# Title:

Color may be the phenomenal dual aspect of two-state quantum systems in a mixed state

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# Abstract

I show that the mathematical description of opponent-colors theory is identical to the mathematical description of two-state quantum systems in a mixed state. Based on the dual-aspect theory of phenomenal consciousness, which suggests that one or more physical entities in our universe have phenomenal aspects that are dual to their physical aspects and therefore predicts an exact correspondence between a system’s phenomenal states and the objective states of its underlying physical substrate, I hypothesize that color sensations are phenomenal dual aspects of two-state quantum systems in a mixed state. Since nothing in this hypothesis suggests that what brings about the phenomenal experience that is dual to two-state quantum systems is the two-dimensionality of these systems, it is natural to generalize this hypothesis and suggest that other types of phenomenal experience (e.g., taste, odor, sound) are phenomenal dual aspects of quantum systems of higher dimensionalities. Finally, I propose that the two-state quantum systems that give rise to color in the brain are two-state ion-channels.

# Preface (or: Taking consciousness seriously)

Like many others, my views on phenomenal consciousness changed dramatically after reading David Chalmers’s attack on physicalism in ‘*The Conscious Mind’* (1996). Chalmers said nothing new, of course. Many thinkers before him, from Leibniz (with his mill) through Huxley (with his djinn) to Nagel (with his bat) and Jackson (with his Mary), had concluded that physicalism is impossible. But Chalmers’s argumentation was so forceful and lucid (and came exactly at a time when people were beginning to realize that neuroscience could not possibly solve what, thanks to Chalmers, has become known as the ‘hard problem of consciousness’) that it has led to a tectonic shift in the philosophy of mind: after being shunned from discussion for many decades, non-physicalist world views on the mind—including ones that had been considered totally crazy, like panpsychism—have become respectable again and are now thriving in the new atmosphere.

At the very beginning of his book, Chalmers urges us to ‘take consciousness seriously’. My understanding of this ‘call for arms’ is that we should accept—*really* accept, not merely as lip service—that there is some process in the brain that gives rise to this wonder of wonders, phenomenal consciousness. Since we can be almost certain that science now understands *all* the physiological, biophysical, chemical, and physical processes that take place in the brain (i.e., it is highly unlikely that some mysterious process that can explain phenomenal consciousness is going to be discovered in the future), the answer to ‘the biggest mystery’ (Chalmers, 1996, p. *xi*) must be within our reach. This is a tantalizing thought: one of those ‘mundane’, almost boring, processes in the brain—processes whose mechanistic details are dryly laid out in our textbooks for undergraduates to memorize—must be responsible for the most bizarre aspect of our universe (excluding, perhaps, the fact that it exists), namely, the existence of phenomenal consciousness. The answer must lie there, in one of those processes, seemingly right at our fingertips. Yet, when we look at those brain processes, all we see is—just as in Democritus’s famous dictum—motion of molecules and atoms. Thus, the famous words of Thomas Huxley—'How it is that anything so remarkable as a state of consciousness comes about as a result of irritating nervous tissue, is just as unaccountable as the appearance of the djinn when Aladdin rubbed his lamp in the story’—are as true today as they were in the 19th century. Although this has led some thinkers to despair of ever solving the ‘hard problem’ (McGinn, 1989), I suggest that there is a glimmer of hope. This glimmer comes from the stipulation of the dual-aspect theory of phenomenal consciousness that there exists an exact correspondence between phenomenal states and their underlying physical processes (Chalmers, 1996, chapters 6 and 8; Cortês et al., 2021; Lockwood, 1989, chapter 11). Here, for example, is G. E. Müller’s 1896-formulation of dual-aspect theory (translated in Boring, 1942, p. 89):

1. The ground of every state of consciousness is a material process, a psychophysical process so-called, to whose occurrence the presence of the conscious state is joined.
2. To an equality, similarity or difference in the constitution of sensations... there corresponds an equality, similarity or difference in the constitution of the psychophysical process, and conversely. Moreover, to a greater or lesser similarity of sensations, there also corresponds respectively a greater or lesser similarity of the psychophysical processes, and conversely.
3. If the changes through which a sensation passes have the same direction, or if the differences which exist between series of given sensations are of like direction, then the changes through which the psychophysical process passes, or the differences of the given psychophysical processes, have like direction. Moreover, if a sensation is variable in *n* directions, then the psychophysical process lying at the basis of it must also be variable in *n* directions, and conversely.

Almost a hundred years later, Lockwood (1989, p. 172) phrased the same idea in an almost identical manner:

Take some range of phenomenal qualities. Assume that these qualities can be arranged according to some abstract $n$-dimensional space, in a way that is faithful to their perceived similarities and degrees of similarities... Then my… proposal is that there exists, within the brain, some physical system, the states of which can be arranged in some $n$-dimensional state space… And the two states are to be equated with each other: the phenomenal qualities are identical with the states of the corresponding physical system.

Thus, according to the dual-aspect theory, if we find a physical system whose states can be shown to have the same structure as the states of some type of phenomenal experience, this physical system should be considered a good candidate to have a phenomenal dual aspect. This is precisely what I do in this paper for what is arguably the simplest of all types of phenomenal experience—color. Specifically, I show that there exists an *exact* correspondence between the mathematical structure of opponent-colors theory and the mathematical structure of two-state quantum systems in a mixed state. This leads me to the conclusion stated in the title of this paper: color may be the phenomenal dual aspect of two-state quantum systems in a mixed state.

# 1. Introduction

As was first noticed by Leonardo da Vinci, there seem to exist six elementary color sensations: red, green, yellow, blue, white, and black (Valberg, 2001). These six sensations are *elementary* because they cannot be broken down into other color sensations, namely, they cannot be described as containing within some color components (compare, for example, with the sensation of orange, which is perceived as some mixture of red and yellow). In his opponent-colors theory, Hering suggested that the six elementary colors are produced by three mechanisms, each containing a pair of *opponent* elementary color processes: a red–green mechanism, a yellow–blue mechanism, and a white–black mechanism (Hurvich & Jameson, 1957). The output of each mechanism results from the difference between the activities of its two constituent processes (which, of course, is why these processes are called opponent). Consequently, the color sensation that each mechanism produces is due to the process that is more active.

This is the standard formulation of opponent-colors theory. It faces two problems. First, while the colors produced by each of the hued mechanisms do not mix to yield a phenomenal intermediate (i.e., there are no greenish reds, reddish greens, yellowish blues, or bluish yellows), the colors produced by the white–black mechanism do have a phenomenal intermediate—gray (Boring, 1942, p. 209; Heggelund, 1974a; Ladd Franklin, 1899). This, of course, raises the question of why the white–black mechanism behaves differently from its hued counterparts, a question that opponent-colors theory doesn’t answer. The second problem facing Hering’s opponent-colors theory is that it predicts that when all three opponent-colors mechanisms are in equilibrium (i.e., when their activities are zero), we should perceive nothing at all, but observation shows that in such a case we perceive gray (Boring, 1942, p. 213; Hurvich & Jameson, 1957; Titchener, 1910, pp. 90–91). (See Hendel, 2023, for a more detailed discussion of the problems in Hering’s theory.)

In a recent paper, based on earlier work by Paul Heggelund (1974a, 1974b, 1991, 1992, 1993), Hendel (2023) provided a modification of Hering’s opponent-colors theory that doesn’t suffer from the above two problems. Here I show that the mathematical description of Hendel’s formulation of opponent-colors theory is identical to the mathematical description of two-state quantum systems in a mixed state. Although a mathematical identity between two theories in such disparate fields might be waved off as a meaningless coincidence, the dual-aspect theory of phenomenal consciousness, which suggests that one or more physical entities in our universe have phenomenal aspects that are dual to their physical aspects (see, e.g., Chalmers, 1996, chapter 8), predicts exactly such a correspondence between a system’s phenomenal states and the objective states of its underlying physical substrate (Chalmers, 1996, chapters 6 and 8; Cortês et al., 2021; Lockwood, 1989, chapter 11; also see G. E. Müller’s famous psychophysical axioms (Boring, 1942, p. 89)). Therefore, following the principles of dual-aspect theory, I suggest that color sensations are phenomenal dual aspects of two-state quantum systems in a mixed state.

