Anti-Realism and Modal-Epistemic Collapse*
Reply to Marton

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Abstract

Marton (2019) argues that that it follows from the standard antirealist theory of truth, which states that truth and possible knowledge are equivalent, that knowing possibilities is equivalent to the possibility of knowing, whereas these notions should be distinct. Moreover, he argues that the usual strategies of dealing with the Church-Fitch paradox of knowability are either not able to deal with his modal-epistemic collapse result or they only do so at a high price. Against this, I argue that Marton's paper does not present any seriously novel challenge to anti-realism not already found in the Church-Fitch result. Furthermore, Edgington (1985)'s reformulated antirealist theory of truth can deal with his modal-epistemic collapse argument at no cost.

Keywords antirealism; truth; possible knowledge; possibility; epistemic collapse; modal collapse; modal-epistemic collapse

1 Modal-epistemic collapse: the equivalence between knowing possibilities and the possibility of knowing

The key idea of anti-realism is that knowability and truth coincide: all truths are knowable and nothing but truths are knowable. Conjoined with an interpretation of knowability as possible knowledge this yields the standard antirealist theory of truth: all truths are possibly known and nothing but truths are possibly known. Using the possibility operator $\diamond$ and the knowledge operator K, one can express this theory as follows:

(A-R) $\phi \rightarrow \diamond K\phi$

($\mathbf{A\text{-}R_{conv}}$) $\diamond K\phi \rightarrow \phi$

The above schemes are the modal-epistemic axioms of the standard antirealist theory of truth.

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In what follows I will also make use of various epistemic and modal principles and epistemic and modal systems. I will use the common labels for modal logical systems and principles and I will add $\Box$ as a subscript. If the subscript K is used instead, then the result of uniformly substituting K for $\Box$ is meant. If the subscript $\Diamond$ is used, then the dual of the common principle is meant. For instance:

$\mathbf{T}_\Box$ $\Box\phi \to \phi$,

$\mathbf{T}_K$ $K\phi \to \phi$,

$\mathbf{T}_\Diamond$ $\phi \to \Diamond\phi$,

$\mathbf{4}_\Box$ $\Box\phi \to \Box\Box\phi$,

$\mathbf{4}_\Diamond$ $\Diamond\Diamond\phi \to \Diamond\phi$.

Modal systems $\mathbf{K}_\Box$, $\mathbf{S4}_\Box$ and $\mathbf{S5}_\Box$ will be used — see Cresswell and Hughes (1996, chapters 2, 3) for the details. Note that the rules of monotonicity for $\Box$ and $\Diamond$ are derivable in $\mathbf{K}_\Box$ (Cresswell and Hughes, 1996, p. 30, pp. 32–33):[1]

$\mathbf{RM}_\Box$ $\vdash \phi \to \psi \quad \Rightarrow \quad \vdash \Box\phi \to \Box\psi$,

$\mathbf{RM}_\Diamond$ $\vdash \phi \to \psi \quad \Rightarrow \quad \vdash \Diamond\phi \to \Diamond\psi$.

According to our stipulations, RM_K is then the following:

$\mathbf{RM}_K$ $\vdash \phi \to \psi \quad \Rightarrow \quad \vdash K\phi \to K\psi$.

Note that the following rule is derivable from $\mathrm{RM}_\Diamond$:

$\mathbf{RE}_\Diamond$ $\vdash \phi \leftrightarrow \psi \quad \Rightarrow \quad \vdash \Diamond\phi \leftrightarrow \Diamond\psi$.

With $\mathbf{S}+\phi_1,\ldots,\phi_n,R$ is meant the smallest theory that results from adding $\phi_1,\ldots,\phi_n$ to the axiomatic base and R to the set of rules of $\mathbf{S}$.

Marton (2019) makes three main claims. His first main claim is that the principles above, in combination with certain other epistemic and modal principles, lead to a modal-epistemic collapse, namely the equivalence between knowing possibilities and the possibility of knowing.

Theorem 1. $\mathbf{K}_\Box + \text{(A-R)}, 4_\Diamond, \mathrm{T}_K \vdash K\Diamond\phi \to \Diamond K\phi$.

Proof. The derivation goes as follows (Marton, 2019, fn. 5):

1.	$K\Diamond\phi \to \Diamond\phi$	T_K
2.	$\phi \to \Diamond K\phi$	(A-R)
3.	$\Diamond\phi \to \Diamond\Diamond K\phi$	$\mathrm{RM}_\Diamond$, 2

[1] Marton calls $\mathrm{RM}_\Diamond$ '$\Diamond$-Elim'.

4. $\Diamond\Diamond K\phi \to \Diamond K\phi$ $\quad 4_\Diamond$

5. $K\Diamond\phi \to \Diamond K\phi$ $\quad$ Taut., 1, 3, 4

$\square$

Theorem 2. $\mathbf{S4}_\square + (A\text{-}R), (A\text{-}R_{conv}) \vdash \Diamond K\phi \to K\Diamond\phi$.

