Partitioning Properly Pursued

Vincent F. Hendricks

March 29, 2007

> 'What the concept of knowledge involves in a purely logical perspective is thus a dichotomy of the space of all possible scenarios into those that are compatible with what I know and those that are incompatible with my knowledge. This observation is all we need for most of epistemic logic.' [Hintikka 03]: 34

Ever since the early 1960's, Hintikka has been emphasizing the significance of the process of partitioning possible scenarios while ascribing propositional attitudes—in particular epistemic ones like knowledge and belief.

The process is, however, not only crucial to epistemic logic but features as *the* key unifying concept running all the way through Hintikka's 'informative epistemology' to his understanding of inquiry, decision-making and action. As

> epistemology seems to enjoy unexpectedly sexy reputation in these days, [Hintikka 07a]: 1

it is hard to imagine a better occassion – this symposium in Hintikka's honor in conjunction wih the forthcoming publication of Hintikka's collected papers in *Socratic Epistemology* [Hintikka 07] – to reflect upon why partitioning must be properly pursued.

1 Partition

Here are a few examples tracking the idea of partitioning in Hintikka's work for almost half a century. In his seminal book *Knowledge and Belief: An Introduction to the Logic of the Two Notions* from 1962 Hintikka introduces the now classical propositional language of knowledge and belief based on an augmentation of the classical language of propositional logic with two unary epistemic operator K_a and B_a such that

$$K_a p \quad \text{reads} \quad \text{'Agent } a \text{ knows } p\text{'}$$

and

$$B_a p \quad \text{reads} \quad \text{'Agent } a \text{ believes } p\text{'}$$

for some arbitrary sentence p. These formalizations of knowledge and belief are roughly interpretations of $\Box p$ in alethic logic reading 'It is necessary that p'. Interpreting modal logic epistemically and doxastically is crudely a reading of modal formulae as epistemic and doxastic statements expressing attitudes of certain agents towards certain propositions. The syntactic augmentation is followed by a semantic interpreration of the unary operators:

$K_a p \approx$ *in all possible scenarios compatible with what a knows it is the case that p*

$B_a p \approx$ *in all possible scenarios compatible with what a believes it is the case that p*

Given the epistemic / doxastic compatibility clauses and accessibility relations, the agent is capable of constructing different 'world-models' using the epistemic(-doxastic) language. The agent is not necessarily required to know which one of the world-models constructed is the real scenario. All the same, the agent does not consider all world-models equally possible or accessible given his epistemic state at that instant. Some world-models may be incommensurable with his current information or background assumptions. These incompatible world-models are excluded. In epistemic logic, as in many other epistemologies, it is typically stipulated that the smaller the set of scenarios an agent considers possible, the smaller his uncertainty. Thus, epistemic logic offers a way of systematically framing the problem of defining the class of scenarios compatible with what someone knows and believes.

Although 'Semantics for Propositional Attitudes' from 1969 is about topics in the theory of meaning and reference, Hintikka again makes the case for partitioning abundantly clear because its use is not only restricted to knowledge and belief:

My basic assumption (slightly ovesimplified) is that an attribution of any propositional attitude to the person in question incvolves a division of all possible worlds (more precisely, all the worlds which we can distinguish in the part of language we use in making the attribution) into two classes: into those possible worlds which are in accordance with the attitude in question and into those worlds which are incompatible with it. The meaning of the division in the case of such attitudes as knowledge, belief, memory, perception, hope, wish, striving, desire etc. is clear enough. For instance, if what we are speaking of are (say) a's memories, then these possible worlds are all the possible worlds compatible with everything he remembers. [Hintikka 69]: 91

Hintikka has since labelled the philosophers' usage of 'possible worlds' as a symptom of intellectual megalomania [Hintikka 03]: 19. As far back as *Knowledge and Belief*, Hintikka used 'model sets' (as formal counterparts to (partial) descriptions of state of affairs) which are not complete linguistic descriptions of possible worlds. The nature of the 'space of options,' as Hintikka calls it is of acute importance to many a central epistemological issue including the nature of the accessibility relation built in the model and thus also for the nature of the forcing relation responsible for partitioning [Arló-Costa 06].

