The Philosophical Quarterly, Vol. 54, No. 214 *January 2004*
ISSN 0031–8094

ON THE SENSE AND REFERENCE OF A LOGICAL CONSTANT

BY HAROLD HODES

Syntax precedes truth-theoretic semantics when it comes to understanding a logical constant. A constant in a language is logical iff its sense is entirely constituted by certain deductive rules. To be sense-constitutive, deductive rules governing a constant must meet certain conditions; those that do so are sense-constitutive by virtue of understanders' conditional dispositions to feel compelled to accept certain formulae. Acceptance is a cognitive formula-attitude. Since acceptance requires understanding, and a formula can contain more than one occurrence of logical constants, this account involves a 'local holism', but no circularity. I argue that no logical constant is ambiguous between a classical and a constructive sense; but I allow that one constant may have distinct classical and constructive 'semantic values'. A logical constant's sense helps to determine its semantic value, but only together with certain constraints on satisfaction and frustration; it seems that the latter must include 'convention T'-style schemata.

Logicism is, roughly speaking, the doctrine that mathematics is fancy logic. So getting clear about the nature of logic is a necessary step in an assessment of logicism. Logic is the study of logical concepts, how they are expressed in languages, their semantic values, and the relationships of these things with the rest of our concepts, their linguistic expressions and their semantic values. A logical concept is what can be expressed by a logical constant in a language. So the question 'What is logic?' drives us to the question 'What is a logical constant?'. Although what follows contains some argument, limitations of space constrain me in large part to express my credo on this topic with the broad brush of bold assertion, and some promissory gestures.

I

Logical expressions are of three sorts: variables, logical constants, and indicators of logical force or speech act. I shall set aside variables, all of which are logical expressions. I shall also set aside indicators (expressions like 'therefore', 'assume that', or 'given a' prefixed to a fresh free variable).

Thesis 1. Logical constants in a language constitute a natural semantic kind. Given a language L and a constant c in L's lexicon, c is logical iff c's sense is entirely constituted by certain of its purely syntactic roles in argumentation in L.

In particular, the distinction between logical and other constants is not merely pragmatic or conventional, to be drawn in the context of some logical or semantic enquiry merely to indicate that one will treat the expressions one calls 'logical' in a distinctive way. I reject the possibility envisaged by Tarski that 'the division of terms into logical and extra-logical' is 'in greater or less degree arbitrary'.[1] (After giving an excellent presentation of the history of the notion of logical-constanthood, Gomez-Torrente concludes that the project of explicating this notion, at least in terms of 'unexplicated semantic or epistemic properties ... may be hopeless'.[2] Thesis 1 is not in these terms: e.g., on my account the 1-place connective 'All widows are female and ...' is not a logical constant.)

Argumentation is reasoning, or expression of reasoning, in language. Can thesis 1 be restricted to demonstrative (i.e., deductive) argumentation? If so, c's role in demonstrative argumentation in L determines c's role in default, statistical and abductive argumentation, and in any other species of non-demonstrative argumentation I have missed. Perhaps this is so, but in this paper I restrict my attention to c's role in demonstrative argumentation.

An expression's sense in a language L is a concept; if not ambiguous, it is uniquely correlated to conditions under which a fluent understander grasps the sense of that expression, as an expression of L. (Fluency here merely rules out understanding by translation into another language.) For my purposes, I shall identify grasping its sense in L with understanding its occurrences in statements in L. Of course this is rough: grasp of sense is the core of linguistic understanding, but is not all of it – there is grasp of connotation, force-indication, indications of non-literal use, and perhaps more. There are degrees of grasp of a sense. I shall say that someone's grasp is 'adequate' if it suffices for day-to-day communicative competence. Grasp of sense is a standing mental state; when one perceives or thinks of an expression whose sense one grasps, that mental state may interact with this perception or thought to produce an occurrent mental state, which I shall refer to as 'comprehension' of that expression.

An argument in L is constructed from inferences, each itself an argument with no proper subarguments in L. An expression's role in argumentation is

[1] See A. Tarski, *Logic, Semantics, Metamathematics* (1935) (Oxford UP, 1956), pp. 419–20.

[2] M. Gomez-Torrente, 'The Problem of Logical Constants', *Bulletin of Symbolic Logic*, 8 (2002), pp. 1–37, at p. 32.

codified by certain rules. I shall understand a rule to be deductive iff it is insensitive to context, content-neutral and indefeasible. Context-insensitivity needs no explanation. Content-neutrality of a rule is a matter of what counts as an instance of that rule in a given language: see §V below. Indefeasibility excludes default rules. Thesis 1 needs further articulation:

Thesis 1′. The sense of a logical constant *c* in *L* is constituted by a [not 'the'] set **R** of syntactic deductive rules that govern *c* in *L*, i.e., for understanders of *L*.

At the risk of sounding like the poor linguist's Christopher Peacocke, I shall say that a rule *R* overtly primitively governs *c* for an *L*-understander *S* iff under normal conditions *S* is disposed to find inferences in *L* which instantiate *R* primitively compelling, by virtue of their being instances of *R*.[3] I shall fill out this definition in the next section. Let *R* tacitly primitively govern *c* for *S* iff, under normal learning conditions, *S* is disposed to learn to find inferences in *L* which instantiate *R* overtly primitively compelling, again by virtue of their being instances of *R*, and without the distinctive cognitive process of adding a homonym to *S*'s lexicon.

Thesis 2. (1) If **R** is the set of rules that constitute *c*'s sense in *L*, fully grasping *c*'s sense in *L* is the mental state that would make its bearers subjects for whom members of **R** overtly primitively govern *c*.

(2) There is a privileged non-empty $\mathbf{R}_0 \subseteq \mathbf{R}$ whose members overtly govern *c*, making $\mathbf{R}_0$ the set of rules that overtly constitute *c*'s sense in *L*. Setting $\mathbf{R}_1 = \mathbf{R} - \mathbf{R}_0$, the members of $\mathbf{R}_1$ tacitly govern *c* in *L*. Adequately grasping *c*'s sense in *L* is the mental state that would make its bearers subjects for whom members of $\mathbf{R}_0$ overtly primitively govern *c*'s sense, and members of $\mathbf{R}_1$ tacitly primitively govern *c*.

(3) $\mathbf{R}_0$ determines $\mathbf{R}_1$ (by a constraint that I shall get to in §IX).

The following further articulates thesis 1.

Thesis 1′′. The following are materially equivalent:

(i) there is a non-empty set **X** of syntactic deductive rules that meets certain conditions (to be specified in §IX below) such that a full grasp of *c*'s sense in *L* is a mental state that would make its bearers subjects for whom members of **X** overtly primitively govern *c*;

(ii) there are disjoint sets $\mathbf{X}_0$ and $\mathbf{X}_1$ of syntactic deductive rules, with $\mathbf{X}_0$ non-empty and meeting certain conditions (to be specified in §IX), such

[3] See C. Peacocke, 'Understanding Logical Constants', *Proceedings of the British Academy*, 73 (1987), pp. 153–200, and *A Study of Concepts* (MIT Press, 1992), pp. 143–5, for the point of the 'by virtue of' clause. Peacocke discusses thought, but his arguments carry over to linguistic understanding.

that an adequate grasp of *c*'s sense in *L* is a mental state that would make its bearers subjects for whom members of $\mathbf{X}_0$ overtly primitively govern *c* and members of $\mathbf{X}_1$ tacitly primitively govern *c*;

(iii) *c* is a logical constant of *L*. (Dropping the 'certain conditions' opens up the possibility that *c* is what some might call a defective logical constant, and others a meaningless expression, e.g., Prior's 'tonk'.)

Ontological relativity is the doctrine that the range of first-order variables is relative to a framework, language, conceptual scheme or postulational situation. Applied to variable-binding logical constants, thesis 1 and its above elaborations are incompatible with ontological relativity, at least if the sense of such a constant uniquely determines the range of the variables it binds. I am inclined to embrace that 'if'-clause, and so to reject ontological relativity.

II

To characterize deductive rules, I must deal with two kinds of inferences.

A formula-inference in *L* goes from a set Δ of formulae to a set Γ of formulae, all in *L*. I shall represent such an inference as $\Delta \Rightarrow \Gamma$; if $\Gamma = \{\phi\}$, I shall omit the curly brackets, as is customary. ('$\Rightarrow$' is a function-constant added to English to form terms that designate inferences when completed by appropriate terms on the left and right. Neither $\Delta \Rightarrow \Gamma$ nor $\Delta \Rightarrow \phi$ is a linguistic expression; so corner-quotes around '$\Delta \Rightarrow \Gamma$' or '$\Delta \Rightarrow \phi$' would be incorrect. One could define inferences to be ordered pairs, so that $\Delta \Rightarrow \Gamma = <\Delta, \Gamma>$ and $\Delta \Rightarrow \phi = <\Delta, \phi>$.) As Gentzen was the first to appreciate, the phenomenon of discharging assumptions in ordinary reasoning makes it useful to consider inferences from formula-inferences to formula-inferences; I shall call them sequent-inferences.

My way of construing what it is to find an inference compelling is sentimental. For *S* to find a single-conclusion formula-inference $\Delta \Rightarrow \phi$ overtly compelling is (1) for *S* to be disposed to feel compelled to accept ϕ given that *S* accepts Δ and comprehends ϕ; and (2) if $\phi \notin \Delta$, for that feeling to be brought about by a process (2.1) initiated by *S*'s acceptance of Δ and *S*'s comprehension of ϕ; and (2.2) not depending on *S*'s prior acceptance of ϕ. Here, to accept a set of formulae Δ is to accept each member of Δ, all at the same time.

The definition of finding $\Delta \Rightarrow \phi$ overtly primitively compelling adds to (2) that the relevant process (2.3) does not involve any further reasoning on *S*'s part. Of course *S*'s feeling compelled to accept ϕ can be overdetermined; the above condition concerns one process that is causally sufficient for feeling compelled to accept ϕ.

