Geometric model of gravity, counterfactual solar mass, and the Pioneer anomalies

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Introduction.

This is a slightly edited paper from 2004. I do not think the situation summarized here has changed significantly in 2014, but I add these extra explanatory notes. This study analyses the predictions of the General Theory of Relativity (GTR) against a slightly modified version of the standard central mass solution (Schwarzchild solution). It is applied to central gravity in the solar system, the Pioneer spacecraft anomalies (which GTR fails to predict correctly), and planetary orbit distances and times, etc (where GTR is thought consistent.)

The modified gravity equation was motivated by a theory originally called '*TFP'* (Time Flow Physics, 2004). This is now replaced by the 'Geometric Model', 2014 [20], which retains the same theory of gravity. This analysis is offered partially as supporting detail for the claim in [20] that the theory is realistic in the solar system and explains the Pioneer anomalies. The overall conclusion is that the model can claim to explain the Pioneer anomalies, contingent on the analysis being independently verified and duplicated of course.

However the interest lies beyond testing this theory. To start with, it gives us a realistic scale on which gravity *might* vary from the accepted theory, remain consistent with most solar-scale astronomical observations. It is found here that the modified gravity equation would appear consistent with GTR for most phenomena, but it would retard the Pioneer spacecraft by about the observed amount (15 seconds or so at time). Hence it is a possible explanation of this anomaly, which as far as I know remains unexplained now for 20 years.

It also shows what many philosophers of science have emphasized: the pivotal role of *counterfactual reasoning.* By putting forward an exact alternative solution, and working through the full explanation, we discover a surprising ‘counterfactual paradox’: *the modified theory slightly weakens GTR gravity – and yet the effect is to slow down the Pioneer trajectory, making it appear as if gravity is stronger than GTR.* The inference that “there must be some tiny extra force…” (Musser, 1998 [1]) is wrong: there is a second option: *“…or there may be a slightly <u>weaker</u> form of gravity than GTR.”*

The reason for this is because *the counterfactual implications* of replacing GTR with the alternative theory is not simply to *replace the equations, and use the same values for the solar mass*. We have to reevaluate all the theoretically-dependant measurements and quantities. It is a holistic system: we have to recalculate mass and distance relations for the solar system bodies, including *revising the mass of the sun*, which is increased by about 1.00000004, or 4 x 10^{-8}. This is the *counterfactual solar mass.* It is chosen so the decreased strength of the *counterfactual gravity law* leaves the sun’s gravitational effect the same at Earth’s orbit.

The change to the Schwarzschild solution simply amounts to replacing the quadratic factor: $k = \left(1 - \frac{2MG}{c^2 r}\right)^{-1/2}$ with the exponential factor: $K = \exp\left(\frac{MG}{c^2 r}\right)$. Expanding these in Taylor series, it is seen they differ only in 2^{nd}-order terms, which are quite tiny. In a scale symmetric theory, *K* is the generalization of *k.*

I think this is the only mathematically coherent modification of the Schwarzschild solution to consider as a possible alternative. It is mathematically sensible in that *mass has a linear addition, with only a small non-linear effect in normal situations,* and it uses the dimensionless combination: (MG/c^2r). The first space differential of *K* is: $dK/dr = (-MG/c^2r^2)K$, which has a linear factor in *M,* with a small non-linearity in *K.* The first space differential of *k* is: $dk/dr = (-MG/c^2r^2)k^3$, which is similar. It is also equivalent to a GTR metric for a Gaussian-like distribution of mass *M* smeared out from the center, instead of the simple central mass that the Schwarzschild solution represents. Hence it makes sense physically, even in GTR.

I should also note that in terms of the broader theory [20], the background constant *G* can change in response to the background mass-energy tensor. It may well undergo periodic perturbations, if there are large ‘free gravitational waves’ in the local solar system or galaxy, which there jolly well could be. I have not considered this, but see Sheldrake 2013 [19], who wonders whether there are regular fluctuations in the absolute value of *G* on Earth, and points out that estimates are based on averages, that there are special measurement uncertainties with *G*, and provides evidence supporting fluctuations. This is possible in the Geometric Universe, where *G* does vary depending on the background mass-energy density provided by the Earth, sun, galaxy. Our position in the Milky Way galaxy has a large effect, increasing *G* for us by about 10^{-5} -10^{-6} parts, compared to *G* in local inter-galactic space. The sun (10^{-8}) and Earth (10^{-9}) make relatively little difference to the background *strain tensor*, compared to the large mass of the galaxy. However I do not think this possibility affects the explanation here of the Pioneer anomalies. Note the phenomena is observed for multiple space craft, traveling in different directions out of the solar system, so a law-like explanation is indicated, and special local influences do not seem likely.