A final example from the forthcoming paper with the initially puzzling and provocative title 'Epistemology Without Knowledge and Without Belief':

> In order to use my knowledge, I must know which possibilities it rules out. In other words, any one scenario must therefore be either incompatible or compatible with what I know, for I am either entitled or not entitled to disregard it. Thus the totality of incompatible scenarios determines what I know and what I do not know, and vice versa. In principle, all that there is to logic of knowledge is this dichotomy between epistemically impossible and epistemically possible scenarios. [Hintikka 07a]: 3

Partitioning, as trivial as it may immediately seem, must be properly pursued because partitioning is responsible for the acquisition of *information* which in turn

- is modularizable in knowledge, belief and other attitudes,

- is the goal of inquiry, and

- is the stuff decisions and actions are based upon.

2   Information

Knowledge is often considered the crown gem in epistemology and an often cited reason is this: If you really know something, then new information should not cause you to change your mind. Knowledge is, in this sense, infallible. The classical conception of knowledge as possessing the property of infallibility is taken to require, that for an agent to have knowledge of some proposition he must be able to eliminate *all* the possibilities of error associated with the proposition in question. The set of all worlds is accordingly considered. However, the set of possible worlds is too rich for knowledge to have scope over. This set includes some rather bizarre worlds in which all knowers are systematically in error in one way or another and it might even be taken to include worlds in which contradictions are true. If these worlds were to be considered relevant, skepticism would have the upper hand all the time.[1] Epistemology has long

---

[1]For a detailed treatment of the relationship between forcing and skepticism, see [Hendricks & Symons 07].

worked to provide a response to skepticism so as to secure the possibility of knowledge. Epistemic logic, as presented by Hintikka form the outset, is in much the same business given the centrality of partitioning of scenarios. The partitioning of scenarios into those that can be legitimately ignored and those that are relevant, of course assumes some account of *legitimacy*. Understanding what legitimacy amounts to here is a deep philosophical problem and Hintikka has suggested that it is equivalent to defining knowledge. However, since some of the central properties of any viable account of knowledge, including prominently infallibility, simply cannot be defined with respect to *all* possible worlds, some partitioning of worlds will be required for epistemology to even begin to get started. The strategy of screening off possibilities of error to secure knowledge is a basic tenet of *Knowledge and Belief*:

> Whoever says "I know that p" proposes to disregard the possibility that further information would lead him to deny that p although he could perhaps imagine (logically possible) experiences which could do just that. [Hintikka 62, 05]: 17.

The 'logically possible' experiences referred to are those pertaining to possibilities of error that any account of knowledge must exclude. These would include conceivable scenarios in which the very possibility of knowledge is undermined: brains in vats, malicious gods and the like. This way of responding to skepticism by limiting the set of citable possible worlds carrying potential error has been dubbed 'forcing' by Hendricks [Hendricks 01] and in particular [Hendricks 06]. When it comes to skeptical arguments that would undermine the very possibility of knowledge, the epistemologist must rely on forcing strategies of various kinds in his or her demonstration that the skeptic's possibilities of error fail to be genuine in the relevant sense. This will be the case no matter what one settles on as a definition of knowledge. In this sense, epistemic logic with its forcing strategy assumes that the skeptic has been defeated and demonstrates the structural manner in which one is obliged to model knowledge.