The definition of finding $\Delta \Rightarrow \phi$ overtly compelling [overtly primitively compelling] by virtue of being an instance of a given rule adds to (2) that the relevant process (2.4) depends on *S*'s sensitivity to the fact that $\Delta \Rightarrow \phi$ is an instance of that rule. (This idea is Peacocke's response to 'Kripkenstein's' worries; see fn. 3 above.)

The corresponding notion for multiple-conclusion formula-inferences involves rejection as well as acceptance; I shall set it aside for this paper. (The key idea for $\Delta \Rightarrow \Gamma$: given that *S* accepts Δ, *S* would feel compelled not to reject all members of Γ.)

This is only a first try, at least if *L* is a social language rather than an idiolect. A fuller characterization will also consider *S*'s dispositions to accept corrections, and recognize others' errors, with regard to the inferences which *S* accepts, where activation of these dispositions also involves sensitivity to the inferences' being instances of given rules.

I shall treat acceptance as occurring in a specious present in which the subject can accept every member, keeping them all 'in mind', with no shift of context. What if Δ is large? Then simultaneous acceptance might be impossible for *S*, as *S* actually is: for example, *S*'s brain might not be big enough. No matter: *S* would be in the triggering-condition provided that *S* were built significantly differently; we need not require it to be feasible for *S* to accept Δ. ('Kripkenstein' might object that we would have no idea what *S* would do if *S* were so different from what *S* actually is that *S* could accept a large Δ. I disagree: extrapolating from what *S* does when accepting small Δs gives us some basis on which to form rational beliefs about what *S* would do if *S* were to accept a large Δ. Be that as it may, the force of the objection is not completely clear if one does not buy an analysis of dispositions in terms of conditionals.[4])

Manifestation of a disposition can be blocked: all sorts of psychological factors may obstruct *S*'s feeling compelled to accept ϕ. In many such cases *S* would at least experience cognitive dissonance. Furthermore, *S* may feel compelled to accept ϕ but still not do so. Does this account imply that if *S* grasps the sense of *L*'s logical constants, *S* will be disposed to feel compelled to accept the conclusion of any complicated deductively correct argument, given that *S* accepts its premises? No. Suppose *S* is disposed to feel compelled to accept ϕ_1 conditionally on accepting ϕ_0, and is disposed to feel compelled to accept ϕ_2 conditionally on accepting ϕ_1. *S* need not be disposed to feel compelled to accept ϕ_2 given that *S* accepts ϕ_0. Suppose *S* does accept ϕ_0, and so feels compelled to accept ϕ_1; *S* might not give in to that feeling, and so might not trigger the second disposition, and so might not feel compelled to accept ϕ_2. Or perhaps *S* does accept ϕ_1, but this somehow destroys

[4] See M. Fara, 'Dispositions and Habituals', forthcoming in *Noûs*.

the second disposition. Or perhaps it merely weakens it, so that *S* is disposed to feel compelled to accept ϕ_2 given that *S* accepts ϕ_0, but this disposition is significantly weaker than the two first-mentioned dispositions; in that case a sufficiently longer chain might not be associated with a disposition of *S* to feel compelled to accept some ϕ_n given that *S* accepts ϕ_0.

Now for a look at acceptance. At its most straightforward, acceptance is an attitude towards statements in a given language, where a statement is a sentence, and so a syntactic object, supplemented with a 'reading', i.e., disambiguated and with indexical parameters tied to appropriate contextually determined values. (Thus a statement has its truth-conditions necessarily.) Of course, acceptance is relative to a language. As I here understand it, acceptance is not a propositional attitude, since propositions are not syntactic objects. When one believes the content of a statement – the proposition it expresses, what it 'says' – one accepts that statement. But there is reason to allow for accepting formulae with free variables. (A formula is usually understood to be an 'open sentence', i.e., either a sentence or the result of replacing some occurrences of constants in a sentence by free occurrences of variables of appropriate type. I shall understand a formula to be an 'open statement', i.e., either a statement or the result of carrying out such replacements on a statement.) In thinking through an argument formalized as a Natural Deduction derivation, one might accept a formula ϕ containing free occurrences of variables (what some call 'parameters') that are not assigned any values; in this case ϕ does not express a proposition. We frequently pretend that we have been 'given', or have ourselves 'fixed', values for variables occurring free in ϕ; but this is heuristic patter. (I reject the thesis that every entry in an argument expresses a proposition; the entries with free variables merely express conditions.) So in full generality, acceptance is an attitude towards formulae.

Acceptance is a cognitive, not a behavioural, relation. One should think of accepting ϕ as consisting in an act of comprehending ϕ, as a formula of *L*, that elicits an act of inward, and perhaps also outward, affirmation directed towards ϕ. I shall suppose that this is unproblematic for atomic formulae. I shall use the notion of acceptance of formulae of *L*, some of which contain occurrences of *c*, to characterize grasping the sense of a logical constant *c* in *L*. So grasp of *c*'s sense in *L* is tied by a 'local holism' to grasp of a range of formulae of *L*. And if *c* is not *L*'s only logical constant, some of the relevant formulae contain other logical constants; so this 'local holism' involves the grasp of the senses of all of *L*'s other logical constants. To show that this is not a vicious circularity, I shall need to Ramseyfy. The details which follow are somewhat digressive; the impatient reader may skip ahead to the last paragraph of the following section.

III

First, I shall suppose that c is the only logical constant in L. Suppose that S grasps the sense of formula ϕ, which I shall abbreviate as 'S s-grasps ϕ'. This is to say that S is in a standing mental state s, s = s-grasp of ϕ, that is relational with respect to ϕ, and perhaps with respect to other things as well. I take it that s either is, or is constituted by, S's being in a bunch of substates which are themselves standing mental states of S, and that among them is S's s-grasp of each constituent of ϕ; if c is a constituent of ϕ, s-grasp of c is a substate of s. Let p be the psychological process-type whose tokens in S would consist of S's thinking of or perceiving ϕ, this event interacting with s, leading S to regard ϕ with inner affirmation. (In this process, S enters the occurrent state of comprehending ϕ.) Let '$\mathrm{M}(x,c)$' abbreviate 'x is a mental state relational with respect to c'. So certainly M(s-grasp of c,c).

Assume that $\mathrm{M}(x,c)$; I shall define a state $\mathbf{s}(x,\phi)$ and then relations $\mathrm{G}(x)$ and $\mathrm{A}(x)$ that might hold between a subject S and formula ϕ. Let $\mathbf{s}(x,\phi)$ be the state obtained by taking s and replacing s-grasp of c by x; so S would be in $\mathbf{s}(x,\phi)$ if S were in a standing mental state as much like being in s as is nomologically possible except that S is in x rather than s-grasping c. (If x is the state of grasping an alternative sense that c might have had, then $\mathbf{s}(x,\phi)$ is a state of grasping a sense that ϕ might have had. But if x is not a state of the former sort, $\mathbf{s}(x,\phi)$ is not a state of the latter sort; in general, $\mathbf{s}(x,\phi)$ may be a state of no psychological interest, one in which x interacts in no interesting ways with s-grasp of the constituents of ϕ other than c.) So $\mathbf{s}(x,\phi)$ is relational with respect to ϕ, and in particular, $s = \mathbf{s}$(s-grasp of c,ϕ). Let $\mathbf{p}(x,\phi)$ be the process obtained by taking p and replacing s-grasp of c by x; so S would undergo $\mathbf{p}(x,\phi)$ if S underwent a process as much like p as is nomologically possible except that S is in x rather than s-grasping c. (If x is the state of grasping an alternative sense for c, $\mathbf{p}(x,\phi)$ would terminate with S regarding ϕ with inward affirmation; otherwise, probably, $\mathbf{p}(x,\phi)$ would not be a coherent process at all.) So $p = \mathbf{p}$(s-grasp of c,ϕ).

With $\mathbf{s}(x,\phi)$ and $\mathbf{p}(x,\phi)$ specified, I shall drop the assumption that S s-grasps ϕ. Let S bear $\mathrm{G}(x)$ to ϕ iff S is in state $\mathbf{s}(x,\phi)$. In particular, the relation of s-grasping between subjects and formulae of L is the relation G(s-grasp of c). For S to bear $\mathrm{G}(x)$ to $\ulcorner c(\psi,\theta)\urcorner$ would be for S to bear $\mathrm{G}(x)$ to both ψ and θ, for S to be in state x, and for these three states to be appropriately interrelated – in whatever way S's s-grasp of ψ and θ would be interrelated to S's s-grasp of c were S to s-grasp $\ulcorner c(\psi,\theta)\urcorner$. $\mathrm{A}(x)$ is defined similarly, so that the relation of acceptance between subjects and formulae of L is the relation A(s-grasp of c). E.g., suppose that ψ and θ are atomic formulae and c is a 2-place connective. For S to bear A(x) to $\ulcorner c(\psi,\theta)\urcorner$ would be for S to bear G(x) to $\ulcorner c(\psi,\theta)\urcorner$, to think of or perceive $\ulcorner c(\psi,\theta)\urcorner$, and for the former states to interact with the latter event so as to initiate $\mathbf{p}(x,\ulcorner c(\psi,\theta)\urcorner)$.

Continuing under the assumption that M(x,c), I can now define S's bearing FOC(x) to $\Delta \Rightarrow \phi$. The idea is that bearing FOC(x) to a formula-inference is to Finding it Overtly Compelling as G(x) and A(x) are to s-grasping and acceptance. For S to bear FOC(x) to $\Delta \Rightarrow \phi$ is (1) for S to be disposed to feel compelled to bear A(x) to ϕ given that S bears A(x) to Δ; and (2) if $\phi \notin \Delta$, for this feeling to be brought about by a process (2.1) initiated at most by S's bearing A(x) to Δ and S's bearing G(x) to ϕ; and (2.2) not depending on S's prior bearing of A(x) to ϕ. So finding $\Delta \Rightarrow \phi$ overtly compelling is bearing FOC(s-grasp of c) to $\Delta \Rightarrow \phi$. The definition of bearing FOPC(x) to $\Delta \Rightarrow \phi$, the analogue with free x of Finding it Overtly Primitively Compelling, adds clause (2.3), requiring the process not to involve further reasoning. Similarly, the definition of bearing FOC(x), or FOPC(x), to $\Delta \Rightarrow \phi$, by virtue of its being an instance of a rule, adds clause (2.4).