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TFP gravity in the solar system and anomalies in Pioneer spacecraft orbits.

Andrew Holster. March 2004, Pukerua Bay, Wellington, New Zealand.

> *"…they did notice that the Pioneers have been slowing down faster than predicted by Einstein's general theory of relativity. Some tiny extra force – equivalent to a ten-billionth of the gravity at Earth's surface – must be acting on the probes, braking their outward motion. ... In 1994 Michael Martin Nieto of Los Alamos National Laboratory and his colleagues suggested that the anomaly was a sign that relativity itself had to be modified."* Musser, 1998 [1].

1. Introduction.

Holster 2004 [2] proposes a new theory of gravity, called *TFP gravity,* based on a new general conception for a unified theory (called *Time Flow Physics: TFP*). This paper compares TFP gravity with the General Theory of Relativity (GTR) for ordinary gravitational fields in the solar system. It is concluded that: (i) TFP gravity makes a detectible difference for the predicted trajectories of the Pioneer spacecraft and the differences are similar to the anomalies observed in these trajectories[1]; (ii) TFP gravity makes small differences to the predicted orbits of the planets; the predicted discrepancies are actually slightly larger than current measurement error reported for mean orbit distances; this appears to disconfirm TFP gravity; but it is not clear that these measurements are direct, and independent of the assumption of GTR in the first place, and this needs to be checked more carefully. It is also observed that (iii) TFP gravity makes only a tiny difference to light trajectories and signaling times on solar system scales.[2]

[1] See references [3-14] for a selection of recent attempts to explain the anomalies; many involve unconventional theories; [7] proposes a conventional explanation, but appears rejected by others, see [9].
[2] The effect on the perihelions of Mercury is not examined here, but is a further key solar system phenomena which requires analysis.

The application of TFP gravity in the solar system is idealized as a simple central mass problem. The TFP gravity solution for this is a simple modification of the usual GTR Schwarzschild solution. Solutions developed for this are here called *K-Gravity*.

One feature is particularly noteworthy in any case: TFP gravity is slightly weaker than GTR gravity, for a fixed mass, and this initially suggests that the Pioneer trajectories should become slightly *faster* on TFP than on GTR as they escape the solar system. But the application of TFP gravity subsequently requires a *reinterpretation of the mass of the sun* – since this mass is ordinarily inferred on the basis of GTR (or its Newtonian limit), along with measurements of distance and periods of orbiting bodies, primarily Earth[3]. There is a reversal of the effect when the mass of the sun is recalculated according to TFP. The mass of the sun is recalculated as slightly larger than on GTR, for consistency with the Earth's orbit; and when this is taken into account, the effect is to slow the predicted trajectories of spacecraft leaving the solar system compared to GTR. So somewhat surprisingly, the weaker theory of gravity predicts that the gravity will appear stronger for an escaping spacecraft. This is an example of the theory dependence of interpretations of astronomical observations.

This is an example of a more general problem raised by Vanderburgh [17]. The same point must also apply to number of other explanations involving modified gravity theories, e.g. Bertolami [4], but this does not appear to be discussed elsewhere. Most proposals for a 'new force' or modification of gravity to explain the Pioneer anomalies seem to assume that modification must generate an additional inward acceleration toward the sun, but this is not necessary.

[3] More exactly, the product: *MG* needs to be adjusted, with *M* the mass of the sun. Measurement errors in *M* and *G* separately are quite large – they are only known to about 6 decimal places [16]. But *MG* is known (on the basis of GTR applied to Earth's orbit parameters) to 10 or 11 decimal places [15]. The proposed adjustment to MG required by K-gravity alters the value by about 4×10^{-8} of the total, i.e. at about the 8th decimal place.