A shop-worn example of the skeptical challenge is the closure condition for knowledge. Hintikka is known to resolve this issue by appeal the distinction between depth and surface information of propositions [Hintikka 07b]. There is another argument for closure that Hintikka could make use of. According to Lewis [Lewis 96] knowledge is closed in uniform contexts. Hintikka may invoke a similar argument given the partitioning of scenarios into the two distinct compartments consisting of those in accordance with the attitude and the scenarios not. The scenarios in accordance with the epistemic attitude may be read in accordance with Lewis' context-sensitive quantifier restriction on knowledge. Then, the demon world, brain-in-a-vat world and other derivatives of global underdetermination are simply excluded from the compatibility partition; these extravagant worlds are not in accordance with the epistemic attitude. Thus, these error-possibilities will not disturb the context, or in Hintikkian terms, will not pass over into the compatibility partition, so knowledge is closed for a given

compatible partition, i.e. uniform context.[2]

Unnatural doubt is however not the only way to start, and in much contemporary work in epistemic logic doubt is probably not a strong motivating factor. Partitioning the possibility space does not necessarily imply a forcing strategy against the skeptic, especially when epistemology is not solely 'defensive' in the Van Fraassenian sense [van Fraassen 89]. The computer scientists that build runs are hardly motivated by Cartesian doubt. They are motivated by avoiding exponential explosion; motivated by avoiding the frame problem, and other conundra. Hintikka agrees with this sentiment in his own way; partitioning is not only about defeating skepticism and gaining knowledge, but just as much about paving the way for deliberation, decision and eventually action—partitioning serves a double purpose all at once:

> To take a simple example, let us suppose that I'm getting ready to face a new day in the morning. How, then, does it affect my actions if I know that it will not rain today? You will not be suprised if I say that what it means is that I'm entitled to behave as if it will not rain, for instance to leave my umbrella at home. [...] The role of knowledge in decision-making is to rule out certain possibilities. [Hintikka 07a]: 3

The concept of knowledge has come under attack in recent. Some say that it is an overrated concept for deliberation, decision and action and that one may make do with measure of opinion as long as the measure is strong enough. Although Hintikka has been one of the pioneers in bringing knowledge to the attention of mainstream and especially formal epistemologists ranging from logicians to computer scientists and information technologists he is also one of the formal philosophers to criticize it the most. It is not that knowledge is overrated as some Bayesians may have it, it is just the wrong concept to place front and center in epistemology. That's the way of the past, that's the way of the present, but it's not the way of the future if Hintikka has his way with replacing the concepts of knowledge, belief, ... with the concept of information or 'information range' as van Benthem and Martinez have recently called it [van Benthem & Martinez 07].

Knowledge is a guide to action, but so are attitudes like certainty, belief, conviction etc. in different degrees and with different constraints enforced for their satisfaction. Additionally, the content of a propositional attitude can be specified independently of discrepancies between the different attitudes—an insight dating back to Husserl while separating the noematic *Sinn* from the thetic ingredient of a noema. If knowledge, certainty, belief and conviction are attitudes towards propositional content, so is doubt, and the content of a doubtful attitude is just as specifiable as the content of any epistemically positive attitude. Being skeptical requires information otherwise there is little to be skeptical about, and being a skeptic is in turn cultivated by doubt as to how the information in question is acquired. The very same goes for the remaining attitudes; 'knowledge' is

---

[2] This argument is spelled out in greater detail in [Hendricks 04].

a label for a particular way of having acquired information; 'belief' is likewise an 'achievement' word wired to a different set of criteria related to when the agent is ready to act on the information in quesion. In sum, knowledge, belief, certainty, conviction,etc. are all derivatives with information as the basis:

> The generic logic of epistemology can be construed as the logic of information. Indeed, what the content of a propositional attitude amounts to can be thought of as a certain item of information. In attributing different attitudes to agents, different things are said about this information, for instance that it is known, believed, remembered and so on. This fits well with the fact that the same content can be known by one person, believeed by another, remembered by a third one, and so on. [...] It seems to me that epistemology would be in a much better shape if instead of the deep word 'knowledge' philosophers cultivated more the ugly foreign word 'information' even though it perhaps does not capture philosophers' profound sense of knowing. [Hintikka 07a]: 10–11