Finally, given that $\mathbf{R}$ and $\mathbf{R}_0$ are as above, fully grasping c's sense in L is the mental state x such that (1) M(x,c); and (2) x would, under normal conditions, dispose any subject in x to bear FOPC(x) to instances of members of $\mathbf{R}$, by virtue of their being instances of those rules. Adequately grasping c's sense in L is the mental state such that (1) M(x,c); (2) x would, under normal conditions, dispose any subject S in x to bear FOPC(x) to instances of members of $\mathbf{R}_0$, by virtue of their being instances of those rules; and (3) x would under normal learning conditions dispose S to learn to bear FOPC(x) to instances of members of $\mathbf{R}_1$, by virtue of their being instances of those rules, given that this learning does not involve adding a homonym to S's lexicon.

If L contains other logical constants d, etc., the above remarks need to be revised as follows. Assume that M(y,d).... In place of the state $\mathbf{s}(x,\phi)$ and relations G(x) and A(x) we define the state $\mathbf{s}(x,y,\ldots,\phi)$, the process-type $\mathbf{p}(x,y,\ldots,\phi)$ and the relations G(x,y,...), A(x,y,...), and then FOPC(x,y,...). Then existential quantifications are added to the preceding condition, thus: full grasp of c's sense in L is the mental state x such that (1) M(x,c); and (2) for some mental states y, ..., M(y,d) and ... and x would, under normal conditions, dispose any subject S in x to bear FOPC(x,y,...) to instances of members of $\mathbf{R}$, by virtue of their being instances of those rules. A similar supplement applies to adequately grasping c's sense.

What if more than one x meet this condition (for full grasp or for adequate grasp)? I take it that a mental state is individuated by its functional role in a subject's psychology; the condition should specify such a role. If it appears that two distinct states satisfy the condition, that shows that they were not individuated at the right 'grain', and that they are merely two ways in which a single mental state is realized.

Obviously it is easier to think about all this in terms of grasping *c*'s sense and acceptance rather than in Ramseyfied terms; so I shall stick to that easier vocabulary from now on.

So much for formula-inferences; now for sequent-inferences. Finding a formula-inference (primitively) compelling is itself a kind of acceptance: when *S* finds a formula-inference $\Delta \Rightarrow \phi$ (primitively) compelling, I shall say that *S* (primitively) c-accepts $\Delta \Rightarrow \phi$ ('c' for 'compelling'). For *S* to find the sequent-inference $<\mathfrak{D};\Delta \Rightarrow \phi>$ compelling by virtue by virtue of its being an instance of a rule is (1) for *S* to be disposed to feel compelled to accept ϕ given that *S* accepts Δ and c-accepts all members of $\mathfrak{D}$; and (2) if $\phi \notin \Delta$, for that acceptance of ϕ to be brought about by a process (2.1) initiated by *S*'s acceptance of Δ, *S*'s c-acceptance of the members of $\mathfrak{D}$, and *S*'s grasp of ϕ's sense; (2.2) not depending on *S*'s prior acceptance of ϕ; (2.3) involving *S*'s sensitivity to the fact that $<\mathfrak{D};\Delta \Rightarrow \phi>$ is an instance of that rule; and (2.4) not involving any further reasoning on *S*'s part. (Does this amount to the following: '*S* is disposed to c-accept $\Delta \Rightarrow \phi$ given that *S* c-accepts all members of $\mathfrak{D}$, and for ... '? I am not sure, but I doubt it. There seems to be a difference between (1) being disposed to γ given α and β, and (2) being disposed to (be disposed to γ given α) given β.)

IV

Thesis 1 says that sense-constituting rules for logical constants are syntactic, making no direct reference to referential or pragmatic relations. If 'semantics' stands for the study of linguistic understanding, rather than the theory of reference and truth, semantics for logical constants is syntactic. A logical constant has its semantic value because of the sense-constitutive rules that govern it, not the converse. My slogan for logic is 'Syntax first'. 'Syntax first' is suggested by remarks of Wittgenstein, Carnap, Gentzen and Popper;[5] Kneale came closest to advocating it clearly: '... formal (or logical) signs are those whose full sense can be given by laying down rules of development for the propositions expressed by their help'.[6] More recently, Powers and Hacking have advocated it.[7]

[5] See K. Popper, 'New Foundations for Logic', *Mind*, 56 (1947), pp. 193–235.

[6] See W.C. Kneale, 'The Province of Logic', in H.D. Lewis (ed.), *Contemporary British Philosophy* (London: George Allen & Unwin, 1956), pp. 237–61, at pp. 254–5. Kneale's rules of development are rules of multiple-conclusion reasoning; in this his proposal differs from mine.

[7] See L.H. Powers, 'Knowledge by Deduction', *Philosophical Review*, 87 (1978), pp. 337–71; I. Hacking, 'What is Logic?', *Journal of Philosophy*, 86 (1979), pp. 285–319.

Carnap went wrong in claiming that any set of rules concerning an expression's role in argumentation could constitute a sense for that expression: '... let any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols'.[8] Dummett seems to think that Carnap's claim 'would necessarily be so' if thesis 2 were true: '... if a grasp of the meaning of a logical constant consisted solely in a readiness to acknowledge as correct those inferences involving it which exemplified one of the rules in some suitable basic set of such rules', then 'any arbitrary (consistent) set of rules of inference admits a range ... of meanings for the logical constants involved under which those and only those rules of inference that are derivable from that set are valid'.[9] He gives no argument for this strong claim, which I think false. Not just any rules for a constant, or even just any introduction and elimination rules, can be constitutive of sense; this is a lesson to be learned from Prior's 'tonk'.[10] (Here I assume that 'tonk' does not express a sense. Perhaps we could as well say that it expresses a defective sense, just as we might take 'true-in-English', as naïvely understood, to express a sense – an incoherent concept that can lead those who possess it into inconsistency. Does anything hang on which we say? I am not sure.) I shall come to the question of which sets of rules are sense-constituting in §IX.

Gentzen went wrong in suggesting that for all the logical constants he discussed, introduction rules have meaning-determining priority over elimination rules. At least I know of no adequate explication of this supposed priority. There is a respect in which the introduction rules for some logical constants, e.g., expressions of negation, disjunction and first-order existence, are cognitively prior to their elimination rules: the former rules overtly govern, and overtly constitute the senses of, such expressions, while for many competent speakers the latter rules only tacitly govern the expressions. But the reverse holds for other logical constants, e.g., expressions of material conditionality and first-order universality. Expressions of conjunction are rather special. For them, there is no such priority either way: all constituting rules are overtly constituting, and ordinary speakers fully grasp the sense of such expressions – 'and' is easy. I shall return to this in §V.

Dummett (p. 363) thinks that thesis 1′ requires that 'the condition for the correctness of an assertion made by means of a sentence containing a logical constant must always coincide with the existence of a deduction, by means of those [sense-constituting] rules to that sentence from correct premises

[8] See the foreword to R. Carnap, *The Logical Syntax of Language* (London: Routledge & Kegan Paul, 1937), p. xv.

[9] See M. Dummett, *Elements of Intuitionism* (Oxford UP, 1977), p. 362.

[10] See A.N. Prior, 'The Runabout Inference Ticket' (1960), repr. in P.F. Strawson (ed.), *Philosophical Logic* (Oxford UP, 1968), pp. 129–31.

none of which contains any ... logical constants'. Peacocke (*A Study of Concepts*, pp. 6–7) thinks that the concepts *conjunction* and *universal quantification over the natural numbers* are constituted by deductive rules. But he goes along with Dummett's requirement; this leads him (*Thoughts*, pp. 91–2) to deny that the concept of *negation* is constituted by deductive rules, maintaining that it is constituted by a broader class of rules that he calls 'transitional' rules. As I have said, I see no reason to accept Dummett's remarkably strong requirement. Of course I reject Peacocke's doctrine about negation.

Logical constants have their truth-relevant properties, including their 'semantic values' (following Dummett's Fregean approach to semantic theorizing), because of their roles in argument, not *vice versa*. This 'because' means 'in part because': certain constraints on truth and the like will matter as well (see §X). I reject the neo-Davidsonian doctrine according to which for a subject S to grasp the sense of an expression of conjunction in L, say, by '&', is for S to know (or if you prefer, cognize) that for any statements ϕ and ψ of L, $\ulcorner\phi \;\&\; \psi\urcorner$ is true in L iff ϕ is true in L and ψ is true in L (or more generally, the corresponding conditions for satisfaction of formulae). This doctrine seems to imply that for young children to come to understand 'and' in English, they first need to bear some cognitively significant relation to the property of being a true statement, or perhaps utterance, in English (perhaps under a mode of presentation of English as 'the language spoken around me'), as well as to material biconditionality, and to universality restricted to statements, or utterances, in English. Perhaps this can be less 'developed' than possession of a concept of being a true statement or utterance in English, of material biconditionality, etc.; this is the point of the fudge-word 'cognize'. But even this seems to ask a lot of an infant learning English – too much, in my opinion.

Davidson himself has been careful to avoid making such a substantive claim about actual linguistic understanding. According to him, the right sort of semantic theory of L is at least part of 'what must be said to give a satisfactory description of the competence of the interpreter'; this implies that 'some mechanism in the interpreter must correspond to the theory'.[11] This second claim, whatever it comes to, seems consistent with 'Syntax first'. The first claim raises the question of whether one 'must say' the important things supported by other things that one 'must say'. I have suggested that a satisfactory description of the competence of an understander (that is, a Davidsonian interpreter) requires us to attribute dispositions to conditional feelings of compelled acceptance. These facts at the level of sense have important consequences at the level of reference, the level described by a

[11] D. Davidson, 'A Nice Derangement of Epitaphs', repr. in A.P. Martinich (ed.), *The Philosophy of Language*, 3rd edn (Oxford: Blackwell, 1996), pp. 465–75, at p. 469.

Davidsonian semantic theory. Davidson's first claim is true of such a theory if a satisfying theoretical description of linguistic understanding must spell out these consequences about reference.