Proof. Note that $\mathbf{S4}_\square$ contains $T_\Diamond$ and $4_\Diamond$. The derivation goes as follows (Marton, 2019, fn. 6):

1. $\Diamond K\phi \to \phi$ $\quad$ ($\mathrm{A\text{-}R_{conv}}$)
2. $\phi \to \Diamond\phi$ $\quad T_\Diamond$
3. $\Diamond\phi \to \Diamond K\Diamond\phi$ $\quad$ (A-R)
4. $\Diamond K\phi \to \Diamond K\Diamond\phi$ $\quad$ Taut., 1, 2, 3
5. $K\Diamond\phi \to \Diamond KK\Diamond\phi$ $\quad$ (A-R)
6. $\Diamond K\Diamond\phi \to \Diamond\Diamond KK\Diamond\phi$ $\quad \mathrm{RM}_\Diamond$, 5
7. $\Diamond\Diamond KK\Diamond\phi \to \Diamond KK\Diamond\phi$ $\quad 4_\Diamond$
8. $\Diamond K\phi \to \Diamond KK\Diamond\phi$ $\quad$ Taut., 4, 6, 7
9. $\Diamond KK\Diamond\phi \to K\Diamond\phi$ $\quad$ ($\mathrm{A\text{-}R_{conv}}$)
10. $\Diamond K\phi \to K\Diamond\phi$ $\quad$ Taut., 8, 9

$\square$

The modal-epistemic collapse is also a corollary of the Church (2009)-Fitch (1963) paradox of knowability, an epistemic collapse result, namely the derivation of the equivalence between truth and knowledge.[2] The paradox is derived using the following consequence of RM_K:

$$K(\phi \wedge \psi) \to (K\phi \wedge K\psi), \tag{1}$$

Lemma 1 (Church-Fitch unknowability lemma)**.**

$$\mathbf{K}_\square + \mathrm{T}_K, (1) \vdash \neg\Diamond K(\phi \wedge \neg K\phi).^3$$

[2] I am grateful to an anonymous reviewer who showed this.

[3] As Mackie (1980) pointed out, it suffices to assume a special case of T_K, namely: $K\neg K\phi \to \neg K\phi$. Jago (2010) has shown that one can still derive epistemic collapse even if one replaces (1) with the following weaker principle: $K(\phi \wedge \psi) \to \Diamond(K\phi \wedge K\psi)$.

Theorem 3 (Church-Fitch paradox of knowability)**.**

$$\mathbf{K}_\Box + \textit{(A-R)}, \mathrm{T}_K, (1) \vdash \phi \leftrightarrow K\phi.^4$$

Corollary 1. $\mathbf{K}_\Box + \textit{(A-R)}, \mathrm{T}_K, (1) \vdash K\Diamond\phi \leftrightarrow \Diamond K\phi.$

Proof.

1. $\phi \leftrightarrow K\phi$ — Theorem 3
2. $\Diamond\phi \leftrightarrow \Diamond K\phi$ — $\mathrm{RE}_\Diamond$, 1
3. $\Diamond\phi \leftrightarrow K\Diamond\phi$ — Theorem 3
4. $K\Diamond\phi \leftrightarrow \Diamond K\phi$ — Taut., 2, 3

□

So, $K\Diamond\phi \leftrightarrow \Diamond K\phi$ can be derived with fewer yet not stronger assumptions. (Admittedly, the derivation depends on (1), but see section 2.)

2 The distinction between knowing possibilities and the possibility of knowing

Marton's second main claim is that there are counterexamples to both entailments between knowing possibilities and the possibility of knowing.

Consider, first, the left-to-right direction, namely $K\Diamond\phi \to \Diamond K\phi$. Given Lemma 1, $\neg\Diamond K(p \wedge \neg Kp)$. Yet, $K\Diamond(p \wedge \neg Kp)$ can be true. Marton suggests to take for p the statement that a virus or an asteroid eradicates the human race, so that there are no human beings left to know that the human race has been extinguished.[5]

Next, consider the right-to-left direction, namely $\Diamond K\phi \to K\Diamond\phi$. Marton argues indirectly against the latter, by first deriving the collapse of possible knowledge of a mathematical truth into knowledge from the modal-epistemic collapse and from:

$$\Diamond\phi \to \phi, \quad \text{for mathematical sentences } \phi. \tag{2}$$

Corollary 2. $\mathbf{S4}_\Box + \textit{(A-R)}, \textit{(A-R}_{conv}\textit{)}, (2), \mathrm{RM}_K \vdash \Diamond K\phi \to K\phi$*, for mathematical sentences* ϕ*.*

Proof.

1. $\Diamond K\phi \to K\Diamond\phi$ — Theorem 2

[4] San (2020) has proved a 'general collapse' theorem: a logic with (A-R), D_K and some 'n-level bridging principle' (e.g., T_K is a one-level bridging principle) with $\mathrm{RN}_\Box$ and RM_K as rules entails a 'nth degree modal collapse'.

[5] If the K-operator means that some human at some time knows, then Marton has to cite scenarios in which no individual human being at some point in time is able to foresee that in the future no human beings are alive anymore.

2. $\Diamond\phi \to \phi$ (2)

3. $K\Diamond\phi \to K\phi$ RM_K,[6] 2

4. $\Diamond K\phi \to K\phi$ Taut., 1, 3

□

Take a mathematical truth p that is provable within one of the known axiomatic theories and indeed there is a possible world in which it is proved[7] and on that basis known. However, in the actual world p is not known. (It may be a mathematical truth that is very complex and of little interest.) This contradicts $\Diamond Kp \to Kp$.