Returning to the idea of partitioning, or the fundamental dicotomy between scenarios in accordance with the attitude and those incompatible with it, every attitudes enforces such a partition and the particular attitude is then cashed out of the general information partition in terms of the additional constraints enforced for the attitude to materialize. When the knowledge attitude is taken to entail infallibility, then all possibilities of error are excluded relative to the set of possibilities in accordance with the attitude. Certainty, say, does not exclude all possibilities of error, but most of them (a probability measure may come into play here [Hintikka 07a]: 5) relative to the set of worlds in accordance with the attitude and so forth for belief, conviction and the other attitudes all they way down to doubt which excludes almost no possibilities of error. Knowledge is modularized information, and that goes for all the other attitudes as well (Figure 1).

Different propositional attitudes are just different modularities of information, modularities that guide action and decision-making. It is then clear that all the propositional attitudes of interest to epistemology belong to *applied* epistemology as opposed to general or theoretical epistemology:

> The criteria of knowledge concern the conditions on which the results of epistemological inquiry can be relied on as a basis of action. It follows that it is an exercise in futility to try to define knowledge in any general epistemological theory. [Hintikka 07a]: 30

What to do in applied epistemology then? Inquire, learn, act! This is exactly the focus of formal epistemology and social software today.
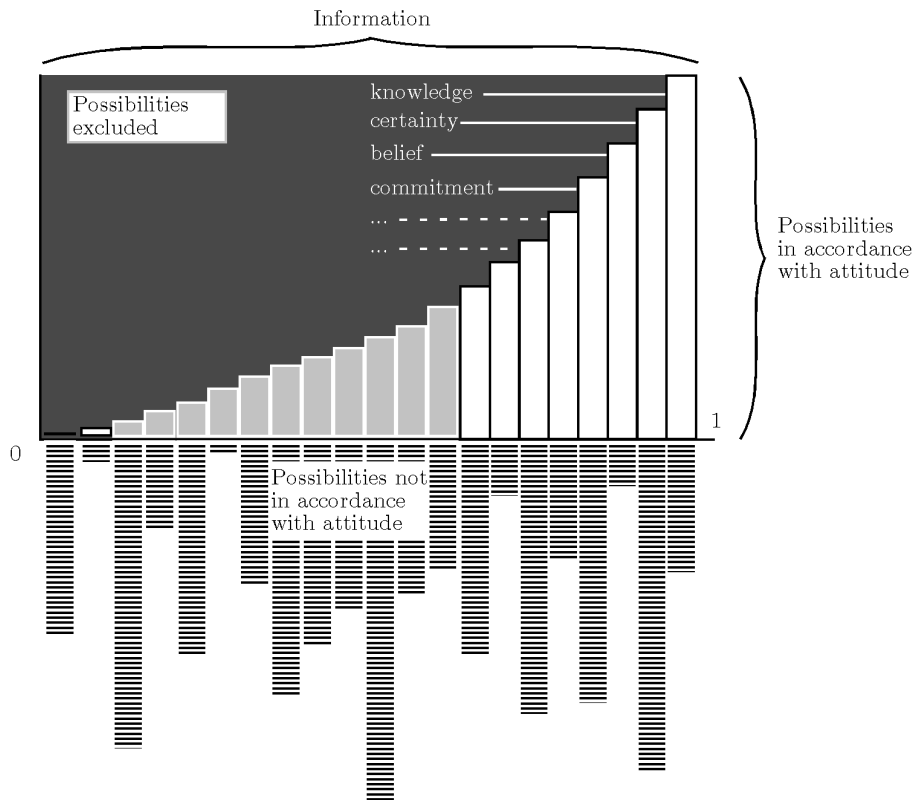
Figure 1: Information, partitions, excluded possibilities and attitudes.