I shall digress to extend 'Syntax first' from logical concepts to our concepts of logical consequence and logical entailment: our 'original' concepts of these relations are also syntactic. In so far as the man in the street has a concept of logical entailment, it is the concept of the existence of a syntactic object: a demonstrative argument – one such that one would find compelling each inference in it – from premises to a conclusion. I do not deny that by the nineteenth century a semantic conception of logical entailment was in circulation among philosophers. But this was the product of proto-mathematical discovery, proto-mathematical in that it looked forward to rigorous semantic definitions (most importantly, the standard model-theoretic definitions) for formal languages that crystallized in Tarski's wake; this was an informative reconception of logical entailment, not the result of mere conceptual analysis. As for the informal, so for the rigorous: the relation between derivability in a Natural Deduction formalization of classical first-order logic and any of several semantic definitions of classical first-order consequence is like that between a formulation of nominal essence, or of the reference-fixing description on which we originally rely in our referential access to a natural kind, and a formulation of its real essence, e.g., between specifying the perceptual and operational properties by which people first fixed the reference of 'gold' and saying that gold is stuff whose atoms each contain 79 protons. (Ian Proops offers evidence that early in his career Russell thought of logical entailment syntactically, at least when he thought of it at all.[12] Proops also discusses a passage in which Frege characterizes what it is for a thought to 'be dependent on' a group of thoughts in terms of an iteration of making logical inferences; though not explicitly syntactic, the reliance on recursion suggests that he too was thinking of this syntactically.)

V

To my knowledge, the literature in logic on rules only considers rules governing particular languages. But it is important to conceive of a deductive rule, and with it of a logical concept, as a language-transcendent object. (This is especially important for variable-binding logical constants, e.g.,

[12] I. Proops, 'The *Tractatus* on Inference and Entailment', in E. Reck (ed.), *From Frege to Wittgenstein: Perspectives on Early Analytic Philosophy* (Oxford UP, 2002), pp. 283–307. Proops discusses Russell's 'Necessity and Possibility', in *The Collected Papers of Bertrand Russell*, ed. A. Urquhart, Vol. IV (London: Routledge, 1994), pp. 507–20, at pp. 513–5, especially paragraph 2 on p. 515.

expressions of quantification. For example, when we introduce a new name, we replace our language by an expanded language, including new instances of universal introduction; this does not mean that we have adopted a new rule of universal introduction.) A logical rule is realized in L by the set of its instances in L; L's assignment of logical constants to their senses and L's argument-conditions determine these realizations for sense-constituting rules.

A word on argument-conditions. To specify a language L as a formal object, one needs to specify the class of deductive arguments which L allows. This involves specifying the overall structure of these arguments, for example, whether L allows for multiple-conclusion formula-inferences. For this paper, this is all that matters regarding L's argument-conditions. (Argument-conditions also constrain an aspect of argument which is something like mood: I shall call it 'mode of acceptance'. To accept a statement is to accept it as actually true – this is the primary mode of acceptance. But in making suppositions, we can also accept a statement as true relative to non-actual possibilities. If bivalence fails, one might accept a statement as non-false rather than as true, either actually or relative to non-actual possibilities. An adequate understanding of intensional logical constants and multi-valued reasoning would require considering multi-modal inferences.[13] For this paper I confine my attention to the primary mode of acceptance.)

Some examples may help. I shall suppose that L allows only for single-conclusion formula-inferences, and that L's lexicon contains familiar constants; I shall consider some well known deductive rules, each involving only a single logical constant. The realization of conjunction introduction in L, $\&$-$intr_L$, is the set of sequent-inferences of the form

$$<\{\Delta_i \Rightarrow \psi_i : i = 0, 1\}; \Delta_0, \Delta_1 \Rightarrow \ulcorner(\psi_0 \ \& \ \psi_1)\urcorner>$$

for any $\Delta_i \subseteq Sent(L)$, $\psi_i \in Sent(L)$, $i = 0, 1$. Similarly the realization of conjunction elimination in L, $\&$-$elim_L$, is the set of sequent-inferences of this form:

$$<\{\Delta, \psi_0, \psi_1 \Rightarrow \psi_2\}; \Delta, \ulcorner(\psi_0 \ \& \ \psi_1)\urcorner \Rightarrow \psi_2>.$$

The realization of conditional introduction in L, $\supset$-$intr_L$, is the set of sequent-inferences of this form:

$$<\{\Delta, \psi \Rightarrow \theta\}; \Delta \Rightarrow \ulcorner(\psi \supset \theta)\urcorner>.$$

The realization of disjunction elimination in L, $\vee$-$elim_L$, is the set of sequent-inferences of this form:

$$<\{\Delta_0, \psi_0 \Rightarrow \phi; \Delta_1, \psi_1 \Rightarrow \phi\}; \Delta_0, \Delta_1, \ulcorner(\psi_0 \vee \psi_1)\urcorner \Rightarrow \phi>.$$

[13] I consider bi-modal arguments in my 'Individual-Actualism and Three-Valued Modal Logics', Part I, *Journal of Philosophical Logic*, 15 (1986), pp. 369–401, Part II, *JPL*, 16 (1987), pp. 17–63; and 'Three-Valued Logics', *Annals of Pure and Applied Logic*, 43 (1989), pp. 99–145.

To represent the language-transcendent introduction and elimination rules instanced here, it suffices to represent their premises schematically. The natural numbers 0 and 1 represent first place and second place for any binary formula connective, and 2 represents a place for the consequent of the conclusion of an elimination-rule for such a connective. In what follows, '/0' is a notation for <{}, 0>, '0, 1/2' for <{0, 1}, 2>, etc. The above language-transcendent rules may be represented thus: conjunction introduction = {/0; /1}; conjunction elimination = {0, 1/2}; conditional introduction = {0/1}; disjunction elimination = {0/2; 1/2}.

A logical concept, the sense of a possible logical constant, is also language-transcendent. In accord with thesis 1′, I suggest that a logical concept is also a mathematical object, one composed, so to speak, of deductive rules. For a constant of *L* to express a logical concept is for the rules making up that concept to constitute that constant's sense in *L* (construed in terms of overt and tacit primitive governance for *L*-understanders). The lexicon of a language *L* assigns each logical constant *c* to a logical concept, and thus to deductive rules **R**, or better, $<\mathbf{R}_0, \mathbf{R}_1>$. The rest of *L*'s lexicon and *L*'s formation-rules then determine the realizations for *L* of the rules in **R**. And now I am ready to propose

Thesis 3. Only rules that are, broadly speaking, introduction rules and elimination rules can constitute the sense of a logical constant.

(I say 'broadly speaking' because I do not know of any fully general characterization of what should count as an introduction or an elimination rule.)

The familiar introduction and elimination rules sit in a natural hierarchy, one that generates a corresponding hierarchy of logical concepts which involve those rules, and thus of corresponding logical constants. The rules of level 0 are distinguished by their 'separability': each concerns a single occurrence of a single constant, the main constant of an instance's 'main' or 'principal' formula. The Big Five connective concepts (absurdity, conjunction, disjunction, material conditionality, material biconditionality), with first-order universality and existence (as usually understood), are of level 0, because their introduction and elimination rules are all of level 0. (The usual rule for surd is an elimination rule; surd has no introduction rule.) Negation is intrinsically more complex than the Big Five: properly speaking, negation introduction involves surd, and its realization in *L* is the set of sequent-inferences of this form:

$$<\{\Delta, \psi_0 \Rightarrow \bot\}; \Delta \Rightarrow \ulcorner\neg\psi_0\urcorner>.$$

Negation, then, along with neither–nor and if–then–else, is of level 1. This step from level 0 to level 1 iterates, generating the mentioned hierarchy.

Do all introduction and elimination rules sit in this hierarchy? Or can there be logical 'local holisms'? What we make of free logics and singular existence-statements depends on this delicate and important question, which I shall put aside. (A predicate of singular existence should, I think, count as a logical constant; and it has an introduction rule that suits certain metaphysical tastes. But its elimination rules seem to be exactly the introduction and elimination rules for expressions of first-order existence and universality in a free logic. There is an interesting issue here.)

Besides introduction and elimination rules, there are rules that shed assumptions in formula-inferences. I call these 'thickening' rules, because adding assumptions to a formula-inference is sometimes called 'thinning'. Such a rule permits us to infer a formula-inference from formula-inferences with the same consequent whose antecedents include formulae not in the antecedent of the conclusion. Excluded middle (EM) and generalized excluded middle (GEM) are thickening rules. Their realizations in L are the sets of sequent-inferences of these forms respectively:

$$<\{\Delta_0, \ulcorner\neg\psi\urcorner \Rightarrow \phi; \Delta_1, \psi \Rightarrow \phi\}; \Delta_0, \Delta_1 \Rightarrow \phi>$$
$$<\{\Delta_0, \ulcorner(\psi \supset \theta)\urcorner \Rightarrow \phi; \Delta_1, \psi \Rightarrow \phi\}; \Delta_0, \Delta_1 \Rightarrow \phi>.$$

So members of EM_L are members of GEM_L with θ taken to be '$\perp$'. (Other thickening rules generate intermediate logics when added to intuitionistic logic.) One thickening rule is of great mathematical importance, but (to my knowledge) has received no attention. I call it the rule of infinite domains (ID). Its realization in L is the set of sequent-inferences of the following form: for a set of formulae Δ, any formula ϕ, any natural number n, any terms $\tau_0, \ldots, \tau_{n-1}$, and any variable v not occurring free in any member of Δ, in ϕ, or in any $\tau_{i<n}$,

$$<\{\Delta, \ulcorner\neg v = \tau_0\urcorner, \ldots, \ulcorner\neg v = \tau_{n-1}\urcorner \Rightarrow \phi\}; \Delta \Rightarrow \phi>.$$

The hierarchy of introduction and elimination rules extends to thickening rules: GEM is of level 0, since it concerns only expressions of material conditionality, which is of level 0; EM and ID are of level 1. (So GEM is more basic than EM; this should undercut the widespread idea that the fundamental proof-theoretic difference between intuitionistic and classical logic concerns negation; rather it concerns material conditionality.)