# 2. The mathematical description of opponent-colors theory

The basic tenets of Hering’s formulation of opponent-colors theory and the two problems that this formulation of the theory faces were described in the *Introduction*. Heggelund (1974a, 1974b, 1991, 1992, 1993) modified Hering’s theory in a way that solved these two problems. Specifically, Heggelund suggested that—in contrast to the assumption that Hering (and the vast majority of other color scientists) had been making—the hueless colors do not lie along a one-dimensional continuum stretching between black and white, but rather require *two* dimensions for their full description: a *lightness dimension* that has the sensations of light and dark (black) at its poles and a *whiteness dimension*. Thus, Heggelund’s theory adds the light sensation as a seventh elementary color. This hueless color sensation, which appears only in objects that are perceived as *emitting* light (e.g., light bulbs or stars in the night sky), forms an opponent pair with dark. The light–dark opponent pair is analogous to the red–green and yellow–blue opponent pairs in that the sensations light and dark are mutually exclusive. By contrast, on Heggelund’s theory, white has no opponent counterpart. Hence, this theory contends that white and black (dark) are not opponent colors, which explains why they mix to yield a phenomenal intermediate (gray).

However, Hendel (2023) showed that although Heggelund’s theory successfully solves the two problems in Hering’s theory, it is not self-consistent. He then revised this theory and obtained a consistent opponent-colors theory. In Hendel’s version of opponent-colors theory, every color is described by the following four-dimensional vector:

$$\begin{array}{c}C=I\hat{x}\_{0}+\left(R-G\right)\hat{x}\_{1}+\left(Y-B\right)\hat{x}\_{2}+\left(L-D\right)\hat{x}\_{3},\#\left(1\right)\end{array}$$

where $\hat{x}\_{μ}$, $μ=0, 1, 2, 3$, is the standard basis of $R^{4}$, namely, $\hat{x}\_{0}=\left(1, 0, 0, 0\right)^{T}$, $\hat{x}\_{1}=\left(0, 1, 0, 0\right)^{T}$, and so on (the superscript $T$ denotes the transpose operation); $I\geq 0$ is the luminance contrast signal (see details below); $R$, $G$, $Y$, $B$, $L$, and $D$ (which are all $\geq 0$) are, respectively, the values of the red, green, yellow, blue, light, and dark processes, and therefore $\left(R-G\right)$, $\left(Y-B\right)$, and $\left(L-D\right)$ are, respectively, the red–green, yellow–blue, and light–dark components of the color.

The *luminance* of a light stimulus is, roughly speaking, the overall amount of light in the stimulus that is available for usage by the visual system (for more precise details, see Hendel, 2023). The *luminance contrast* of a light stimulus is the difference between the luminance of the stimulus itself and the luminance of its spatial surroundings. The visual system holds a representation of luminance contrast and uses it in its processing of stereoscopic depth, form, and motion (Livingstone & Hubel, 1987). The luminance contrast *signal*, $I$, which is the $\hat{x}\_{0}$-component of the color vector $C$, is taken to be this representation of luminance contrast in the visual system. Hendel (2023) showed that the value of $I$ is related to (a) the values of $R$, $G$, $Y$, $B$, $L$, and $D$ in the following manner:[[1]](#footnote-2)

$$\begin{array}{c}I=R+G=Y+B=L+D,\#\left(2\right)\end{array}$$

and to (b) the magnitudes of the three opponent-colors components, $\left(R-G\right)$, $\left(Y-B\right)$, and $\left(L-D\right)$, through the following inequality:

$$\begin{array}{c}\left(R-G\right)^{2}+\left(Y-B\right)^{2}+\left(L-D\right)^{2}\leq I^{2}.\#\left(3\right)\end{array}$$

Finally, the level of whiteness in the color, $W$, is determined by the value that completes the inequality in Eq. (3) to an equality, namely,

$$\begin{array}{c}W^{2}=I^{2}-\left(R-G\right)^{2}-\left(Y-B\right)^{2}-\left(L-D\right)^{2}.\#\left(4\right)\end{array}$$

If we are only interested in the colorful dimensions of a light stimulus, namely, if we are willing to disregard the stimulus’s luminance contrast, we can normalize the vector $C$ in Eq. (1) by the luminance contrast signal $I$ and represent the color of the light stimulus by the following three-dimensional vector:

$$c=\frac{\left(R-G\right)}{I}\hat{x}\_{1}+\frac{\left(Y-B\right)}{I}\hat{x}\_{2}+\frac{\left(L-D\right)}{I}\hat{x}\_{3},$$

where the now constant $\hat{x}\_{0}$-component was discarded. From Eq. (3) we immediately see that $\left‖c\right‖\leq 1$. Hence, if we ignore luminance, we can represent all colors as vectors inside, or on the surface of, a unit sphere. Figure 1 illustrates this three-dimensional phenomenal color space.



**Figure 1.** Phenomenal color space. If we ignore luminance, all colors can be represented as vectors inside, or on the surface of, a unit sphere. One example, the vector $c$, is shown in the figure. The $\hat{x}\_{1}$-axis has at its poles unique (elementary) red and unique (elementary) green. These are the elementary hues generated by the opponent processes $R$ and $G$, respectively. Similarly, the $\hat{x}\_{2}$-axis has unique yellow and unique blue at its poles. These are the elementary hues generated by the opponent processes $Y$ and $B$, respectively. The hue circle is the continuum of hues that goes through these four elementary hues. The $\hat{x}\_{3}$-axis has light and dark at its poles. These are the elementary hueless colors generated by the opponent processes $L$ and $D$, respectively. (Notice that in contrast to the other poles of the canonical axes, no representative color sample is shown at the light pole. This is because of the impossibility of faithfully representing a light-emitting stimulus on the page.) The color white is located at the origin of phenomenal color space, namely, it corresponds to the vector $c=0$.

# 3. The mathematical description of two-state quantum systems in a mixed state

The purpose of this section is to provide a brief review of the basic mathematical description of two-state quantum systems in a mixed state. This description can be found in many standard textbooks on quantum mechanics (e.g., Blum, 1981, chapter 1).

## 3.1 Two-state quantum systems

Two-state quantum systems are quantum systems that exhibit two physically distinguishable states relative to some measurement. Some common examples of two-state quantum systems are the spin state of spin-1/2 particles, the polarization state of photons, and atomic systems that can be approximated as effectively having only two electronic levels (Altepeter et al., 2004). The two physical states of a two-state quantum system will be represented by the two Hilbert-space vectors $\left.\left|+\right.\right⟩,\left.\left|-\right.\right⟩\in C^{2}$.[[2]](#footnote-3) These two vectors are taken to be orthogonal and therefore constitute a basis for the two-dimensional Hilbert space $C^{2}$. A two-state quantum system is then fully described by a *state vector*, denoted $\left.\left|ψ\right.\right⟩$, given by the following superposition (i.e., linear combination):

$$\begin{array}{c}\left.\left|ψ\right.\right⟩=a\left.\left|+\right.\right⟩+b\left.\left|-\right.\right⟩,\#\left(5\right)\end{array}$$

where $a,b\in C$ and $\left|a\right|^{2}+\left|b\right|^{2}=1$ (Blum, 1981, chapter 1). A measurement conducted to determine the state of the system described by Eq. (5) has a probability of $\left|a\right|^{2}$ of finding the system in the state $\left.\left|+\right.\right⟩$ and a probability of $\left|b\right|^{2}$ of finding it in the state $\left.\left|-\right.\right⟩$.