3   Inquiry

Information is acquired via interrogative inquiry—new information is the result of answers to questions that an inquirer directs to some suitable source of information. In this inquiry process epistemic logic plays a paramount role in conjunction with its close allied game theory:

> Another main requirement that can be addressed to the interrogative
> approach – and indeed to the theory of any goal-directed activity –
> is that it must do justice to the strategic aspects of inquiry. This
> requirement can be handled most naturally by doing what Plato
> already did to the Socratic *elenchus* and by construing knowledge-
> seeking by questioning as a game that pits the questioner against
> the answerer. Then the study of those strategies of knowledge ac-
> quisition becomes another application of the mathematical theory
> of games, which perhaps ought to be called strategy theory in the
> firsts place [Hintikka 07a]: 13.

Epistemic logic is really a logic of questions and answers and the search for the best questions to ask [Hintikka 99], [Hintikka 03]. In this new setting, epistemic logic augmented with an independence-friendly logic constitute the basis for an interrogative theory of inquiry.[3] Answers to questions are in essence requests for knowledge, information or epistemic imperatives. Hintikka's approach rests on the recognition that questions are essentially epistemic, insofar as they express epistemic aims. A question's epistemic aim can be presented as a statement specifying the epistemic state which the answer will bring about; the *desideratum* of a particular question.

Consider, for example, the desideratum of the following question:

1. Is Milton in the kitchen, the living room, or the garden?

is simply:

2. I know that Milton is in the kitchen or I know that Milton is in the living room or I know that Milton is in the garden.

But of course, this is equivalent to stating that

3. I know whether Milton is in the kitchen, the dining room or the garden.

Hintikka reduces the study of questions to the study of their desiderata. Desiderata can, of course be studied by using our usual traditional logical methods. Desiderata differ from their corresponding direct questions insofar as they crucially involve the term "know" in such a way as to make any viable logic of questions and answers ineliminably epistemic.

Hintikka understands his interrogative model as a game against nature, or against any source of answers to queries. He distinguishes two different kinds of rules or principles characteristic of a game. The *definitory* rules define the game. In a game of chess, for instance, the definitory rules tell us which moves are permitted and which not, what checkmate, castling, mean, and so on. These rules define the game of chess. If a player makes a move not allowed by the definitory rules, say by moving a pawn three spaces forward, it is not a chess move and the player must take it back. One may thus describe the definitory rules of any game or rule-governed, goal-oriented activity. However, knowing the definitory rules of a game does not mean you know how to play. One must also know what Hintikka calls the *strategic* rules (or principles) of a game. In chess, for instance, you must plan your moves, select the best course of action, make judgments as to which moves will serve you better than others, and so on. These rules are not merely heuristic. They can be formulated as precisely as the

---

[3]Independence-friendly logic (or IF-logic for short) is a first-order logic augmented with an independence operator '/'. The slash notation for a quantified statement of the form Q_2y/Q_1x expresses the independence of the two quantifiers. This independence may be captured by game-theoretical semantics as informational independence in the sense that the move performed or mandated by Q_2y is independent of the move performed Q_1x. Introducing the independence operator then allows for the unequiviocal formulation of a fan of questions and answers without scope ambiguity, cross-world identity problems etc.

definitory rules. This is well explained by the crucial role of *complete strategies* in von Neumann's game theory.

The results of applying Hintikka's distinction to the interrogative 'games' of inquiry reveals the following: First, the standard rules of an interrogative game—the rules for logical inference moves as well as interrogative moves—are definitory. They say little to nothing about what to do in a logical or epistemological game. The rules for making both logical inference moves and interrogative moves merely define the game. For example, the so-called rules of inference in deductive logic are neither descriptive nor prescriptive but merely permissive, in so far as they do not tell us which particular inference or set of inferences we should draw from a given number of potential premises [Hintikka 07b]. What is needed, if inquiry is going to be successful, is more than the definitory rules of inquiry. *Strategic rules* are needed. Indeed, the better the strategic rules, the better our inquiry. The best player in a game of inquiry is the player with the best strategy, which corresponds in game theory to what happens where values, i.e., "utilities," are associated not with moves themselves but, rather, with *combinations* of *strategies*, as in von Neuman's game theoretical notion of a *complete strategy*.