2. The K-gravity metric.

K-gravity is the solution (to a very fine approximation) of the more general theory of TFP gravity, for a spherically symmetric central mass. To introduce K-gravity we first consider the ordinary Schwarzschild solution GTR in its usual line-element form:

$$(1) \qquad \mathrm{d}\tau^2 = \frac{dt^2}{k^2} - \frac{dr^2k^2}{c^2} - \frac{r^2d\theta^2}{c^2} - \frac{(r\sin\theta d\phi)^2}{c^2}$$

where the factor *k* ('little k')is defined by:

$$(2) \qquad k = \left(1 - \frac{2MG}{c^2r}\right)^{-1/2}$$

For K-gravity, we simply exchange *k* in (2) for the quantity *K* ('big K') defined by:

$$(3) \qquad K = \exp\left(\frac{MG}{c^2r}\right)$$

replacing this in (1) to obtain:

$$(4) \qquad \mathrm{d}\tau^2 = \frac{dt^2}{K^2} - \frac{dr^2K^2}{c^2} - \frac{r^2d\theta^2}{c^2} - \frac{(r\sin\theta d\phi)^2}{c^2}$$

Note that *k* represents a series approximation to *K*. Consider the quantity:

$$(5) \qquad \frac{1}{k^2} = 1 - \frac{2MG}{c^2r}$$

and the series expansion for $1/K^2$:

$$(6) \qquad \frac{1}{K^2} = 1 - \frac{2MG}{c^2r} + \frac{1}{2!}\left(\frac{2MG}{c^2r}\right)^2 - \frac{1}{3!}\left(\frac{2MG}{c^2r}\right)^3 + \ldots$$

Because the term $2MG/c^2r$ is very small in ordinary gravity, the higher-order terms in $1/K^2$ are very small, and the difference with GTR is very small in ordinary fields. Because this term is dimensionless, it is (logically) possible to expand from *k* to *K*. The alteration to the exponential function *K* is required by the underlying TFP model, but this general model is not discussed here[4].

[4] There are some dramatic differences in strong gravity; the GTR event horizon disappears in K-gravity, and there are no longer GTR-type black holes; the theory is conservative, but not gauge symmetric; and while TFP models Special Relativity, TFP gravity (and cosmology) is non-covariant.

The K-gravity solution, represented by (4), is not complete without adopting some principle to play the role of the usual geodesic or action principles of GTR, which give the metric equations their physical implications. The meaning of the TFP gravity metric is ultimately interpreted via a principle of energy conservation. But fortunately, for the central mass problem, the solutions can be obtained (to a very fine approximation) by treating (4) as if it was just a special GTR metric equation. The possibility of doing this can be seen by observing that (4) provides a consistent GTR metric for a spherically symmetric mass distribution – not the central mass singularity, but one in which a total mass, *M*, is slightly 'smeared out' in space around the central point in a spherically symmetric mass-density distribution. (It must be smeared out to an indefinitely large radius from *M*, and infinitely finely, although only a tiny amount of mass is smeared out beyond the small central region.) This smearing of the point-mass into a continuous mass-density, when treated in GTR, slightly weakens the gravitational effect on the metric obtained from a point-mass.

We can therefore turn directly to calculating the effective differences on trajectories between the metrics (4) and (1) regarded as alternative GTR metrics.

3. Derivation of radial trajectory solutions.

The main solution of interest here is for a radial trajectory, with non-relativistic velocities. Radial light trajectories and circular orbits are derived subsequently. The metrics (1) and (4) are static, and we can apply the geodesic principles directly. We begin by simplifying (1) to a reduced metric for a specific trajectory, where we introduce local orthogonal coordinates at the field point ***r***, represented by: (r,y,z), and for the specific trajectory we are considering at this point, we choose the directions of y and z so that: $dz = 0$ on the trajectory. We will subsequently set $dy = dz = 0$ for the radial trajectory, but we leave the dy term in for the moment. Thus we reduce the metric (1) to the special simple form:

(7) GTR: $$\mathrm{d}\tau^2 = \frac{dt^2}{k^2} - \frac{dr^2 k^2}{c^2} - \frac{dy^2}{c^2}$$

Equivalently, the form (4) reduces to:

These features will not be discussed here; the aim of this paper is only to apply TFP gravity to simple solar system phenomena. See [2] for more details of the general conception.