## 3.2 The Pauli operators

The Pauli operators, which are commonly denoted $\hat{σ}\_{i}$, $i=1, 2, 3$, are a set of three two-dimensional linear operators (i.e., linear operators that act on vectors in $C^{2}$) that obey a certain algebra (see Blum, 1981, chapter 1, for this algebra). In addition, the two-dimensional identity operator is often referred to as the zeroth Pauli operator, $\hat{σ}\_{0}$. The Pauli operators are Hermitian, i.e., $\hat{σ}\_{μ}=\hat{σ}\_{μ}^{†}≡\left(\hat{σ}\_{μ}^{\*}\right)^{T}$, $μ=0, 1, 2, 3$ (*ibid*.). This means that these operators are quantum observables operating on states of two-state quantum systems. I will therefore often refer to these operators as the Pauli observables. The set $\left\{\hat{σ}\_{0},\hat{σ}\_{1},\hat{σ}\_{2},\hat{σ}\_{3}\right\} $constitutes an orthogonalbasis for the vector space of $2×2$ operators (Aerts & Sassoli de Bianchi, 2017).[[3]](#footnote-4) Therefore, this basis, to which I will refer as the Pauli basis, also spans the space of all two-dimensional Hermitian operators, which is the space of all observables of two-state quantum systems.

It is easy to show that the eigenvalues of the three Pauli operators $\hat{σ}\_{i}$, $i=1, 2, 3$, are $\pm 1$.[[4]](#footnote-5) For each Pauli operator $\hat{σ}\_{i}$, $i\in \left\{1, 2, 3\right\}$, we will denote the eigenvectors associated with these eigenvalues by $\left.\left|\pm \hat{x}\_{i}\right.\right⟩\in C^{2}$, namely,

$$\begin{array}{c}\hat{σ}\_{i}\left.\left|\pm \hat{x}\_{i}\right.\right⟩=\pm \left.\left|\pm \hat{x}\_{i}\right.\right⟩.\#\left(6\right)\end{array}$$

Because the Pauli operators are Hermitian, the vectors in each of the pairs $\left.\left|\pm \hat{x}\_{i}\right.\right⟩$, $i\in \left\{1, 2, 3\right\}$, are orthogonal to each other and therefore constitute a basis of $C^{2}$. If we take the two vectors $\left.\left|\pm \hat{x}\_{3}\right.\right⟩$ as the basis of $C^{2}$, it is straightforward to show that the vectors $\left.\left|\pm \hat{x}\_{1}\right.\right⟩$ and $\left.\left|\pm \hat{x}\_{2}\right.\right⟩$ can be expanded in this basis in the following manner:

|  |  |
| --- | --- |
| $$\left.\left|\pm \hat{x}\_{1}\right.\right⟩=\frac{1}{\sqrt{2}}\left(\left.\left|+\hat{x}\_{3}\right.\right⟩\pm \left.\left|-\hat{x}\_{3}\right.\right⟩\right),$$ | $$\left(7a\right)$$ |
| $$\left.\left|\pm \hat{x}\_{2}\right.\right⟩=\frac{1}{\sqrt{2}}\left(\left.\left|+\hat{x}\_{3}\right.\right⟩\pm i\left.\left|-\hat{x}\_{3}\right.\right⟩\right).$$ | $$\left(7b\right)$$ |

We can employ the spectral decomposition theorem to obtain the following expression for each Pauli operator $\hat{σ}\_{i}$, $i\in \left\{1, 2, 3\right\}$:

$$\begin{array}{c}\hat{σ}\_{i}=\left.\left|+\hat{x}\_{i}\right.\right⟩\left⟨\left.+\hat{x}\_{i}\right|\right.-\left.\left|-\hat{x}\_{i}\right.\right⟩\left⟨\left.-\hat{x}\_{i}\right|\right.,\#\left(8\right)\end{array}$$

where the vectors $\left⟨\left.+\hat{x}\_{i}\right|\right.$ and $\left⟨\left.-\hat{x}\_{i}\right|\right.$ are, respectively, the transposed complex conjugates of the vectors $\left.\left|+\hat{x}\_{i}\right.\right⟩$ and $\left.\left|-\hat{x}\_{i}\right.\right⟩$. Therefore, Eq. (8) shows that each Pauli operator $\hat{σ}\_{i}$, $i\in \left\{1, 2, 3\right\}$, is composed of two underlying operators, $\left.\left|+\hat{x}\_{i}\right.\right⟩\left⟨\left.+\hat{x}\_{i}\right|\right.$ and $\left.\left|-\hat{x}\_{i}\right.\right⟩\left⟨\left.-\hat{x}\_{i}\right|\right.$, that operate in an *opponent* manner to each other. By applying linearity to Eq. (8) it is easy to show that the expectation values of the Pauli observables $\hat{σ}\_{i}$, $i=1, 2, 3$, are given by

$$\begin{array}{c}\left〈\hat{σ}\_{i}\right〉=\left〈\left.\left|+\hat{x}\_{i}\right.\right⟩\left⟨\left.+\hat{x}\_{i}\right|\right.\right〉-\left〈\left.\left|-\hat{x}\_{i}\right.\right⟩\left⟨\left.-\hat{x}\_{i}\right|\right.\right〉\#\left(9\right)\end{array}$$

(Altepeter et al., 2004).

Just as we did above for the three Pauli operators $\hat{σ}\_{i}$, $i=1, 2, 3$, we can use the eigenvalue decomposition theorem on the Pauli operator $\hat{σ}\_{0}$ (which, as will be recalled, is the identity operator). Since *any* two-dimensional vector is an eigenvector of the identity operator with an eigenvalue of $1$, the result of eigenvalue decomposition in this case is the following:

$$\hat{σ}\_{0}=\left.\left|+\hat{n}\right.\right⟩\left⟨\left.+\hat{n}\right|\right.+\left.\left|-\hat{n}\right.\right⟩\left⟨\left.-\hat{n}\right|\right.,$$

where $\left.\left|+\hat{n}\right.\right⟩$ and $\left.\left|-\hat{n}\right.\right⟩$ are any two orthogonal unit vectors in $C^{2}$. This is known as the *completeness relation*. In our case, we will use the completeness relation with the pair of orthogonal vectors $\left.\left|+\hat{x}\_{i}\right.\right⟩$ and $\left.\left|-\hat{x}\_{i}\right.\right⟩$,$i\in \left\{1, 2, 3\right\}$, namely,

$$\begin{array}{c}\hat{σ}\_{0}=\left.\left|+\hat{x}\_{i}\right.\right⟩\left⟨\left.+\hat{x}\_{i}\right|\right.+\left.\left|-\hat{x}\_{i}\right.\right⟩\left⟨\left.-\hat{x}\_{i}\right|\right..\#\left(10\right)\end{array}$$

By applying linearity to Eq. (10) we obtain that the expectation value of the Pauli observable $\hat{σ}\_{0}$ is given by

$$\begin{array}{c}\left〈\hat{σ}\_{0}\right〉=\left〈\left.\left|+\hat{x}\_{i}\right.\right⟩\left⟨\left.+\hat{x}\_{i}\right|\right.\right〉+\left〈\left.\left|-\hat{x}\_{i}\right.\right⟩\left⟨\left.-\hat{x}\_{i}\right|\right.\right〉,\#\left(11\right)\end{array}$$

$ i\in \left\{1, 2, 3\right\}$.

## 3.3 Two-state quantum systems in a mixed state and the density matrix

A *mixture* of quantum systems is an ensemble of quantum systems, each described by a state vector, in which the quantum states of the systems do not coherently interfere with each other (Blum, 1981, chapter 1). The systems are then said to be in an incoherent state, or, as it is referred to more commonly, in a *mixed state*. Because the quantum systems that comprise a mixture do not interfere with each other, the mixture cannot be described by a state vector. Instead, it is described by a Hermitian operator (i.e., a quantum observable), referred to as the *density operator* (*ibid*.). By contrast, quantum systems that *can* be described by state vectors, namely, quantum systems whose component do coherently interfere with each other (see, e.g., Eq. (5)), are said to be in a *pure state*. Density operators have several unique properties that characterize them, but these will not be of interest to us here (for details, see *ibid*.). What will be of interest to use here is the fact that a quantum system’s density operator *fully* describes the physical properties of the system. This means the following: the density operator allows the calculation of the probability of finding the system in *any* particular pure state upon measurement of any observable (*ibid*.).