4   Action

Hintikka once noted what one initially may count as an insignificant syntactical mismatch between the formalization of knowledge and what the formalization is intended to mean:

> Epistemic logic begins as a study of the logical behavior of the expression of the form '*b* knows that.' One of the main aims of this study is to be able to analyze other constructions in terms of 'knows' by means of '*b* knows that.' The basic notation will be expressed in the notation used here by 'K_b.' This symbolization is slightly misleading in that a formula of the form K_bS the term b for the agent (knower) is intended to be outside the scope of K, not inside as our notation might suggest. [Hintikka & Halonen 98], p. 2.

There is only one role left to agents in the 'first generation epistemic logic' as Hintikka recently dubbed the tradition of epistemic logic descending directly from *Knowledge and Belief* [Hintikka 03]. The agents here serve as indices on the accessibility relation between possible scenarios. Epistemic-logical principles or axioms building up modal systems are relative to an agent who may or may not validate these principles. Indices on accessibility relations will not suffice for epistemological and cognitive pertinence simply because there is nothing particularly epistemic about being indices. The agents are *inactive* in the first generation epistemic logic.

In *Knowledge and Belief* the following principle holds in multi-agent systems:

$$K_a K_b p \rightarrow K_a p. \tag{1}$$

If John knows that Pat knows p, then John knows p. The principle holds as a simple iterated version of Axiom T with different agent indicies and as long as a and b index the same scenarios the implication follows through immediately. But who says that agents always index the same scenarios? Chances are that in many situations they do not index the same scenarios especially in modal contexts [Lewis 96]. The point may also be cast in methodological or strategical terms: There is no guarantee that different agents behave in the same way in response to the evidence received, or apply the same methods for gaining information, or strategies of questioning. Thus, unless the different agents can duplicate each others behavior or apply strategies of inquiry yielding the same outcome there is no guarantee that 1 holds in all generality. That is agent dependent [Hendricks 06].

Reference to the agent is sometimes dropped in the formalism of epistemic logic such that $K_a p$ becomes Kp and is read 'It is known that p' exactly due to the inactive nature of first generation agents. See for instance [Hintikka 03]. But if epistemic logics are not to be pertinent to the knower who are they to be pertinent to? An agent may have knowledge which is valid up to an including **S4** say. The interesting question is however is *how* the agent has to *behave* in order to gain the epistemic strength that he allegedly has. We need to activate the agents in order to make epistemic logic pertinent to epistemology, computer science, artificial intelligence and cognitive psychology. The original symbolic notation of a knowing agent also suggests this: An agent should be inside the scope of the knowledge operator—not outside as Hintikka correctly notes. Inquiring agents are agents who read data, change their minds, interact or have common knowledge, act according to strategies and play games, have memory and act upon it, follow various methodological rules, expand, contract or revise their knowledge bases, or ask questions in particular ways all in the pursuit of knowledge or information. Inquiring agents are *active agents* [Hendricks 03].

This is admittedly an interpretation of one of the characterizing features, and great virtues of, what Hintikka calls the 'second generation epistemic logic' [Hintikka 03]: The realization that the agents of epistemic logic should play an active role in the information acquisition, validation and maintenance processes. Hintikka again observes this obligation by emphasizing the strategies for his new application of epistemic logic as a logic of questions and answers and the search for the best questions to ask.

Epistemic logic is one of the main toolboxes for conducting epistemology, and Hintikka initially let the way. Active agency is one of the main areas of research in formal epistemology and social software today and Hintikka independently had something to say. No wonder then, that a little partitioning takes you a long way.

5   Acknowledgements

Vincent F. Hendricks
Department of Philosophy and Science Studies
Roskilde University, Denmark
vincent@ruc.dk

References

[Arló-Costa 06]   Arló-Costa, H. (2006). 'Review of Mainstream and Formal Epistemology', *Notre Dame Review of Books*, October. http://ndpr.nd.edu/review.cfm?id=7044.

