Decoherence, the measurement problem, and interpretations of quantum mechanics

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Introduction

Consider a tiny dust-particle of diameter 0,01 mm, floating around in vacuum, and colliding with surrounding photons, illustrated in Figure 1:

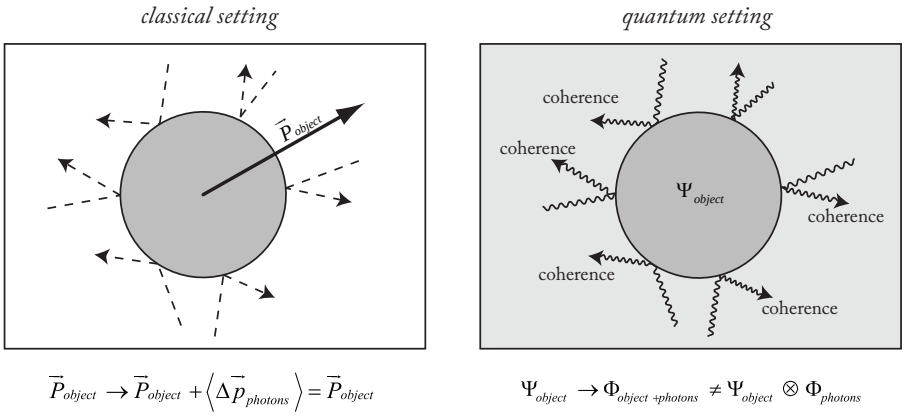

Figure 1: The different influence of the environment on the system in the classical and quantum settings, for a small dust particle immersed in light (photons) from all directions. Left: In the classical case, the interaction with the photons does not alter the motion of the object. Right: In the quantum case the photons become entangled with the object by the interaction, which causes a delocalisation of the coherence in the object, making quantum effects such as interference patterns unobservable at the level of the system. After Schlosshauer[1]

In a classical setting, when we consider the movement of the particle, it is perfectly safe to ignore the scattering of the photons. The amount of momentum transferred from the photon to the particle per collision is very small, but even when the interaction is strong, the incident photons are usually distributed isotropically in position and direction, thus averaging out the momentum transfer to zero. However, in a quantum setting, considering the state of the particle Ψ, for instance in the position basis $\Psi(x) = \langle x|\Psi\rangle$, generally every collision interaction entangles a photon with the particle. In this case the photon distribution does not matter: the initially local coherent state of the particle becomes more and more entangled with its environment of photons. The photons, flying off after scattering, thus delocalise the coherence, which makes quantum effects (such as interference) *unobservable at the level of the system.* Clearly, in this

case we cannot just ignore the environment!

This is the basic underlying idea of the theory of *environment-induced decoherence*. It is the purpose of this essay to review the theory of decoherence, and its implications for the traditional problem of quantum measurement, intimately related to the emergence of the classical world from a quantum reality. Additionally we discuss how decoherence fits in with a number of traditional and more recent interpretations of quantum theory.

Roughly, the outline of this essay is as follows:

- In Chapter 1 I will discuss the physics and mathematics of environment-induced decoherence, give a few examples of models to which it is applicable, and discuss recent experiments.
- In Chapter 2 I define the measurement problem (in different ways), discuss why quantum mechanics needs an interpretation, and describe a few mainstream interpretations of quantum mechanics.
- Next, combining these two chapters I will discuss the implications of the decoherence program for the measurement problem itself in Chapter 3; What parts of the measurement problem does the theory (claim) to solve (if any)? Which other interpretations of quantum mechanics connect well with results from decoherence theory?
- Finally I will summarise in chapter 4, including a small outlook from my own perspective.

Most of chapter 1 is based on the extensive book on Decoherence published by M. Schlosshauer in 2008 [1], and the later chapters use H. Janssen's 2008 master's thesis [3] as the main source of information.

Chapter 1

The theory of environment-induced decoherence

1.1 What is Decoherence?

Decoherence is the term used to describe the destruction of phase relations in the state of a quantum mechanical system, as a result of a dynamical process. According to the Superposition Principle, any two state vectors in a Hilbert space of a quantum mechanical system, can be linearly added together to form another valid state of the system: for $|\psi\rangle, |\phi\rangle \in \mathcal{H}$

$$|\Psi\rangle = a\,|\psi\rangle + b\,|\phi\rangle \in \mathcal{H} \tag{1.1}$$

where $a, b \in \mathbb{C}$. This causes the occurrence of many purely quantum mechanical effects, such as interference in the double slit experiment (see section 1.4.2), entanglement of quantum systems, and is one of the key reasons a quantum computer might be advantageous compared to a classical one [2]. In practice however, it is very hard to keep a system in a coherent superposition due to interactions with its environment, causing interference effects and entanglement correlations to vanish quickly. In this basic form, decoherence is then an unwanted but unavoidable fact from perspective of the quantum physicist, in his attempt to exploit these phase relations in experiments.[1]

However, "decoherence" is now often used for a much more general idea, namely that of the environment-induced decoherence program, referring not only to the effect of decoherence itself, but also referring to

- its main cause, the ubiquitous and almost unavoidable interaction of a quantum system with its environment;
- its physical implications, expressed in predictions for empirically verifiable experiments;

[1] In textbooks related to quantum computation, decoherence is often called quantum noise, or, in the field of quantum information theory, described by a phase damping channel, a process in which information is lost without a loss of energy. See for instance [2].

- its conceptual implications, on for instance the traditional problem of quantum measurement, and the emergence of the classical world from a quantum reality.

It is important to distinguish between these last two points, because although the relevance of environment-induced decoherence on empirical outcomes is widely acknowledged, its conceptual implications are subject to much more controversy. Opinions range from solving (part of) the measurement problem, as founding decoherence theorists used to claim, to denying any conceptual implications apart from those illustrated any other quantum mechanical calculation.

Nevertheless, decoherence theory is a well-established subject, and many currently popular philosophical interpretations of quantum theory either use results from decoherence theory to propagate their ideas, or are entirely based on these results.

1.2 Basic formalism

This section contains the basic formalism that we need to describe decoherence in a mathematical formulation of quantum theory. I will describe (very) briefly the mathematical framework of quantum theory, and the theory of mixed states and density matrices. We will also look at von Neumann's Measurement scheme, as it plays a significant role in the philosophical discussion later. Other more extensive reviews can be found in any quantum theory textbook; this section is largely based on the Quantum Information Theory textbook by Nielsen and Chuang (2000) [2], and lecture notes by Nilanjana Datta (2009) [4].

1.2.1 Mathematical quantum theory; Postulates of quantum mechanics

Quantum Mechanics is a physical theory that replaces Newtonian mechanics and Classical Electromagnetism at the atomic and subatomic level. Its mathematical framework can be used to make predictions about the behaviour of particular physical systems, and the laws they must obey. The connection between the physical system and a workable mathematical abstraction of it, is made through a few basic postulates, that were basically derived by a long process of "trail and error". Note that these postulates are therefore not proven from any more fundamental principles. When we come to discuss interpretations of quantum theory and the measurement problem in chapter 2, we have to be careful not to take these postulates as some kind of absolute truth. However most discussions about interpretation of quantum theory and extensions to it are based on these shared assumptions and we will need them to formulate our description of decoherence.

Postulate 1. Any physical system is described by a state vector $|\psi\rangle$ which lives in a Hilbert space, a complex vector space equipped with an inner product $\langle\phi\,|\psi\rangle$. A state vector has unit norm $\langle\psi\,|\psi\rangle = 1$.

- In most cases we will encounter, our Hilbert space will simply be $\mathbb{C}^n$, with $|\psi\rangle$ being a unit n-dimensional column-vector, with the usual inner product.

- This means that any superposition of two state vectors,

$$|\Psi\rangle = a\,|\psi\rangle + b\,|\phi\rangle\,, \tag{1.2}$$

with $a, b \in \mathbb{C}$, and $|a|^2 + |b|^2 = 1$, is also a valid state of our system.

- A property of our physical system that can be measured is called an observable and is represented by a linear operator $\hat{A}$ acting on $\mathcal{H}$, that is Hermitian (self-adjoint): $\hat{A} = \hat{A}^\dagger$. $\hat{A}$ then has a spectral decomposition $\hat{A} = \sum_i a_i\,|\psi_i\rangle\,\langle\psi_i|$, where $\{|\psi_i\rangle\}$ denotes a complete orthonormal set of eigenvectors of $\hat{A}$ with corresponding eigenvalues $\{a_i\}$. We can define the orthogonal projectors $\hat{P}_i$ onto the eigenspace of $\hat{A}$

$$\hat{P}_i = |\psi_i\rangle\,\langle\psi_i|\,, \tag{1.3}$$

where by definition

$$\hat{P}_i\hat{P}_j = \delta_{ij}\hat{P}_i, \quad \hat{P}_i{}^\dagger = \hat{P}_i, \quad \sum_i \hat{P}_i = \hat{\mathbb{I}}, \tag{1.4}$$

with δ_{ij} the Kronecker delta, and $\hat{\mathbb{I}}$ the identity operator. Note that the projectors themselves have eigenvalues 0 and 1. We will see the significance of these projectors when we discuss quantum measurement at the third postulate.

Postulate 2. The time-evolution of an isolated (closed) quantum system is described by a unitary transformation. $|\psi(t)\rangle = \hat{U}(t_0, t)\,|\psi(t_0)\rangle$, where $\hat{U}$ is a unitary operator acting on $\mathcal{H}$.

- The unitary transformation $\hat{U}$ is determined by solving the Schrödinger equation,

$$i\hbar\frac{d\,|\psi\rangle}{dt} = \hat{H}\,|\psi\rangle\,, \tag{1.5}$$

where $\hbar$ is Planck's constant, and $\hat{H}$ is a linear Hermitian operator acting on $\mathcal{H}$ called the Hamiltonian. If $\hat{H}$ is time independent, we have

$$\hat{U}(t_0, t) = \exp\left[\frac{-i}{\hbar}\hat{H}(t - t_0)\right]|\psi\rangle\,. \tag{1.6}$$

- If we are able to construct a general Hamiltonian, we can perform arbitrary unitary transformations on our system.

- The above relation only holds for isolated systems. However, when an experiment is done to find out properties of a system, we have to let the system interact with our experimental equipment, so the system is no longer closed and the evolution no longer unitary. The following posulate describes the evolution of a system under a measurement.

Postulate 3. This postulate is also known as the *collapse postulate*. Quantum measurements are described by a collection $\{\hat{M}_m\}$ of linear measurement operators, acting on $\mathcal{H}$, which satisfy the relation

$$\sum_m \hat{M}_m^\dagger\hat{M}_m = \hat{\mathbb{I}}. \tag{1.7}$$

The index m refers to the measurements outcomes that may occur in the experiment.

- Suppose the system under measurement is in state $|\psi\rangle$ just before the measurement. Then the probability of obtaining result m from the measurement is given by

$$p(m) = \langle\psi| \hat{M}_m^\dagger \hat{M}_m |\psi\rangle , \tag{1.8}$$

and the state after measurement is given by

$$|\psi\rangle \longrightarrow |\psi'\rangle = \frac{\hat{M}_m |\psi\rangle}{\sqrt{\langle\psi| \hat{M}_m^\dagger \hat{M}_m |\psi\rangle}}. \tag{1.9}$$

- Note that the projectors $\hat{P}_i$ we defined for observable $\hat{A}$ of a system in postulate 1 in equation (1.3) satisfy the relation for measurement operators, equation (1.7). All measurements with operators satisfying both (1.7) and (1.4) form an important special case of the general measurement postulate, called *projective measurements*[2].

Postulate 4. The state space of a composite quantum system made up of two (or more) distinct physical systems is the tensor product of the state spaces of the component physical systems. Suppose we have systems numbered 1 through n, with system number i prepared in the state $|\psi_i\rangle$, then the composite system is described by a state vector $|\Psi\rangle$ which lives in a Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes ... \otimes \mathcal{H}_n$, and is itself given by a tensor product:

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes ... \otimes |\psi_n\rangle . \tag{1.10}$$

These four postulates are all we need to define our mathematical quantum theory. In the next section we will consider a reformulation of the theory in the language of *density operators*. This alternate formulation is mathematically equivalent, but much more convenient to work with in many scenarios encountered in quantum mechanics, in particular, decoherence.

1.2.2 Density operator formalism

Instead of formulating quantum theory in the language of state vectors, we can formulate it in the language of *density operators*. The density operator formalism is advantageous when we are dealing with

- an ensemble of states, for instance a system whose state is not exactly known;
- the description of individual subsystems of a composite quantum system.

Suppose a quantum system is in one of the states $|\psi_i\rangle \in \mathcal{H}$, with respective probabilities p_i. So the system is physically in one of the states $|\psi_i\rangle$, we are just ignorant, and don't know for sure which one.[3] This could for instance be the case if we have a reservoir of quantum systems, that contains different

[2] Projective measurements are actually the only kind of measurement we know how to directly implement experimentally, but combined with the ability to perform unitary transformations (Postulate 2) and the ability to combine physical system to a composite system (Postulate 4), we can perform a generalised measurement on a system as stated above.

[3] See however section 1.2.3.

proportions of quantum systems in state $|\psi_i\rangle$ and we pick one at random. We call $\{p_i, |\psi_i\rangle\}$ an ensemble of pure states. The density operator is then defined as

$$\rho := \sum_i p_i \left|\psi_i\right\rangle \left\langle\psi_i\right| . \tag{1.11}$$

Note that $\rho = \hat{\rho}$ is an operator acting on $\mathcal{H}$. A density matrix is called *pure* if and only if it can be written $\rho = |\phi\rangle \langle\phi|$ for some $|\phi\rangle \in \mathcal{H}$, otherwise it is called *mixed*. In the finite case the density operator is often called the *density matrix*.

The density operator has two important properties:

1. Its trace is equal to one

$$Tr[\rho] = \sum_i p_i Tr[|\psi_i\rangle \langle\psi_i|] = \sum_i p_i = 1, \tag{1.12}$$

   since the probabilities sum to one.

2. It is a positive operator: For any $|\phi\rangle \in \mathcal{H}$

$$\langle\phi| \rho |\phi\rangle = \sum_i p_i \langle\phi|\psi_i\rangle \langle\psi_i|\phi\rangle = \sum_i p_i |\langle\phi|\psi_i\rangle|^2 \geq 0 \tag{1.13}$$

Now note that any operator $\hat{O}$ acting on $\mathcal{H}$ that has the above two properties, also defines a density operator: Since $\hat{O}$ is positive it has a spectral decomposition

$$\hat{O} = \sum_j \lambda_j |\chi_j\rangle \langle\chi_j| \tag{1.14}$$

with $\{|\chi_j\rangle\}$ its orthonormal eigenvectors, and λ_j the accompanying real, non-negative eigenvalues. From the trace condition we now have $\sum_j \lambda_j = 1$. Therefore $\hat{O}$ defines an ensemble $\{\lambda_j, |\chi_j\rangle\}$, with a density operator $\rho_O = \hat{O}$ by definition.