In the particular case of two-state quantum systems, the density operator is a two-dimensional operator, namely, it acts on vectors that belong to $C^{2}$. It turns out that density operators that describe two-state quantum systems can be expanded in the Pauli basis of observables, $\left\{\hat{σ}\_{0},\hat{σ}\_{1},\hat{σ}\_{2},\hat{σ}\_{3}\right\}$, in the following manner (where, as is customary, we denote the density operator by $\hat{ρ}$):

$$\begin{array}{c}\hat{ρ}=\frac{1}{2}\sum\_{μ=0}^{3}\left〈\hat{σ}\_{μ}\right〉\hat{σ}\_{μ}.\#\left(12\right)\end{array}$$

In Eq. (12), the brackets $\left〈⋅\right〉$ denote the expectation value of a quantum observable (i.e., the mean value of this observable gleaned from many measurements). Thus, the expectation values that Eq. (12) contains are those of the four Pauli observables. While $\left〈\hat{σ}\_{1}\right〉,\left〈\hat{σ}\_{2}\right〉, \left〈\hat{σ}\_{3}\right〉\in Z$, $\left〈\hat{σ}\_{0}\right〉\in N$ and carries a special meaning—it is the number of two-state systems in the mixture (*ibid*.).[[5]](#footnote-6) The expectation values of the Pauli observables appearing in Eq. (12) obey the following constraint:

$$\begin{array}{c}\left〈\hat{σ}\_{1}\right〉^{2}+\left〈\hat{σ}\_{2}\right〉^{2}+\left〈\hat{σ}\_{3}\right〉^{2}\leq \left〈\hat{σ}\_{0}\right〉^{2}\#\left(13\right)\end{array}$$

(*ibid*.).

Often, the number of two-state systems in the mixture is immaterial to us. In such cases it is convenient to normalize Eq. (12) by this number, namely, by $\left〈\hat{σ}\_{0}\right〉$ (notice that this entails the normalization of Eq. (13) by $\left〈\hat{σ}\_{0}\right〉^{2}$). With this normalization it is readily realized that there is a *one-to-one correspondence* between the set of all density operators describing two-state quantum systems and the set of vectors that reside inside, or on the surface of, the unit sphere in $R^{3}$. These vectors are commonly referred to as the Bloch vectors and the unit sphere that they reside in is referred to as the Bloch sphere (Aerts & Sassoli de Bianchi, 2017; Aletepeter et al., 2004). From Eqs. (12) and (13) it is easy to show that the Bloch vectors are given by

$$b=\frac{\left〈\hat{σ}\_{1}\right〉}{\left〈\hat{σ}\_{0}\right〉}\hat{x}\_{1}+\frac{\left〈\hat{σ}\_{2}\right〉}{\left〈\hat{σ}\_{0}\right〉}\hat{x}\_{2}+\frac{\left〈\hat{σ}\_{3}\right〉}{\left〈\hat{σ}\_{0}\right〉}\hat{x}\_{3},$$

where $\left‖b\right‖\leq 1$ (note that Bloch vectors for which $\left‖b\right‖=1$, namely, those that are on the surface of the Bloch sphere, represent two-state quantum systems in a pure state, like the one described in Eq. (5)). Figure 2 provides an illustration of the Bloch sphere.



**Figure 2.** The Bloch sphere. There is a one-to-one correspondence between the set of all density operators describing two-state quantum systems in a mixed state (Eq. (12)) and the set of vectors that reside inside, or on the surface of, the unit sphere in $R^{3}$. Thus, the vector $b$ shown in the figure ($\left‖b\right‖<1$) represents a two-state system in a mixed state. The unit vector $\hat{b}$, which resides on the surface of the Bloch sphere, represents a two-state system in a pure state.

## 3.4 The Hamiltonian of two-state quantum systems

The time evolution of a quantum state is determined by the Hamiltonian operator $\hat{H}$. In the case of systems that are described by state vectors, the time-evolution equation is called the Schrödinger equation; in the case of systems that are described by density matrices (i.e., systems in a mixed state), the time-evolution equation is called the Liouville equation (Blum, 1981, chapter 2). Here we will only be interested in the Hamiltonian operators of two-state quantum systems. We will denote the eigenvectors of the Hamiltonian operator, which are commonly referred to as the energy eigenstates, by $\left.\left|\pm \right.\right⟩$. If the Hamiltonian is constant in time, it is easy to show that the eigenvector/eigenvalue equation for the Hamiltonian can always be written as

$$\begin{array}{c}\hat{H}\left.\left|\pm \right.\right⟩=\left.\left(E\_{0}\pm ε\right)\left|\pm \right.\right⟩,\#\left(14\right)\end{array}$$

where $E\_{0}, ε\in R$.

# 4. Color is the phenomenal dual aspect of two-state quantum systems in a mixed state

## 4.1 Establishing an identity between the mathematical descriptions of colors and two-state quantum systems

Let us postulate the following set of correspondences between the six expectation values $\left〈\left.\left|\pm \hat{x}\_{i}\right.\right⟩\left⟨\left.\pm \hat{x}\_{i}\right|\right.\right〉$, $i=1, 2, 3$, and the six variables $R$, $G$, $Y$, $B$, $L$, and $D$:

|  |  |  |
| --- | --- | --- |
| $\left〈\left.\left|+\hat{x}\_{1}\right.\right⟩\left⟨\left.+\hat{x}\_{1}\right|\right.\right〉⟷R$, | $$\begin{array}{c}\left〈\left.\left|-\hat{x}\_{1}\right.\right⟩\left⟨\left.-\hat{x}\_{1}\right|\right.\right〉⟷G,\end{array}$$ | $$(15a)$$ |
| $\left〈\left.\left|+\hat{x}\_{2}\right.\right⟩\left⟨\left.+\hat{x}\_{2}\right|\right.\right〉⟷Y$, | $$\begin{array}{c}\left〈\left.\left|-\hat{x}\_{2}\right.\right⟩\left⟨\left.-\hat{x}\_{2}\right|\right.\right〉⟷B,\end{array}$$ | $$(15b)$$ |
| $\left〈\left.\left|+\hat{x}\_{3}\right.\right⟩\left⟨\left.+\hat{x}\_{3}\right|\right.\right〉⟷L$, | $$\begin{array}{c}\left〈\left.\left|-\hat{x}\_{3}\right.\right⟩\left⟨\left.-\hat{x}\_{3}\right|\right.\right〉⟷D.\end{array}$$ | $$(15c)$$ |

Using this set of correspondences, we immediately see that the following correspondence exists between $\left〈\hat{σ}\_{0}\right〉$ in Eq. (11) and $I$ in Eq. (2):

$$\begin{array}{c}\left〈\hat{σ}\_{0}\right〉=\left〈\left.\left|+\hat{x}\_{i}\right.\right⟩\left⟨\left.+\hat{x}\_{i}\right|\right.\right〉+\left〈\left.\left|-\hat{x}\_{i}\right.\right⟩\left⟨\left.-\hat{x}\_{i}\right|\right.\right〉⟷I=R+G=Y+B=L+D,\#\left(16\right)\end{array}$$

$i=1, 2, 3$. Given Eq. (15), we also obtain that the following correspondences hold:

|  |  |
| --- | --- |
| $$\left〈\hat{σ}\_{1}\right〉=\left〈\left.\left|+\hat{x}\_{1}\right.\right⟩\left⟨\left.+\hat{x}\_{1}\right|\right.\right〉-\left〈\left.\left|-\hat{x}\_{1}\right.\right⟩\left⟨\left.-\hat{x}\_{1}\right|\right.\right〉⟷R-G,$$ | $$(17a)$$ |
| $$\left〈\hat{σ}\_{2}\right〉=\left〈\left.\left|+\hat{x}\_{2}\right.\right⟩\left⟨\left.+\hat{x}\_{2}\right|\right.\right〉-\left〈\left.\left|-\hat{x}\_{2}\right.\right⟩\left⟨\left.-\hat{x}\_{2}\right|\right.\right〉⟷Y-B,$$ | $$(17b)$$ |
| $\left〈\hat{σ}\_{3}\right〉=\left〈\left.\left|+\hat{x}\_{3}\right.\right⟩\left⟨\left.+\hat{x}\_{3}\right|\right.\right〉-\left〈\left.\left|-\hat{x}\_{3}\right.\right⟩\left⟨\left.-\hat{x}\_{3}\right|\right.\right〉⟷L-D$. | $$(17c)$$ |

