dummett regarded Frege's criticism and rejection of psychologism — the view that logic is a branch of psychology — as among the most important and far-reaching of his contributions to philosophy. That makes Dummett's interest in mathematical intutionism in some ways surprising. But it is, oddly enough, actually a consequence of how Dummett understands Frege's rejection of psychologism. Dummett took Frege to have shown, but never fully to have appreciated, that the relation between a word and the concept it expresses cannot be 'private' in Wittgenstein's sense, but must somehow be manifest in the use one makes of that word. Dummett then proceeds to argue, first, that the applicability of classical reasoning to mathematics requires that our understanding of such propositions as Goldbach's conjecture should guarantee that they are either true or false (*i.e.*, that bivalence holds) and, second, that no such understanding *could* be manifest in the use we make of the sentences expressing such propositions. Rather, Dummett claims, our understanding of Goldbach's conjecture consists simply in our appreciation of what would constitute a proof of it, in which case classical logic is not applicable in mathematics and an alternative must be sought, which Dummett took to be intuitionistic logic. Dummett thus proposes to ground intuitionism not in psychology but, much as Kreisel had, in a theory of constructions.

This sort of argument has become known as the 'manifestation argument'. An extremely condensed version appears in one of Dummett's earliest publications, the fascinating but difficult paper 'Truth' [1978e]. A developed version appears in the legendarily complex but still rewarding paper 'The philosophical basis of intuitionistic logic' [1978b], which was first published in *Logic Colloquium '73*. But Dummett was aware from the outset that the argument would extend far beyond mathematics, to encompass any domain of inquiry in which we cannot, even in principle, determine the truth or falsity of claims within it, for example, claims about the past [Dummett, 1978d], and the argument itself concerns the foundations of semantics more than of mathematics. As a result, the most sustained developments of the argument are in such papers as 'What is a theory of meaning? (II)' [Dummett, 1993] and in the book *The Logical Basis of Metaphysics* [Dummett, 1991b], which is based upon Dummett's William James Lectures, which were delivered at Harvard in 1976.

The manifestation argument has often been criticized as depending upon a crude form of behaviorism. My own view is that the charge is unjustified, though understandable [Heck, 2005]. But, however that may be, Dummett's argument for 'semantic anti-realism', as the view is often known, has few defenders nowadays. In recent years, though, more and more philosophers have learned to look beyond the seemingly outrageous conclusion for which Dummett was arguing to be able to appreciate the profound insights about language on which that argument was based. What makes the charge of behaviorism understandable is Dummett's initial commitment to a view on which understanding is a 'purely practical ability'. But that view is short-lived, and, from the late 1970s into the 1990s, Dummett works very hard to articulate a different view on which understanding has a crucial cognitive component: on which understanding is knowledge of meaning, and where knowledge is not just know-how. And that view, despite Dummett's own antipathy towards the program, fits very naturally with the cognitive conception of linguistic competence that emerges from contemporary linguistics, as done in the shadow of Chomsky. In my own view, this is an extremely important convergence, one we have yet fully to appreciate or exploit.

Dummett's interest in intuitionism extended beyond its philosophical basis, however, and into the details of intuitionistic mathematics, logic, and semantics. The fruits of that interest may be found in his book *Elements of Intuitionism* [1977]. Though somewhat idiosyncratic in its choice of topics, Dummett's book remains the most accessible survey of the logic and mathematics of intuitionism I know. The second edition [Dummett, 2000] remedies many of the problems with the first (which means, most importantly, the fact that the first was not typeset but basically photocopied from a typescript) and adds some new material.

What was most important about Dummett's defense of intuitionism, however, was what it did to legitimize logical revisionism. It had of course been suggested before that the 'correct' logic might not be the one bequeathed to us by Boole, Frege, and Russell. But Dummett was one of the first to show how this sort of idea might emerge naturally within a larger philosophical framework. As Graham Priest wrote in remembrance of Dummett: 'Perhaps his greatest achievement ... was to demonstrate beyond doubt the intellectual respectability of a fully-fledged philosophical position based on a contemporary heterodox logic' [Lepore, 2012].

Dummett on Indefinite Extensibility

As is well-known, Frege's attempt to reduce arithmetic to logic ran aground on the shoals of Russell's Paradox. Dummett argues in *FPM* that the contradiction reveals a fundamental flaw in Frege's philosophy, in particular, in his conception of how reference to abstract entities might be secured. But, of course, it raises another question, too, namely, as Dummett puts it, how the serpent entered Eden, that is, what is responsible for the set-theoretic paradoxes.

Dummett makes two sorts of proposals in answer to this question. The first is that, contrary to what is usually supposed, the culprit is *not* Frege's infamous Basic Law V, which for our purposes may be taken to be:

$$\hat{x}Fx = \hat{x}Gx \equiv \forall x(Fx \equiv Gx).$$

The easiest way to derive Russell's Paradox from Law V is first to define membership

$$a \in b \equiv \exists F(b = \hat{x}Fx \wedge Fa)$$

and then to ask whether $\hat{x}(x \notin x)$ is or is not a member of itself. The argument to a contradiction is then a simple exercise, but it is easy to see that it depends upon the availability of the comprehension axiom:

$$\exists F \forall x[Fx \equiv x \in x]$$

which is impredicative, because of the presence of the second-order existential quantifier in the definition of $\in$. Dummett therefore suggests that the inconsistency of Frege's formal system is due to the presence of second-order quantification or, more precisely, of *impredicative* second-order quantification [Dummett, 1991a, pp. 217–22], much as Russell had.