(8) K-gravity: $$\mathrm{d}\tau^2 = \frac{dt^2}{K^2} - \frac{dr^2 K^2}{c^2} - \frac{dy^2}{c^2}$$

The geodesic equations for these are:

(9) $$\frac{d}{ds}\left(\frac{g_{tt}(dt/d\tau)}{L(s)}\right) = 0$$

(10) $$\frac{d}{ds}\left(\frac{g_{rr}(dr/d\tau)}{L(s)}\right) = 0$$

(11) $$\frac{d}{ds}\left(\frac{g_{yy}(dy/d\tau)}{L(s)}\right) = 0$$

(12) $$L(s) = \sqrt{g_{tt}(dt/d\tau)^2 + g_{rr}(dr/d\tau)^2 + g_{yy}(dy/d\tau)^2}$$

We choose the parameter s as the proper time, τ, as usual, so that $L = 1$.

Radial trajectories.

On assuming that $dy/d\tau = 0$ for a radial trajectory, the metric terms are alternatively:

(13) GTR: $g_{tt} = 1/k^2$ $g_{rr} = -k^2/c^2$ $g_{yy} = -1/c^2$

(14) K-gravity: $g_{tt} = 1/K^2$ $g_{rr} = -K^2/c^2$ $g_{yy} = -1/c^2$

We wish to compare the ordinary velocities, dr/dt, generated by the alternative metrics. We can write these as the full differentials, dr/dt, *in the special central frame of reference for the specific radial trajectory*, because we can adopt t as the trajectory parameter, i.e. we can transform from: $r(\tau) \rightarrow r(t)$ to represent these trajectories[5]. Similarly we can take the partial differentials: $\partial k/\partial r$ as equivalent to full differentials: dk/dr, since in this coordinate frame, k is a function of r only.

[5] This is possible as long as there is an invertible function: $t = t(\tau) \leftrightarrow \tau = \tau(t)$, where these functions are specific to each trajectory, as in the present examples of sub-luminal trajectories, and as long as we remain outside the event horizon. We cannot do this for light signals.

The first key equation is obtained by setting $dy = dz = 0$ in (7) and rearranging to:

(15) $$\left(\frac{dr}{dt}\right)^2 = \frac{c^2}{k^4} - \frac{c^2}{k^2}\left(\frac{d\tau}{dt}\right)^2$$

The ordinary radial velocity, $dr/dt = v$ in this case is the total ordinary velocity. The second key equation is:

(16) $$\frac{1}{k^2}\frac{dt}{d\tau} = N \qquad \text{or:} \qquad \frac{dt}{d\tau} = Nk^2$$

where N is constant. This follows directly from integrating (9). Note that no operation has yet been performed on the function k. These two equations let us solve for $v = dr/dt$:

(17) $$\left(\frac{dr}{dt}\right)^2 = \frac{c^2}{k^4} - \frac{c^2}{N^2k^6}$$

We can define v_0 as the velocity 'at infinity', where k→1, given by:

(18) $$v_0 =_{\lim k\to 1} \frac{dr}{dt} = c^2 - \frac{c^2}{N^2}$$

For the Pioneer trajectories we assume that this is positive (they have escape velocity). This gives:

(19) $$\frac{1}{N^2} = \frac{c^2 - {v_0}^2}{c^2}$$

Putting this in (17) and rearranging gives:

(20) $$\left(\frac{dr}{dt}\right)^2 = c^2\left(\frac{1}{k^4} - \frac{1}{k^6}\right) + \frac{{v_0}^2}{k^6}$$

Finally, we can insert the specific function for k in (20) to obtain the solution for GTR.