Notice that the equalities on the left-hand side of Eq. (17) are from Eq. (9). Equation (17) matches pairs of opponent processes on the right-hand side with pairs of operators operating in an opponent manner to each other on the left-hand side. Now, from Eqs. (16) and (17) we find that there exists an exact correspondence between $\hat{ρ}$ in Eq. (12) and $C$ in Eq. (1):

$$\begin{array}{c}\hat{ρ}=\frac{1}{2}\sum\_{μ=0}^{3}\left〈\hat{σ}\_{μ}\right〉\hat{σ}\_{μ}⟷C=I\hat{x}\_{0}+\left(R-G\right)\hat{x}\_{1}+\left(Y-B\right)\hat{x}\_{2}+\left(L-D\right)\hat{x}\_{3},\#\left(18\right)\end{array}$$

where, it will be noticed, the correspondence $\hat{σ}\_{μ}⟷\hat{x}\_{μ}$, $μ=0, 1, 2, 3$, is implicitly assumed. Another exact correspondence that follows from Eqs. (16) and (17) is one between Eq. (13) and Eq. (3):

$$\begin{array}{c}\left〈\hat{σ}\_{1}\right〉^{2}+\left〈\hat{σ}\_{2}\right〉^{2}+\left〈\hat{σ}\_{3}\right〉^{2}\leq \left〈\hat{σ}\_{0}\right〉^{2}⟷\left(R-G\right)^{2}+\left(Y-B\right)^{2}+\left(L-D\right)^{2}\leq I^{2}.\#\left(19\right)\end{array}$$

Finally, from Eqs. (18) and (19) it is easy to see that the Bloch space of Fig. 2 corresponds to the phenomenal color space of Fig. 1.

Overall, Eqs. (15)–(19) show that there exists an identity between the mathematical description of the proposed opponent-colors theory and the mathematical description of two-state quantum systems in a mixed state. Notice that there is no need to find a parallel to Eq. (4) of the proposed theory (which is the equation for whiteness) in the quantum description of mixtures of two-state systems because this equation is merely definitional (that is, the level of whiteness in a color is simply *assigned* the value of the right-hand side of Eq. (4)).

## 4.2 A privileged basis for Hilbert space

In recent decades several notable researchers have advocated the hypothesis that the physical world is the result of a gigantic computation (Fredkin, 2003; Lloyd, 2007; ‘t Hooft, 2016; Wolfram, 2002; Zuse, 1982). A similar hypothesis, which elegantly explains the mysterious success that mathematics has in describing the physical world (Wigner, 1960), is that the universe is an abstract mathematical model (Carroll, 2022; Tegmark, 2008; also see Woit, 2015). The two hypotheses—namely, the world as computation and the world as a mathematical model—are, in fact, essentially identical. This is because mathematical models are inherently atemporal; hence, for a mathematical model to constitute an implementation of the physical universe it must be run in time. But this turns it into a computation.

In this paper I will adopt the general metaphysical framework suggested by the above hypotheses, namely, that the physical world is an implementation in time of a mathematical model. Moreover, I will assume that quantum mechanics provides a correct description of the physical world. Taking these two assumptions together, we arrive at the world picture suggested by Lloyd (2007), ‘t Hooft (2016), and Carroll (2022), in which the physical world is a computation that follows the rules of quantum mechanics.

In quantum theory it is often emphasized that a quantum state, i.e., a Hilbert-space vector, can be represented in any basis of Hilbert space (e.g., position space, momentum space, etc.). However, when one carries out *computations* with Hilbert-space vectors, one must commit to a definite representation of these vectors, namely, one must choose a particular basis for Hilbert space. Since in the metaphysical world view adopted here, the physical world results from a computation, we must stipulate that there exists a specific basis for Hilbert space in which all quantum states in the universe are represented. ‘t Hooft (2016), for example, refers to this privileged basis as the ‘ontological basis’. Carroll (2022) suggests that the privileged basis for Hilbert space is given by the energy eigenstates, namely, the set of eigenvectors of the Hamiltonian. This choice is based on the fact that the Hamiltonian is the most fundamental operator in quantum theory since it is what moves the quantum world in time.

Here I will follow Carroll (2022) in assuming that in the computation that gives rise to our universe quantum states are represented relative to the basis of the energy eigenstates. In the case of two-state quantum systems, these energy eigenstates are the vectors $\left.\left|\pm \right.\right⟩$ that satisfy Eq. (14). We will denote this privileged basis of two-dimensional Hilbert space by $P$, namely, $P=\left\{\left.\left|+\right.\right⟩ ,\left.\left|-\right.\right⟩\right\}$. *Ex hypothesi*, the quantum states of all two-state quantum systems in the universe are represented in the privileged basis $P$. For example, the six vectors $\left.\left|\pm \hat{x}\_{i}\right.\right⟩$, $i=1, 2, 3$, i.e., the eigenvectors of the Pauli operators $\hat{σ}\_{i}$ (see Eq. (6)), are represented by $\left[\left.\left|\pm \hat{x}\_{i}\right.\right⟩\right]\_{P}$ (the subscripted brackets $\left[⋅\right]\_{P}$ denote a vector coordinatized relative to the basis $P$ (Lipschutz & Lipson, 2009, chapter 6)). Notice that the representation of the basis vectors $\left.\left|\pm \right.\right⟩$ themselves relative to the basis $P$ yields the standard basis of $C^{2}$, namely, $\left[\left.\left|+\right.\right⟩\right]\_{P}=\left(1,0\right)^{T}$ and $\left[\left.\left|-\right.\right⟩\right]\_{P}=\left(0,1\right)^{T}$.

Since quantum observables (i.e., Hermitian operators) operate on quantum states, consistency requires that if the latter are represented relative to some basis, the former must be represented relative to the same basis as well. Thus, in the quantum computational universe hypothesized here, quantum observables are represented as matrices relative to the privileged basis of Hilbert space, namely, the basis of energy eigenstates. Let us see how this works in the case of two-state quantum systems, which is the case that interests us here. A trivial example is the two-dimensional Hamiltonian, $\hat{H}$. As is easy to see from Eq. (14), when $\hat{H}$ is represented in privileged basis $C^{2}$, which is the basis of its two eigenvectors, $\left.\left|\pm \right.\right⟩$, is becomes a $2×2$ diagonal matrix with the energy eigenvalues along the diagonal, namely,

$$\left[\hat{H}\right]\_{P}=\left[\begin{matrix}E\_{0}+ε&0\\0&E\_{0}-ε\end{matrix}\right].$$

Another important example is the Pauli observables $\hat{σ}\_{μ}$, $μ=0, 1, 2, 3$. On our hypothesis here these operators have a privileged representation as the matrices $\left[\hat{σ}\_{μ}\right]\_{P}$. These are the known as the Pauli matrices. The set of Pauli matrices $\left\{\left[\hat{σ}\_{μ}\right]\_{P}\right\}$, $μ=0, 1, 2, 3$, constitutes a basis for the space of $2×2$ Hermitian matrices. We can decompose the Hamiltonian matrix $\left[\hat{H}\right]\_{P}$ in this basis. Based on the algebra of the Pauli matrices it can be shown that this decomposition is given by:

$$\begin{array}{c}\left[\hat{H}\right]\_{P}=E\_{0}\left[\begin{matrix}1&0\\0&1\end{matrix}\right]+ε\left[\begin{matrix}1&0\\0&-1\end{matrix}\right].\#\left(20\right)\end{array}$$