The formulation of Quantum theory now takes exactly the same form as described in section 1.2.1, with minor changes to the four postulates, which are the following:

- In Postulate 1, instead of a vector, the state of a system is now completely characterised by a density operator ρ acting on $\mathcal{H}$.
- In Postulate 2, the time-evolution of ρ is $\rho(t) = \hat{U}(t, t_0)\rho(t_0)\hat{U}^\dagger(t, t_0)$.
- In Postulate 3, the probability of outcome m becomes $p(m) = Tr[\hat{M}_m^\dagger \hat{M}_m \rho]$, and the state transition after measurement is: $\rho \longrightarrow \rho' = \frac{\hat{M}_m \rho \hat{M}_m^\dagger}{Tr[\hat{M}_m^\dagger \hat{M}_m \rho]}$.
- In Postulate 4, the joint state of the composite system becomes $\varrho = \rho_1 \otimes \rho_2 \otimes ... \otimes \rho_n$.

You might ask whether there are other formulations of the same mathematical quantum theory that may be even more useful. There is however a very nice theorem proven by Gleason [5], that states that in fact the density operator ρ acting on a state space $\mathcal{H}$ is the most general way to assign consistent probabilities to all possible orthogonal projections in a $\mathcal{H}$, and therefore to all possible measurements of observables.

The reduced density operator

Consider a composite quantum system, made up of physical systems A and B, in the state $\rho^{AB} \in \mathcal{H}_A \otimes \mathcal{H}_B$. Given that we are only interested in results of measurements done on system A (we might for instance be unable to do measurements on B, because it is not under our control), our measurement operators will be of the form

$$\hat{M}_m^{AB} = \hat{M}_m^A \otimes \hat{\mathbb{I}}^B. \tag{1.15}$$

We would then like a description of the different outcome probabilities only in terms of the state of system A. Such a description is made through the *reduced density operator*. With the definitions

$$\rho^A := Tr_{\mathcal{H}_B}[\rho^{AB}], \qquad Tr_{\mathcal{H}_B}[|a_1\rangle\langle a_1| \otimes |b_1\rangle\langle b_1|] := |a_1\rangle\langle a_1| \, Tr[|b_1\rangle\langle b_1|] \tag{1.16}$$

we have for the outcome probabilities[4].

$$\begin{aligned} p(m) &= Tr[(\hat{M}_m^{AB})^\dagger \hat{M}_m^{AB} \rho^{AB}] \\ &= Tr[(\hat{M}_m^A \otimes \hat{\mathbb{I}}^B)^\dagger (\hat{M}_m^A \otimes \hat{\mathbb{I}}^B) \rho^{AB}] \\ &= Tr[\hat{M}_m^A Tr_{\mathcal{H}_B}[\rho^{AB}]] \\ &= Tr[\hat{M}_m^A \rho^A] \end{aligned} \tag{1.17}$$

Finally, suppose that someone, unknown to us, takes the system B away and does some measurement on it. Does this change our description ρ^A of the properties of system A that we had previously? It turns out this is not the case. If A and B are separated, nothing happening to B - neither Hamiltonian evolution nor measurements - affects our predictions for the physics of A that we had obtained before with our reduced density operator ρ^A, *unless* we actually obtain information about the results of a measurement on B. To see why this is so we take

$$\hat{M}_m^{AB} = \hat{\mathbb{I}}^A \otimes \hat{M}_m^B \tag{1.18}$$

and the state after measurement becomes:

$$\rho^{AB} \longrightarrow \rho'^{AB} = \frac{\hat{M}_m^{AB} \rho^{AB} (\hat{M}_m^{AB})^\dagger}{\text{Prob}(m)}. \tag{1.19}$$

Now because presume we are ignorant of the measurement outcome, the reduced density operator after the measurement is the probability-weighted sum of the different possible final states:

$$\begin{aligned} \rho^A \longrightarrow \rho'^A &= \sum_m \text{Prob}(m) Tr_{\mathcal{H}_B}[\rho'^{AB}] \\ &= Tr_{\mathcal{H}_B}[\sum_m (\hat{M}_m^{AB})^\dagger \hat{M}_m^{AB} \rho^{AB}] \\ &= Tr_{\mathcal{H}_B}[\rho^{AB}] = \rho^A \end{aligned} \tag{1.20}$$

Where we have used the cyclicity of the trace to get to the second line. This also implies that we cannot send any kind of information through the quantum state from 'A' to 'B', for instance by performing measurements on one of the systems, a result that is known as the *quantum no signalling theorem.*

[4]To see why this is so, not that in the finite case we can write any $\rho^{AB} = \sum_{i,j,\alpha,\beta} c_{i\alpha;j\beta} |\phi_i\rangle\langle\phi_j| \otimes |\chi_\alpha\rangle\langle\chi_\beta|$ for i, j, α, β indexes, and $c_{i\alpha;j\beta}$ complex coefficients. In the other (infinite) Hilbert spaces a similar argument holds.

1.2.3 Proper vs. Improper mixtures

In our derivation of the reduced density operator, we have skipped over an important detail. In our definition of the density operator at the beginning of this section, we stated that it was our ignorance that caused our system to be in a mixture of states. The system was actually in a well defined quantum state vector, we just did not know which one. This is different from the case where our system is described by a mixed reduced density operator, as a subsystem of an ensemble in a pure state. To clarify this difference, consider the following example:

Suppose we have quantum systems living in a Hilbert space $\mathcal{H}$ with orthonormal basis $\{|\uparrow_z\rangle, |\downarrow_z\rangle\}$, and we prepare three systems as follows:

1. We prepare the superposition $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle)$ which gives the density matrix

$$\rho_1 = |\psi_1\rangle\langle\psi_1| = \frac{1}{2}(|\uparrow_z\rangle\langle\uparrow_z| + |\uparrow_z\rangle\langle\downarrow_z| + |\downarrow_z\rangle\langle\uparrow_z| + |\downarrow_z\rangle\langle\downarrow_z|). \quad (1.21)$$

2. We pick a random system from a reservoir of systems of which half of the systems is in state $|\uparrow_z\rangle$, and the other half is in $|\downarrow_z\rangle$;

$$\rho_2 = \sum_i p_i |\updownarrow_z\rangle_i \langle\updownarrow_z|_i = \frac{1}{2}(|\uparrow_z\rangle\langle\uparrow_z| + |\downarrow_z\rangle\langle\downarrow_z|). \quad (1.22)$$

3. We prepare a composite of two systems A, B, $|\Psi_3\rangle \in \mathcal{H}\otimes\mathcal{H}$ in the superposition state $|\Psi_3\rangle = |\uparrow_z\rangle_A|\uparrow_z\rangle_B + |\downarrow_z\rangle_A|\downarrow_z\rangle_B$, and remove B from our control. This leaves system A in the state

$$\rho_3 = Tr_{\mathcal{H}_B}[|\Psi_3\rangle\langle\Psi_3|] = \frac{1}{2}(|\uparrow_z\rangle_A\langle\uparrow_z|_A + |\downarrow_z\rangle_A\langle\downarrow_z|_A). \quad (1.23)$$

There are a number of things to be said about this example.

Firstly, note system 1 is in a pure state, whereas systems 2,3 are in a mixed state.

Secondly, measurements of the three systems in the z-basis would all yield ±1 with probabilities 1/2, but system 1 is the only system in a superposition state. If we measure in any other basis, system 2,3 will always yield ±1 with probabilities 1/2, whereas system 1 will produce outcome $+1$ with certainty for some measurement bases. Specifically, interpreting the systems as spin-systems (as the notation suggests) and measuring them in a Stern-Gerlach experiment in x-direction (projectors $\hat{P}_1 = |\uparrow_x\rangle\langle\uparrow_x|$, $\hat{P}_2 = \hat{\mathbb{I}} - |\uparrow_x\rangle\langle\uparrow_x|$), yields the outcome $+1$ for system 1 with certainty, and outcomes ±1 with probabilities 1/2 for system 2,3. Another way of saying this is stating that ρ_1 has interference - off diagonal - terms, whereas ρ_2 and ρ_3 do not. In that sense one would be inclined to say that both systems 2 and 3 are now in a classical distribution of states.

However, note that even though $\rho_2 = \rho_3$, their physical interpretation is not quite the same. System 2 is in a definite deterministic physical state, whereas system 3 is part of a composite superposition state. Its physical state is truly undetermined, as long as no measurement is performed on "part B" of system 3 (that we removed from our control). System 2 is said to be a *proper* mixture, versus system 3 which is in a *improper* mixture. When a measurement

is performed on the discarded part B of system 3, but we are not told of the outcome (ignorance), system 3 reduces to a proper mixture, and systems 2 and 3 are then physically identical.

In the previous section we (Gleason[5]) proved that the density operator is the most general description of measurement statistics of a quantum system, and yet we just stated that although $\rho_2 = \rho_3$, they are physically not the same state. How can this be so? The answer is that indeed we are unable to discern systems 2,3 by any local measurement, since *at the level of the systems* 2,3 the density operator is indeed the most fundamental object describing the measurement outcome probabilities, but looking at the global state including the discarded part B of system 3, we find that systems 2,3 are different.

This difference between proper and improper mixtures will play an important role when we come discuss the implications of environment-induced decoherence on the quantum to classical transition in chapter 3.

1.2.4 Von Neumann ideal measurement scheme

Von Neumann devised a scheme for describing a quantum measurement in his 1932 book on mathematical quantum theory. The scheme is based on entanglement between the quantum system under measurement and the measuring apparatus used. Von Neumann therefore treated not only the system but also the apparatus as a quantum-mechanical object.[5] We will see later how von Neuman's scheme relates to the kind of measurement we have defined above (using projectors $\hat{P}_i$). The scheme is as follows:

Define a quantum system $\mathcal{S}$ (usually microscopic), with Hilbert space $\mathcal{H}_\mathcal{S}$ and an orthonormal basis $\{|s_i\rangle\}$ (of the observable to be measured), and a measurement apparatus (possibly macroscopic) $\mathcal{A}$, with Hilbert space $\mathcal{H}_\mathcal{A}$ and orthonormal basis $\{|a_i\rangle\}$. The apparatus is now to measure the state of the system. Suppose the apparatus has some kind of pointer that moves to position i, corresponding to its state $|a_i\rangle$, if the system is measured to be in the state $|s_i\rangle$. Taking the state of the apparatus before the measurement to be some initial 'ready' state $|a_r\rangle$, the dynamical (unitary) measurement interaction is then of the form

$$|s_i\rangle\,|a_r\rangle \longrightarrow |s_i\rangle\,|a_i\rangle \tag{1.24}$$

for all i. Here the initial and final states live the Hilbert space $\mathcal{H}_\mathcal{S} \otimes \mathcal{H}_\mathcal{A}$ of the total $\mathcal{SA}$ system.

Now in general the initial state of the system need not be an eigenstate, it can be in any superposition

$$|\psi_\mathcal{S}\rangle = \sum_i c_i\,|s_i\rangle\,, \tag{1.25}$$

with $c_i \in \mathbb{C}$, and $\sum |c_i|^2 = 1$. Then the measurement interaction implies, by linearity of the Schrödinger equation,

$$|\psi_\mathcal{S}\rangle\,|a_r\rangle = \left(\sum_i c_i\,|s_i\rangle\right)|a_r\rangle \longrightarrow \sum_i c_i\,|s_i\rangle\,|a_i\rangle \tag{1.26}$$

[5]This is in sharp contrast with the so called Copenhagen Interpretation, that postulated the existence of purely classical measurement-apparatuses.

Note that the von Neumann measurement interaction has left the system and apparatus in an entangled state, we can no longer describe them by a state vector on their separate Hilbert spaces. The superposition in the system has been amplified to the level of the (macroscopic) apparatus.

To see how this scheme, sometimes called a pre-measurement, connects to our previously (third postulate) defined measurement, suppose we proceed to measure the combined $\mathcal{SA}$ system using the set of projectors

$$\hat{P}_j = \hat{\mathbb{I}} \otimes |a_j\rangle \langle a_j| \tag{1.27}$$

where $\hat{\mathbb{I}}$ denotes the identity operator on $\mathcal{H}_\mathcal{S}$. This leaves the combined system in the state

$$\frac{\hat{P}_j \left(\sum_i c_i |s_i\rangle |a_i\rangle\right)}{\sqrt{\text{Prob}(j)}} = \frac{\sum_i c_i \, \hat{\mathbb{I}} |s_i\rangle \otimes |a_j\rangle \langle a_j | a_i\rangle}{\sqrt{\text{Prob(j)}}} = |s_j\rangle |a_j\rangle . \tag{1.28}$$

The system and apparatus are no longer entangled, and the state of the system has collapsed in the eigenstate corresponding to eigenvalue j. So this would be equivalent to measuring the system using projectors without the intermediate step (pre-measurement) of entangling it with the apparatus.

Implementing a Von Neumann measurement

Such a von Neumann measurement interaction would in real experiments usually correspond to bringing the system and apparatus very close together and letting them interact for a certain time. As a simple example, suppose we want to measure a two-level spin system, by letting it interact with our apparatus in the form of another two-level spin under our control; (to complete the measurement, we could then measure our apparatus-spin in for instance a Stern-Gerlach experiment). A typical interaction Hamiltonian would then take the form

$$\hat{H}^{\mathcal{SA}} = g(\hat{\sigma_z}^{\mathcal{S}} \otimes \hat{\sigma_z}^{\mathcal{A}}) = g \left(|\uparrow_z\rangle \langle\uparrow_z|_\mathcal{S} - |\downarrow_z\rangle \langle\downarrow_z|_\mathcal{S}\right) \otimes \left(|\uparrow_z\rangle \langle\uparrow_z|_\mathcal{A} - |\downarrow_z\rangle \langle\downarrow_z|_\mathcal{A}\right), \tag{1.29}$$

which says in essence that the energy is minimised when the spins are aligned. Now in accordance to equation (1.25), we take the state at time $t = 0$:

$$|\Psi(t=0)\rangle = |\psi_\mathcal{S}\rangle |a_r\rangle = \left(c_1 |\uparrow_z\rangle_\mathcal{S} + c_2 |\downarrow_z\rangle_\mathcal{S}\right) |\uparrow_x\rangle_\mathcal{A} , \tag{1.30}$$

where we have chosen without loss of generality $|a_r\rangle = |\uparrow_x\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle)$. The state at time $t > 0$ then becomes

$$\begin{aligned} |\Psi(t)\rangle &= \exp\left(\frac{-i}{\hbar}\hat{H}^{\mathcal{SA}} t\right) |\Psi(0)\rangle \\ &= \frac{c_1}{\sqrt{2}} |\uparrow_z\rangle_\mathcal{S} \left(e^{-\frac{ig}{\hbar}t} |\uparrow_z\rangle_\mathcal{A} + e^{\frac{ig}{\hbar}t} |\downarrow_z\rangle_\mathcal{A}\right) \\ &\quad + \frac{c_2}{\sqrt{2}} |\downarrow_z\rangle_\mathcal{S} \left(e^{\frac{ig}{\hbar}t} |\uparrow_z\rangle_\mathcal{A} + e^{-\frac{ig}{\hbar}t} |\downarrow_z\rangle_\mathcal{A}\right), \end{aligned} \tag{1.31}$$

so to implement our measurement scheme we can for instance choose to let the systems interact for a time $t = \pi\hbar/2g$, which leaves the state,

$$|\Psi(t = \pi\hbar/2g)\rangle = c_1 |\uparrow_z\rangle_\mathcal{S} |\uparrow_x\rangle_\mathcal{A} + c_2 |\downarrow_z\rangle_\mathcal{S} |\downarrow_x\rangle_\mathcal{A} = \sum_i c_i |s_i\rangle |a_i\rangle , \tag{1.32}$$

clearly in the form of equation (1.26).