Equally, we can insert K in (20) to obtain the alternative solution for K-gravity, because the reasoning so far does not depend upon the choice of k or K. We use the useful identities:

(21) GTR: $$\frac{1}{k^4}-\frac{1}{k^6}=\left(\frac{1-2MG}{c^2r}\right)\left(\frac{2MG}{c^2r}-4\left(\frac{MG}{c^2r}\right)^2\right)$$

$$=\frac{2MG}{c^2r}-8\left(\frac{MG}{c^2r}\right)^2+8\left(\frac{MG}{c^2r}\right)^3$$

(22) K-gravity:

$$\frac{1}{K^4}-\frac{1}{K^6}=\left(1-\frac{4MG}{c^2r}+\frac{1}{2!}\left(\frac{4MG}{c^2r}\right)^2-...\right)-\left(1-\frac{6MG}{c^2r}+\frac{1}{2!}\left(\frac{6MG}{c^2r}\right)^2-...\right)$$

$$=\frac{2MG}{c^2r}-10\left(\frac{MG}{c^2r}\right)^2+...$$

(The '…' terms are cubic or higher-order, and ignored in the approximate solutions). The speed function calculated on GTR will be denoted *v(r)*, and the alternative speed function calculated on K-gravity will now be denoted *v*(r)*. Substituting the identities (21) and (22) in (20) we obtain the main radial velocity solutions:

(23) GTR: $$v(r,M)^2=\frac{2MG}{r}-8c^2\left(\frac{MG}{c^2r}\right)^2+8c^2\left(\frac{MG}{c^2r}\right)^3+\frac{v_0{}^2}{k^6}$$

(24) K-gravity: $$v*(r,M)^2=\frac{2MG}{r}-10c^2\left(\frac{MG}{c^2r}\right)^2+...+\frac{v_0{}^2}{k^6}$$

We take these relations at the two different radii, r_1 and r_2, to obtain the approximations, up to second order terms:

(25) GTR: $$v_1{}^2-\frac{2MG}{r_1}+8c^2\left(\frac{MG}{c^2r_1}\right)^2-\frac{v_0{}^2}{k_1{}^6}\approx v_2{}^2-\frac{2MG}{r_2}+8c^2\left(\frac{MG}{c^2r_2}\right)^2-\frac{v_0{}^2}{k_2{}^6}$$

The last two terms on each side effectively cancel, because: $v_0^2(1/k_1^6 - 1/k_2^6)$ is extremely small compared to other terms, for non-relativistic velocities, and hence to a very close approximation:

(26) GTR: $$v_1{}^2-\frac{2MG}{r_1}+8c^2\left(\frac{MG}{c^2r_1}\right)^2\approx v_2{}^2-\frac{2MG}{r_2}+8c^2\left(\frac{MG}{c^2r_2}\right)^2$$

Similarly, in K-gravity we obtain:

(27) K-gravity: $$v_1 *^2 - \frac{2MG}{r_1} + 10c^2\left(\frac{MG}{c^2 r_1}\right)^2 \approx v_2 *^2 - \frac{2MG}{r_2} + 10c^2\left(\frac{MG}{c^2 r_2}\right)^2$$

We will use these to compare differences in the Pioneer trajectories predicted on the two theories in Section 5.

4. Central accelerations for circular orbits.

We can derive radial accelerations for non-relativistic circular trajectories from the GTR relation above:

(27) $$\left(\frac{dr}{d\tau}\right)^2 = c^2 N^2 - \frac{c^2}{k^2}$$

Differentiating by τ gives:

(28) $$2\left(\frac{dr}{d\tau}\right)\left(\frac{d^2 r}{d\tau^2}\right) = -c^2\left(\frac{dr}{d\tau}\right)\frac{d}{dr}\left(\frac{1}{k^2}\right)$$

Differentiating *k* gives:

(29) $$\frac{dk}{dr} = -k^3 \frac{MG}{c^2 r^2}$$

Giving the *GTR Radial Trajectory Acceleration*:

(30) $$\frac{d^2 r}{d\tau^2} = -\frac{MG}{r^2}$$

We obtain the solution for K-gravity by using *K* instead of *k*. We get a similar result, the difference resulting from the fact that:

(31) $$\frac{dK}{dr} = -K \frac{MG}{c^2 r^2}$$

Since $k/K \approx 1$, we see that: $dk/dr \approx k^2 dK/dr$. This difference between the first-order divergences of *k* and *K* gives the key difference in the solutions. With K-gravity we get:

(32) $$\frac{d^2 r}{d\tau^2} = -\frac{MG}{r^2 K^2}$$ *K-gravity Radial Trajectory Acceleration.*

This is how the main difference between the two theories arises for weak gravitational fields: through the additional factor of $1/K^2$ in the K-gravity accelerations. For non-relativistic speeds $v<<c$, essentially the same difference of $1/K^2$ carries through, to a very close approximation, to the ordinary accelerations: d^2r/dt^2, (for any orbits). This is how the *modified mass*: $M^* = M/K^2$ introduced below is derived.[6]

5. Differences between GTR and K-Gravity for radial free-fall.

The key problem treated here is to determine the time taken by a spaceship in free-fall (regarded as a point-particle), traveling on a radial trajectory outwards from a central mass, M (the sun), from an initial point, r_1 to a final point, r_2.[7] The initial speed, $v_1 = v(r_1)$, is assumed to be known. The speed functions must be determined for the two distinct theories, GTR and K-gravity, and the time-lapse difference calculated, to determine the discrepancy that can be expected on the alternative theory.

The speed function calculated on GTR is denoted $v(r)$, and the alternative speed function calculated on K-gravity is denoted $v^*(r)$. The total time for the journey from r_1 to r_2 calculated by GTR is denoted T_{12}, and the corresponding time calculated by K-gravity is denoted T_{12}^*.

(33) GTR

$$T_{12} = \int_{r_1}^{r_2} \frac{1}{v(r)} dr$$

(34) K-gravity

$$T_{12}^* = \int_{r_1}^{r_2} \frac{1}{v^*(r)} dr$$

The difference is then given by:

(35)

$$\Delta T = T_{12} - T_{12}^* = \int_{r_1}^{r_2} \frac{1}{v(r)} dr - \int_{r_1}^{r_2} \frac{1}{v^*(r)} dr$$

The functions $v(r)$ and $v^*(r)$, are obtained from (26) and (27) above, and a numerical integration will give the desired result. First I give the Newtonian solution, denoted

[6] A second method of deriving the results of K-gravity from more fundamental principles of TFP gravity is given in [2].
[7] We can neglect the angular momentum in this situation.

$v_N(r)$. In Newtonian theory, $v_N(r)$ is given by fixing r_1 and $v_{N1} = v_N(r_1)$ as constants at some special point, and using the general Newtonian energy relationship:

Newtonian Gravitational Potential for free-fall

(36)
$$E = \frac{1}{2} m_0 v_N(r)^2 - \frac{m_0 MG}{r} = \frac{1}{2} m_0 {v_{N1}}^2 - \frac{m_0 MG}{r_1}$$

Thus we get the classical velocity function for radial free-fall:

Newtonian velocity

(37)
$$v_N(r)^2 = {v_1}^2 - \frac{2MG}{r_1} + \frac{2MG}{r}$$

The second-order approximation for GTR radial free-fall with non-relativistic velocities is obtained by rearranging (26) and taking $v = v_2$:

GTR radial free-fall velocity approximation (v << c).

(38)
$$v(r)^2 \approx {v_1}^2 - \frac{2MG}{r_1} + \frac{8MG}{r_1}\frac{MG}{c^2 r_1} + \frac{2MG}{r} - \frac{8MG}{r}\frac{MG}{c^2 r}$$

Similarly, rearranging (27) shows that K-gravity modifies the GTR solution, for a common mass M, and fixed v_1, to:

K-gravity radial free-fall velocity approximation (v << c).

(39)
$$v^*(r)^2 \approx {v_1}^2 - \frac{2MG}{r_1} + \frac{10MG}{r_1}\frac{MG}{c^2 r_1} + \frac{2MG}{r} - \frac{10MG}{r}\frac{MG}{c^2 r}$$

Thus K-gravity modifies the GTR velocity by:

(40) $$v^*(r)^2 \approx v(r)^2 + \frac{2MG}{r_1}\frac{MG}{c^2 r_1} - \frac{2MG}{r}\frac{MG}{c^2 r}$$

Thus, K-gravity weakens the effect of GTR gravity on non-relativistic velocities by the second order factor of: *2(MG/r)(MG/c*^{2}*r)*, and the effective difference between GTR and K-gravity is given by the integral (35) by setting:

(41) $$v^*(r) \approx \sqrt{v(r)^2 + \frac{2MG}{r_1}\frac{MG}{c^2 r_1} - \frac{2MG}{r}\frac{MG}{c^2 r}}$$

This is the convenient form of relationship to generate a numerical approximation for the difference between GTR and K-gravity. However, before we can apply this solution, there is an additional critical feature that must be taken into account: it relates to the *theory-dependence of the estimation of the central mass, M.* This gives an unexpected twist to the situation.