The first matrix on the right-hand side of Eq. (20) is the $2×2$ identity matrix, i.e., $\left[\hat{σ}\_{0}\right]\_{P}$. It is customary to refer to the second matrix on the right-hand side of Eq. (20) as the *third* Pauli matrix, namely, $\left[\hat{σ}\_{3}\right]\_{P}=\left[\begin{matrix}1&0\\0&-1\end{matrix}\right]$. It can easily be shown that this necessarily means that

$$\begin{array}{c}\left[\left.\left|\pm \hat{x}\_{3}\right.\right⟩\right]\_{P}=\left[\left.\left|\pm \right.\right⟩\right]\_{P}.\#\left(21\right)\end{array}$$

Given Eq. (21) and given that the coordinatization of the vectors $\left[\left.\left|\pm \right.\right⟩\right]\_{P}$ yields the standard basis of $C^{2}$ (see above), we conclude that the matrices $\left[\hat{σ}\_{μ}\right]\_{P}$, $μ=0, 1, 2, 3$, are the Pauli matrices in their well-known standard representation (e.g., Aerts & Sassoli de Bianchi, 2017), namely,

$$\left[\hat{σ}\_{0}\right]\_{P}=\left[\begin{matrix}1&0\\0&1\end{matrix}\right], \left[\hat{σ}\_{1}\right]\_{P}=\left[\begin{matrix}0&1\\1&0\end{matrix}\right],\left[\hat{σ}\_{2}\right]\_{P}=\left[\begin{matrix}0&-i\\i&0\end{matrix}\right],\left[\hat{σ}\_{3}\right]\_{P}=\left[\begin{matrix}1&0\\0&-1\end{matrix}\right].$$

Finally, recall from *Subsection 3.3* that a quantum system in a mixed state is described by a density operator $\hat{ρ}$. On our hypothesis, the universe represents any two-state quantum system in a mixed state by a $2×2$ matrix $\left[\hat{ρ}\right]\_{P}$. From Eq. (12) we see that this matrix can be expanded in the Pauli basis $\left[\hat{σ}\_{μ}\right]\_{P}$, $μ=0, 1, 2, 3$, in the following manner:

$$\left[\hat{ρ}\right]\_{P}=\frac{1}{2}\sum\_{μ=0}^{3}\left〈\left[\hat{σ}\_{μ}\right]\_{P}\right〉\left[\hat{σ}\_{μ}\right]\_{P}.$$

## 4.3 Color is the phenomenal dual aspect of two-state quantum systems in a mixed state

*Subsection 4.1* established an identity between the mathematical description of two-state quantum systems in a mixed state and the mathematical description of opponent-colors theory (see Eqs. (15)–(19)). Such a correspondence between the states of a physical system and phenomenal states is exactly what the dual-aspect theory of phenomenal consciousness predicts. However, we are still falling short of the requirements of dual-aspect theory because the values on the left-hand sides of the correspondences in Eqs. (15)–(19) change with the choice of basis for Hilbert space. To clearly see the problem, consider, for example, Eq. (15). The expectation values $\left〈\left.\left|\pm \hat{x}\_{i}\right.\right⟩\left⟨\left.\pm \hat{x}\_{i}\right|\right.\right〉$, $i=1, 2, 3$, on the left-hand sides of the correspondences delineated by this equation depend on the specific basis that was chosen for $C^{2}$. Hence, different bases for $C^{2}$ will lead to different expectation values. But on Eq. (15), these expectation values correspond to the fundamental color sensations, $R$, $G$, $Y$, $B$, $L$, and $D$. Thus, as it stands, Eq. (15) makes the prediction that our color experiences should depend on the particular basis chosen for $C^{2}$. This, of course, is absurd. To solve this problem with Eqs. (15)–(19), we invoke the hypothesis of *Subsection 4.2* that the universe represents quantum states in a privileged basis. On this hypothesis, from nature’s point of view, the mathematical description of two-state quantum systems given in Eqs. (15)–(19) exists in *a specific form*, which is the representation relative to the privileged basis of $C^{2}$, $P$. For concreteness, let us rewrite Eqs. (15)–(19) in this privileged representation. We begin with Eq. (15), which establishes the basic correspondence between the realm of two-state quantum systems and the realm of opponent-colors theory. This equation now becomes

|  |  |  |
| --- | --- | --- |
| $\left〈\left[\left.\left|+\hat{x}\_{1}\right.\right⟩\left⟨\left.+\hat{x}\_{1}\right|\right.\right]\_{P}\right〉⟷R$, | $$\begin{array}{c}\left〈\left[\left.\left|-\hat{x}\_{1}\right.\right⟩\left⟨\left.-\hat{x}\_{1}\right|\right.\right]\_{P}\right〉⟷G,\end{array}$$ | $$(22a)$$ |
| $\left〈\left[\left.\left|+\hat{x}\_{2}\right.\right⟩\left⟨\left.+\hat{x}\_{2}\right|\right.\right]\_{P}\right〉⟷Y$, | $$\begin{array}{c}\left〈\left[\left.\left|-\hat{x}\_{2}\right.\right⟩\left⟨\left.-\hat{x}\_{2}\right|\right.\right]\_{P}\right〉⟷B,\end{array}$$ | $$(22b)$$ |
| $\left〈\left[\left.\left|+\hat{x}\_{3}\right.\right⟩\left⟨\left.+\hat{x}\_{3}\right|\right.\right]\_{P}\right〉⟷L$, | $$\begin{array}{c}\left〈\left[\left.\left|-\hat{x}\_{3}\right.\right⟩\left⟨\left.-\hat{x}\_{3}\right|\right.\right]\_{P}\right〉⟷D.\end{array}$$ | $$(22c)$$ |

The next step is to convert the correspondences listed in Eqs. (16) and (17) into

$$\begin{array}{c}\left[\hat{σ}\_{0}\right]\_{P}=\left〈\left[\left.\left|+\hat{x}\_{i}\right.\right⟩\left⟨\left.+\hat{x}\_{i}\right|\right.\right]\_{P}\right〉+\left〈\left[\left.\left|-\hat{x}\_{i}\right.\right⟩\left⟨\left.-\hat{x}\_{i}\right|\right.\right]\_{P}\right〉⟷I=R+G=Y+B=L+D\#\left(23\right)\end{array}$$

for $i=1, 2, 3$, and

|  |  |
| --- | --- |
| $$\left[\hat{σ}\_{1}\right]\_{P}=\left〈\left[\left.\left|+\hat{x}\_{1}\right.\right⟩\left⟨\left.+\hat{x}\_{1}\right|\right.\right]\_{P}\right〉-\left〈\left[\left.\left|-\hat{x}\_{1}\right.\right⟩\left⟨\left.-\hat{x}\_{1}\right|\right.\right]\_{P}\right〉⟷R-G,$$ | $$(24a)$$ |
| $$\left[\hat{σ}\_{2}\right]\_{P}=\left〈\left[\left.\left|+\hat{x}\_{2}\right.\right⟩\left⟨\left.+\hat{x}\_{2}\right|\right.\right]\_{P}\right〉-\left〈\left[\left.\left|-\hat{x}\_{2}\right.\right⟩\left⟨\left.-\hat{x}\_{2}\right|\right.\right]\_{P}\right〉⟷Y-B,$$ | $$(24b)$$ |
| $\left[\hat{σ}\_{3}\right]\_{P}=\left〈\left[\left.\left|+\hat{x}\_{3}\right.\right⟩\left⟨\left.+\hat{x}\_{3}\right|\right.\right]\_{P}\right〉-\left〈\left[\left.\left|-\hat{x}\_{3}\right.\right⟩\left⟨\left.-\hat{x}\_{3}\right|\right.\right]\_{P}\right〉⟷L-D$. | $$(24c)$$ |
| Using Eqs. (23) and (24) we can represent the left-hand side of Eq. (18) relative to the basis $P$ to obtain |  |

$$\begin{array}{c}\left[\hat{ρ}\right]\_{P}=\frac{1}{2}\sum\_{μ=0}^{3}\left〈\left[\hat{σ}\_{μ}\right]\_{P}\right〉\left[\hat{σ}\_{μ}\right]\_{P}⟷C=I\hat{x}\_{0}+\left(R-G\right)\hat{x}\_{1}+\left(Y-B\right)\hat{x}\_{2}+\left(L-D\right)\hat{x}\_{3}.\left(25\right)\end{array}$$