Vague Conjecture 1. An adequate account of introduction, elimination and thickening rules will show that they suffice to characterize uniquely the role of a logical constant in demonstrative argumentation.

All the rules considered above are purely syntactic. What rules are not? An example: 'true-in-English' may be thought of as a constant predicate, characterized by certain introduction and elimination rules. One might conceive of the realization of these rules in English as sets of sentence-inferences with members like the following, where ‘**a**’ names ‘Snow is white’:

$$\langle\{\Delta \Rightarrow \text{‘Snow is white’}\}; \Delta \Rightarrow \text{‘}\mathbf{a}\text{ is true-in-English’}\rangle$$
$$\langle\{\Delta, \text{‘Snow is white’} \Rightarrow \theta\}; \Delta, \text{‘}\mathbf{a}\text{ is true-in-English’} \Rightarrow \theta\rangle.$$

In full generality, the realization of these rules in English leads to inconsistency: ‘true-in-English’ is a defective. Various ways of constructing consistent semantics for ‘true-in-English’ amount to proposals to replace it with a non-defective constant. But the important point is this: these characterizing introduction and elimination rules are not purely syntactic, because whether a sequent-inference is an instance of these rules depends on semantic information. For the above example, we need to specify that ‘**a**’ designates ‘Snow is white’. ‘True-in-English’ is what I shall call a semi-logical constant.

For any L that we can translate into English, we can introduce the predicates ⌜true-in-L⌝ and ⌜false-in-L⌝ into English, governed by corresponding introduction and elimination rules. If L is well enough behaved, e.g., if it lacks semantic vocabulary, these constant predicates are not defective. The above point applies to satisfaction and frustration as well as to truth and falsity. For any formula ϕ of L, let $trans_\phi$ be its translation into English. The introduction and elimination rules for ⌜satisfies-in-L⌝ has instances like these, for any variable assignment A, a singular term in English σ referring to ϕ, a singular term α referring to A, and $trans'_\phi$ formed by replacing each free occurrence of each variable v free in $trans_\phi$ by a fresh singular term designating $A(v)$:

$$\langle\Delta \Rightarrow \text{trans}'_\phi; \Delta \Rightarrow \ulcorner\alpha \text{ satisfies-in-L } \sigma\urcorner\rangle$$
$$\langle\{\Delta, \text{trans}'_\phi \Rightarrow \theta\}; \Delta, \ulcorner\alpha \text{ satisfies-in-L } \sigma\urcorner \Rightarrow \theta\rangle.$$

Again these rules are not purely syntactic: in the generalization to satisfaction, we need to specify that σ and α designate ϕ and A respectively. Predicates like ⌜true-in-L⌝, ⌜satisfies-in-L⌝, etc., are also semi-logical constants.

If we restrict the introduction and elimination rules for ‘true-in-English’ and ‘false-in-English’ to instances in English, and require the terms of which these predicates are predicated in these instances to be quote-names, we would obtain purely syntactic rules, since we could state these restricted rules without attaching riders like ‘“**a**” designates “Snow is white”’. From this one might conclude that in a way ‘true-in-English’ and ‘false-in-English’ are logical constants after all. But this is an illusion. These rules would not constitute the sense of the predicate ‘true-in-English’; they would constitute the sense of a connective written in an odd way (attaching to a sentence by prefixing that sentence with a left quotation mark, and appending it with a right quotation mark followed by 'is true-in-English'). This connective would express the 1-place redundant operator, with the introduction rule {/0} and the elimination rule {0/1}. There is no purely syntactic way to make quote-names of sentences into singular terms designating the sentences within the quotation marks.

Thesis 4. Semi-logical constants of a language constitute a natural, though quite small, semantic kind; their senses are constituted at least in part by partially semantic introduction and elimination rules. They are all predicates.

One could replace the introduction and elimination rules for ⌜true-in-*L*⌝ and ⌜false-in-*L*⌝ by the instances of Tarski's schema Tr and the corresponding schema Fa, the latter with instances like

b is false-in-English iff snow is not white

where '**b**' designates 'Snow is white'. I think the rules are more fundamental: these rules could govern ⌜true-in-*L*⌝ even in a meta-language so impoverished that it had no way to express material conditionality or biconditionality. But the schematized biconditionals are needed by those who prefer theories in which all theorems are provable by purely syntactic rules – rules of logic properly so-called. This preference is widespread and understandable. Later it will be useful to have available the following schemata for satisfaction and frustration, corresponding to schemata Tr and Fa. For A, α, ϕ, σ and $trans'_{\phi}$ as above,

Sat. α satisfies-in-L σ iff trans'_{ϕ}
Fr. α frustrates-in-L σ iff it is not the case that trans'_{ϕ}.

VI

The literature with which I am acquainted identifies a logic with a theory in a particular language, one closed under a generous sort of substitution, or (marginally better) a similarly closed consequence relation on a particular language. This will not do. As with rules, a language-transcendent conception of a logic would be better. A logic is a four-tuple: (1) a set of types for lexical categories, e.g., the types formula, individual constant, individual variable, *n*-place predicate constant, *n*-place formulae-to-formula operator that does not bind variables (i.e., connectives), or that does (e.g., quantifiers); (2) a set of argument-conditions (details would take me far afield, but suffice it to say that this component will determine whether the logic allows multiple-conclusion inferences); (3) a set of logical concepts, each of a unique type such that it would make sense for a logical constant of that type to express that concept; (4) a perhaps empty set of additional rules involving only logical concepts in the third set.

A language *L* realizes a logic **L** iff (1) the types in **L**'s first component correspond to non-empty lexical categories of *L*; (2) *L* has argument-conditions that accord with **L**'s second component; (3) *L* has logical constants of the appropriate categories that express the concepts in **L**'s third component; and (4) the additional rules in **L**'s fourth component govern the logical constants expressing the logical concepts involved in those rules. **L** determines the set of provable formula-inferences in *L*, provable using only the rules provided by the third and fourth components of **L**. Such proofs can be 'formatted' in a sequent-calculus or a Natural Deduction system; at this level of abstraction, a logic is neutral between such formats.

Setting aside issues of vagueness, I propose that *L* realizes a unique 'basic' logic, whose concepts are exactly those expressed by *L*'s logical constants and whose fourth component is empty: so all the rules built into *L*'s basic logic are sense-constitutive. *L* also realizes a unique 'total' logic, obtained from the basic logic by adding to its empty fourth component all the other rules primitively governing *L*'s logical constants. I conjecture that these are all thickening rules.

One might object that a language need not have one total logic, since different kinds of discourse in it might be subject to different rules. Perhaps an English-speaking mathematician does constructive mathematics during the week and relaxes by doing classical mathematics at weekends. The objection is well taken (assuming that English allows only for single-conclusion inferences): strictly speaking, what realizes a logic is a practice or type of discourse. For convenience, I shall retreat to a technical notion of languagehood, according to which our mathematician works in constructive mathematical English during the week but in classical mathematical English at weekends. The basic logics for constructive and for classical mathematical English are identical: for most purposes we can take it to be standard first-order intuitionistic logic. The total logic for constructive mathematical English is obtained from its basic logic by adding at least the rule of infinite domains to its fourth component. The total logic for classical mathematical English is obtained by also adding EM or GEM.

Concepts of truth and falsity for statements are, of course, language-relative. I have built a logical practice into the identity of a language: we might have two languages that differ merely in whether their logical practices (*viz* their total logics) are constructive or classical: for example, constructive English and classical English. This opens room for a distinction between concepts of constructive and classical truth for statements belonging to both languages. This is *not* a distinction between different conceptions of truth, or better, between different philosophical theories of truth.

VII

Whether a purely syntactic rule overtly governs a constant in *L* is a matter of *L*'s syntax. This enlarges the scope of syntax in three respects. First, it concerns the syntactic structure of arguments, rather than merely that of single sentences or formulae. Secondly, whether a deductive rule governs certain expressions is a conditional matter, concerning conditional feelings of compelled acceptance. Grammaticality of sentences lacks this conditional structure. Thirdly, whether a rule overtly governs a logical constant for *S* involves facts about *S*'s dispositions to accept statements, and so also facts about *S*'s understanding of the statements. In contrast, it is been claimed that whether a string of phonemes is grammatical in *L* (or *S*'s idiolect) involves only facts to which *S*'s understanding of that string is irrelevant.

In spite of these differences, there are continuities between argumentative syntax and the linguist's 'sentential' syntax. For one thing, the last claim might suggest that a native speaker classifies a string of phonemes as grammatical 'directly' from its phonological properties. But no one does this; for most strings, a speaker (or better, a speaker's 'understanding module') must first parse it into recognized words and assign these words to grammatical categories. These processes do not require sense-grasping, but they do bring the speaker's lexicon into play. So the third gap between argumentative and sentential syntax is not as deep as it might initially seem.

Nor is the difference all that deep between the kinds of evidence at issue. The syntactician's most basic evidence about which strings of phonemes in *L* are grammatical is information about which strings speakers of *L* produce and respond to. The syntactician in the field can get further evidence by asking a native for information about which strings of phonemes 'sound OK' to him. We cannot expect speakers to have the concept of grammaticality at 'the personal level', even if speakers' language-processing modules might, in some sense, have this concept. Similarly the evidence of what rules overtly govern *L* is how speakers of *L* reason in *L*, including what sorts of criticism of reasoning they accept and give. Here the syntactician's basic evidence is information about whether the natives actually accept particular formulae conditionally on their acceptance of particular sets of formulae; we cannot expect speakers to have the concept of logical entailment. Of course, acceptance plays no role in the 'sounds OK' response. But even here there are some commonalities. The logical syntactician will have to form hypotheses about whether responses occur because of sensitivity to structural properties of statements involved. The linguistic syntactician will have to form corresponding hypotheses about the 'sounds OK' response – to assess whether informants respond thus merely because of sensitivity to the syntactic properties of a phonemic string, or because they understand what would be meant by someone who uttered that string (after all, we can understand a wide range of quite ungrammatical statements), or because they agree with the thought the string expresses, or like its prosodic features. One cannot avoid psychological hypotheses if one is to describe the sentential syntax or the argumentative syntax in play in a population.