We can improve on Marton's counterargument by appealing once more to the Church-Fitch paradox of knowability (Theorem 3).

Corollary 3. $\mathbf{K}_\Box + (A\text{-}R), (2), \mathrm{T}_K, (1) \vdash \Diamond K\phi \to K\phi$, *for any mathematical sentence* ϕ.

Proof.

1. $K\phi \to \phi$ T_K

2. $\Diamond K\phi \to \Diamond\phi$ $\mathrm{RM}_\Diamond$, 1

3. $\Diamond\phi \to \phi$ (2)

4. $\phi \to K\phi$ Theorem 3

5. $\Diamond K\phi \to K\phi$ Taut., 2, 3, 4

□

So, $\Diamond K\phi \to K\phi$ (for mathematical sentences ϕ) can be derived using fewer yet not stronger assumptions. (Admittedly, the derivation depends on (1),[8] but (1) is weaker than RM_K, which is used in the proof of Corollary 2 and, moreover, it is used in the derivation of Lemma 1, which Marton uses to argue against $K\Diamond\phi \to \Diamond K\phi$.)

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[6] Marton does not use RM_K explicitly. He seems to tacitly appeal to K_K (i.e., $K(\phi \to \psi) \to (K\phi \to K\psi)$) or perhaps on a weakening of K_K and RM_K, i.e., $\vdash \phi \to \psi \Rightarrow \vdash K(\phi \to \psi) \to (K\phi \to K\psi)$.

[7] See Horsten (1994) for the following inference rule for a system of modal-epistemic arithmetic: for every provable formula it is possible that there is a mathematician who has a proof of it.

[8] Or weaker versions of those principles (cf. footnote 3).

3 Hard to avoid modal-epistemic collapse?

Marton's third main claim is that some of the standard strategies used to avert the Church-Fitch paradox are either powerless against the modal-epistemic collapse argument or they come at a significant cost. He discusses three kinds of strategies and some of their implementations for avoiding the Church-Fitch paradox provable from the standard antirealist theory of truth. These strategies are the revision strategy, including the intuitionistic revision of Williamson (1982) and the paraconsistent revision of Beall (2000), the restriction strategy, implemented by Tennant (1997) and Dummett (2001), and the reformulation strategy of Edgington (1985).[9] Obviously, any strategy that blocks the Church-Fitch paradox of knowability (Theorem 3) also blocks Corollaries 1 and 3. Let us see how Marton's results fare.

First, there is Theorem 1. Recall that Marton's argument against $K\Diamond\phi \rightarrow \Diamond K\phi$ was targeted at the instantiation $K\Diamond(p \wedge \neg Kp) \rightarrow \Diamond K(p \wedge \neg Kp)$ and it invoked the Church-Fitch unknowability lemma (Lemma 1). As a result, any of the revision and restriction strategies that (i) block the proof of Lemma 1 and/or (ii) block the instantiation of (A-R) with $p \wedge \neg Kp$ in the proof of Theorem 1 also block Marton's counterargument. These include the paraconsistent revision strategy of Beall (2000) and the restriction strategies of Tennant (1997) and Dummett (2001), but not the intuitionistic revision strategy of Williamson (1982).

Second, there is Theorem 2 and Corollary 2. The proofs of Theorem 2 and Corollary 2 make use of neither double negation elimination nor *reductio ad absurdum*, so it is immune to the intuitionistic and the paraconsistent revision strategies. The proof of Theorem 2 makes uses of instantiations of (A-R) with $\Diamond\phi$ and $K\Diamond\phi$ and (A-$\mathrm{R_{conv}}$) with $K\Diamond\phi$, with ϕ a mathematical sentence for the purpose of Corollary 2. This makes it vulnerable to a restriction strategy discussed by Tennant (2009), namely the restriction of (A-$\mathrm{R_{conv}}$) to non-epistemic formulas for a reason that will become clear in a moment. In any case, the obvious line of attack against Theorem 2 and Corollary 2 is to deny (A-$\mathrm{R_{conv}}$).

Williamson (1992, p. 67) gives a counterexample to (A-$\mathrm{R_{conv}}$) and he offers a diagnosis:

> For consider a statement p which in a broad sense is contingent and decidable: for example, that the number of tennis balls in my garden today, 4 July 1990, is even. In some broadly possible situations p is true and verified in the sense of K; in others its negation is. If $\Diamond$ were read correspondingly, $\Diamond Kp \,\&\, \Diamond K\neg p$ would hold; but [(A-$\mathrm{R_{conv}}$)] reduces it to the contradition $p \,\&\, \neg p$. The problem arises because situations in which the number of tennis balls is other than it actually is

[9] I mention only the approaches explicitly discussed by Marton. There are other approaches, e.g., the dynamic-epistemic approach of van Benthem (2004) and Balbiani et al (2008), which is a combination of the restriction and the reformulation strategy. Note that on this approach knowability (understood as known after an announcement) is not factive, which Marton presumably reckons to be pricey. See footnote 14 for another example of the reformulation strategy.