Finally, when the left-hand side of Eq. (19) is represented relative to $P$ we get

$$\begin{array}{c}\left〈\left[\hat{σ}\_{1}\right]\_{P}\right〉^{2}+\left〈\left[\hat{σ}\_{2}\right]\_{P}\right〉^{2}+\left〈\left[\hat{σ}\_{3}\right]\_{P}\right〉^{2}\leq \left〈\left[\hat{σ}\_{0}\right]\_{P}\right〉^{2}⟷\left(R-G\right)^{2}+\left(Y-B\right)^{2}+\left(L-D\right)^{2}\leq I^{2}.\#\left(26\right)\end{array}$$

Like their progenitors in Eqs. (15)–(19), Eqs. (22)–(26) establish an identity between the mathematical description of two-state quantum systems in a mixed state and the mathematical description of opponent-colors theory. However, in contrast to Eqs. (15)–(19), the mathematical description of two-state quantum systems delineated in Eqs. (22)–(26) is given relative to the privileged basis of Hilbert space. The most important correspondence in the set of correspondences listed in Eqs. (22)–(26) is the one given in Eq. (25). This equation establishes that for every state of a two-state quantum system, where the state is represented relative to the privileged basis of $C^{2}$, there corresponds a specific color experience. Based on this exact correspondence and the principles of the dual-aspect theory of phenomenal consciousness, we reach the main hypothesis of this paper:

$H$: Color is the phenomenal dual aspect of a two-state quantum system in a mixed state.

## 4.4 The hypothesis $H$ explains several fundamental phenomenal properties of color

The hypothesis $H$ explains several fundamental phenomenal properties of color. First, it explains why there exist seven elementary colors. As can is evident from Eq. (22), six of those colors are associated with the six eigenvectors of the three Pauli matrices $\left[\hat{σ}\_{i}\right]\_{P}$, $i=1, 2, 3$, namely, with the vectors $\left[\left.\left|\pm \hat{x}\_{3}\right.\right⟩\right]\_{P}$. The seventh elementary color, white, is the dual aspect of the zero vector in $C^{2}$. Second, light and dark, which are the color sensations that are associated with the two *privileged* basis vectors of $C^{2}$, $\left[\left.\left|\pm \hat{x}\_{3}\right.\right⟩\right]\_{P}$, (see Eq. (21)), are perceived as more *fundamental* than the other elementary color sensations, which are associated with the vectors $\left[\left.\left|\pm \hat{x}\_{1}\right.\right⟩\right]\_{P}$ and $\left[\left.\left|\pm \hat{x}\_{2}\right.\right⟩\right]\_{P}$. Third, $H$ explains the age-old puzzle (Purves & Yegappan, 2017; Shepard, 1994) of why the hues can be ordered in a closed continuum (the hue circle; see Fig. 1). To see this clearly, we first express Eq. (7) relative to the privileged basis of $C^{2}$, $P$:

|  |  |
| --- | --- |
| $$\left[\left.\left|\pm \hat{x}\_{1}\right.\right⟩\right]\_{P}=\frac{1}{\sqrt{2}}\left(\left[\left.\left|+\hat{x}\_{3}\right.\right⟩\right]\_{P}\pm \left[\left.\left|-\hat{x}\_{3}\right.\right⟩\right]\_{P}\right),$$ | $$\left(27a\right)$$ |
| $$\left[\left.\left|+\hat{x}\_{2}\right.\right⟩\right]\_{P}=\frac{1}{\sqrt{2}}\left(\left[\left.\left|+\hat{x}\_{3}\right.\right⟩\right]\_{P}\pm i\left[\left.\left|-\hat{x}\_{3}\right.\right⟩\right]\_{P}\right).$$ | $$\left(27b\right)$$ |

As is evident from Eq. (27), the vectors $\left[\left.\left|\pm \hat{x}\_{1}\right.\right⟩\right]\_{P}$ and $\left[\left.\left|\pm \hat{x}\_{2}\right.\right⟩\right]\_{P}$, whose dual aspects are the four fundamental hues (see Eq. (22a) and Eq. (22b)), can all be converted to each other by varying the *relative phase* between the basis vectors $\left[\left.\left|\pm \hat{x}\_{3}\right.\right⟩\right]\_{P}$ (e.g., the vector $\left[\left.\left|+\hat{x}\_{1}\right.\right⟩\right]\_{P}$ is converted to the vector $\left[\left.\left|+\hat{x}\_{2}\right.\right⟩\right]\_{P}$ by adding a phase of ${π}/{2}$ radians to the basis vector $\left[\left.\left|-\hat{x}\_{3}\right.\right⟩\right]\_{P}$ in Eq. (27a), i.e., multiplying $\left[\left.\left|-\hat{x}\_{3}\right.\right⟩\right]\_{P}$ by $e^{i\frac{π}{2}}=i$). On $H$, the phenomenal dual aspect of this relative phase between the vectors of the privileged basis of $C^{2}$ is the continuum of hues along between the four elementary hues. Fourth, as can be easily seen from Eq. (24), $H$ elegantly explains why color sensations result from the operation of three pairs of opponent processes.

# 5. Discussion

## 5.1 The differences between the hypothesis suggested here and previous suggestions relating consciousness and quantum mechanics

There have been many suggestions that consciousness and quantum mechanics might be related (e.g., Chalmers & McQueen, 2022; Hameroff & Penrose, 1996; Lockwood, 1989; Okon & Sebastián, 2020; Stapp, 1993; Wigner, 1961). The hypothesis made in this paper, i.e., that color sensations are phenomenal dual aspects of two-state quantum systems in a mixed state, is distinct from these past suggestions in three important respects. First, the motivation for the relationship between consciousness and quantum mechanics in the case of the current hypothesis stems from the identical mathematical description of opponent-colors theory and two-state quantum systems conjoined with the principles of the dual-aspect theory of phenomenal consciousness. By contrast, past suggestions that consciousness might be related to quantum mechanics were mostly motivated by ‘the fact that the problems in quantum mechanics seem to be deeply tied to the notion of observership, crucially involving the relation between a subject's experience and the rest of the world’ (Chalmers, 1996, p. 333). Namely, past suggestions have mostly attempted to relate the problem of consciousness with the problem of measurement in quantum mechanics. Second, whereas past suggestions relating consciousness and quantum mechanics were vague in what they meant by ‘consciousness’, the current hypothesis is very precise—it addresses a specific aspect of phenomenal consciousness, namely, color experience. Third, past suggestions of a relationship between consciousness and quantum mechanics all required that the brain be able to sustain a quantum *coherent* state. Given the physical properties of the brain, this is *prima facie* highly unlikely. Indeed, Tegmark (2000) has calculated that any coherent quantum state in the brain will decohere in a time that is orders of magnitude too short to be of any use for conscious processes. By contrast, quantum decoherence doesn’t pose a problem for the suggestion put forth here because on this suggestion color is a phenomenal dual aspect of a *mixture* of quantum systems, which is a population of quantum systems in an *incoherent* state.

## 5.2 Generalizing the argument to other types of phenomenal experience

On the hypothesis $H$, *any* two-state quantum system should have color experience as its phenomenal dual aspect. Since the world contains a vast number of such systems (e.g., the spin state of spin-1/2 particles or the polarization state of photons), the proposed hypothesis is clearly panpsychist in nature. Moreover, nothing in the hypothesis that color sensations are the phenomenal dual aspects of the states of two-state quantum systems suggests that what brings about the phenomenal experience is the two-dimensionality of these systems. It is therefore natural to generalize $H$ and suggest that

$H^{\*}$: All types of phenomenal experience are phenomenal dual aspects of quantum systems in a mixed state. Each specific type of phenomenal experience (color, taste, odor, sound, etc.) is ‘attached’ to a quantum system with a certain dimensionality.