To bring this out, suppose that there is a tribe which speaks a regimented first-order language L of the sort beloved by logicians; that its members engage in a significant amount of demonstrative argumentation already formalized into standard first-order intuitionistic logic – many of them are mathematicians; and that they are quite competent with all its rules. These are the sophisticated constructivists. Suppose a radical translator sets out to translate the logical constants of L. The syntax of L's sentences will be easy to discern. The next step is to determine what rules overtly govern expressions of L. I suggest that if the translator can tell when speakers make deductive inferences, can re-identify statements, or more generally formulae, can detect comprehension and acceptance reasonably well, and can form reasonable hypotheses about the psychological processes behind such responses, he has the ball rolling, even without any understanding of L beyond that. In particular, I suggest that the translator will not need to translate any non-logical constants of L in order to translate L's logical constants (apart from those needed to detect comprehension and acceptance).

VIII

By itself, thesis 3 takes no position on whether the basic logic for a language L is classical or constructive. That depends on argument-formation in L, specifically on whether argumentative practice among speakers of L involves only single-conclusion formula-inferences. I think that actual argumentative practice among English speakers, in fact among all actual people, involves only single-conclusion formula-inferences, i.e., one argument-condition of any natural human language is that each argument has a single conclusion. We can represent classical reasoning as multiple-conclusion reasoning, but this is not a direct characterization of actual classical reasoning (multiple conclusions are understood disjunctively). If this psychological speculation is right, thesis 3 implies that our basic logic is constructive.

I am not in this committing myself to any so-called 'anti-realist' theses, e.g., that truth is constituted by knowledge or justified belief, or that it *a priori* implies knowability. A mathematician might even believe that every proposition is either true or false, but still take no interest in classical mathematics because it is insufficiently computationally informative. (So I reject Tennant's objection to M-realism: 'One cannot simply give up the classical rules and carry on thinking like a realist. McDowell has failed to appreciate just what is involved, by way of semantic and philosophical foundations, in being an intuitionistic logician.'[14]) I have no objection to classical logic, even though it is not our basic logic.

Thesis 5. (1) The distinction between constructive and classical argumentation originates from a distinction between a more and a less demanding standard for reasonable belief for disjunctive and existential statements.

(2) No logical constant is ambiguous between a constructive and a classical sense.

(Well, at least not in the way many have supposed: e.g., expressions of negation are not ambiguous in this way. In a bimodal logic accommodating truth-value gaps, there is room for a kind of disjunction that forms a truth even though the disjuncts lack a truth-value, and room for a kind that does not. It seems appropriate to call the former 'non-constructive' and the latter 'constructive'. Perhaps 'or' in English is ambiguous between these connectives, e.g., in statements about future contingencies.)

The distinction between standards mentioned in (1) leads to a distinction between standards for non-conditional acceptance. If the assertions of mathematicians have intentional contents, it also leads to a distinction between the proposition which a given statement constructively expresses and the one it classically expresses.

I actually have an argument for part (2). Things are clearest regarding the material conditional. Suppose we have two expressions, $\supset_I$ and $\supset_K$, the first with the constructive sense for the material conditional, the second with the purported classical sense. So $\supset_I$ is governed by $\supset_I$-intr and $\supset_I$-elim, and $\supset_K$ is governed by $\supset_K$-intr, $\supset_K$-elim and GEM. It is easy to see that then $\supset_I$ is also governed by GEM. One might object that this merely shows that the constructive and classical senses for expressions of material conditionality cannot live in the same language. But if there really are two such senses, and we assign a distinct expression to each, how could that be impossible? It might be urged that there is no possible language in which 'water' expresses

[14] See N. Tennant, *The Taming of the True* (Oxford UP, 2002), p. 239.

its usual English-language sense and another word, say 'twater', expresses the sense which 'water' expresses in twin English. If this claim has any basis at all, it is because grasping these senses would involve being in incompatible relations to external reality. But according to 'Syntax first', grasping the sense of a logical constant is a matter largely internal to the understander of *L*, the only external element being the expressions of *L*. Perhaps oil (or twater) and water cannot mix; but there is no reason to think that distinct logical concepts cannot be expressed in a single language.

When classical and constructive mathematicians disagree, it may seem that they are really talking past each other, that what the classical mathematician asserts on the basis of a non-constructive proof is not what the constructive mathematician refuses to assert. This ecumenical content-pluralism should be appealing, at least to those who dislike disagreement. But – and this is the crucial point – classical content is not determined purely compositionally. The source of the misguided popular doctrine of the ambiguity of logical constants is blind faith in compositionality. Among single-conclusion reasoners, constructive content is determined purely compositionally. But a speaker operating under a classical logic makes assertions with classical content because at a second stage the logic kicks in, collapsing the constructive content to classical content.

One might think that if one is to use EM, or other rules that are not constitutive of the senses of the logical constants they govern, one needs a powerful justification. I think weak pragmatic justification suffices: such rules make mathematics easier. Be that as it may, gentlemen, and gentlewomen, prefer constructive proofs, because they are more informative than proofs which make non-constructive inferences.

IX

What combinations of introduction and elimination rules can constitute the sense of a logical constant? And how does the part of the sense of a logical constant that a speaker adequately grasps determine the complete sense of that constant? According to 'Syntax first', this is a syntactic question, though its answer has consequences for truth.[15]

For a logical constant *c* of language *L*, let *c*'s introduction package [elimination package] in *L* be the set of language-transcendent introduction rules [elimination rules] governing *c* in *L*. We do need sets; an expression of disjunction has a two-membered introduction package, and an expression

[15] For some influential related thoughts, see N.D. Belnap, 'Tonk, Plonk and Plink', repr. in P.F. Strawson (ed.), *Philosophical Logic* (Oxford UP, 1967), pp. 132–7.

of biconditionality has a two-membered elimination package; an expression of surd has the empty introduction package. Let *c*'s package-pair in *L* be the ordered pair of its introduction package and its elimination package. Properly speaking, this is the logical concept that *c* expresses in *L*. Supplementing thesis 3, I propose

Thesis 3′. If *c* is a logical constant in *L*, either all of *c*'s introduction rules are among those that overtly constitute *c*'s sense, or all of *c*'s elimination rules are.

The question now is: what package-pairs are logical concepts? Most obviously, *c*'s elimination package must invert its introduction package. This generalizes Prawitz's 'inversion principle', an explication of one of Gentzen's ideas, the one behind both cut-elimination for sequent calculi and normalization for ND systems: if one reasons properly, one gains nothing by introducing a logical constant only to eliminate it. The rigorous idea is best expressed algebraically; I shall forgo details here. Of course the elimination packages for the Big Five and for the standard quantifiers invert their corresponding introduction packages. Prior's 'tonk' does not express a logical concept because 'tonk'-elim does not invert 'tonk'-intr.

Indeed, I think that we need perfect inversion: *c*'s elimination package is the maximum inverter of *c*'s introduction package, and the latter is the maximum inverter of the former (Tennant, pp. 316, 321, calls this 'the requirement of harmony'). The ordering here is the natural ordering by strength on the appropriate sets of packages. I shall call a package-pair meeting these conditions 'perfect'. The package-pairs for the Big Five and the universal and existential quantifiers are perfect.

Along with thesis 3′, perfect inversion helps to secure whatever constitutive rules tacitly govern *c*, on the basis of those overtly governing *c*. For if *c*'s introduction [elimination] rules are among its overtly sense-constituting rules, this introduction [elimination] package uniquely determines the rest of *c*'s sense-constituting rules: they are the members of its maximum inverter [invertee]. Contrast the sophisticated constructivists with another tribe, the unsophisticated constructivists (for this discussion, their constructivism is not relevant). They use the 'non-proviso' rules, universal elimination and existential introduction, without problems; for them, only these rules overtly govern '∀' and '∃'. But (like many students in introductory logic courses) they have not really got the hang of universal introduction or existential elimination, rules that involve those nasty provisos. In other words, the latter rules do not overtly govern '∀' and '∃' in their language. They even have difficulties with disjunction elimination (again like some students), or conditional introduction. Some of their great mathematicians managed to use the problematic quantifier-rules correctly in proofs which others of the tribe could come to find persuasive, but without having achieved any explicit formulation of these rules. This tribe is rather like the Europeans of the late eighteenth century; in fact, one of their famous philosophers attributed the apparent cogency of these proofs to 'pure intuitions', experiences which this philosopher said were essential parts of understanding these proofs.

The radical translator might have a harder time with these unsophisticated constructivists. But if the translator is also a logician, he has reason to think that in their language universal introduction and existential elimination tacitly govern '∀' and '∃': the former is the maximal invertee of universal elimination, and the latter is the maximal inverter of existential elimination. This tacit governance among actual logic students is shown by the fact that many such students at first find universal introduction and existential elimination puzzling and *ad hoc*; but with proper teaching, they come to find them natural, even primitively compelling, and do not think that they have been taught new meanings for old words. (Universal introduction and universal elimination form a perfect pair, and universal elimination overtly primitively governs the unsophisticated constructivists' use of '∀'. Do these two facts suffice to make universal introduction tacitly govern '∀' among the unsophisticated constructivists? I do not rule this out, though my characterization of what it is for a rule tacitly to govern a constant contained the clause concerning the disposition to learn, in order to avoid ruling it in.)

Still, perfection is not enough. I shall call a package-pair $<I,E>$ 'definitive' iff for any two constants c and c' in any language L, if L's lexicon assigns both to the package-pair $<I,E>$, then c and c' are provably equivalent using only rules in $I \cup E$. E.g., if the package-pair is designed for n-place formula connectives, equivalence means that for any formulae $\psi_0, ..., \psi_{n-1}$ of such a language, $\ulcorner c(\vec{\psi}) \Rightarrow c'(\vec{\psi})\urcorner$ and $\ulcorner c'(\vec{\psi}) \Rightarrow c(\vec{\psi})\urcorner$ are provable. With the notion of definitiveness on the table, I shall stick my neck far out and suggest:

Thesis 6. Perfection and definitiveness are necessary and sufficient for a package-pair to be a logical concept.