> have been counted as possible. Roughly speaking, only those situations count towards $\Diamond$ in which the facts (the number of tennis balls) are the same but knowledge of them may differ. Since our knowledge of a fact is itself a fact, the distinction is not an easy one to draw ...

Similarly, Tennant (2009, p. 225) states that:

> ... to the extent that $\Diamond K$ is factive, $\Diamond$ is not to be analyzed as the familiar alethic modal operator. Its contribution to truth- or assertability-conditions of sentences in which it is prefixed to K will have to be elucidated in terms of possibilities of investigative outcomes, at future times, within the actual world. Those possibilities will be strongly constrained by relevant contingencies in the actual world.

Given that acquiring knowledge often requires certain investigative acts (e.g., going into the garden and collecting all the tennis balls there), the non-epistemic facts are often going to be different (e.g., the investigator is going to be at a different time and place). If $\Diamond$ ranges only over situations in which only epistemic facts differ, then those situations are outside the scope of $\Diamond$. However, this makes (A-R) extremely implausible: it requires the existence of situations in which knowledge magically appears without any non-actual investigative acts that lead to that knowledge. If $\Diamond$ ranges over situations that may differ (if only slightly) with respect to the non-epistemic facts, then counterexamples to (A-$\mathrm{R_{conv}}$) can be given (e.g., one could know that one is doing certain things in a process of investigation that one is not actually doing).

Marton is right to point out that giving up on (A-$\mathrm{R_{conv}}$) comes with a price tag, since knowability (understood as possible knowledge) and truth no longer coincide. At this point the reformulation strategy comes into view. It promises to restore the coincidence of knowability (*not* understood as possible knowledge) and truth.

With the help of the actuality operator A, Edgington (1985) has proposed to reformulate (A-R) and (A-$\mathrm{R_{conv}}$) as follows:

(VA) $A\phi \to \Diamond KA\phi$

($\mathbf{VA_{conv}}$) $\Diamond KA\phi \to A\phi$

Note that ($\mathrm{VA_{conv}}$) is a theorem, given that $\Diamond A\phi \to A\phi$ is a theorem and that we have T_K and $\mathrm{RM}_\Diamond$.

Let us begin with some good news for Marton: we do have a counterpart to Theorem 1.

Theorem 4. $\mathbf{K}_\Box + (\mathrm{VA}), 4_\Diamond, \mathrm{T}_K \vdash K\Diamond A\phi \to \Diamond KA\phi$.[10]

Proof.

[10] Alternatively, replace (VA) and $4_\Diamond$ with $\Diamond A\phi \to A\phi$, $\mathrm{T}_\Diamond$ and RM_K.

1. $K\Diamond A\phi \to \Diamond A\phi$ $\quad$ T_K

2. $A\phi \to \Diamond KA\phi$ $\quad$ (VA)

3. $\Diamond A\phi \to \Diamond\Diamond KA\phi$ $\quad$ $\mathrm{RM}_\Diamond$, 2

4. $\Diamond\Diamond KA\phi \to \Diamond KA\phi$ $\quad$ $4_\Diamond$

5. $K\Diamond A\phi \to \Diamond KA\phi$ $\quad$ Taut., 1, 3, 4

$\square$

However, Marton's argument against $K\Diamond\phi \to \Diamond K\phi$ does not carry over to $K\Diamond A\phi \to \Diamond KA\phi$. Recall that his idea was to replace ϕ with $p \wedge \neg Kp$. But $\Diamond KA(p \wedge \neg Kp)$ is not contradictory, so we do not have a counterpart for the Church-Fitch unknowability lemma (Lemma 1).

Now comes the bad news. Marton (2019, fn. 19) says that:

> It seems fairly obvious that VA (and $\mathrm{VA}_{\mathrm{conv}}$) cannot prevent the collapse between $\Diamond KAp$ and $K\Diamond Ap$. But the real issue here is still the collapse between $\Diamond Kp$ and $K\Diamond p$.

However, VA (and $\mathrm{VA}_{\mathrm{conv}}$) *can* prevent the collapse between $\Diamond KAp$ and $K\Diamond Ap$ and between $\Diamond Kp$ and $K\Diamond p$. In order to show this I will use a model on which Edgington's antirealist theory of truth holds yet the aforementioned equivalences fail.

I will make use of simplified versions of the models developed by Rabinowicz and Segerberg (1994).[11] A Rabinowicz and Segerberg (1994)-style model or RS-model is a tuple $\langle W, R_E, R_M, V\rangle$, with W a non-empty set of worlds, R_M a two-place relation on W, and V a function from sentence letters and worlds to truth-values, and with R_E a two-place reflexive relation on $W \times W$ such that

$$\forall w, w', v, v' \left((\langle w, w'\rangle R_E \langle v, v'\rangle \wedge w = w') \to v = v' \right). \tag{3}$$

The relation $\mathfrak{M}, \langle w, w'\rangle \models \phi$ is defined inductively as follows:

- if ϕ is a sentence letter, then $\mathfrak{M}, \langle w, w'\rangle \models \phi$ iff $V(\phi, w) = 1$;
- if $\phi = \Diamond\psi$, then $\mathfrak{M}, \langle w, w'\rangle \models \phi$ iff $\mathfrak{M}, \langle v, w'\rangle \models \psi$ for at least one $v \in W$ such that $wR_M v$;

[11] The simplified versions can be found in: Heylen (2016, 2020a). One of the simplification is the following: Rabinowicz and Segerberg (1994) define V as a function from sentence letters and *pair of worlds* to truth values, whereas here V is defined as a function from sentence letters and *worlds* to truth values. The function V could instead have been defined in the same way as Rabinowicz and Segerberg (1994) do and then it could have been stipulated that V should agree on all pairs of worlds, if the first elements of the pairs of worlds are the same. In other words, the models used here are a special case of the models described by Rabinowicz and Segerberg (1994).