Whether or not one accepts this diagnosis, it sparked an investigation that has proved quite fruitful. The predicative fragment of Frege's theory is an extremely natural system. The obvious questions, however, are whether it is actually consistent and, if so, how much mathematics can be developed within it. The first question was answered affirmatively by the present author [Heck, 1996], with further investigations by John P. Burgess [2005], Mihai Ganea [2007], and Albert Visser [2009] eventually culminating in an answer to the second question, as well. It turns out that the theory is very weak,[3] but one might well think that a properly predicative theory should be quite weak.

Dummett does not suggest that we retreat to a predicative version of Frege's system, however. He proposes instead that the paradoxes result from a failure to recognize, and to understand how properly to reason with, what he calls 'indefinitely extensible concepts' [Dummett, 1991a, p. 316].[4] These are essentially what Russell called 'self-reproductive' properties: ones for which, 'given any class of terms all having such a property, we can always define a new term also having the property ... ' [Russell, 1906, p. 36]. The most natural (putative) example of such a concept is *ordinal number*.

People nowadays tend to think of ordinals as order types, but, in ordinary usage, ordinal numbers are ones that are used to answer questions about an object's position in a certain sort of ordering (namely, a well-ordering). That is to say, just as cardinal numbers answer the question 'How many?', ordinals answer the question 'The which$^{\text{th}}$ one?' Now consider all the ordinals there are. Under their natural ordering, the ordinals are themselves well-ordered, so we can ask, of a given ordinal: Which position in this ordering does it occupy? The answer will be given by that very ordinal.[5] For example, ω occupies the ω^{th} place in the ordering of ordinals. Now put me at the end of this ordering. *i.e.*, consider the ordering of the ordinals plus me in which $x \prec y$ if either x and y are ordinals and $x < y$, in the usual sense, or x is an ordinal and y is me. Which position in the ordering do I occupy? None of our original ordinals specifies that position. One might therefore want to say that there simply no answer to this question. But how can there just *not be* an answer? How could our making such a minor modification to the original ordering cause such chaos? Is it not perfectly determinate what position I occupy in this ordering?

Dummett's answer is: No, it is not perfectly determinate. Our error, he would have us say, was to suppose that the ordinals form a determinate

[3] Let T_0 be Robinson arithmetic, Q; and let T_{n+1} be $T_n + \mathrm{Con}(T_n)$. Then what Visser showed is that ramified predicative second-order logic plus Basic Law V is mutually interpretable with $T_\omega = \bigcup T_n$.

[4] The idea first appears, in Dummett's writings, in his paper 'The philosophical significance of Gödel's Theorem' [Dummett, 1978c], first published in 1963.

[5] Of course, this assumes we started with one, not zero. But there is no zero$^{\text{th}}$ position in any ordering, so there is not really an ordinal number zero$^{\text{th}}$.

collection onto the end of whose natural ordering I can be stuck. Given anything purporting to be all the ordinals there are, he wants to claim, we will always be able to construct an ordering from them and then ask a perfectly sensible question of the form, ‘Which position in this ordering does that object occupy?’ which question *cannot* be answered using any of those ordinals, which implies that there is at least one ordinal not included in the original collection. So the Burali-Forti paradox is the result of our trying to reason about ‘all ordinals’ as if the ordinals constituted a completed collection rather than one that is forever growing.

The analogy between the notion of an indefinitely extensible concept and the more familiar notion of potential infinity should be obvious. In a sense, then, it is not surprising that Dummett should suggest that the correct logic to use when reasoning about indefinitely extensible concepts is not classical logic, but intuitionistic logic [Dummett, 1991a, p. 319]. What Dummett has offered us, then, is a very different sort of argument for intuitionism, one that is much closer to Brouwer’s original motivations than Dummett’s ‘semantic’ argument is, and one that is specific to mathematics rather than almost completely general.

It is, of course, another question whether Dummett’s diagnosis of the Burali-Forti paradox is really satisfying, and even whether the notion of an indefinitely extensible concept can properly be explained. But the argument seems to me, at least, to be of substantial interest and to be worthy of continued attention.

Closing

Sir Michael Dummett was undoubtedly one of the most important British philosophers of the twentieth century. But he was also much more. He did important work on the theory of voting, publishing several articles and two books on the subject, and inventing the Quota Borda voting system. He was one of the world’s foremost experts on the history of tarot (more the cards and the game than the occult practice), authoring or co-authoring several books on that topic. He was a dogged opponent of racism and a tireless advocate of the rights of immigrants, which was the lifelong devotion of his wife of sixty years, Ann, who died just six weeks after Sir Michael. He was also a veteran, having served from 1943–47 in the Royal Artillery and Intelligence Corps. And he was a devoted husband and father, fiercely proud of his family.

More than anything, though, I will always remember Sir Michael as a teacher. When I went to Oxford in 1985, it was in the hope of studying with him. The opportunity to do so for two years shaped my life, confirmed my passion for philosophy, and gave me the best possible foundation on which to begin a serious study of Frege. Sir Michael had a similar influence on several generations of students, and he helped shape what ‘Oxford philosophy' was for almost sixty years. And yet, what is most striking about him is how deeply, and how broadly, he was loved and admired. The remembrances collected by Ernie Lepore and published on *The Stone* [Lepore, 2012] will give those who did not know him a reasonable sense of why.

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