1.3 Environment-induced decoherence

In this section I use the tools described in the previous section to derive the effects of a surrounding environment on our combination system + measurement-apparatus, as is done in Zurek's original 1981 and 1982 papers [6][7], and is discussed by Schlosshauer (2008) [1]. Many of the derivations are not very mathematically rigorous, and are not meant to make general statements about quantum-mechanics in general, but the idea will be quite clear. As Janssen(2008) remarks in her thesis on the subject:

> The literature about decoherence can be difficult to grasp. The reason for this is twofold. First, there is a large amount of rather sophisticated technical literature about decoherence that does not touch upon the foundational issues (and has no such intentions). [...] Second, there also exists another kind of literature [...] that does aim to address the kind of questions I posed in the previous chapter. The problem with this kind of literature is that it is full of claims that are not really substantiated, that it is nowhere clearly stated what the questions are that are being addressed, and that it suffers from a thorough lack of self-criticism. ([3], pp. 61)

1.3.1 Interaction with the environment and local suppression of interference

The idea is simple. Postulate 2 says that a closed quantum system evolves unitarily according to the Schrödinger equation. *Open* systems however, do not evolve unitarily. An open system is a system interacting with an environment which is not included in the describing quantum state, but does affect the dynamics: Working again with the same definitions for our quantum system of interest $\mathcal{S}$ and measurement-apparatus $\mathcal{A}$ as in the von Neumann measurement scheme (section 1.2.4), let us assume $\mathcal{S}$ is initially in a superposition of eigenstates $|s_i\rangle$ (just like in equation (1.25)):

$$|\psi_{\mathcal{S}}\rangle = \sum_i c_i |s_i\rangle . \tag{1.33}$$

We take the initial state of the apparatus to be again the ready state $|a_r\rangle$, and we now introduce a third system, the environment $\mathcal{E}$ with Hilbert space $\mathcal{H}_{\mathcal{E}}$. We assume the environment is initially in the pure[6] state $|e_0\rangle$.

$$|\psi_{\mathcal{SAE}}\rangle_{initial} = \left(\sum_i c_i |s_i\rangle\right) |a_r\rangle |e_0\rangle . \tag{1.34}$$

After the pre-measurement interaction the final state becomes an entangled state of not only system and apparatus, but also the environment:

$$|\psi_{\mathcal{SAE}}\rangle_{final} = \sum_i c_i |s_i\rangle |a_i\rangle |e_i\rangle . \tag{1.35}$$

[6] We may always assume the environment is in a pure state, by using *purification*, a method that is based on the *Schmidt decomposition* (see for instance [2], section 2.5, pp. 109). Basically the method tells you to enlarge your Hilbert space by introducing a reference system, such that the combined system is in a pure state.

Remember, we could implement such a von Neumann pre-measurement (section 1.2.4) by choosing a certain interaction Hamiltonian between the system and apparatus, and letting it evolve for a certain time. However, this time we also have to consider the interaction Hamiltonian with the environment, which we cannot choose. As we shall see in the next section, this imposes certain conditions on the states $|s_i\rangle$ and $|a_i\rangle$ that evolve in the manner of equation (1.35). In the meantime, we assume that we have designed our apparatus such that the states $\{|a_i\rangle\}$ are orthonormal (we want to be able to distinguish between the possible measurement outcomes), but the states $\{|e_i\rangle\}$ of the environment are not necessarily (nor even likely to be) orthogonal.

The density matrix of our final state is

$$\rho^{\mathcal{SAE}} = \sum_{ij} c_i c_j^* \left|s_i\right\rangle \left\langle s_j\right| \otimes \left|a_i\right\rangle \left\langle a_j\right| \otimes \left|e_i\right\rangle \left\langle e_j\right|. \tag{1.36}$$

Now the crucial step (and assumption) is that since we only care about the state of our system and apparatus $\mathcal{SA}$ and since the environment is out of our control anyway, for all practical purposes of measurement predictions we can trace out the environment and look only at the subsystem $\mathcal{SA}$. Using the reduced density operator formalism we get

$$\rho^{\mathcal{SA}} = Tr_{\mathcal{H}_\mathcal{E}}[\rho^{\mathcal{SAE}}] = \sum_{ij} c_i c_j^* \left\langle e_j\right| \left.e_i\right\rangle \left|s_i\right\rangle \left\langle s_j\right| \otimes \left|a_i\right\rangle \left\langle a_j\right|. \tag{1.37}$$

Now if we could show $\left\langle e_j\right| \left.e_i\right\rangle$ were (or would become) orthogonal, this would reduce the system-apparatus state to an incoherent ensemble (mixture),

$$\rho^{\mathcal{SA}}_{decohered} = \sum_i |c_i|^2 \left|s_i\right\rangle \left\langle s_i\right| \otimes \left|a_i\right\rangle \left\langle a_i\right|. \tag{1.38}$$

This is exactly what different models developed by decoherence theorists try to predict; how fast and under which circumstances the entangled environment states $\{|e_i\rangle\}$ become orthogonal. This generally depends on the nature of the assumed interaction between the environment and the system/apparatus. But most models show that this orthogonality is achieved very rapidly due in part to the high dimension of $\mathcal{H}_\mathcal{E}$, i.e. the fact that there are so many environment states.

1.3.2 Environment-induced superselection

We now wish to proceed and consider the interaction with the environment in more detail, to find out under what circumstances decoherence will take place. Basically what we will find is that for some states of the system decoherence is more effective than for others, which leads to the definition of so called *pointer observables*; observables whose eigenstates are stable with respect to the decohering effect of the environment.

For convenience, following Schlosshauer (2008)[1], we temporarily drop the description in terms of the apparatus, and focus just on the system and environment.[7] The composite system-environment Hamiltonian can be decomposed

[7]This is not necessary: in Zurek's original 1981 paper[6], the apparatus is explicitly kept in the derivation, but there it is assumed that the system-apparatus interaction only takes a very brief time, and that the system-environment interaction can be neglected. This basically splits the derivation in two (in the time domain), making it equivalent to ours.

like

$$\hat{H}^{\mathcal{SE}} = \hat{H}^{\mathcal{S}} + \hat{H}^{\mathcal{E}} + \hat{H}^{int}, \tag{1.39}$$

where $\hat{H}^{\mathcal{S}}$ and $\hat{H}^{\mathcal{E}}$ are the self-Hamiltonians of the system and environment, and $\hat{H}^{int}$ is the interaction Hamiltonian between them.

We then identify three different regimes in which decoherence occurs:

(i) *The quantum measurement limit*, in which the interaction between system and environment is sufficiently strong so that the intrinsic dynamics of the system and environment are negligible in comparison with the evolution induced by the interaction, i.e. $\hat{H}^{\mathcal{SE}} \approx \hat{H}^{int}$.

(ii) *The quantum limit of decoherence*, when the spacing between energy levels of the system is large compared to the frequencies present in the environment $\hat{H}^{\mathcal{S}} \gg \hat{H}^{int}$.

(iii) *The intermediate regime*, where both the internal dynamics and the interaction govern the evolution of the system.

1.3.3 The quantum measurement limit

Beginning with the simplest case of the quantum measurement limit, we demand that the initial state of the system remains unchanged by the action of $\hat{H}^{\mathcal{SE}} \approx \hat{H}^{int}$. Clearly this means,

$$|\Psi(t)\rangle = \exp\left(\frac{-i}{\hbar}\hat{H}^{int}t\right)|\psi_{\mathcal{S}}\rangle\,|e_0\rangle = |\psi_{\mathcal{S}}\rangle \exp\left(\frac{-i}{\hbar}\hat{H}^{int}t\right)|e_0\rangle \tag{1.40}$$

so that $|\psi_{\mathcal{S}}\rangle$ must be an eigenstate of the interaction Hamiltonian $\hat{H}^{int}$. Restating this in the language of observables, we have found the *pointer observables* of system $\mathcal{S}$, that is the observables whose eigenstates are stable with respect to the decohering effect of the environment, namely those observables $\hat{O}^{\mathcal{S}}$ that commute with $\hat{H}^{int}$,

$$\left[\hat{O}^{\mathcal{S}}, \hat{H}^{int}\right] = 0 \tag{1.41}$$

This then determines the condition on the states $|s_i\rangle$ and $|a_i\rangle$ of equation (1.35) we mentioned in the derivation of the diagonalization of the reduced density matrix in equation (1.38) on page 14. They must be the eigenstates of an observable $\hat{O}^{\mathcal{S}}$ that satisfies equation (1.41). States $|s_i\rangle$ are robust against decoherence, but superpositions $\sum_i c_i\,|s_i\rangle$ are not, which will be reduced to a mixed incoherent state, as in the previous section.

In many cases of interest, we can write the interaction Hamiltonian $\hat{H}^{int}$ in tensor product form:

$$\hat{H}^{int} = \hat{S} \otimes \hat{E} \tag{1.42}$$

with $\hat{S}$ and $\hat{E}$ operators acting on the system and environment Hilbert spaces respectively. Now the pointer observables will be those that commute with $\hat{S}$. If $\hat{S}$ is Hermitian, it represents simply the quantity that is monitored by the environment, of which a frequently encountered example is position, where $\hat{H}^{int} = \hat{x} \otimes \hat{E}$. This causes the environment to perform an effective non-demolition pre-measurement in position basis of the system. If we denote

the eigenstates of $\hat{x}$ by $|x\rangle$, the explicit Schrödinger time evolution of such an eigenstate becomes

$$e^{-i\hat{H}^{int}t}\,|x\rangle\,|e_0\rangle = |x\rangle\,e^{-ix\hat{E}t}\,|e_0\rangle = |x\rangle\,|e_x(t)\rangle \tag{1.43}$$

where the subscript 'x' in $|e_x(t)\rangle$ denotes the fact that the state of the environment now contains information about the position of the system.

Position

Summarising, the point is that the basis with respect to which decoherence takes place - i.e. superpositions of eigenstates of this basis decohere into a improper mixture of these eigenstates - is determined by the form of the system/apparatus-environment interaction Hamiltonian. Therefore the 'classical' observables, the ones that we perceive as classical, are exactly those determined by this basis. One of the consequences of this is that any interaction described by a potential $V(r)$, is diagonal in position, and therefore position is always the pointer observable measured by the interaction. many interactions in nature are described by such a potential $V(R)$.

A simple model for decoherence

Let us work out the simplest of models for decoherence, which Zurek used in his 1982 paper [7]. The model is analogous to the example we set up for implementing a von Neumann pre-measurement in section 1.2.4, and the system is again represented by a simple two level spin system whose state lives in a Hilbert space $\mathcal{H}_{\mathcal{S}}$ with basis states $\{|\uparrow_z\rangle, |\downarrow_z\rangle\}$. The environment is defined as N two level spin systems with respective Hilbert spaces $\mathcal{H}_{\mathcal{E}_i}$ with bases $\{|\Uparrow_z\rangle_i, |\Downarrow_z\rangle_i\}$, $i = 1...N$. Our system-environment state $\left|\Psi^{\mathcal{SE}}\right\rangle$ now lives in a Hilbert space

$$\mathcal{H}_{\mathcal{SE}} = \mathcal{H}_{\mathcal{S}} \otimes \mathcal{H}_{\mathcal{E}_1} \otimes \mathcal{H}_{\mathcal{E}_2} \otimes ... \otimes \mathcal{H}_{\mathcal{E}_N}. \tag{1.44}$$

The interaction Hamiltonian is taken to be

$$\begin{aligned}\hat{H}^{int} &= \frac{1}{2}\hat{\sigma_z}^{\mathcal{S}} \otimes \sum_{k=1}^{N}\left[g_i\hat{\sigma_z}^{\mathcal{E}_i}\bigotimes_{i'\neq i}\hat{\mathbb{I}}^{\mathcal{E}_{i'}}\right] \\ &= \frac{1}{2}(|\uparrow_z\rangle\langle\uparrow_z| - |\downarrow_z\rangle\langle\downarrow_z|) \otimes \sum_{k=1}^{N}\left[g_i(|\Uparrow_z\rangle_i\langle\Uparrow_z|_i - |\Downarrow_z\rangle_i\langle\Downarrow_z|_i)\bigotimes_{i'\neq i}\hat{\mathbb{I}}_{i'}\right]\end{aligned} \tag{1.45}$$

This Hamiltonian is very much like the one in equation (1.29), and basically the spin states of the system are linearly coupled to each degree of freedom of the environment with coupling strengths g_i. Despite its simplicity the model illustrates the mechanisms at work and also seems to be quite realistic in certain cases (like NMR). We immediately see that the pointer observable is going to be $\hat{\sigma}_z$, as it clearly commutes with the interaction Hamiltonian (this is easy to see as the Hamiltonian is already diagonal). Writing the general initial state

$$\left|\Psi^{\mathcal{SE}}\right\rangle(0) = (c_1\,|\uparrow_z\rangle + c_2\,|\downarrow_z\rangle)\bigotimes_{i}^{N}\left(\epsilon_{1(i)}\,|\Uparrow_z\rangle_i + \epsilon_{2(i)}\,|\Downarrow_z\rangle_i\right) \tag{1.46}$$

where $|c_1|^2 + |c_2|^2 = 1$ and $|\epsilon_{1(i)|^2+|\epsilon_{2(i)}|^2=1}$, we get for the time evolution:

$$\left|\Psi^{\mathcal{SE}}\right\rangle(t) = \exp\left(\frac{-i}{\hbar}\hat{H}^{\mathcal{SE}}t\right)|\Psi(0)\rangle = c_1\left|\uparrow_z\right\rangle\left|\mathcal{E}_\uparrow\right\rangle(t) + c_2\left|\downarrow_z\right\rangle\left|\mathcal{E}_\downarrow\right\rangle(t) \tag{1.47}$$

which is exactly of the form we found in (1.35) and where we have defined the different environmental states that are correlated with the measurement outcome as:

$$\left|\mathcal{E}_\uparrow\right\rangle(t) := \bigotimes_i^N \left[e^{\frac{i}{\hbar}g_i t}\epsilon_{1(i)}\left|\Uparrow_z\right\rangle_i + e^{\frac{-i}{\hbar}g_i t}\epsilon_{2(i)}\left|\Downarrow_z\right\rangle_i\right] \tag{1.48}$$

$$\left|\mathcal{E}_\downarrow\right\rangle(t) := \bigotimes_i^N \left[e^{\frac{-i}{\hbar}g_i t}\epsilon_{1(i)}\left|\Uparrow_z\right\rangle_i + e^{\frac{i}{\hbar}g_i t}\epsilon_{2(i)}\left|\Downarrow_z\right\rangle_i\right]. \tag{1.49}$$

The reduced system density operator becomes, in accordance with equation (1.37):

$$\begin{aligned}\rho^{\mathcal{S}}(t) &= Tr_{\mathcal{H}_\mathcal{E}}[\rho^{\mathcal{SE}}] \\ &= |c_1|^2\left|\uparrow_z\right\rangle\left\langle\uparrow_z\right| + |c_2|^2\left|\downarrow_z\right\rangle\left\langle\downarrow_z\right| \\ &\quad + c_1c_2^* r(t)\left|\uparrow_z\right\rangle\left\langle\downarrow_z\right| + c_2c_1^* r(t)\left|\downarrow_z\right\rangle\left\langle\uparrow_z\right|\end{aligned} \tag{1.50}$$

where $\rho^{\mathcal{SE}} = \left|\Psi^{\mathcal{SE}}\right\rangle\left\langle\Psi^{\mathcal{SE}}\right|$, and the time dependent function $r(t)$ now determine the size of the interference (off-diagonal) terms. It is given by

$$r(t) = \left\langle\mathcal{E}_\downarrow\right|\left.\mathcal{E}_\uparrow\right\rangle = \prod_i^N \left[|\epsilon_{1(i)}|^2 e^{\frac{2i}{\hbar}g_i t} + |\epsilon_{2(i)}|^2 e^{\frac{-2i}{\hbar}g_i t}\right]. \tag{1.51}$$

This function is periodic for any finite N, but decays exponentially very quickly. With the help of some probability theory it can be shown that after dropping to zero, fluctuations are suppressed and become very rare, with a period comparable to the age of the universe, even for relatively small N. See [7] for numerical examples and a random walk-based analysis. Note also that for an environment Hamiltonian with a continuous spectrum, the recurrence time would be truly infinite.