For example, as this paper has shown in detail, color is attached to two-state quantum systems. Thus, on $H^{\*}$, the various types of phenomenal experience that we humans (and presumably other creatures) have are not generated *de novo* by the brain. Rather, these phenomenal experiences are a fundamental part of our universe. Presumably, brains have evolved to tie these phenomenal experiences to various types of sensory information.

The generalized hypothesis $H^{\*}$ allows us to derive a prediction that can be used to test it. The mixed states of an $N$-state quantum system can be represented by vectors residing inside, or on the surface of, a $\left(N^{2}-1\right)$-dimensional hypersphere (Bertlmann & Krammer, 2008).[[6]](#footnote-7) We saw an example of this for two-state quantum systems, i.e., systems for which $N=2$: the states of these systems can be represented by vectors inside, or on the surface of, a sphere in three dimensions ($2^{2}-1=3$), namely, the Bloch sphere of Fig. 2. On $H^{\*}$, the dimensionality of the phenomenal space that is dual to the states of an $N$-state quantum system will also be $\left(N^{2}-1\right)$-dimensional. Again, we saw an example for this in the case of the phenomenal space that is dual to two-state quantum systems, namely, the phenomenal space of color, which is schematized in Fig. 1. Hence, $H^{\*}$ predicts that the dimensionalities of *all* types of phenomenal experience should be given by $\left(N^{2}-1\right)$ for various values of $N$.

Let us look at odor space as an example. Mamlouk and Martinetz (2004) concluded that odor space is at least 32-dimensional but is no more than 68-dimensional. Since on $H^{\*}$ the dimensionalities of phenomenal spaces are quantized to $N^{2}-1$, the only dimensionalities that are in line with the results of Mamlouk and Martinetz are 35, 48, and 63 (for $N=6, 7, 8$, respectively). However, since Mamlouk and Martinetz showed that the quality of fit of their model does not rise appreciably beyond 32 dimensions, we remain with a prediction of 35-dimensional odor space (i.e., $N=6$). Interestingly, Weiss et al. (2012) showed that different odorant mixtures smell alike once the number of molecular components in them exceeds ~30. They concluded that ‘a common olfactory percept, “olfactory white,” is associated with mixtures of ∼30 or more equal-intensity components that span stimulus space...’ (p. 19959). Since a minimum of ∼30 odorants that span odor space was required to reach ‘olfactory white’, we can conclude that odor space is ~30-dimensional. A more detailed look at the results of Weiss et al. shows that for a mixture of odorants to be identified as ‘olfactory white’ with a probability of 50%, the mixture needs to contain between 35 and 40 components (see their Fig. 3). These results are consistent with the prediction made above of a 35-dimensional odor space. In summary, then, when we combine the hypothesis that odor sensations are the phenomenal dual aspects of some $N$-state quantum system with the experimental results on odor space that were cited above, we arrive at a quantitative prediction on this space: it should be exactly 35-dimensional. To test this prediction, the procedure described by Meister (2015, p. 9) may be used.

## 5.3 Two-state ion channels may be the two-state quantum systems that give rise to color

This paper proposes that color sensations are phenomenal dual aspects of two-state quantum systems in a mixed state. On this hypothesis, our color sensations arise from a mixture of two-state quantum systems that exists somewhere in the brain. What is the identity of these two-state quantum systems? Clearly, these systems must somehow be linked to color-related processes in the visual system. Further, since we are looking for a *quantum* system, it is likely to be found in the molecular level. Taking these two considerations together, I suggest that *two-state ion channels*—or, more likely, some component in them—are the two-state quantum systems whose phenomenal dual aspect is color. These channels, which lie at the very basis of neuronal activity, are large membrane-spanning proteins that transition between two discrete molecular conformations in response to the binding/release of a ligand or the presence/absence of voltage (for a review of ion channels, see Siegelbaum & Koester, 2000). The two molecular conformations of two-state ion channels correspond to two functional states: an open state, in which channel-specific ions are free to cross from the extracellular side to the intracellular side (or vice versa), and a closed state, in which no ions can pass through the channel. Since ion channels are huge protein molecules, and since the transition from one molecular conformation to the other entails many structural changes in the amino acids that comprise the channels (see details in DaCosta & Baenziger, 2013), it is unlikely that a two-state ion channel *as a whole* can be considered to be a two-state quantum system. Rather, it is more likely that the sought-for two-state quantum system is some *component* of the ion channel. Specifically, I suggest that the two-state quantum system that gives rise to color is the molecular component that resides at the orthosteric site, namely, the site where the channel’s agonist operates (Changeux & Christopoulos, 2016). The interaction with the channel’s agonist (e.g., the docking of a ligand) causes some conformational change in the orthosteric site which, in turn, cascades into the many structural changes that constitute the conformational change of the entire ion-channel molecule.

A well-established example of a conformational change in the orthosteric site of a receptor protein is given by rhodopsin, which is the photoreceptor molecule of rod cells in the retina (Tessier-Lavigne, 2000). The rhodopsin molecule is composed of a large protein, opsin, that is covalently bonded to a small, light-absorbing molecule—retinal. ‘In its nonactivated form rhodopsin contains the 11-*cis* isomer of retinal. Absorption of light by 11-*cis* retinal causes a rotation around the 11-*cis* double bond. As retinal returns to its more stable all-*trans* configuration, it brings about a conformational change in the opsin portion of rhodopsin, which triggers the other events of visual transduction’ (*ibid*., p. 511).[[7]](#footnote-8) If the *cis* and *trans* configurations of retinal constitute the only two physical configurations of this molecule, then the configuration state of the retinal molecule is a two-state quantum system. Therefore, on the hypothesis suggested here, the quantum states of the retinal molecule have color sensations as phenomenal dual aspects. (Of course, I do not claim that our color sensations stem from retinal rods; rather, our color experiences arise from somewhere in the visual cortex (see below). That is, for some unknown reason, the color sensations presumably generated by retinal molecules in our retinas are not used by the brain for *our* conscious color experience.)

Where can we expect to find the hypothesized two-state ion channels that give rise to color? The recent work by Li et al. (2022) has shown that there exist cone-opponent functional domains in the primary visual cortex (V1). The hue preferences in these functional domains are geometrically organized into so-called ‘pinwheels’. It is natural to suggest that the first place to look for two-state ion channels that give rise to color experience is in the neurons within these cone-opponent functional domains.

# 6. References

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1. Hendel showed that this expression is one of two different, yet mathematically equivalent, expressions for $I$. For our purposes here, the expression in Eq. (2) will be more useful. [↑](#footnote-ref-2)
2. For our purposes here, Hilbert space simply means the vector space $C^{n}$, where $n\in \left\{2, 3, …\right\}$. [↑](#footnote-ref-3)
3. The orthogonality of the Pauli operators is defined with respect to the inner product $\left〈\hat{σ}\_{μ},\hat{σ}\_{ν}\right〉=tr\left(\hat{σ}\_{μ}^{†}\hat{σ}\_{ν}\right)$, $μ, ν\in \left\{0, 1, 2, 3\right\}$, where $tr(⋅)$ is the trace operation. [↑](#footnote-ref-4)
4. To show this, we represent the Pauli operators as matrices relative to some basis of $C^{2}$ and then compute the eigenvalues using the standard procedure. [↑](#footnote-ref-5)
5. Equation (12) gives the unnormalized version of the density operator. Usually, this operator is given with all values normalized by $\left〈\hat{σ}\_{0}\right〉$ (Altepeter et al., 2004; Blum, 1981, chapter 1). [↑](#footnote-ref-6)
6. This disregards the dimension that holds the number of systems in the mixture. If we include this dimension, then the hypersphere is $N^{2}$-dimensional. [↑](#footnote-ref-7)
7. Notably, rhodopsin is not an ion channel, but rather a G-protein-coupled receptor. However, the biophysical principles behind its conformational changes are similar to those in ion channels (Changeux & Christopoulos, 2016). [↑](#footnote-ref-8)