X

So far I have considered logical constants with regard to their sense. But a theory of sense needs what Peacocke calls a determination theory to characterize how the sense of an expression, or better, the conditions for grasp of that sense, contribute to determining the expression's 'referent' or, perhaps less misleadingly, its semantic value. Peacocke coined the phrase 'determination theory' with regard to concepts, not linguistic expressions; but language as well as thought needs a determination theory, even if language somehow inherits its determination theory from thought.

I shall assume that a semantic theory, whatever else it does, assigns linguistic expressions to semantic values, and that this assignment captures how that expression contributes to determining at least the truth- and falsity-conditions of statements in which it occurs. (In a loose sense, this Fregean picture of semantic theory is 'realistic'. But it carries no commitment to thinking that concepts of truth and falsity are *the* central concepts of any plausible semantic theory, or to the thesis that understanding every truth-apt statement consists in 'knowing its truth-conditions'. It is not obvious that this Fregean framework applies to a language whose total logic is constructive; here I merely proceed on the hypothesis that it does.) Much is unclear about what semantic values should be, especially for a language whose total logic is constructive. It is conceivable that an unambiguous logical constant has distinct constructive and classical semantic values; perhaps this is the kernel of truth behind the popular view which thesis 5(2) contests. This would not compromise thesis 5(2), since there is no road back from reference to sense. (If the best determination theories for constructive and for classical discourse have this result, it seems likely that the classical semantic values will be 'restrictions' or special cases of the constructive semantic values.)

A truth- or falsity-condition can be treated as a function, perhaps partial, from possible situations (or 'worlds of evaluation') to truth-values. To handle statements containing variable-binding constructions, we need to look beyond truth and falsity to satisfaction and frustration. So, given a variable-assignment, I shall say that there are (at least) two satisfaction-values; given a variable-assignment and a possible situation or world, the semantic value of a formula will determine a satisfaction-value for that formula relative to these givens. We demand at least this of the semantic value of an expression: it must capture how that expression contributes to the satisfaction- and frustration-conditions (hereafter the 'pre-alethic' conditions) for formulae in which that expression occurs.

I shall set aside the deep question of how best to conceive of semantic values, and consider what might be a narrower question: how do the rules constituting the sense of a logical constant help to determine its contribution to the pre-alethic conditions for formulae in which it is the main logical constant? I shall call this aspect of its semantic value its 'contributory value'. By themselves, a logical constant's sense-constituting rules do not determine its contributory value. They do so only together with certain constraints on satisfaction and frustration.

Here are some appealing constraints for any language that people might use for communication or thought:

> No formula is both satisfied and frustrated
> Some formula is frustrated
> If two formulae express the same sense, one is satisfied iff the other is
> Ditto for frustration.

Suppose that the translator has settled enough of the determination theory regarding speakers of *L* to specify the pre-alethic conditions for the 'logic-free' formulae of *L*, those containing no logical constants; and suppose that this specification honours the above constraints. The translator now aims to extend that theory to the remaining formulae of *L*.

Let a substitution instance of a formula-inference $\Delta \Rightarrow \phi$ be a formula-inference obtainable from $\Delta \Rightarrow \phi$ by uniform substitution of expressions of appropriate type for non-logical constants, and by restrictions of bound variables for variable-binding operators. Let $\Delta \Rightarrow \phi$ be sound [cosound] iff each of its substitution instances preserves satisfaction [non-frustration], i.e., if all members of its premises (i.e., antecedent) are satisfied [non-frustrated], then so is its conclusion (i.e., consequent). (This follows the mathematical usage of 'sound'. Many philosophers use 'valid' to mean what I shall mean by 'sound'; but others mean something different, e.g., counterfactual preservation of warranted assertability, or of knowledge, or something else. At least mathematical usage has been fairly unambiguous.) Let a sequent-inference be sound [cosound] iff it preserves soundness [cosoundness], i.e., iff if its premises are all sound [cosound] then so is its conclusion. These semantic properties apply also to entire arguments, in the obvious way. A rule is sound [cosound] in *L* iff all inferences in *L* that instantiate that rule are sound [cosound].

Let basic soundness [cosoundness] be this requirement on satisfaction and frustration in *L*:

> Every argument constructed using only sense-constituting deductive rules that govern logical constants of L is sound [cosound].

Let total soundness [cosoundness] be the corresponding requirement for arguments constructed using any deductive rules that govern logical constants in *L*. Basic soundness and cosoundness strike me as compelling constraints on how a determination theory assigns pre-alethic conditions to formulae of *L*. I am less confident that we must insist on total soundness and cosoundness. Of course if *L*'s logic is classical and *L* is sufficiently expressive, anti-realists of a Dummettian stripe will say that satisfaction in *L* will not satisfy total soundness: *L*'s speakers are in a state of philosophical error, *L* is not bivalent, and EM_L is not sound. (If L contains logical constants expressing negation and disjunction and L's total logic is classical, total soundness implies that L is bivalent: for any statement ϕ in L, classicality ensures that there is a proof of $\ulcorner\phi \vee \neg\phi\urcorner$; soundness requires that $\ulcorner\phi \vee \neg\phi\urcorner$ is true-in-L; constructive reasoning shows that either ϕ is true-in-L or $\ulcorner\neg\phi\urcorner$ is true-in-L, which implies that ϕ is either true-in-L or false-in-L.) Still, one might conjecture this: if satisfaction and frustration honour total soundness and cosoundness as well as the obvious constraints, then the sense-constituting rules for any logical constant will suffice to fix that constant's contributory value uniquely. A stronger conjecture replaces 'total' by 'basic'.

As long as sense-constituting rules are restricted to the familiar introduction and elimination rules for L's logical constants, these conjectures are false. Without assuming bivalence, &-intr$_L$ and &-elim$_L$ are sound both for weak Kleene (a.k.a. Fregean) and strong Kleene conjunction; so these rules do not uniquely determine the contributory value of '&'. To avoid this trivialization, without considering 'bi-modal' rules, one could weaken the above conjectures by adding the constraint that L must be bivalent:

> Any formula is either satisfied or frustrated.

I shall argue that even thus weakened, these conjectures are false.

I shall look at the simplest sort of logical constant: an n-place extensional connective c. Here extensionality is a proof-theoretic property. First, for any set Δ of formulae, let any formulae ϕ and ϕ' be equivalent mod Δ iff ϕ' is derivable from $\Delta \cup \{\phi\}$ and ϕ is derivable from $\Delta \cup \{\phi'\}$, using only sense-constitutive rules. Let c be extensional iff for any such Δ and any formulae $\phi_0, ..., \phi_{n-1}$ and $\phi'_0, ..., \phi'_{n-1}$, if ϕ_i and ϕ'_i are equivalent mod Δ for each $i \in n$, so are $\ulcorner c(\phi_0, ..., \phi_{n-1})\urcorner$ and $\ulcorner c(\phi'_0, ..., \phi'_{n-1})\urcorner$.

Suppose c is an extensional logical constant in L. Rather than require specification of c's contributory value, I shall merely ask that an acceptable determination theory should imply that c is weakly truth-functional ('satisfaction-functional' is more accurate, but I shall stick with the more familiar phrase): for any variable-assignment, any possible situation, and any formulae $\phi_0, ..., \phi_{n-1}$ in L,

> If each of $\phi_0, ..., \phi_{n-1}$ has a satisfaction-value, then these values uniquely determine $\ulcorner c(\phi_0, ..., \phi_{n-1})\urcorner$'s satisfaction-value.

(Strong truth-functionality requires, in addition to weak truth-functionality, that if $\ulcorner c(\phi_0, ..., \phi_{n-1})\urcorner$ has a satisfaction-value, then $\phi_0, ..., \phi_{n-1}$ must have one of the distributions of satisfaction-values that determine $\ulcorner c(\phi_0, ..., \phi_{n-1})\urcorner$ to have that satisfaction-value. Weak truth-functionality with bivalence ensures strong truth-functionality; without bivalence, it does not. In intuitionistic logic each of the standard connectives is extensional and weakly truth-functional, but conditionality is not strongly truth-functional: e.g., $(\phi \supset \phi)$ is true though ϕ may be neither true nor false. Also, without bivalence the weak Kleene connectives are strongly truth-functional, but strong Kleene conjunction and disjunction are not.) I can now formulate a well defined test: if the sense-constituting rules for an extensional connective *c*, together with the general constraints on satisfaction and frustration, determine *c*'s contributory value, then they must imply that *c* is weakly truth-functional. Focusing on familiar extensional connectives, do they do that?

XI

To make the issues vivid, I shall return to my radical translator. He has determined the pre-alethic conditions for logic-free formulae of *L*, and now wants to determine them for the rest. For generality, I shall not allow him to assume bivalence for *L*.

For conjunction, matters are straightforward: regardless of what other logical constants *L* contains, soundness and cosoundness ensure that '&' is weakly truth-functional. Other connectives are more problematic. It is useful to consider negation; from its weak truth-functionality one can show the weak truth-functionality of other familiar connectives expressible in *L*. Suppose we are given a variable-assignment. By the second constraint, it frustrates some formula; suppose this is θ. The cosoundness of $\perp \Rightarrow \theta$ implies that $\perp$ is frustrated. Then the cosoundness of ϕ, $\ulcorner\neg\phi\urcorner \Rightarrow \perp$ requires that either ϕ or $\ulcorner\neg\phi\urcorner$ is frustrated. The first constraint gives these principles: if ϕ is satisfied then $\ulcorner\neg\phi\urcorner$ is frustrated; if $\ulcorner\neg\phi\urcorner$ is satisfied then ϕ is frustrated.

But if the determination theory is to declare '$\neg$' to be weakly truth-functional, it had better provide this crucial principle: if ϕ is frustrated then $\ulcorner\neg\phi\urcorner$ is satisfied. Peacocke recognizes that this involves a step 'beyond the primitively obvious', that this 'raises the question of how the thinker knows such principles', and that 'the issue deserves extended attention'.[16] He considers a thinker reflecting on his own concepts, not a radical translator; still, the issue is the same. (One might prefer to consider connectives simpler than negation, i.e., of level 0 in the hierarchy. The corresponding non-obvious principles for '$\vee$' and '$\supset$' are these: if ϕ and ψ are frustrated then so is $\ulcorner(\phi \vee \psi)\urcorner$; if ϕ is frustrated then $\ulcorner(\phi \supset \psi)\urcorner$ is satisfied.)