1.3.4 The quantum limit of decoherence and intermediate regime

The second regime, the quantum limit of decoherence, will cause a correlation between the environment and the energy of the system, as this is the only non-oscillating quantity that can be monitored by the environment. Analysed by Paz and Zurek in 1999 [9], this causes the pointer observable to be exactly the self-Hamiltonian of the system. Energy eigenstates of the system will be robust against decoherence, but superpositions thereof will still decohere.

The third intermediate case, will represent a compromise between the first two cases. This more complex case has an application in the model of quantum Brownian motion, which we shall not discuss here[8]. However it does illustrate why we should consider a more general way of determining the pointer observables.

[8] But see Schlosshauer [1] section 5.2 for an introductionary discussion on this. Interestingly enough the pointer states, i.e. those most resistant to decoherence, are localised in both position and momentum.

1.3.5 Predictability sieve

In the case of the quantum measurement limit we found above that the pointer observables were those that commuted with the interaction Hamiltonian between the system and environment. In the simple example shown, we found we could exactly determine what those observables were. However for more realistic interactions, and in the other two regimes we considered we cannot expect to find observables that satisfy this criterion exactly. To this end a more general method is introduced by Zurek, Habib and Paz (1993)[8], called "the predictability sieve". The idea is to just test all states in the Hilbert space of the system, evolving them with the interaction Hamiltonian for a fixed time t_0, measuring how much they have decohered, and sorting them accordingly. A convenient measure is to minimize the von Neumann entropy (remember the von Neumann entropy is 0 for a pure state, and becomes maximal for a maximally mixed (least predictable) state - it is a good measure for loss of predictability)

$$S(\rho_{t_0}) = Tr[\rho_{t_0} \ln \rho_{t_0}], \tag{1.52}$$

where ρ_{t_0} is the reduced density operator of the system after evolving with the environment for a time t_0. The states on top of the list, that have least decohered are then the prime candidate for the pointer states. These states do not necessarily form a complete orthogonal basis however, in fact it appears that they usually define an overcomplete basis, and therefore do not necessarily define a Hermitian observable.

1.3.6 Decoherence free subspaces

In relation to the field of quantum computing, some recent work has been done under the heading of *decoherence free subspaces* (DFS). In equation (1.42) on page 15 we decomposed the interaction Hamiltonian H^{int} in two parts, but a more general decomposition that is always possible is given by

$$\hat{H}^{int} = \sum_{\alpha} \hat{S}_\alpha \otimes \hat{E}_\alpha \tag{1.53}$$

where $\hat{S}_\alpha$ and $\hat{E}_\alpha$ are (not necessarily Hermitian) operators acting on the system and environment Hilbert spaces respectively. In this case a sufficient condition for a set of states $\{|s_i\rangle\}$ to be pointer states is requirement that $|s_i\rangle$ are simultaneous eigenstates of the operators $\hat{S}_\alpha$:

$$\hat{S}_\alpha |s_i\rangle = \lambda_i^{(\alpha)} |s_i\rangle \qquad \forall \alpha, i, \tag{1.54}$$

in which case:

$$\begin{aligned} e^{-i\hat{H}^{int}t} |s_i\rangle |e_0\rangle &= e^{-i(\sum_\alpha \hat{S}_\alpha \otimes \hat{E}_\alpha)t} |s_i\rangle |e_0\rangle \\ &= |s_i\rangle e^{-i(\sum_\alpha \lambda_i^{(\alpha)} \hat{E}_\alpha)t} |e_0\rangle \\ &= |s_i\rangle |e_i(t)\rangle , \end{aligned} \tag{1.55}$$

so that states belonging to this set do not become entangled with the environment. Now for the set of states $\{|s_i\rangle\}$ (or a subset of these) to form a DFS, they must form an orthonormal basis for a subspace of $\mathcal{H}_\mathcal{S}$, and satisfy an even

stronger condition than equation (1.54), namely that there are simultaneous *degenerate* eigenstates of each $\hat{S}_\alpha$:

$$\hat{S}_\alpha |s_i\rangle = \lambda^{(\alpha)} |s_i\rangle \qquad \forall \alpha, i, \tag{1.56}$$

so that any state $|\phi\rangle$ belonging to the subspace can be written $|\phi\rangle = \sum_i c_i |s_i\rangle$, and evolves as

$$\begin{aligned} e^{-i\hat{H}^{int}t} |\phi\rangle |e_0\rangle &= e^{-i(\sum_\alpha \hat{S}_\alpha \otimes \hat{E}_\alpha)t} \left(\sum_i c_i |s_i\rangle \right) |e_0\rangle \\ &= \left(\sum_i c_i |s_i\rangle \right) e^{-i(\sum_\alpha \lambda^{(\alpha)} \hat{E}_\alpha)t} |e_0\rangle \\ &= |\phi\rangle |e_\phi(t)\rangle , \end{aligned} \tag{1.57}$$

In a more general case, not in the quantum measurement limit, an additional condition would be that the states $|s_i\rangle$ remain in the subspace they span under evolution by the system self-Hamiltonian $\mathcal{H}_\mathcal{S}$.

One might ask if all these conditions can ever be satisfied in realistic models; however in many cases the decomposition of the interaction Hamiltonian in equation (1.53) contains only a few terms, and in fact experimental proof of principle was reported in several studies in 2000. The first qubit encoded in a DFS was reported by Kielpinski at al. in 2001, and later Viola at al. succeeded in creating a three-qubit DFS in NMR qubits.

1.4 Physical example, realistic models, experimental tests/results

So far our discussion of decoherence has been very abstract, studying the effect of decoherence in terms of mathematical objects such as the reduced density matrix. If we are to discuss the contribution of the decoherence program to a solution of the measurement problem, and its conceptual relevance to the quantum-to-classical transition, it is a good idea to get a feel for what environment-induced decoherence is supposed to predict in some more realistic physical situations.

In this section I shall therefore first (in section 1.4.1) present the model for particle localisation due to environmental scattering (the example we used in the introduction), as discussed in Schlosshauer's 2008 book [1] and originally worked out by Joos and Zeh in 1985. I shall not go through the entire derivation in detail, but merely state the main ideas, assumptions and results (for detail see [1] pp. 115-151). This example is particularly important, since together with emission of thermal radiation, environmental scattering (air molecules, light, background radioactivity, cosmic muons, 3K background radiation, etc.) is the main process for decoherence in the macroscopic domain. Also in the end, what we actually measure in a quantum measurement in the laboratory is usually (if not always) position of some kind (see the discussion in chapter 3).

Then (in section 1.4.2) I will give an account of the intuitive example of the double-slit experiment, and I will conclude (in section 1.4.3 with some recent actual results on the double-slit experiment, where predictions due to the environmental-induced decoherence are put to the test.

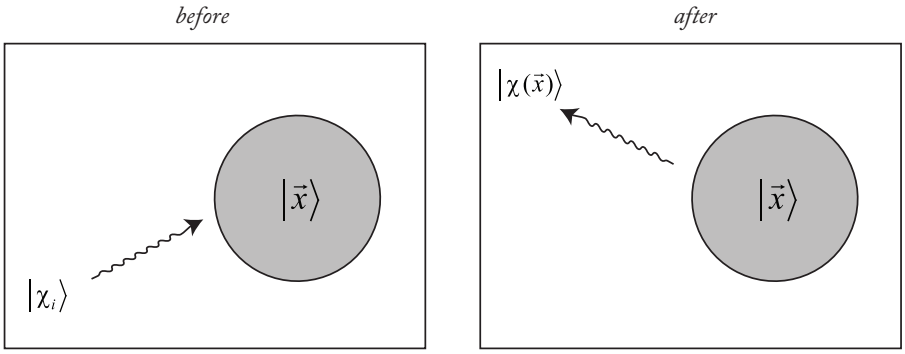

Figure 1.1: A single scattering event, assuming that the system particle is much more massive than the environmental particle. The interaction is then of the kind $|\vec{x}\rangle\,|\chi_i\rangle \longrightarrow |\vec{x}\rangle\,|\chi(\vec{x})\rangle$. After Schlosshauer[1]

1.4.1 Free particle localisation due to environmental scattering

So far we have been describing decoherence in finite dimensional Hilbert spaces. When considering a free particle, our state becomes a continuously indexed vector (function) using a continuously infinite basis. For instance in position basis:

$$|\psi(t)\rangle = \int_{-\infty}^{\infty} \mathrm{d}^3\vec{x}\,[\psi(\vec{x},t)\,|\vec{x}\rangle] \tag{1.58}$$

where $\psi(\vec{x},t)$ is the continuous index function in position space. Our density operator becomes

$$\hat{\rho}_\psi(t) = \int_{-\infty}^{\infty} \mathrm{d}^3\vec{x} \int_{-\infty}^{\infty} \mathrm{d}^3\vec{y}\,[\rho(\vec{x},\vec{y},t)\,|\vec{x}\rangle\,\langle\vec{y}|] \tag{1.59}$$

where $\rho(\vec{x},\vec{y},t)$ is now the continuous index in position space.

We now want to investigate the effect of a collection of environmental particles $\mathcal{E}$ scattering off our system $\mathcal{S}$, as in the right hand side of figure 1. The outline of the derivation with accompanying assumptions is as follows:

- We consider the effect of a single scattering event on a position eigenstate of our system $|\vec{x}\rangle$, as illustrated in figure 1.1. Denoting the initial state of the environmental particle $|\chi_i\rangle$ and making the following assumptions;

  (i) $\mathcal{S}$ and $\mathcal{E}$ are initially uncorrelated $\hat{\rho}(0) = \hat{\rho}^{\mathcal{S}}(0) \otimes \hat{\rho}^{\mathcal{E}}(0)$;

  (ii) the scattering interaction is invariant under translations of the joint $\mathcal{SE}$ system;

  (iii) the scattering system is much more massive than the evironmental scatterd particle (no recoil);

  the interaction can be written

$$|\vec{x}\rangle\,|\chi_i\rangle \longrightarrow \hat{S}\,|\vec{x}\rangle\,|\chi_i\rangle = |\vec{x}\rangle\,e^{-i\hat{\vec{q}}\cdot\vec{x}/\hbar}\hat{S}_0 e^{i\hat{\vec{q}}\cdot\vec{x}/\hbar}\,|\chi_i\rangle =: |\vec{x}\rangle\,|\chi(\vec{x})\rangle \tag{1.60}$$

where $\hat{\vec{q}}$ is the momentum operator for the scattered environmental particle and $\hat{S}_0$ the quantum mechanical scattering matrix for a scattering centre located at the origin. Thus the density operator of our system becomes after one scattering event

$$\hat{\rho}_1^{\mathcal{S}} = Tr_{\mathcal{H}_\mathcal{E}}[\hat{\rho}_1] = \int_{-\infty}^{\infty} \mathrm{d}^3\vec{x} \int_{-\infty}^{\infty} \mathrm{d}^3\vec{y} [\rho(\vec{x},\vec{y},t) \left|\vec{x}\right\rangle \left\langle\vec{y}\right| \left\langle\chi(\vec{y})\right| \chi(\vec{x})\rangle] \quad (1.61)$$

which means that the scattering-induced evolution of the reduced density operator in position basis can be summarised as

$$\rho^{\mathcal{S}}(\vec{x},\vec{y},0) \quad \xrightarrow{\text{1 event}} \quad \rho^{\mathcal{S}}(\vec{x},\vec{y},0) \left\langle\chi(\vec{y})\right| \chi(\vec{x})\rangle, \quad (1.62)$$

which is very similar to the result we found in equation (1.37) on page 14, for the finite dimensional model studied there.

- To proceed, the overlap $\left\langle\chi(\vec{y})\right| \chi(\vec{x})\rangle$ must be calculated using quantum mechanical scattering theory, and a time scale must be introduced, analogous to the example discussed on page 17.

  (iv) Assuming that the typical wavelength λ_0 of the incoming particles is much longer than the maximum extent of our system wavefunction $\lambda_0 >> \Delta x := |\vec{x} - \vec{y}|$

one finds an evolution of the reduced density operator in position basis described by the differential equation

$$\frac{\partial \rho^{\mathcal{S}}(\vec{x},\vec{y},t)}{\partial t} = -\Lambda \times (\vec{x} - \vec{y})^2 \times \rho^{\mathcal{S}}(\vec{x},\vec{y},t) \quad (1.63)$$

where the scattering constant Λ is given by, assuming spatially isotropically distributed incoming particles (particles coming from all directions equally likely);

$$\Lambda := \int_0^{\infty} \mathrm{d}q \varrho(q) v(q) \frac{q^2}{\hbar^2} \sigma_{\text{eff}}(q) \quad (1.64)$$

where $\varrho(q)$ is the number density of incoming particles with momentum $q = |\vec{q}|$; and $v(q)$ is the speed of particles with momentum q; ($v(q) = q/m$, $v(q) = c$ for massive, massless particles respectively, with c the speed of light); and $\sigma_{\text{eff}}(q)$ is the effective total cross section for our system, which is of the same order as the total cross section.[9] The differential equation (1.63) has a simple solution

$$\rho^{\mathcal{S}}(\vec{x},\vec{y},t) = \rho^{\mathcal{S}}(\vec{x},\vec{y},0) e^{-\Lambda(\vec{x}-\vec{y})^2 t}. \quad (1.65)$$

which suggest introducing a decoherence timescale

$$\tau(\Delta x) = \frac{1}{\Lambda(\Delta x)^2}, \quad (1.66)$$

which is similar to the half-life time scale in nuclear physics.

[9] Specifically, $\sigma_{\text{eff}}(q) = \frac{2\pi}{3} \int \mathrm{d}\cos\Theta \ (1 - \cos\Theta) |f(q, \cos\Theta)|^2$, where $|f(q,q')|^2$ is the scattering form factor, determined from the differential cross section $f \sim \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$.

Table 1.1: Estimates of decoherence timescales $\tau(\Delta x)$ (in seconds) for the suppression of spatial interference over a distance Δx for a particle of size a. $\Delta x = a = 10^-3$ cm for the dust grain, and $\Delta x = a = 10^-6$ cm for the large molecule. After Schlosshauer (2008) [1] and Joos, Zeh (1985) [14]

Environment	Dust grain	Large molecule
Cosmic background radiation	1	10^{24}
Photons at room temperature	10^{-18}	10^{6}
Best laboratory vacuum	10^{-14}	10^{-2}
Air at atmospheric pressure	10^{-31}	10^{-19}

- The final step is to assign realistic values to the variables $\varrho(q)$, $v(q)$, $\sigma_{\text{eff}}(q)$, that define the kind of environment our system is submerged in, like thermal photons or air molecules. Table 1.1 contains estimates gained from the model for certain environments and systems.