- if $\phi = K\psi$, then $\mathfrak{M}, \langle w, w' \rangle \models \phi$ iff $\mathfrak{M}, \langle v, v' \rangle \models \psi$ for every v, v' such that

$$\langle w, w' \rangle R_E \langle v, v' \rangle;$$

- if $\phi = A\psi$, then $\mathfrak{M}, \langle w, w' \rangle \models \phi$ iff $\mathfrak{M}, \langle w', w' \rangle \models \psi$;
- the other clauses are as expected.

A formula ϕ is strongly valid iff it is true at every pair of worlds in every RS-model, whereas a formula is only weakly valid iff it is true at every pair of identical worlds in every RS-model.

Note that ($\mathrm{VA}_{\mathrm{conv}}$) is strongly valid relative to these models, as it should be. Assume that $\mathfrak{M}, \langle w, w' \rangle \models \Diamond KA\phi$, with $\mathfrak{M}$ an RS-model and with $w, w' \in W$. Then $\mathfrak{M}, \langle v, w' \rangle \models KA\phi$, for at least one $v \in W$ with $wR_M v$. Given the reflexivity of R_E, it follows that $\mathfrak{M}, \langle v, w' \rangle \models A\phi$. Consequently, it is the case that $\mathfrak{M}, \langle w', w' \rangle \models \phi$. This entails that $\mathfrak{M}, \langle w, w' \rangle \models A\phi$. The strong validity of (VA) corresponds to the following frame condition (Heylen, 2020a):

$$\forall w, w' \exists v (wR_M v \wedge \forall u, u' (\langle v, w' \rangle R_E \langle u, u' \rangle \rightarrow u' = w')) . \tag{4}$$

The strong validity of the axioms of $\mathbf{S5}_K$ (which include T_K) and $\mathbf{S5}_\Box$ corresponds to respectively R_E and R_M being equivalence relations. (The rules RN_K and $\mathrm{RN}_\Box$ preserve strong validity, regardless of the frame conditions.)

Theorem 5. *There is an RS-model based on a frame on which (VA), (VA_{conv}) and the axioms of $\mathbf{S5}_K$ and $\mathbf{S5}_\Box$ are strongly valid and the model shows that*

1. *$\Diamond KA\phi \rightarrow K\Diamond A\phi$ is not weakly valid,*
2. *$\Diamond K\phi \rightarrow K\Diamond\phi$ is not weakly valid,*
3. *$K\Diamond\phi \rightarrow \Diamond K\phi$ is not strongly valid.*

Proof. Consider an RS-model $\mathfrak{M}$ with W containing exactly four worlds, w_1, w_2, w_3 and w_4. Let R_M be a reflexive relation on W and it is universal in the subsets $\{w_1, w_2\}$ and $\{w_3, w_4\}$.[12] (No other pairs of worlds belongs to the extension of R_M.) Next, let R_E be a reflexive relation on $W \times W$ and, in addition, let $\langle w_1, w_1 \rangle R_E \langle w_3, w_3 \rangle$ and $\langle w_3, w_3 \rangle R_E \langle w_1, w_1 \rangle$. (No other pair of pairs of worlds belongs to the extension of R_E.) So, $\langle w, w' \rangle R_E \langle u, u' \rangle$ iff (i) $w = u$ and $w' = u'$ or (ii) $w = w' = w_1$ and $u = u' = w_3$ or (iii) $w = w' = w_3$ and $u = u' = w_1$.

- Condition (3) is satisfied in cases (i), (ii) and (iii). In case (i), if $w = w'$, then by the symmetry and transitivity of identity it follows that $u = u'$. In cases (ii) and (iii), $w = w'$ and $u = u'$, so the condition is trivially satisfied.
- Condition (4) is satisfied for any $w \in W$.

[12] I am grateful to an anonymous reviewer for a suggestion that turned my original model in one based on a frame on which $\mathbf{S5}_K$ is valid.