[van Benthem & Martinez 07] Benthem, J.v. and Martinez, M. (2007). The Stories of Logic and Information, forthcoming in *Handbook of the Philosophy of Information.*

[van Fraassen 89]   Fraassen, B.v. (1989) *Laws and Symmetry.* Oxford: Oxford University Press.

[Hendricks 01]   Hendricks, V. F. (2001). *The Convergence of Scientific Knowledge – A View From the Limit.* Dordrecht: Springer.

[Hendricks 03]   Hendricks, V. F. (2003). 'Active Agents', ΦNEWS (2002), vol. **2**: 5–40. A revised version of the paper is published in a special volume of the *Journal of Logic, Language, and Information*, van Benthem, J. and van Rooy, R. (eds.), vol. **12**, Autumn, no. 4: 469–495.

[Hendricks 04]   Hendricks, V. F. (2004). 'Hintikka on Epistemological Axiomatizations', in *Quantifiers, Questions and Quantum Physics: Essays on the Philosophy of Jaakko Hintikka.* Kolak, D, and Symons, J. (eds.). Kluwer Academic Publishers: 3–34.

[Hendricks 06]   Hendricks, V. F. (2006). *Mainstream and Formal Epistemology.* New York: Cambridge University Press.

[Hendricks et al. 03]   Hendricks, V.F., Pedersen, S.A. og Jørgensen, K.F. (eds.) (2003). *Knowledge Contributors.* Synthese Library, vol. 322. Springer. Contributions from H.v. Ditmarsch, R. Fagin, J. Halpern, J. Hintikka, W. v. d. Hoek, B. Kooi, W. Lenzen, Y. Moses, H. Rott, J. Sowa, M. Vardi, and R. Wójcicki.

[Hendricks & Symons 06]   Hendricks, V.F. and Symons, J. (2006): 'Where is the Bridge? Epistemic Logic and Epistemology,' *Philosophical Studies*, vol. 128/1, March (I): 137–167.

[Hendricks & Symons 07]   Hendricks, V.F. and Symons, J. (2007): 'Limiting Skepticism,' *forthcoming*.

[Hintikka 62, 05]   Hintikka, J. (2005). *Knowledge and Belief: An Introduction to the Logic of the Two Notions*. Reprint edition prepared by Hendricks, V.F. and Symons, J. London: King's College Publications.

[Hintikka 69]   Hintikka, J. (1969). 'Semantics for Propositional Attitudes', in *Models for Modalities*, Dordrecht: D. Reidel Publishing Company: 87–111.

[Hintikka 99]   Hintikka, J. (1999). *Inquiry as Inquiry: A Logic of Scientific Discovery*. Kluwer Academic Publishers.

[Hintikka 03]   Hintikka, J. (2003). 'A Second Generation Epistemic Logic and its General Significance', in [Hendricks et al. 03]: 33–55.

[Hintikka 07]   Hintikka, J. (2007). *Socratic Epistemology*. New York: Cambridge University Press.

[Hintikka 07a]   Hintikka, J. (2007). 'Epistemology Without Knowledge and Without Belief', in [Hintikka 07].

[Hintikka 07b]   Hintikka, J. (2007). 'Who has Kidnapped the Notion of Information', in [Hintikka 07].

[Hintikka & Halonen 98]   Hintikka, J. and Halonen, I. (1998). 'Epistemic Logic', *Routledge Encyclopedia of Philosophy*, volume 1.

[Hintikka 07c]   Hintikka, J. (2007). 'The Place of the A Priori in Epistemology', in [Hintikka 07].

[Hintikka 07d]   Hintikka, J. & Symons, J. (2007). 'Systems of Visual Identification and Neuroscience: Lessons from Epistemic Logic', in [Hintikka 07].

[Lewis 96]   Lewis, D. (1996). "Elusive Knowledge", reprinted in *Papers in Metpahysics and Epistemology*. New York: Cambridge University Press.