Illustrations of localisation

To illustrate the explicit dynamics of the decoherence process, let us consider a simple one-dimensional example, a free particle (wave-packet), that starts out in the state

$$\Psi(x, t = 0) = \left(\frac{1}{\sqrt{\pi}\sigma_0}\right)^{1/2} \exp\left[-\frac{x^2}{2\sigma_0^2}\right], \tag{1.67}$$

illustrated in figure 1.2 (left). The time evolution of this state under the free-particle Hamiltonian $\hat{H}(x, p) = \frac{\hbar^2\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$ becomes

$$\Psi(x, t) = \left(\frac{1}{\sqrt{\pi}\sigma(t)}\right)^{1/2} \exp\left[-\frac{x^2}{2\sigma(t)^2}\right], \tag{1.68}$$

with

$$\sigma(t) = \sigma_0\sqrt{1 + \hbar^2 t^2/(m^2\sigma_0^4)}. \tag{1.69}$$

The effect of this evolution is illustrated in figure 1.2 (right). Schrödinger already realised that this spreading behaviour was a bit of a problem (note that if the wave packet describes a particle with the mass of an electron, and has initial width $\sigma_0 = 1$ Å, the evolution spreads to a width σ of order 1000 km in a second). To illustrate the effect of decoherence, we must move to the density operator picture, we have

$$\rho(x, y, t) = \Psi(y, t)\Psi(x, t)^*, \tag{1.70}$$

which is illustrated in figure 1.3. We now introduce some terminology in figure 1.4. A big *coherence length* ℓ means a large probability of finding a coherent superposition of separated positions (e.g. by performing an interference experiment), and a large *ensemble width* ΔX corresponds to a large range of possible positions in which the system can be found upon measurement of its position.

To include the effect of environmental scattering, we combine the free-particle evolution (using the Liouville-von Neumann equation) and the evolution of the density operator we found in equation (1.63):

$$\begin{aligned}\frac{\partial \rho^{\mathcal{S}}(x,y,t)}{\partial t} &= -\frac{i}{\hbar}\left[\hat{H},\rho^{\mathcal{S}}\right] - \Lambda(x-y)^2\rho^{\mathcal{S}}(x,y,t) \\ &= -\frac{i}{2\hbar m}\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2}\right)\rho^{\mathcal{S}}(x,y,t) - \Lambda(x-y)^2\rho^{\mathcal{S}}(x,y,t). \quad (1.71)\end{aligned}$$

I will not derive an explicit solution to this equation (for a derivation see Schlosshauer (2008)[1] pp. 140-146 and Joos, Zeh (1985)[14]), but the solution is of a Gaussian form

$$\rho^{\mathcal{S}}(x,y,t) = \exp\left[-A(t)(x-y)^2 - iB(t)(x-y)(x+y) - C(t)(x+y)^2 - D(t)\right], \quad (1.72)$$

where we identify $\ell(t) := \frac{1}{\sqrt{8A(t)}}$ as the decoherence length, and $\Delta X(t) := \frac{1}{\sqrt{8C(t)}}$ as the ensemble with. The results for the same initial condition (equation (1.67)) is shown in figure 1.5. Note that the coherence length reduces quickly compared to the free case, but that the ensemble width actually increases faster.

Finally, we can do the same derivation with as initial state a superposition of two equal weight Gaussian wave packets of the same form (1.67), centred around $x = \pm x_0$,

$$\Phi(x, t=0) = \left(\frac{1}{\sqrt{\pi}\sigma_0}\right)^{1/4}\left(\exp\left[-\frac{(x-x_0)^2}{2\sigma_0^2}\right] + \exp\left[-\frac{(x+x_0)^2}{2\sigma_0^2}\right]\right). \quad (1.73)$$

The results are depicted in figure 1.6. Note that the interference terms on the off-diagonal become quickly damped.

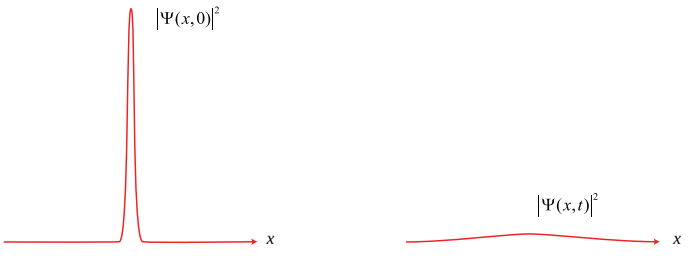

Figure 1.2: Unitary time evolution of the probability density function $|\Psi|^2$ for a free-particle Gaussian wave packet

Figure 1.3: Unitary time evolution of density operator index function $\rho(x, y, t)$ for a free-particle Gaussian wave packet

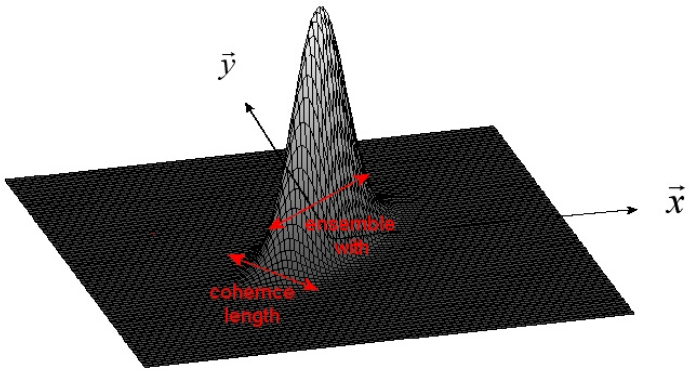

Figure 1.4: Definitions of *coherence length* and *ensemble width*. Note that the coherence length lies along the axis $y = -x$ (off-diagonal), and the ensemble with along $y = x$ (diagonal). A big coherence length means a large probability of finding a coherent superposition of separated positions (e.g. by performing an interference experiment), and a large ensemble width corresponds to a large range of possible positions in which the system can be found upon measurement of its position.

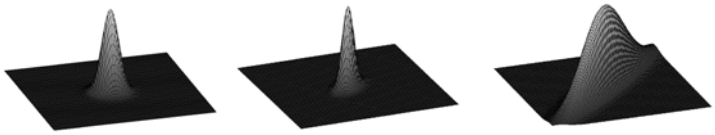

Figure 1.5: Time evolution of density operator index function $\rho(x, y, t)$ for a free-particle Gaussian wave packet under influence of environmental scattering

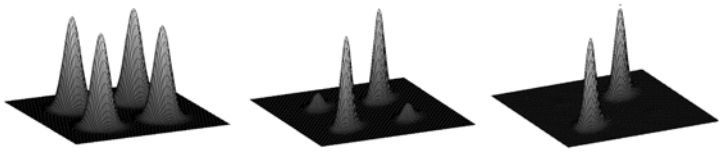

Figure 1.6: Time evolution of density operator index function $\rho(x, y, t)$ for a superposition of two free-particle Gaussian wave packets, centred around $x = \pm x + 0$, under influence of environmental scattering

1.4.2 Double-slit interference

The double-slit experiment is considered one of the most beautiful experiments in physics [16]. It demonstrates simply and completely the strange features of quantum mechanics. It is also the experiment where the predictions of decoherence are most easily tested. The intuitive setup is very simple (figure 1.7), although the actual setup used in decoherence double slit experiments are much more complicated (next section). We will use this simple picture to

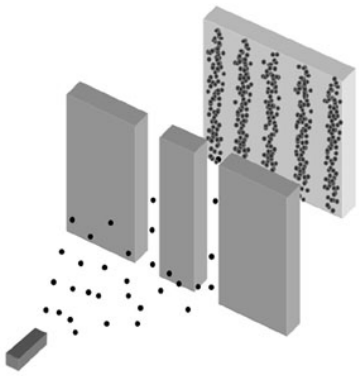

Figure 1.7: Illustration of the double-slit experiment

explain the ideas. We describe the state of the particle just after the two slits:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle), \tag{1.74}$$

where $|\psi_1\rangle$ denotes the state of the particle that has gone through slit 1, and $|\psi_2\rangle$ the particle through slit 2. The particle then travels to the detection screen, but on the way interacts with environmental particles (air molecules, light, background radioactivity, cosmic muons, 3K background radiation, etc.) as described in the previous section. We shall not go into the details, but just assume the environment entangles with the state of the particle, and thus acts as a "which-path monitor":

$$|\Psi\rangle\,|e_0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|\psi_1\rangle\,|e_1\rangle + |\psi_2\rangle\,|e_2\rangle). \tag{1.75}$$

the reduced density operator of just the particle (trace over the environment) then becomes

$$\hat{\rho}_{\text{particle}} = \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2| + |\psi_1\rangle\langle\psi_2|\langle e_2|\,e_1\rangle + |\psi_2\rangle\langle\psi_1|\langle e_2|\,e_1\rangle), \tag{1.76}$$

which gives a particle density ϱ at a position z along the screen

$$\varrho(z) = \langle z|\,\hat{\rho}_{\text{particle}}\,|z\rangle = \frac{1}{2}|\psi_1(z)|^2 + \frac{1}{2}|\psi_2(z)|^2 + \text{Re}\{\psi_1(z)\psi_2^*(z)\,\langle e_2|\,e_1\rangle\}. \tag{1.77}$$

The last term causes the appearance of an interference pattern on the screen, and we see that it is weighted by the extend in which the environmental states overlap, $\langle e_2|\,e_1\rangle$. This overlap of course depends on the dynamics of the environmental scattering as described in the previous section, like the pressure of environmental molecules present, or the temperature of the particle.

1.4.3 Experimental tests

The predictions of decoherence theory on the disappearance of interference due to interactions with the environment have been verified for a number of setups. For a extensive review see Chapter 6 of Schlosshauer (2008) [1].

(i) The first experiments date from 1996 and use mesoscopically separated states of a radiation field (10 photons), created by letting an atom in an energy superposition state interact with a photon field in a cavity. Decoherence was then observed for these states, and the time dependence found to (qualitatively) depend on the extent to which the states were distinguishable in the first place, and the number of photons in the superposition.

(ii) The most extensively tested setup is the analog of the double-slit experiment described in the previous section. This interferometry experiment uses C70-molecules (figure 1.8) and three diffraction gratings that together form a setup that can measure an interference pattern of the C70-molecules. Quantitative measurements were done on the effects of pressure (decoherence due to environmental scattering with air molecules) and molecule temperature (decoherence due to thermal radiation). The results were found to agree well with the theoretical predictions.

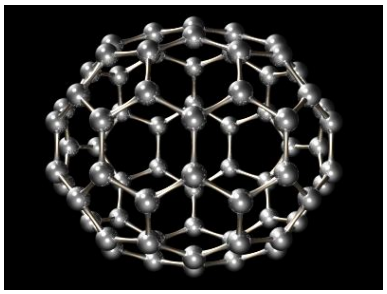

Figure 1.8: Schematic illustration of a C_{70} molecule used in the matter-wave interference experiments. Seventy carbon atoms are arranged in a the shape of a stretched bucky ball with a diameter of about 1 nm.

(iii) Decoherence has also been studied in superconducting quantum interference devices, or SQUIDs and other super conducting qubit systems. Decoherence timescale were measured using Ramsey interferometry. Current experimental evidence shows that the main source of decoherence is the presence of intrinsic defects in the Josephson junctions and the superconductor itself, rather than the interaction with external environment such as the external circuit used to control the loop junction setup.

(iv) Other experiments include decoherence measured in overlapping Bose-Einstein condensates, and quantum-electromechanical systems (QEMS). Decoherence in QEMS has not been measured, but provides a potential to measure decoherence in truly mechanical "Schrödinger kittens" involving billions of atoms in a superposition of two well-distinguishable positions in space.

Summarising, on the scales investigated thus far, there is no evidence pointing to a funcdamental limitation of the superposition principle, and there is a good empirical basis for the models devised within decoherence theory to describe disappearance of interference on the level of the system.

1.5 Quantum Darwinism

The quantum Darwinism program is based on the idea that in everyday life we infer properties from objects by observing parts of the environment that have interacted with it. A common example is of course the visual registration of photons scattered off an object of interest. In this case the environment we described in our different models of decoherence no longer acts merely as a sink that carries away information and coherence from the system, but actually becomes the source of the information. The quantum Darwinism program then proceeds to quantify the amount of information an observer can infer from observing part of the environment using tools from information theory, such as the mutual information $I(X:Y)$,

$$I(X:Y) = H(X) + H(Y) - H(X,Y). \tag{1.78}$$

Here $H(X)$ is the Shannon entropy of the stochastic variable X that takes values x in an alphabet J_X with probability $p_X(x)$;

$$H(X) = \sum_{x \in J_X} p_X(x) \log p_X(x). \tag{1.79}$$

$H(X,Y)$ is then the joint entropy,

$$H(X,Y) = \sum_{x \in J_X} \sum_{y \in J_Y} p_{X,Y}(x,y) \log p_{X,Y}(x,y), \tag{1.80}$$

with $p_{X,Y}(x,y)$ the joint probability function.

Some other measures of information can be used such as the quantum mutual information that is based on Von Neumann entropies, and the *redundancy* of information recorded in the environment. See for instance [11] and [10].

As an example and illustration, let us look at the simple model we worked out in section 1.3.2. We found a time evolution for the total composite system state $\left|\Psi^{\mathcal{SE}}\right\rangle(t)$ given in equation (1.47) on page 17. Suppose now, instead of taking the trace over the environment, we actually measure *part of the environment*, and we want to know how much information we can infer about an observable of the system $\hat{O}^{\mathcal{S}}$. Specifically, suppose we measure $1 \leq m \leq N$ spins of the environment, where we measure each spin in a random direction, i.e. we measure them with an observable

$$\hat{\sigma}_{\vec{n}_i} = \begin{pmatrix} \hat{\sigma}_x \\ \hat{\sigma}_y \\ \hat{\sigma}_z \end{pmatrix}^T \vec{n}_i, \tag{1.81}$$

where $\vec{n}_i$ is a random unit vector in three dimensions. So summarising, we measure the composite state living in the Hilbert space in equation (1.44), using the observable[10]

$$\mathbb{I}^{\mathcal{S}} \otimes \hat{O}^{\mathcal{E}} = \mathbb{I}^{\mathcal{S}} \otimes \hat{\sigma}_{\vec{n}_1} \otimes \hat{\sigma}_{\vec{n}_2} \otimes ... \otimes \hat{\sigma}_{\vec{n}_m} \otimes \hat{\mathbb{I}}_{m+1} \otimes ... \otimes \hat{\mathbb{I}}_N \tag{1.82}$$

[10]Here we measure the *first* m spins of the environment, but as we will choose the coupling strengths g_i randomly as well, it makes no difference which spins we measure.