Here is the crucial point: soundness and cosoundness, with the other above-mentioned constraints, do not ensure this non-obvious principle;

[16] In Peacocke, 'Understanding Logical Constants', *Proceedings of the British Academy*, 73 (1987), pp. 153–200.

adding bivalence does not help.[17] So the sense-constituting rules for '$\neg$' together with these constraints do not imply that '$\neg$' is weakly truth-functional, let alone determine a unique contributory value for '$\neg$'.

A cheap proof. Let V be a truth-assignment (i.e., V maps the set of sentence constants into $\{0,1\}$ with 0 representing falsity and 1 representing truth) on the sentence constants of a sentential formal language respecting the standard truth-tables; suppose V('P') = 0. We construct a 'truth'-assignment V' on the set *Sent* of sentences, one with respect to which all classical truth-functional derivations are sound but with V'('P') = V'('$\neg P$') = 0. Let V_0 be the usual extension of V to *Sent*. From each minimal set Δ classically implying '$\neg P$' with $V_0 \vDash \Delta$, select a $\phi \in \Delta$ and set $V_1(\phi) = 0$. (Such $\Delta \neq \{\}$.) For all other $\phi \in$ *Sent*, set $V_1(\phi) = V_0(\phi)$. From each minimal set Δ classically implying some ψ so that $V_1(\psi) = 0$ but for all $\phi \in \Delta$ $V_1(\phi) = 1$, select a $\phi \in \Delta$ and set $V_2(\phi) = 0$, etc. Let $V' = lim V_{n \in \omega}$. For any Δ and any ψ, if Δ classically implies ψ and for all $\phi \in \Delta$ $V'(\phi) = 1$, then $V' \vDash \psi$, by the construction of V'. Since $dom(V') =$ *Sent*, bivalence is satisfied. So soundness and cosoundness are satisfied.

The difficulty here would have been a rather good reason for Peacocke to retreat from deductive rules to rules of transition in his discussion of negation in *Thoughts*, a reason better than the one he gives, *viz* respect for Dummett's requirement (mentioned in §IV). But I am unpersuaded that retreat is necessary. What further constraints should be imposed?

Here is a bad idea. In addition to its set of provable formula-inferences in L, a logic realized in L determines a set of provable sequent-inferences of L, those of the form $<D, \Delta \Rightarrow \phi>$ for which $\Delta \Rightarrow \phi$ is derivable from D. Let a variable assignment satisfy a formula-inference $\Gamma \Rightarrow \phi$ iff it either frustrates some member of Γ or satisfies ϕ, and let it frustrate $\Gamma \Rightarrow \phi$ iff it both satisfies all members of Γ and frustrates ϕ. Let super-soundness [super-cosoundness] be the constraint that provable sequent-inferences preserve satisfaction [non-frustration]. If a variable assignment frustrates ϕ, it satisfies the inference $\{\phi\} \Rightarrow \bot$, from which $\Rightarrow \ulcorner\neg\phi\urcorner$ is derivable. Assuming super-soundness, $\Rightarrow \ulcorner\neg\phi\urcorner$ is satisfied; since no member of $\{\}$ is frustrated, $\ulcorner\neg\phi\urcorner$ is satisfied.

This approach treats formula-inferences as if they were formulae; it replicates Russell's unfortunate confusion of conditionality and implication properly so-called, *viz* entailment. An inference is not true; so calling one satisfied or frustrated is mistaken. In fact, satisfaction should not conform to super-soundness! '$\forall$'-introduction permits us to infer $\Rightarrow$'$\forall xPx$' from $\Rightarrow$'Px'

[17] When I first noticed this, I thought it was a great discovery; a few weeks later I read Peacocke's 'Understanding Logical Constants'. Mark Brown told me that years ago he also thought he had discovered this point, but later found it discussed in notes he had taken years earlier for a class taught by Gerald Massey. I would be grateful for any information about publications prior to Peacocke's that make this point.

(since 'x' does not occur free in any member of {} or in '$\forall xPx$'). But satisfaction of the latter should not require satisfaction of the former.

Now for a better idea. I shall first consider English. How can we justify this proposition: if 'That dog is sleeping' (accompanied by demonstration of my dog) is false-in-English then 'That dog is not sleeping' is true-in-English? Assume the if-clause. Using 'false-in-English' elimination, we can conclude that my dog is not sleeping. By 'true-in-English' introduction, we can conclude to the then-clause. Applying conditional introduction, we are done. This is argument by semantic descent followed by ascent.

Now suppose English is the radical translator's home language. Suppose that ϕ is a statement of L, so truth and falsity may replace satisfaction and frustration; suppose it is atomic. The translator understands ϕ, setting $trans_{\phi}$ to be the statement made in his current context by 'That dog is asleep', accompanied by a pointing gesture towards a dog. The translator may reason as follows. 'Assume that ϕ is false-in-L. Thus, using "false-in-L" elimination, that dog is not asleep. "That dog is not asleep" is a negation of "That dog is asleep". I have determined that "$\neg$" is governed in L by the same package-pair as governs expressions of negation in my English. A philosopher has persuaded me that this ensures that they express the same sense; so I can translate "$\neg$" as an expression of negation. So $trans_{\neg\phi}$ is the statement expressed in my current context by "That dog is not asleep". Since that dog is not asleep, $\ulcorner\neg\phi\urcorner$ is true-in-L, by "true-in-L" introduction.'

This pattern of argument generalizes to any atomic formula that the translator understands. Suppose for the moment that the only logical constants in L are extensional connectives. Such arguments then will give the translator the pre-alethic conditions for formulae of L of logical depth 1. Iterating by depth will secure the desired principle for any formula of L.

I have suggested that the translator can in principle figure out the sense of a logical constant of L without understanding a single formula of L. In contrast, the above justification for the non-obvious principle requires the translator to understand the formulae of L to which an extensional connective applies, well enough to translate them, starting with the atomic formulae and bootstrapping up. But semantic descent and ascent would be available no matter how ϕ is translated – even if mistranslated! The translator's reliance on such arguments does not force us to say that the semantic value of '$\neg$' depends on the senses of the atomic formulae of L or on their semantic values.

Still, the above argument may produce suspicion. Is this argument non-explanatory? Does it push from L to English the problem posed by the non-obvious principle, or even to Thought? If it assumed that an expression of negation in English reversed truth-value, this charge would stick. But it uses no claims about the truth- or falsity-conditions of English sentences or of the translator's thoughts. Its dialectical status is delicate. One might think this: if we are unsure whether the fact that '¬' in *L* is governed by the rules governing expressions of negation in English gives us a good reason to translate '¬' as an expression of negation, then we would look to the determination theory to settle whether we should translate '¬' thus. But if the determination theory is supported in part by arguments like the above, that would be illegitimate. This seems right; but I do not think that we should be unsure in the way the if-clause suggests; so I do not think we need a determination theory to justify the translation of '¬'. It *is* legitimate to use one's theoretical beliefs about senses to support one's preferred determination theory. The latter theory for a language *L* must be tailored to an account of sense-grasping for expressions of *L*; so what could be wrong with relying on the latter account in one's justifications for principles of a determination theory? 'Syntax first' is a part of an account of sense-grasping; and it supports a linguist's translation of '¬' as an expression of negation. I submit that the fact that 'Syntax first' helps to justify the non-obvious principle, and others like it, constitutes abductive support for 'Syntax first'.

Enough for connectives; what about variable-binding logical constants? The notion of extensionality generalizes straightforwardly to them. And the notion of weak truth-functionality can be extended to such constants; I shall refrain from details. Suffice it to say that even with the weak truth-functionality of the familiar connectives in place, the difficulty with negation has analogues for expressions of first-order universal or existential quantification. For example, this principle is non-obvious: if every v-variant of a variable-assignment satisfies ψ, then that assignment satisfies $\ulcorner\forall v\psi\urcorner$. It can be proved that soundness and the other obvious constraints cannot deliver this non-obvious principle, even assuming bivalence. An argument by semantic descent and ascent can secure this principle; I shall spare the reader the details. (Peacocke claims in *A Study of Concepts*, p. 7, that universal quantification over the natural numbers is 'the unique second-level concept' whose possession-condition is this: finding primitively compelling universal elimination using substitution with appropriate numerical concepts. This passage seems to confuse the concept of universality over the natural numbers with its semantic value, the universal quantifier restricted to the natural numbers. It immediately follows a discussion of conjunction; I am not sure whether Peacocke meant to suggest that a determination theory's account of the concept of universality over the natural numbers would be as straightforward as its account of the concept of conjunction. Suffice it to say that this is not so.)

Vague Conjecture 2. Once we have settled what sort of semantic values logically atomic expressions have, total soundness and cosoundness, with the obvious constraints on satisfaction and frustration, and (here is the crucial point) all instances of the Sat and Fr schemata (and thus the Tr and Fa schemata), will suffice to determine the semantic values, or at least the contributory values, for *L*'s logical constants.

Should this conjecture be strengthened by replacing 'total' by 'basic'? This strengthening would imply that logical constants have the same semantic values for both classical and constructive discourse. (More fully: if L and L' differ merely in that the total logic for one is classical and for the other is constructive, then each logical constant in either (i.e., in both) has the same semantic value in L and in L'.) I have no settled opinion regarding this strengthened conjecture, though I am inclined to reject it.[18]

Cornell University

[18] This is a much expanded version of the first two thirds of my talk on logicism at the Arché conference on the philosophy of mathematics, held at St Andrews in August 2002; that was the portion primarily on the philosophy of logic. The remaining third focused on the philosophy of mathematics. Thanks to audiences at Notre Dame, Syracuse University, St Andrews and in my own department, and particularly Michael Detlefsen, Michael Fara, Michael Kremer, Jeff Roland and Zoltán Szabó, for valuable comments and discussions.