- First, take any $w \in \{w_1, w_2\}$ and $w' \in W$. Let $v = w_2$. Since R_M is the universal relation on $\{w_1, w_2\}$, it follows that wR_Mv, for any $w \in \{w_1, w_2\}$. Moreover, by the stipulation on R_E, it follows that $\langle w_2, w'\rangle R_E \langle u, u'\rangle$ iff (i) $w_2 = u$ and $w' = u'$ or (ii) $w_2 = w' = w_1$ and $u = u' = w_3$ or (iii) $w_2 = w' = w_3$ and $u = u' = w_1$. Cases (ii) and (iii) are ruled out, because $w_2 \neq w_1$ and $w_2 \neq w_3$. It follows that $\langle w_2, w'\rangle R_E \langle u, u'\rangle$ iff $w_2 = u$ and $w' = u'$. Hence, if $\langle w_2, w'\rangle R_E \langle u, u'\rangle$, then $u' = w'$.
- Second, take any $w \in \{w_3, w_4\}$ and $w' \in W$. Let $v = w_4$. Since R_M is the universal relation on $\{w_3, w_4\}$, it follows that wR_Mv, for any $w \in \{w_3, w_4\}$. By the stipulation on R_E, it follows that $\langle w_4, w'\rangle R_E \langle u, u'\rangle$ iff (i) $w_4 = u$ and $w' = u'$ or (ii) $w_4 = w' = w_1$ and $u = u' = w_3$ or (iii) $w_4 = w' = w_3$ and $u = u' = w_1$. Cases (ii) and (iii) are ruled out, because $w_4 \neq w_1$ and $w_4 \neq w_3$. It follows that $\langle w_4, w'\rangle R_E \langle u, u'\rangle$ iff $w_4 = u$ and $u' = w'$. Hence, if $\langle w_4, w'\rangle R_E \langle u, u'\rangle$, then $u' = w'$.

- It is trivially the case that R_E and R_M are equivalence relations.

Finally, let $V(p, w_1) = V(p, w_2) = 1$ and $V(p, w_3) = V(p, w_4) = 0$.

1. $\mathfrak{M}, \langle w_1, w_1\rangle \models \Diamond KAp$ but $\mathfrak{M}, \langle w_1, w_1\rangle \not\models K\Diamond Ap$.

   (a) Note that $\mathfrak{M}, \langle w_2, w_1\rangle \models Ap$, because $\mathfrak{M}, \langle w_1, w_1\rangle \models p$, given the stipulation of V. Moreover, $\mathfrak{M}, \langle w_2, w_1\rangle \models KAp$ iff $\mathfrak{M}, \langle w_2, w_1\rangle \models Ap$, since

   $$\langle w_2, w_1\rangle R_E \langle u, u'\rangle$$

   iff $w_2 = u$ and $w_1 = u'$. Hence, $\mathfrak{M}, \langle w_2, w_1\rangle \models KAp$. Furthermore, due to the universality of R_M on $\{w_1, w_2\}$, it is the case that $w_1 R_M w_2$. Therefore, $\mathfrak{M}, \langle w_1, w_1\rangle \models \Diamond KAp$.

   (b) Note that $\mathfrak{M}, \langle w_3, w_3\rangle \not\models p$, given the stipulation of V. Therefore, there is no $w' \in W$ for which it is the case that $\mathfrak{M}, \langle w', w_3\rangle \models Ap$. Consequently, there is no $w' \in W$ such that $w_3 R_M w'$ and for which $\mathfrak{M}, \langle w', w_3\rangle \models Ap$. Hence, $\mathfrak{M}, \langle w_3, w_3\rangle \not\models \Diamond Ap$. Since $\langle w_1, w_1\rangle R_E \langle w_3, w_3\rangle$, it follows that $\mathfrak{M}, \langle w_1, w_1\rangle \not\models K\Diamond Ap$.

2. $\mathfrak{M}, \langle w_1, w_1\rangle \models \Diamond Kp$ but $\mathfrak{M}, \langle w_1, w_1\rangle \not\models K\Diamond p$.

   (a) $\mathfrak{M}, \langle w_1, w_1\rangle \models \Diamond Kp$, because $w_1 R_M w_2$ and $\mathfrak{M}, \langle w_2, w_1\rangle \models Kp$, which follows from $\mathfrak{M}, \langle w_2, w_1\rangle \models p$ and $\langle w_2, w_1\rangle R_E \langle u, u'\rangle$ iff $w_2 = u$ and $w_1 = u'$.

   (b) $\mathfrak{M}, \langle w_1, w_1\rangle \not\models K\Diamond p$, because $\langle w_1, w_1\rangle R_E \langle w_3, w_3\rangle$ and $\mathfrak{M}, \langle w_3, w_3\rangle \not\models \Diamond p$. The latter is the case since $\mathfrak{M}, \langle w_3, w_3\rangle \not\models p$ and $\mathfrak{M}, \langle w_4, w_3\rangle \not\models p$ (due to the fact that $V(p, w_3) = V(p, w_4) = 0$) and since $w_3 R_M v$ iff $v = w_3$ or $v = w_4$.

3. $\mathfrak{M}, \langle w_2, w_1\rangle \models K\Diamond(p \wedge \neg Kp)$ but $\mathfrak{M}, \langle w_2, w_1\rangle \models \neg\Diamond K(p \wedge \neg Kp)$.

(a) Note that $\mathfrak{M}, \langle w_1, w_1\rangle \models (p \wedge \neg Kp)$, because $V(p, w_1) = 1$ and $V(p, w_3) = 0$ with $\langle w_1, w_1\rangle R_E \langle w_3, w_3\rangle$. Therefore, $\mathfrak{M}, \langle w_2, w_1\rangle \models \Diamond(p \wedge \neg Kp)$. Since

$$\langle w_2, w_1\rangle R_E \langle u, u'\rangle$$

only if $u = w_2$ and $u' = w_1$, it follows that $\mathfrak{M}, \langle w_2, w_1\rangle \models K\Diamond(p \wedge \neg Kp)$.