Now to calculate the mutual information between the (stochastic) outcomes from measuring part of the environment and those from an observable $\hat{O}^{\mathcal{S}}$ on the system, note that we can write the probabilities of the different outcomes $o_j^{\mathcal{S}}$ for the system as

$$p(o_j^{\mathcal{S}}) = Tr\left[(\hat{P}_j^{\mathcal{S}} \otimes \hat{\mathbb{I}}^{\otimes N})\rho^{\mathcal{SE}}(t)\right], \tag{1.83}$$

where $\rho^{\mathcal{SE}}(t) = \left|\Psi^{\mathcal{SE}}\right\rangle\left\langle\Psi^{\mathcal{SE}}\right|$, and $\hat{P}_j^{\mathcal{S}}$ are the eigenprojectors (equation (1.3) on page 6) of $\hat{O}^{\mathcal{S}}$. Similarly for the probabilities of outcomes $o_k^{\mathcal{E}}$ of observable $\hat{O}^{\mathcal{E}}$ defined above,

$$p(o_k^{\mathcal{E}}) = Tr\left[(\hat{\mathbb{I}} \otimes \hat{P}_k^{\mathcal{E}})\rho^{\mathcal{SE}}(t)\right] \tag{1.84}$$

and the joint probabilities

$$p(o_j^{\mathcal{S}}, o_k^{\mathcal{E}}) = Tr\left[(\hat{P}_j^{\mathcal{S}} \otimes \hat{P}_k^{\mathcal{E}})\rho^{\mathcal{SE}}(t)\right]. \tag{1.85}$$

These are all we need to calculate the mutual information I in equation (1.78). In figure 1.9 you can see a plot of this quantity versus the number of environmental spins observed, m and the angle μ between the system observable $\hat{O}^{\mathcal{S}}$ and the pointer observable for the system $\hat{\sigma}_z$ (for instance $\hat{O}^{\mathcal{S}} = \cos(\mu)\hat{\sigma}_z + \sin(\mu)\hat{\sigma}_x$). In this plot the interaction strengths g_i are chosen randomly, for a fixed time, or equivalently: $g_i t$ randomly chosen from $[0, 4\pi]$, and $N = 50$. It can be seen that only information about the pointer states is recorded redundantly in the environment that has interacted with the system. Observing a part of this environment allows one to infer information about the state of the system without perturbing it (further): but only information about pointer observables.

Of course this is a rather simple model, and one should work this out for a more realistic case, such as the environmental scattering of an object like in section 1.4.1, but this is currently ongoing research.

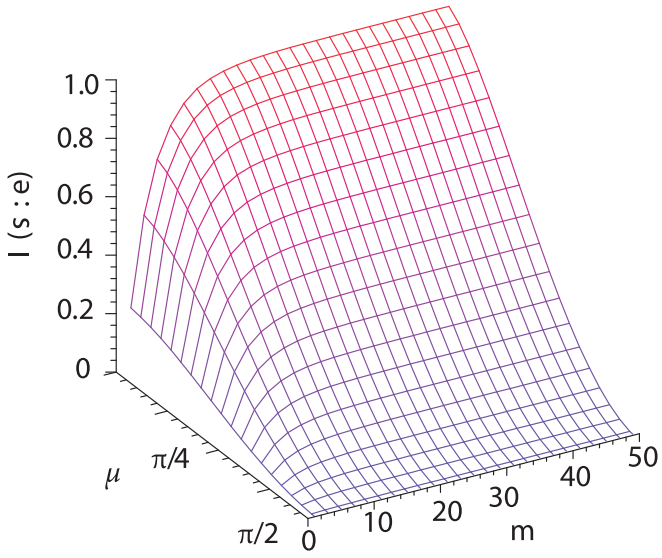

Figure 1.9: Information about an observable $\hat{O}^{\mathcal{S}} = \cos(\mu)\hat{\sigma}_z + \sin(\mu)\hat{\sigma}_x$ of the system extracted by an observer restricted to local random measurements on m environmental subsystems, for the model defined on page 17. The interaction action $g_i t$ is randomly chosen from $[0, 4\pi]$. It can be seen that only information about observables close to the pointer observable can be inferred via the environment. From [10]

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Chapter 2

The measurement problem

Since the birth of quantum theory (QT, henceforth), its conceptual interpretations have been subject to endless debate. Already early on it was clear that reconciling the indeterministic nature and, the particle wave duality, of QT with our everyday experiences and classical Newtonian physics was a daunting task.

QT's main interpretative problem is generally agreed to be the measurement problem. This focus on measurement is of course quite natural since early on the interaction between our classical world of outcomes and the quantum world of superpositions was only through actual experimental measurements. But the conceptual measurement problem naturally extends far beyond the experiments we do in laboratories, and forms part of this more general task of defining the relationship between classical and quantum theory, or recovering the successful predictions of the classical in terms of suitable limits of the QT. A successful interpretation of QT - usually considered as interpreting the mathematical formalism described in section 1.2.1 (or parts of it) in terms of physical and conceptual meaning - must then somehow solve the measurement problem.

In this chapter I will describe the measurement problem in detail, and in the next chapter list a few interpretations of QT.

The measurement problem arises quite naturally from QT's success in describing the realm of microscopic particles and allowing them to have indefinite values for quantities like position and momentum. The problem is that there is nothing in QT that forbids the same indefiniteness from occurring for macroscopic objects like books, tables or cats, which does not agree with our perception.

Describing the problem in more (mathematical) precise terms, and dividing it into sub-problems, quickly becomes dependent on which interpretation one adheres to. Different interpretations might not recognise all the postulates mentioned in 1.2.1. We will discuss further interpretations in section 3.2, but for now we take the formalism we derived in the last chapter to be correct (although we shall question part of the third postulate on page 6 concerning measurements)[1]. The effect is that some of the (sub-)problems I describe might not applicable to

[1]This is similar to the standard, or orthodox interpretation, which is not really an interpretation at all, but just a pragmatic implementation of the formalism as it is used by the practising physicist.

all interpretations (or in other words they would solve some problems by taking away the postulates that cause them). Whenever this effect occurs I will try to make a remark about it.

Specifically, looking at von Neumann's scheme[2] of measurement we introduced in 1.2.4, we found in equation (1.26) that an initial superposition of the system states lead to a superposition of the apparatus pointer states:

$$\left(\sum_i c_i \left|s_i\right\rangle\right) \left|a_r\right\rangle \longrightarrow \sum_i c_i \left|s_i\right\rangle \left|a_i\right\rangle . \tag{2.1}$$

But if the apparatus pointer is in fact an actual macroscopic pointer on a display of our apparatus, this means that the pointer is in multiple positions at the same time! This is of course nothing other than a formal description of the infamous Schrödinger Cat Paradox, where the system takes the form of a superposition of an either decayed or undecayed unstable atom, and the pointer takes the form of an either alive or dead cat of which Schrödinger (1935) remarked:

> The psi-function of the total system would yield an expression for all this in which the living and the dead cat are (pardon the expression) blended or smeared out in equal measure. (Schrödinger 1935)

So summarising, the measurement problem is then how to make sense of equation (2.1).

2.1 Dividing the problem

We now wish to decompose the measurement problem into a number of parts, so that we can address them separately. Such a decomposition is not straightforward, and one might ask for instance if a specific decomposition covers all the problems contained in equation (2.1). The decomposition I shall describe (roughly) follows Janssen[3] and Schlosshauer [1]. The measurement problem is proposed to be composed of five parts, which we shall describe in more detail in the following sections:

(i) The problem of outcomes: Why does one perceive a single outcome among the many possible ones in equation (2.1)?

(ii) The problem of the collapse: What kind of process causes the state of the system to 'collapse' to the outcome one perceived (in the sense that a repeated measurement yields the same answer)?

(iii) The problem of interference: Why do we not observe quantum interference effects on macroscopic scales?

(iv) The problem of the preferred basis (general): What determines the limited set of quantities that appear to be definite for macroscopic objects?

[2]This confines us to interpretations that recognise the eigenstate-eigenvalue-link (part of the third postulate in section 1.2.1) for projections, i.e. the idea that the system has a value for a given quantity if and only if it is an eigenstate of an observable. Interpretations that do not recognise this link are for instance hidden variable theories or modal interpretations.

(v) The problem of the preferred basis (decomposition): The final state after pre-measurement in equation (2.1) can equally well be described in a different basis:[3]

$$\sum_i c_i \left|s_i\right\rangle \left|a_i\right\rangle = \sum_i c'_i \left|s'_i\right\rangle \left|a'_i\right\rangle . \tag{2.2}$$

However that would imply that we are measuring two different quantities of the system (possibly non-commuting quantities, which is not allowed by QT), and we are doing so with the same measurement apparatus (which is contrary to our experience that we build a specific apparatus to measure a certain quantity). What determines the 'right' basis?

The problems might be overlapping, a solution to the one might even imply a solution to the other. Before going further into the separate parts in more detail, I would like to the following cautionary remark.

The above decomposition is a specifically convenient one in relation to environment-induced decoherence: as we shall see in chapter 3, the decoherence program seems to have some chances at solving precisely the last three parts of the decomposition above. Specifically, parts (iv) and (v), the preferred basis problem, have been separated out of the general measurement problem by none other than one of our main decoherence program theorists, viz. Zurek in his 1981 paper[6].

2.2 The problems of outcomes and of collapse: (i) & (ii)

The final state in equation (2.1) is the state after the von Neumann pre-measurement. Yet we must somehow have[4]:

$$\sum_i c_i \left|s_i\right\rangle \left|a_i\right\rangle \quad \xrightarrow{\text{'collapse'}} \quad \left|s_1\right\rangle \left|a_1\right\rangle \text{ or } \left|s_2\right\rangle \left|a_2\right\rangle \text{ or ... or } \left|s_n\right\rangle \left|a_n\right\rangle \tag{2.3}$$

in which we perceive[5] the 'or' to be mutually exclusive. To illustrate the problem further, we take a quick preview of the next section, and take two straightforward solutions one might pose to the problem - corresponding to the standard and Copenhagen interpretation of QT.

If we just take postulate 3 on page 6 to be correct, and apply its formalism (orthodoxy), we indeed find after measurement one of the outcomes i with corresponding probability $|c_i|^2$, and the state of the combined system-apparatus becomes $\left|s_i\right\rangle \left|a_i\right\rangle$. However this shifts the problem to the new problem of what constitutes a measurement. The mere fact that 'measurement' is mentioned in one of the postulates of QT, must mean that a measurement must be something outside this theory. The Copenhagen interpretation then argues that the world of measurement outcomes, and what we perceive, must always be described classically. There is therefore a strict dualism between the system under measurement (to be described by QT) and the apparatus/observer (obeying classical

[3] Under certain circumstances; see section 2.4.

[4] This again presupposes the eigenstate-eigenvalue link, see footnote 2 on the page 31.

[5] You could interpret the 'or' in varous ways: in relative-state interpretations, which we shall describe later, in fact *all* outcomes are realised, the 'or' is not mutually exclusive. However one must still explain why we *perceive* there to be only one outcome.

physics), divided by the so called "Heisenberg cut". Stepping over the division constitutes a measurement. However this again shifts the problem, for *where*, on the way from our macroscopic system to an ever larger ensemble of atoms of the apparatus, does one draw the "Heisenberg cut"? J.S. Bell remarks

> Thus in contemporary quantum theory it seems that the world must be divided into a wavy 'quantum system', and a remainder which is in some sense 'classical'. The division is made one way or another, in a particular application, according to the degree of accuracy and completeness aimed at. For me it is the indispensability, and above all the shiftiness, of such a division that is the big surprise of quantum mechanics. (Bell (1987), pp. 188 [12])

In the next section we will see some other strategies to solve the problem of outcomes and collapse; but the above should convince you that it is a fundamental problem that should be explained by any successful interpretation of QT.

2.3 The problem of interference: (iii)

The density operator of the final state in equation (2.1) is given by

$$\rho = \sum_{ij} c_i c_j^* \left|s_i\right\rangle \left\langle s_i\right| \otimes \left|a_i\right\rangle \left\langle a_j\right| . \tag{2.4}$$

Now the probability of the event corresponding to reading off an outcome k on the apparatus pointer display (projector $P_k = \hat{\mathbb{I}} \otimes \left|a_k\right\rangle \left\langle a_k\right|$) is, according to the formalism: Prob(outcome k) $= p(k) = |c_k|^2$. Imagine we have another copy of the system and apparatus, evolved again to the same final state. The probability of reading off another outcome m on the pointer display of this second apparatus is then Prob(outcome m) $= p(m) = |c_m|^2$. Imagine now a third copy of the system/apparatus again in final state as above. We would expect that if we were to calculate the probability for the event that we see the pointer to be in at least one of the two positions corresponding to outcomes k and m (projector $P_{k+m} = \hat{\mathbb{I}} \otimes \left|a_k\right\rangle \left\langle a_k\right| + \left|a_m\right\rangle \left\langle a_m\right|$), this would be the sum of the two probabilities above. But as our density matrix has non-zero off diagonal terms, we get some extra terms Prob(outcome $k \vee$ outcome m) $= p(k) + p(m) + 2Re(c_m c_k^*)$.

The terms of course correspond to the fact that our state is in a quantum superposition state not a classical distribution.

This problem is best illustrated with the example of the double slit experiment. The physical setup suggests that summing the probability distribution obtained with only either one of the slits opened should result in the probability distribution for both slits opened. In the case of electrons as particles the probability distribution of course differs, but for a similar setup using macroscopic particles it does not.

2.4 The problem of the preferred basis: (iv) & (v)

General

In the case where we do not specifically design an apparatus to measure the quantum system: what determines the limited set of quantities that appear to be definite for macroscopic objects? An example of this is our own vision, we perceive objects, chairs etc to be in definite positions - what singles out position as the preferred variable in this case? See also section 1.5 for a more extensive discussion of this picture where one infers information via the environment.

Decomposition

We say that we can write the final state in equation (2.1) as

$$\sum_i c_i \left|s_i\right\rangle \left|a_i\right\rangle = \sum_i c'_i \left|s'_i\right\rangle \left|a'_i\right\rangle . \tag{2.5}$$

If we assume, like we did before, that the states $\{|s_i\rangle\}$ are orthonormal (we are measuring an observable) and that we have designed our apparatus such that the states $\{|a_i\rangle\}$ are orthonormal (we want to be able to distinguish between the possible measurement outcomes), the decomposition of the final state is in fact unique (by the Schmidt decomposition), unless two or more of the coefficients c_i are equal.

To emphasise the problem with this lack of uniqueness, take another look at the example of an implementation of a Von Neumann measurement on page 12. We specially designed the interaction Hamiltonian $\hat{H}^{\mathcal{SA}} = g(\hat{\sigma_z}^{\mathcal{S}} \otimes \hat{\sigma_z}^{\mathcal{A}})$ to measure the observable $\hat{\sigma_z}$ of the system. But note that the final outcome in equation (1.32), in the case of $c_1 = c_2$, can be written in two equivalent bases;

$$|\Psi(t = \pi\hbar/2g)\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_{\mathcal{S}} |\uparrow_x\rangle_{\mathcal{A}} + |\downarrow_z\rangle_{\mathcal{S}} |\downarrow_x\rangle_{\mathcal{A}}) = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle_{\mathcal{S}} |\uparrow_z\rangle_{\mathcal{A}} + |\downarrow_x\rangle_{\mathcal{S}} |\downarrow_z\rangle_{\mathcal{A}}) \tag{2.6}$$

which implies we have measured the observable $\hat{\sigma_x}$ of the system. So which has the apparatus (designed to measure $\hat{\sigma_z}$ of the system) actually measured?

2.5 Quantum to classical transition

A solution to the measurement problem does not immediately give one a description of how a classical macrorealm is deduced from QT. For instance, in the case of particle trajectories, a mere solution to the different parts of the measurement problem defined above would not automatically yield classical particle trajectories: Not only position must then be well-behaved, but also momentum. In particular, particles must follow a path described by the laws of classical mechanics.

However a solution would clear up the relation between classical theories and QT, and would ideally explain how in certain examples, in some limit, calculations from QT agree with those from a classical theory.