6. The mass adjustment for the sun.

We have seen that K-gravity is slightly weaker than ordinary GTR gravity *when we apply both theories to a common central mass, M.* In the case of trajectories in the solar system, *M* is the estimated mass of the sun. But before we can directly apply the results to solar system trajectories, we have to take into account that *the mass M of the sun is inferred from observations on the basis of the adopted theory.* This is normally GTR (or just Newtonian gravity). But to apply K-gravity properly, we must *reinterpret the entire solar system using K-gravity,* rather than simply substituting equation (41) into (35).

On the assumption of K-gravity, we cannot arrive at the same value for the initial mass, *M*, of the sun, that we obtain on GTR. The mass of the sun is calculated by applying our accepted theory of gravity to directly measured quantities of the period of rotation and distance of the Earth or other orbiting bodies from the sun. These measured quantities tell us the acceleration the sun's gravity is generating, and we estimate the mass of the sun to satisfy this acceleration by assuming the preferred theory of gravity.

If K-gravity is true, the assumption of GTR must lead us to underestimate the real mass of the sun, since GTR overestimates the strength of gravity generated by a given mass, M (comparing (30) and (32)). To apply K-gravity, we consequently have to re-evaluate the mass of the sun, and assign it a slightly larger measured mass. I will continue to use M to denote the mass of the sun estimated through GTR, and M^* to denote the mass of the sun estimated through K-gravity. We require $M^*>M$ to obtain consistency with the measured orbits and the laws of K-gravity.[8]

Note also that GTR and K-gravity *converge at large radius from a given mass* M^* (the difference in the second-order terms of r becoming negligible). Hence K-gravity predicts that, at large r, the trajectories will *converge to ordinary GTR solutions, but for an increased central mass, M*, which is larger than M as measured on the assumption of GTR.* This will make it *appear* to the GTR theorist who observes a spacecraft leaving the solar-system in free-fall that there is an additional inward force slowing the space-craft down: but it is not really an additional force, *it is the effect of extra mass of the sun*.

The observation that the Pioneer spacecraft have slowed down more than they should have has led to speculation that there is an unknown extra force acting *inwards* to slow them down, or that gravity is slightly stronger than previously thought - but this is not necessary. It depends on how a proposed 'extra force' acts at the distance of Earth's orbit compared to larger orbits – if it already acts at Earth's orbit, and *becomes weaker at larger orbits*, then we require the force to be an outwards force; but if the force only comes into play at large distances, it needs to be an inward force.

Thus the explanation obtained from K-gravity for solar-system orbits requires simultaneously reevaluating the mass of the sun (or more exactly, MG), along with the modified laws of gravity.

Assuming that M is determined most accurately from measurements of the period and radius of the orbit of the Earth, this leads to a correction of:

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[8] More exactly, we need to correct the factor: MG, i.e. we should really make the transformation from *MG-using-GTR* to *(MG)*-using-K-gravity*. Note that MG is known much more accurately than M or G separately. The proposed adjustment in M is smaller than the accuracy to which M is known by about two orders of magnitude, but larger than the accuracy to which MG is known, by about two orders of magnitude. But the correction is made to M rather than MG here, to emphasize that G can be determined independently of the mass of the sun, whereas M is dependant on our theory of gravity. This makes no practical difference to the calculations since the terms M and G occur inseparably in the equations.

K-gravity modification of estimated mass of the sun.