(b) $\mathfrak{M}, \langle w_2, w_1\rangle \models \neg\Diamond K(p \wedge \neg Kp)$: left to the reader.

$\square$

So, neither do (VA) and ($\mathrm{VA}_{\mathrm{conv}}$) imply the equivalence of $\Diamond KAp$ and $K\Diamond Ap$ nor do they imply the equivalence of $\Diamond Kp$ and $K\Diamond p$, even when the very strong systems $\mathbf{S5}_K$ and $\mathbf{S5}_\Box$ are in the background. Edgington's antirealist can avoid modal-epistemic collapse, without any restrictions or logical revisions and without giving up on factivity.[13] This undercuts Marton's third main claim.[14]

4 Marton's notion of epistemic truth

Marton (2019, section 5) introduces a new kind of 'truth' operator T, which he defines as follows:

(MAR) $T\phi \leftrightarrow (\phi \wedge \Diamond K\phi)$.

People can give stipulative definitions as much as they like, but what is missing here is a theory. Marton mentions that one can avoid both the Church-Fitch epistemic collapse result and his own modal-epistemic collapse result. Of course, adding a stipulative definition like the one above to a given theory, which includes $\mathbf{K}_\Box$ extended with T_K, is conservative: one cannot prove more with such a definition than one can prove without it. Indeed, adding the above stipulative definition to a hard-core realist theory (extending the base theory) will not make any difference in terms of what that theory entails.

Let us contrast this with Dummett (2001)'s work. He also introduces a new 'truth' operator Tr operator, which he defines inductively as follows:[15]

[13] Williamson (1987, 2000) questions the existence of non-trivial non-actual knowledge about the actual world, while Edgington (2010) thinks that there is. Heylen (2020a) objects to Edgington's theory, because it comes with possible omniscience: there is an accessible state at which all truths are known. Schlöder (2019) reformulates Edgington's theory to address these concerns. Heylen (2020b) argues that Schlöder's version of the knowability thesis overgenerates knowledge.

[14] Another reformulation strategy has been pursued by Fuhrmann (2014), who uses the notion of 'potential knowledge'. This notion is expressed with the help of a primitive, unanalyzed operator, $\langle K\rangle$. With no modal ($\Diamond$) and no epistemic (K) operators as syntactical components of this operator, a modal-epistemic collapse result with that operator is not possible. One can also transform the model used in the proof of Theorem 5 into a model for potential knowledge, showing that Fuhrmann's theory is also free of modal-epistemic collapse. In the light of footnote 13, it is noteworthy that potential knowledge is an 'intra-world affair'.

[15] I am leaving out the clauses for the quantifiers.

1. $Tr(\phi)$ iff $\Diamond K\phi$, if ϕ is a 'basic statement';

2. $Tr(\phi$ and $\psi)$ iff $Tr(\phi) \wedge Tr(\psi)$;

3. $Tr(\phi$ or $\psi)$ iff $Tr(\phi) \vee Tr(\psi)$;

4. $Tr($if ϕ then $\psi)$ iff $Tr(\phi) \to Tr(\psi)$;

5. $Tr($ it is not the case that $\phi)$ iff $\neg Tr(A)$,

with the rules of intuitionistic logic governing the operators on the right-hand-side. The above is nothing but a definition. We could compare the definition of T with the definition of Tr, but the main point that I want to make is that Dummett does not stop by giving a definition of a new truth operator. Indeed, Dummett follows it up with a substantial thesis:

(+) $\phi \to Tr(\phi)$.

It is not that Marton thinks that $p \to Tp$ should be valid. If it were, one could derive $p \to \Diamond Kp$ as well, and the Church-Fitch paradox would re-emerge.

5 Conclusion

Throughout the paper I have addressed the three main claims made by Marton (2019).

Marton's first main claim is that (A-R) and (A-$\mathrm{R}_{\mathrm{conv}}$), in combination with certain modal and epistemic principles, entail the equivalence between $K\Diamond\phi$ and $\Diamond K\phi$ (Theorem 1 and Theorem 2). We have seen that this equivalence is already a corollary (Corollary 1) of the Church-Fitch paradox of knowability (Theorem 3), which depends on fewer yet not stronger assumptions or assumptions. (Admittedly, the derivation depends on (1), but Marton makes use of that principle or a principle that entails it to back up his second main claim.)

Marton's second main claim is that there are counterexamples to the equivalence of $K\Diamond\phi$ and $\Diamond K\phi$. We have seen that Marton's argument against $K\Diamond\phi \to \Diamond K\phi$ depends on the Church-Fitch lemma of unknowability (Lemma 1). Marton argues against $\Diamond K\phi \to K\Diamond\phi$ by deriving from it and the necessity of mathematical truth that $\Diamond K\phi \to K\phi$, for mathematical sentences ϕ. Again this result can also be obtained by appealing to to the Church-Fitch paradox of knowability (Corollary 3), while using fewer yet not stronger assumptions. (The derivation depends on (1), but the proof of corollary 2 makes use of RM_K, which entails (1), and Marton makes use of (1) to argue against $K\Diamond\phi \to \Diamond K\phi$.)