Note that a solution to the measurement problem is allowed to be an approximate solution, in the sense that in some limit QT is approximate to the classical theory. This view is nicely illustrated by Wallace (2001)[18], who compares our view of definiteness with the existence of certain patterns. Those patterns exist to the extent that the classical theory they are embedded in has explanatory power and predictive reliability. As an example, any state of a pattern of a cat is actually a member of a Hilbert space containing all possible macroscopic objects made out of the cat's sub-atomic constituents, which includes, according to Wallace, a dead cat, a dead dog, etc. Wallace then argues:

> Patterns can be imprecise[...], a pattern can tolerate a certain amount of noise or imprecision whilst still remaining the same pattern. (A tiger which loses a hair is still the same tiger). Beyond a certain point the noise is such that the pattern can no longer be said to be present, but there is no reason to expect there to be any precise point where this occurs. (It may sometimes be convenient to define such a point by fiat: the biologist sometimes introduces an exact moment when one species becomes another; the astrophysicist defines an exact radius at which the suns atmosphere starts. But neither believes that any deep truth is captured by this exactness.) (Wallace 2001 [18] pp. 9)

It is in this sense that we can argue that for instance a selection of a preferred basis (on macroscopic scale), does not have to be exact (defining each eigenstate exactly).

Chapter 3

Interpreting quantum theory

In this chapter I shall first discuss how the effects of environment-induced decoherence influence different parts of the measurement problem defined in the last chapter. Can we build a satisfiable interpretation of QT just out of results form the decoherence program?

Next I shall list a few mainstream interpretations of QT. The reason for restricting the scope of the discussion to the ones below, and for discussing some more extensively than others, is partially due to the limits in time and space, and partially because some connect particularly well, or are particularly influenced by the decoherence program. To bring some order into the discussion I shall categorise the interpretations first with respect to their view of reality.

Instrumentalist QT is merely supposed to give explanations and predictions of phenomena we find in experiments.

SciReal or scientific realism: There exists a mind independent reality, and QT should describe (part of) that reality.

For the interpretations that adhere a **SciReal** picture, another categorisation can be made due to Butterfield (2001) [13].

- A choice of the kind of definiteness the interpretation wishes to recover:

  Objective definiteness: One secures actual definite values for quantities of objects in the macroscopic world.

  Apparent definiteness: We still allow an indefiniteness for macroscopic objects, and solve the measurement problem by explaining why they *appear* definite. This needs some kind of quantum-theoretic description of the brain or mind.

- A choice of strategy to solve the measurement problem (if the interpretation aims to do so):

  Dynamics We postulate a new dynamical law instead of the Schrödinger equation for a completely isolate quantum system, that ensures the 'collapse'.

ExtraValues We ascribe some extra values to quantities of a quantum mechanical system, that determine definite values. Such a strategy must avoid the various hidden variable proofs.

Many other categories can be distinguished, but I will limit myself to describing other properties such as locality, Lorentz-invariance where necessary.

In this section I shall predominantly focus on the literature mentioned in the introduction.

3.1 Decoherence as an interpretation (?)

In some early, more elementary discussions, environment-induced decoherence is sometimes stated to solve the measurement problem, or to constitute an interpretation of quantum mechanics by itself. As we shall see, this is of course not quite the case, for many reasons, but most importantly because - as is generally agreed now - it does not solve the problem of the outcomes.

We shall now proceed to analyse the decoherence argument more thoroughly. In the description of decoherence in chapter 1, we (sometimes implicitly) made assumptions so as to proceed with the argument. For instance, we naturally assumed that postulates 1,2 and 4 in section 1.2.1 were correct, but further assumptions are necessary if one aims to solve the full measurement problem. What assumptions are needed, and what are the implications if one accepts these assumptions? Listing these assumptions (after [3], pp. 72):

(1) **Interaction**: System-(apparatus-)environment interactions are faithfully represented by the interaction models (Hamiltonians) used.

(2) **Local observer**: We do not observe the environmental degrees of freedom. Stated otherwise, we can observe only locally, in mathematical terms: measurements are of the form $\hat{S} \otimes \hat{A} \otimes \hat{\mathbb{I}}$ on $\mathcal{H}_{\mathcal{S}} \otimes \mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{E}}$, with $\mathcal{S}, \mathcal{A}, \mathcal{E}$ the system, apparatus and environment respectively.

(3) **Infinite time**: "For all practical purposes" - determined by how fast we can do an experiment versus the characteristic decoherence time scale (equation (1.66)) - we can take the limit $t \to \infty$.

(4) **Ignorance interpretation**: The mixed states we find by taking the partial trace over the environment can be interpreted as a *proper* mixture. Note that this is essentially a collapse postulate.

Assuming one or more of the above allows one to use certain parts of decoherence theory, aimed to solve certain parts of the measurement problem. The arguments (or parts of decoherence theory), and which part of the measurement problem they are aimed at are listed in table 3.1.

Table 3.1: Consequences of making the different assumptions listed in section 3.1

Assumptions		Results according to decoherence theory	Part of the measurement problem aimed to solve
(1)	A.	Interactions with the environment lead to a selection of a preferred basis of pointer observables, by the argument that only the states $\|a_i\rangle$ of the apparatus that *do not become entangled* with the environment *remain* correlated with the system states.	The preferred basis problem, composition (iv) (section 2.4)
(1) and (2)	B.	A sufficient condition for a selection of a preferred basis as in A. is the 'purity' of the reduced density operator of system and apparatus $\rho^{\mathcal{SA}}$ (equation (1.37)), for instance measured by the von Neumann entropy (this is the predictability sieve of section 1.3.5).	The generalised preferred basis problem (iv) (section 2.4)
(1), (2) and (3)	C.	Interactions with the environment lead to disappearance of interference terms from the local reduced density operator of the system-apparatus $\rho^{\mathcal{SA}}$, in the basis selected by A. or B. I.e. $\|\psi_{\mathcal{SAE}}\rangle_{final}$ in equation (1.35), may be replaced by $\rho^{\mathcal{SA}}_{decohered}$ in (1.38).	The problem of interference (iii) (section 2.3)
(1), (2), (3) and (4)	D.	Interactions with the environment explain the apparent definiteness of measurement outcomes.	The problem of the outcomes (i) (section 2.2)

We shall now discuss the extent to which the different arguments listed in table 3.1 succeed in their aim of solving certain parts of the measurement problem. Note that the problem of the collapse is not addressed by the Decoherence interpretation.

A. This point generally is not very controversial, but it is not obvious that solving the preferred basis problem for the specific case of a degenerate decomposition is very relevant to the measurement problem in general.

B. This point requires the locality assumption, which presumes that there is a clear definition of a subspace in the total Hilbert space, that defines the system of interest. In many cases this need not be a problem, and we can proceed to select a preferred basis via methods like the predictability sieve.

   The question remains whether the method of the predictability sieve (section 1.3.5) is a stable enough method to yield an exact definition of a preferred basis (remember that the bases retrieved this way are in no way necessarily complete or orthonormal). However in the light of only needing an approximate solution to the measurement problem as described in section 2.5, this

might not be an issue. Basically this argument explains why we perceive position to be definite for macroscopic objects, since most interactions with the environment in nature are diagonal in position ($V(r)$), and the pointer states are therefore usually position eigenstates. These pointer states are the states that are stable against the effect of decoherence, so remain definite under its action. We can explain more thoroughly why we perceive the pointer states selected by the predictability sieve to be definite, in the picture of Quantum Darwinism (see section 1.5).

Of course this does not yet explain the definiteness of macroscopic objects that were initially in a superposition, that requires arguments C. and D. (and more).

Finally, one might ask *why* most interactions in nature *are* diagonal in position (assumption (1)), but decoherence takes this as a given and explains why position is the preferred basis.

C. It certainly seems that the third assumption is a valid one, as the decoherence timescales calculated for everyday settings are extremely small (see table 1.1). It is therefore generally agreed that decoherence solves the problem of interference, and its predictions have some experimental verification as seen in section 1.4.2. However some question the conceptual relevance of this, as Janssen remarks:

> That the empirical predictions of decoherence can be verified empirically simply indicates that it is good physics - that quantum mechanics is an accurate physical theory that, if properly used, yields the right kind of empirical predictions. (H. Janssen 2008 [3] pp. 35)

But argument C. does clarify why one does not observe interference for macroscopic objects for a specific example so at least explains something for the quantum to classical transition.

D. Of the premises listed above the last one is clearly the most controversial. Recalling the discussion about proper vs. improper density operators in section 1.2.3, the system-apparatus are still entangled with the environment, which means that it is not in a definite state. Thinking back to the example of the double slit experiment, the disappearance of the interference pattern means that the phase relations of the superposition of the particle going through the two slits has disappeared. It does not mean that each particle path is determinate. Key here is equation (2.3). As Bell remarked:

> The idea that elimination of coherence, in one way or another, implies the replacement of 'and' by 'or', is a very common one among solvers of the 'measurement problem'. It has always puzzled me. (Bell 1990 [15] pp. 36)

Postulating that although the system-apparatus is in an improper mixed state, we can interpret it as a proper mixed state superficially solves the problem of outcomes, but does not explain why this happens, how or when. This kind of interpretation is sometimes called the ensemble-, or ignorance

interpretation. Although the state $|\Psi_{\mathcal{SA}}\rangle$ is supposed to describe an individual quantum system, one claims that since we can only infer probabilities from multiple measurements, the reduced density operator $\rho^{\mathcal{SA}}$ is supposed to describe an ensemble of quantum systems, of which each member is in a definite state.

Decoherence theorists have generally come to accept the criticisms above, and accept that decoherence alone does not solve the problems of outcomes, and therefore leaves the most essential question untouched. Some authors even think environment-induced decoherence *aggravates* the measurement problem. Indeed in the case of particle localisation due to environmental scattering, we found that the ensemble width increased faster, under the influence of decoherence, so that without a collapse postulate the particles position seems to have become indeterminate faster under the influence of decoherence.

However this does not yet mean that decoherence has no conceptual consequences at all. As we shall see in the next section, it may be an important supplement to various existing interpretations of QT; and some new interpretations originated from decoherence theory. The technical argument of environmental decoherence itself seems to be compatible with both interpretations that use the **Dynamics** strategy defined above, and those that use the **ExtraValues** strategy to solve the measurement problem.

3.2 Other interpretation of quantum theory and the role of decoherence in them

3.2.1 Standard and Copenhagen interpretation

We have already seen a few examples of the standard (orthodox) and Copenhagen interpretations.

The standard or orthodox interpretation is the only **instrumentalist** interpretation, and therefore not really an interpretation at all: instead it merely couples the mathematical theory to possible experimental settings. It includes the measurement-collapse postulate (postulate 3 on page 6), but makes no attempt at explaining its physical mechanism or what a measurement exactly is.

The Copenhagen interpretation, on the other hand, does propose the **SciReal** view, by postulating that the classical is not to be derived from quantum theory, but exists objectively, therefore recovering **objective** definiteness. As mentioned before, this creates a problem of where this fundamental boundary between the quantum world and classical realm is drawn.

Both the standard and Copenhagen interpretation argue that it is the observer (or user of a measurement apparatus, in a strict Copenhagen view), that selects the specific observable being measured. This of course runs counter to the notion of an observer-independent reality, which is problematic for a realist Copenhagen interpretation.

Role of decoherence in the Copenhagen interpretation

Insights gained from decoherence strongly suggest that the Copenhagen postulate of classicality that cannot be derived from QT, cannot be upheld. Environment-

induced superselection and suppression of interference shows that robust states can emerge, using just QT, for a broad range of microscopic and macroscopic objects. Moreover, results show that some subspaces of a Hilbert space of macroscopic objects are 'decoherence free', so that even for macroscopic objects superpositions can exist. This is confirmed by recent experiments (section 1.4.3), where the potential Heisenberg cut is moved further and further toward the apparatus (which is supposed to be described classically).

If we take a positive position and anticipate decoherence being embedded in some additional interpretive structure (see the options below), that could lead to a consistent definite macroworld derived from QT (and maybe other postulates). This would make the postulate of an independent classical reality unnecessary.

3.2.2 Wigner/von Neumann quantum mind/body interpretation

Von Neumann already realised that an axiomatic structure of QT should not contain any terms that are themselves to be described by the theory. So there must be something else in one's ontology that falls outside the scope of the theory. Von Neumann argued that it is a matter of convention where one draws the "Heisenberg cut" in the Copenhagen interpretation, as long as it is drawn somewhere. Indeed as we have seen in section 1.2.4, it does not matter whether one applies the collapse postulate to just the system, or the system entangled with the apparatus.

For von Neumann, the notion of consciousness fitted perfectly well into this ontology. In this way, it is at the level of the consciousness of the observer, that a measurement finally takes place, which seems to imply that it is the act of observation itself that *creates* a definite macroworld, and the fact being observed, thus securing a **Definite** macro realm (but only when looked at).

This rather radical move was and is wholly unacceptable to a number of physicists, such as Bohr and Bell, who wondered when exactly an observer is conscious enough to cause a collapse:

> Was the wave function waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer for some highly qualified measurer - with a PhD? (Bell 1981, [12] pp. 117)

Indeed this interpretation, like the Copenhagen and Orthodox interpretation leaves a few questions of the measurement problem unanswered, but it has regained some recent interest in the form of the Many-Minds Interpretation, see section 3.2.3 below.

Eugene Wigner identified this interpretation as QT's version of the traditional mind-body problem[20].

Role of decoherence in the quantum mind/body interpretation

One may also ask the opposite question: instead of consciousness creating a definite world by collapsing quantum superpositions, may quantum coherence be associated with the emergence of consciousness? The question is not directly

related, but certainly an interesting one. It has recently been argued[19] that decoherence theory answers "No" to this question.

Indeed: independently of the vague notion of consciousness and its potential role in the theory of physics, one could ask if quantum superpositions could possibly play a role in our brain processes. The brain is typically modelled in completely classical terms as a massively parallel interconnected web of neurons, which act as on-off switches, in turn depending on particular, often nonlinear activation functions. For quantum coherent states, superpositions, to play a role in this model, they must be able to exist for a long enough period of time in the relevant structures in the brain, to allow for some kind of "quantum computation". The neurons and microtubules connecting them together, while small on a biological scale, are still macroscopic and complex on the scale generally considered in QT. Indeed the models from decoherence show a decoherence timescale of the order 10^{-20} seconds for neurons and 10^{-13} seconds for microtubules, where typical cognitive processes take around $10^{-3} - 10^{0}$ seconds.

3.2.3 Relative state interpretations

The relative state approach, encompassing several different interpretations[1], was first described by H. Everett in 1957. Its main contribution is to take seriously the idea of superpositions at the macroscopic level, thereby leaving the mathematical structure of QT intact, not needing extra postulates, hidden values or modifications to the dynamics. However, one still needs some extra values, as I shall argue below. The three main ideas of this approach are as follows:

(i) There exists a total quantum state $|\Psi\rangle$ of the entire universe. Note that this automatically rules out external observers since by definition there are no observers outside the universe.

(ii) The Schrödinger equation is universally valid.

(iii) At the completion of a measurement, all terms in the expansion of the total state $|\Psi\rangle$ in the eigenbasis of the measured observable (i.e. the basis of possible measurement-outcomes), each corresponding to a definite outcome, are actual. That is, no 'outcome'-state is singled out, formally or physically.

These states can be thought of as relative;

(a) to the other part of the composition (this is called the relational, or relative interpretation); or

(b) to a particular 'branch' of the universe that has split (this is the many-worlds interpretation, MWI); or

(c) to a particular 'mind', among the possible minds of the conscious observer (this is the many-minds interpretation, MMI).

[1] I shall not discuss the "Existential Interpretation" here, mainly due to Zurek, as it partly overlaps with the Many Worlds Interpretation; but see Janssen (2008)[3] for a review, or Zureks own 1998, 2002 and 2003 papers on the subject.

Relative state interpretations can be categorised as a **SciReal, ExtaValues** interpretation. The extra values take the form of perspectival or even mental values, defining which branch, state, or mind "you" are in. An analogy between the branches and the various slices of spacetime (spacelike slices) is described by many authors (see for instance [13], pp. 34, and [17] for a point by point comparison). One can then argue that the different branches can be parameterized by an extra value analogous to time in the spacetime picture.

The MWI interprets the terms in a system-apparatus(-observer) superposition to represent two or more distinct macroscopic worlds(branches), so that the total state represents a multiplicity of worlds, each of which is macroscopically definite. The MWI can thus be seen as aiming for a **Definite** (though multiplied) macroscopic realm. In the MMI on the other hand, the definiteness is **Apparent** to one of the observer's minds.

A claimed advantage of this interpretation is that it would fit well into a relativistic description of measurements in QT, although no such description currently exists in any detail.

Role of decoherence in Relative state interpretations

One problem concerns defining probabilities. If every outcome occurs, how can there be a sense of probabilities conforming to the Born rule? We will not go into this here, but recent derivations in this approach were shown using information-theoretic formalism, by Deutch (1999), Wallace (2002 - 2009) and Saunders (2004). Zurek (2005) also derived the Born rule in his relative state approach, replacing Deutsch's decision theoretic assumptions with decoherence arguments.

Another main problem is determining in which decomposition the total state is expanded, and therefore split. This links directly to the preferred basis problem we discussed above. Environment-induced decoherence is called upon by many relative state authors. The branches are then defined by the decomposition defined by arguments A. or B. on page 38. The fact that the basis produced might not be complete or exact, can be seen as problematic, but we again refer to the analogy with spacetime mentioned above: When treating a specific problem in relativity (for instance everyday circumstances), there is an approximately best choice of foliation of the slices of spacetime at each time t, but the details are not important, especially regarding the foliation of spacetime far away. When treating some specific problem in QT (for instance everyday circumstances), there is a similar approximately defined best choice of basis, dependent on the physical constitution for the considered system. And again the details are arbitrary, especially regarding events for systems other than those one aims to describe. An exact basis would even have undesirable physical consequences according to Wallace's view, for instance choosing position as the once-and-for-all preferred quantity would prevent superpositions of position eigenstates crucial for chemical bonds holding molecules together.

Also the fact that it takes a finite time of interaction with the environment before decoherence selects the basis, need not be a problem according to the conception of a definite macrorealm as emergent patterns. Wallace says:

> During the decoherence period the wavefunction is best regarded as some sort of quantum soup which does not lend itself to a classical

> description but since the decoherence timescale $\tau(\Delta x)$ is incredibly short compared to any timescale relevant at the cat level of description, this need not worry us. Put another way, the cat description is only useful when answering questions on timescales far longer than $\tau(\Delta x)$, so whether or not quantum splitting is occurring, it just doesn't make sense to ask questions about cats that depend on such short timescales. (Wallace 2001[18], pp. 11)

To conclude: decoherence seems to play a vital role in the relative state interpretations.

3.2.4 Modal interpretations

Modal interpretations, first suggested by Van Fraassen, weakens the standard eigenvalue-eigenstate link mentioned earlier. They are of the type **SciReal, Definite, ExtraValues**. Besides the normal dynamical quantum state, physical systems posses a number of well-defined physical properties, i.e. definite extra values of physical quantities. Which physical quantities are thus defined, and which values they take, may however change in time. The dynamical state determines the set of possible value states and their possible time evolutions. However a system may have a sharp value of an observable even if the dynamical state is not an eigenstate of that same observable.

Note that therefore modal interpretations assume only half of the eigenvalue-eigenstate-link, specifically, if a system is in an eigenstate of an observable $|\Psi\rangle = |\psi_i\rangle$, upon measurement we will get outcome i, but if we measure outcome i this does not necessarily mean that the system is in the corresponding eigenstate.

Of course the attribution of these definite properties must satisfy certain requirements. The probabilities for outcomes of measurements should be consistent with the Born probabilities of standard QT, and we wish to recover the apparent (in this case actual) definiteness of macroscopic objects, thereby solving the measurement problem.

Role of decoherence in Modal interpretations

Different interpretations then proceed to define different rules for property assignments. One of the suggested methods is to use environment-induced decoherence for the property assignment. See for instance [21].

3.2.5 de Broglie - Bohm Pilot wave

This interpretation is of the same type as the modal one just described, **SciReal, Definite, ExtraValues**, and it can in some sense be seen as a modal interpretation itself. In this picture every particle is attributed (as extra value) a definite position $x(t)$. Then, arguing that

> in physics the only observations we must consider are position observations, if only the positions of instrument pointers (Bell (1993)[12] pp. 166),

measurements also have definite results. Quantum systems still have a state $|\psi\rangle$, governed by the Schrödinger equation, and the dynamics of the position $x(t)$ is then determined by other precise *guiding equations* that depend on $|\psi\rangle$.

Problems arise with relation to the fundamental ontological status attributed to particles, as this makes an extension of the theory to a relativistic quantum theory inherently more difficult.

Also, in certain cases, guiding equations used in non-relativistic wave mechanics yield particle trajectories that are non-classical.

Role of decoherence in the de BroglieBohm interpretation

Interestingly, the connection with decoherence seems to be a double one. On the one hand, decoherence seems to support the idea that position should be the preferred value which to assign a definite value $x(t)$. But on the other hand, decoherence makes the postulation of a determinate position seem unnecessary, since for most measurements the reduced local density operator is already diagonal in position basis. Of course, as we have seen, a certain position is in no way singled out (no solution to the problem of outcomes), so that the de Broglie-Bohm interpretation certainly adds something to the description, since it *does* solve the problem of outcomes.

One can also address the problem of non-classical trajectories using decoherence. Environment-induced decoherence results in quasi-classical trajectories for particles by a localisation in the phase space of the quantum state (see Chapters 4,5, especially section 5.2.5 of Schlosshauer (2008)[1]). Using these decohered states in the guidance equation of the de Broglie-Bohm interpretation might lead to quasi-classical trajectories for $x(t)$ as well. Some work on this has been done (Appleby (1990), Allori (2001), Allori and Zangh (2004)); and under certain circumstances decoherence effects can play the role of preserving classical properties of Bohmian trajectories.

3.2.6 Physical collapse theories

Physical collapse theories introduce an explicit modification of the Schrödinger equation to solve both the problem of outcomes and interference. So it is an interpretation of the type**SciReal, Definite, Dynamical**.

Early models by Pearle (1976) and Gisin (1984) introduced a white noise term to the Schrödinger equation (*stochastic dynamical reduction*), causing the coefficients $|c_i|$ in the state-vector expansion

$$|\Psi(t)\rangle = \sum_i c_i(t)\,|\psi_i\rangle \tag{3.1}$$

to fluctuate in time, while the state remains normalised. Eventually one amplitude approaches unity, while the others go to zero (due to Huygen's gambler's ruin corollary), where the probability of a specific c_k 'winning' is equal to $|c_k(t=0)|^2$, to ensure agreement with the Born rule.

However this model suffers from a severe version of the preferred basis problem: for what selects the form of the expansion (3.1)? If this were random as well, why do we not perceive superpositions of macroscopic object states? Secondly, this model does not explain why the collapse is more effective on macroscopic scales.

Motivated by these problems, Ghirardi, Rimini and Weber (1986) developed a *spontaneous localisation model*. This relies again, like the Broglie-Bohm interpretation in the previous section, on the argument that position is all we

ever really measure. The N-particle wavefunction $|\psi(x_1, ..., x_N)\rangle$ is at random intervals multiplied by a Gaussian

$$|\psi(x_1, ..., x_N)\rangle \longrightarrow |\psi(x_1, ..., x_N)\rangle \times A \exp\left[-\frac{(x_i - \mathbf{X})^2}{2\Delta^2}\right] \tag{3.2}$$

with Δ small, A a constant, $i \in \{1, ..., N\}$ randomly chosen. $\mathbf{X}$ is a stochastic variable with a probability distribution $\text{Prob}[\mathbf{X} = y] \sim |\langle\psi(y)|\psi(y)\rangle|^2$; so its probability distribution is proportional to the square of the wave function at that position, to conform with the Born rule. The frequency of localisation events $f_l = \alpha N$, is chosen so that for microscopic systems unitarity is almost always preserved (i.e. α is very small), but for macroscopic objects, where N is very large, spatial superpositions disappear on timescales too short to observe realistically.

Note that no physical explanation is given for the localisation events, no real solution to the collapse problem is given. The collapse is just postulated (but then again, so is much of QT).

Role of decoherence in Physical Collapse theories

The same discussion holds here as for the postulation of position as the preferred basis, in the de Broglie Bohm interpretation (see section 3.2.5).

However for collapse theories, the similarities in formalism compared to decoherence theory extend further than that. Empirical evidence shows the validity of the predictions of environmental decoherence, so given the collapse theory, decoherence will always be present as well. Assuming the two theories act in the same direction, i.e. select the same preferred basis (position), one can ask the question which of these two effects dominates the evolution of the system. Thus, if the collapse occurs on a shorter timescale than the environment-induced superselection of a preferred basis and the suppression of local interference, decoherence will have little influence in most cases. Conversely, if decoherence acts quicker, the interaction with the environment selects an improper mixture of quasi-classical robust states. Remember though, that although in this case decoherence dominates the selection of states, decoherence alone does not solve the problem of the outcomes, as emphasised in section 3.1. An actual *outcome* is subsequently selected by the localisation according to the collapse theory. Comparing specific timescale values for environmental scattering theory and GRW-theory shows that the latter scenario is more likely: decoherence theory will typically dominate the selection.

So is there experimental evidence for the postulated collapse? In principle the deviation from the Schrödinger dynamics could be tested. However it is the presence of decoherence that makes this difficult, since one would have to find an experiment in which no significant suppression due to decoherence arises, but involves enough particles to observe the effect of the collapse theory. The required shielding of the system from the environment is then a considerable technical challenge. However, as experiments become more precise, and interference effects are observed for ever increasing sized physical systems, this will impose stronger bounds on the parameters (like α) used in collapse models.

Chapter 4

Summary

I hope to have given a relatively objective introductory review of the different aspects of the decoherence program. I have of course described only a tiny part of the research that has been published on this subject. I wish to mention a few things that I should certainly have discussed, but did not:

- There is another argument that explains the uniqueness of the preferred basis in certain Hilbert space settings. It involves the tri-decompositional theorem, that states that a tri-decomposition of a state is unique if it exists. The argument is only applicable in certain simple cases, as the decomposition's existence is not trivial in more complex cases, and requires a lot of assumptions.
- Master equation formulations of decoherence, quantum browninan motion, and spin models. All are extensively discussed in chapters 4 and 5 of Schlosshauer's 2008 book [1].
- Zurek devised his own extension to the relative state interpretation, Zurek's existential interpretation. A much more detailed discussion of conceptual issues regarding decoherence and the relative state-, and existential interpretations can be found in Hanneke Janssen's thesis on the subject[3].

Below I wish to make some summarising remarks, split into two categories: looking first at decoherence theory as a framework for describing physics, and secondly at its conceptual relevance. This corresponds perhaps to a division in looking at the decoherence program from the phenomenological perspective and the ontological perspective.

Decoherence as a framework

The formalism stated in section 1.3 might not seem a very mathematically rigorous framework. Rather than a framework, the formalism seems to consist of a multitude of worked examples (see also the quote from Janssen quote on page 13). However one could argue that this is exactly the point; that for each experimental setup or situation, one needs to consider the specific interactions with the system *and* the environment. Listing our findings from chapter 1:

- Interaction with the environment leads to a selection of states of the system that are robust against this very interaction, in the sense that they become least entangled with the environmental states (sections 1.3.2-5).
- In certain cases, subspaces of the Hilbert space of a system may be found that are decoherence free, which, if nothing else, could be very useful for the field of quantum computation/control (section 1.3.6).
- If a system under interaction with the environment is considered locally, the off diagonal terms in its density operator (in the pointer basis) responsible for interference effects are efficiently suppressed (sections 1.3.1 and 1.3.3) .
- Only information about the pointer states is recorded redundantly in the environment that has interacted with the system. Observing a part of this environment allows one to infer information about the state of the system without perturbing it (further), but only about pointer observables (section 1.5).

Conceptual relevance of decoherence

The decoherence program is relevant to interpreting quantum theory in two ways. Firstly, by solving parts of the measurement problem described in chapter 2 by itself. Secondly by "assisting" current or new interpretations of QT, where decoherence is incorporated in the interpretation's framework. Listing our findings from chapter 3 we have:

- If nothing else, the decoherence program has certainly given a boost to research in foundational issues concerning the measurement problem in quantum mechanics.
- In experiments that involve interference measurements, such as the infamous double slit experiment, environment-induced decoherence explains disappearance of the interference on macroscopic scales, therefore solving the problem of interference (section 2.3 and argument C. in table 3.1 in section 3.1).
- The early claims that environment-induced decoherence would by itself solve the measurement problem were grossly over-simplified: in fact, as we have seen it makes the problem more acute in certain cases. The most important reason for this is that without some kind of collapse postulate or relative state interpretation (which both come with their own problems), the problem of outcomes and collapse (section 2.2) is not sufficiently addressed. No outcome is singled on any scale, therefore restoring neither **Objective** or **Apparent** definiteness in the macroscopic realm (section 3.1).
- However the decoherence framework has proved an important tool to support other interpretations with some dynamical physical background, especially with relation to the preferred basis problem (sections 2.4 and 3.2).

Personal outlook

It seems naive to think that we already know everything there is to quantum mechanics, and that we can currently come up with a perfect interpretation of it. Inevitably new underlying theories will be formed. As Wallace remarks, we should

> consider vibrations in a (quantum-mechanical) crystal [;..] the collective excitations are called quasi-particles - so crystal vibrations are described in terms of phonons, waves in the magnetisation direction of a ferromagnet in terms of magnons, collective electron waves in a plasma in terms of plasmons, and so on. [...] In fact, although this account of quasi-particles represents them as structures in an ontology of real particles, the description in terms of nonrelativistic particle mechanics is itself effective, and derives from a description in terms of quantum field theory - there is every reason to believe particles like quarks and electrons to be patterns in the underlying quantum field in almost exactly the same sense that quasi-particles are patterns in the underlying crystal. It is interesting to ask whether the existence of some underlying stuff is essential, or whether we can continue this chain of theories forever (Wallace 2001[18] pp. 8,9).

However, it appears to me that environment-induced decoherence will at least form a part of any kind of solution to the measurement problem.

Personally I would not be surprised if an interaction with some hidden degrees of freedom - whatever those may be (folded dimensions in string theory, dark matter/energy) - was found to physically collapse a photon in a (approximate) position eigenstate, with some minute probability. Such a physical collapse in combination with environment-induced decoherence, and Bell's idea that position is everything we ever really observe, would seem to me to be the most down-to-earth and sufficient future solution to the measurement problem.

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