The upshot of my first two responses is that Marton's paper does not present any seriously novel challenge to anti-realism not already found in the Church-Fitch result.

Marton's third main claim is that the usual strategies to deal with the Church-Fitch paradox either do not work against his modal-epistemic collapse argument or only offer a pricey solution. It turns out that Edgington (1985)'s (VA) and ($\mathrm{VA}_{\mathrm{conv}}$), which are reformulations of (A-R) and ($\mathrm{A\text{-}R}_{\mathrm{conv}}$), entail $K\Diamond A\phi \to \Diamond KA\phi$ (Theorem 4), but Marton's counterargument does not work against this consequence. Moreover, there can be neither a proof of the entailment of $K\Diamond Ap$ from $\Diamond KAp$ nor a proof of the equivalence of $\Diamond Kp$ and $K\Diamond p$ from Edgington (1985)'s (VA) and ($\mathrm{VA}_{\mathrm{conv}}$) extended with $\mathbf{S5}_K$ and $\mathbf{S5}_\Box$ (Theorem 5). This result is without logical revisions, restrictions or giving up factivity.

Finally, Marton's own contribution is just a stipulative definition that can be conservatively added to a any theory containing a base theory for modality and knowledge, even if it is a hard-core realist theory (section 4).

References

Balbiani P, Baltag A, van Ditmarsch H, Herzig A, Hoshi T, de Lima T (2008) 'Knowable' as 'Known After an Announcement'. Review of Symbolic Logic 1(3):305–334, DOI 10.1017/s1755020308080210

Beall JC (2000) Fitch's proof, verificationism, and the knower paradox. Australasian Journal of Philosophy 78(2):241–247, DOI 10.1080/00048400012349521

van Benthem J (2004) What one may come to know. Analysis 64(2):95–105, DOI 10.1093/analys/64.2.95

Church A (2009) Referee reports on Fitch's "Definition of Value". In: Salerno J (ed) New Essays on the Knowability Paradox, Oxford University Press, pp 13–20

Cresswell MJ, Hughes GE (1996) A New Introduction to Modal Logic. Routledge

Dummett M (2001) Victor's error. Analysis 61(1):1–2, DOI 10.1093/analys/61.1.1

Edgington D (1985) The paradox of knowability. Mind 94(376):557–568, DOI 10.1093/mind/XCIV.376.557

Edgington D (2010) Possible knowledge of unknown truth. Synthese 173(1):41–52, DOI 10.1007/s11229-009-9675-9

Fitch F (1963) A logical analysis of some value concepts. Journal of Symbolic Logic 28(2):135–142, DOI 10.2307/2271594

Fuhrmann A (2014) Knowability as potential knowledge. Synthese 191(7):1627–1648, DOI 10.1007/s11229-013-0340-y

Heylen J (2016) Counterfactual theories of knowledge and the notion of actuality. Philosophical Studies 173(6):1647–1673, DOI 10.1007/s11098-015-0573-3

Heylen J (2020a) Factive knowability and the problem of possible omniscience. Philosophical Studies 177(1):65–87, DOI 10.1007/s11098-018-1180-x

Heylen J (2020b) Counterfactual knowledge, factivity, and the overgeneration of knowledge. Erkenntnis pp 1–21, DOI 10.1007/s10670-020-00300-w

Horsten L (1994) Modal-epistemic variants of Shapiro's system of Epistemic Arithmetic. Notre Dame Journal of Formal Logic 35(2):284–291, DOI 10.1305/ndjfl/1094061865

Jago M (2010) Closure on knowability. Analysis 70(4):648–659, DOI 10.1093/analys/anq067

Mackie JL (1980) Truth and knowability. Analysis 40(2):90–92, DOI 10.1093/analys/40.2.90

Marton P (2019) Knowing possibilities and the possibility of knowing: A further challenge for the anti-realist. Erkenntnis pp 1–12, DOI 10.1007/s10670-019-00115-4

Rabinowicz W, Segerberg K (1994) Actual truth, possible knowledge. Topoi 13(2):101–115, DOI 10.1007/BF00763509

San WK (2020) Fitch's paradox and level-bridging principles. Journal of Philosophy 117(1):5–29, DOI 10.5840/jphil202011711

Schlöder JJ (2019) Counterfactual knowability revisited. Synthese pp 1–15, DOI 10.1007/s11229-019-02087-y

Tennant N (1997) The Taming of the True. Oxford University Press

Tennant N (2009) Revamping the restriction strategy. In: Salerno J (ed) New Essays on the Knowability Paradox, Oxford University Press

Williamson T (1982) Intuitionism disproved? Analysis 42(4):203–7, DOI 10.1093/analys/42.4.203

Williamson T (1987) On the paradox of knowability. Mind 96(382):256–261, DOI 10.1093/mind/XCVI.382.256

Williamson T (1992) On intuitionistic modal epistemic logic. Journal of Philosophical Logic 21(1):63–89

Williamson T (2000) Knowledge and its Limits. Oxford University Press

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