(42) $\Delta M/M = (M^*-M)/M \approx 4\text{x}10^{-8}M.$

The solution is obtained by comparing (30) and (32), to obtain:

(43) $$M^* = \frac{M}{K(r_{Earth})^2}$$

M generates the same acceleration using K-gravity at Earth orbit as M generates using GTR-gravity.*

To confirm this proposal of modifying the sun's estimated mass, we must also consider whether using this modified mass *M** in K-gravity would be detectible through observations of planetary orbits. The analysis of circular orbits given below suggests that this correction is just beyond the current accuracy of planetary observations. (The effect might show up in the precession of the perihelions of Mercury, but this is not analysed here).

To complete the application of the theory, we continue by requiring that *v*(r)* in the integrals (34) and (35), and the solutions represented in (39)-(41) are obtained by taking the K-gravity solution for *the modified mass, M*.* We will represent this mass transformation by writing *k* and *K* and *v* and *v** explicitly as functions of both radius and mass, and writing the distinct terms: *k(r,M), k(r,M*), K(r,M), K(r,M*), v(r,M), v(r,M*).* Thus we convert (41) to:

(44) $$v^*(r,M^*) \approx \sqrt{v(r,M^*)^2 + \frac{2M^*G}{r_1}\frac{M^*G}{c^2 r_1} - \frac{2M^*G}{r}\frac{M^*G}{c^2 r}}$$

Note that we set:

(45) $$v^*(r_1) = v^*(r_1,M^*) = v(r_1,M^*) = v(r_1,M) = v(r_1)$$

as the boundary condition for *the initial observed velocity,* because this is independently known. We then use the use the terms:

(46) $$v(r) = v(r,M)$$

in (33), and the term:

(47) $$v^*(r) = v^*(r, M^*)$$

in (34), to obtain the time-delay equation (35) as:

(48) $$\Delta T = T_{12} - T_{12}^* = \int_{r_1}^{r_2} \frac{1}{v(r,M)} dr - \int_{r_1}^{r_2} \frac{1}{v^*(r,M^*)} dr$$

We are now in a position to calculate a numerical approximation for the solutions, using parameters appropriate for the Pioneer spacecraft orbits, summarized next.

7. Approximate Numerical Solutions for the Pioneer Trajectories.

I have calculated numerical approximations of the differences between GTR and K-gravity for a range of parameters approximating the Pioneer trajectories.[9] The results are quite sensitive to the trajectory parameters, but the results indicate a good match with the empirical data.

We can just use the Newtonian approximation, (37), for $v(r)$, throughout the calculations, because although the values for $v(r)$ using (37) are not particularly accurate as absolute velocities, the velocity *differences* between GTR and K-gravity predictions generated by using this approximation for $v(r)$ in (41) and (35) are accurate. I.e. there is no practical need to obtain $v(r)$ any more accurately than in the Newtonian approximation for the comparison of the GTR and K-gravity trajectories.

The results are graphed in Figure 1, which shows the variation in the time lapse for a journey to $r_2 = 80$ A.U., plotted against variations in the radial parameter r_1 from $r_1 = 1$ A.U. to 20 A.U., using three different velocity parameters approximately fitting the real Pioneer data. The critical parameters needed to make the predictions are: (a) the radial distances, r_1 and r_2, between which the free-fall trajectory has been measured; and (b) the initial velocity $v_1 = v_1^*$ at r_1. The effect is sensitive to the

[9] To make a numerical approximation in a spreadsheet, I have: (i) generated a column of ordinary GTR velocities, $v(r,M)$, at discrete points of r (dividing r into 100 equal increments) (ii) generated a similar column of mass-modified K-gravity velocities, $v(r,M^*)$, using the alternative mass: $M^* = M/K(r_{Earth})^2$; (iii) used (44), with the mass-modified values $v(r,M^*)$ for $v(r)$ in (38), to obtain the difference between GTR and K-gravity using the mass M*; (iv) approximated (48) by numerically summing and subtracting the differences between the incremental times for journey increments, using the GTR-velocities based on $v(r,M)$, and the K-gravity velocities based on $v^*(r,M^*)$.

velocity parameter, and I have duplicated the calculations for three values of initial velocity, assuming free-fall by a spacecraft with a radial speed equal to Earth orbit speed plus (alternatively) 50,000, 51,000 and 52,000 km/hr. The results show a good initial match with the reported anomalies.

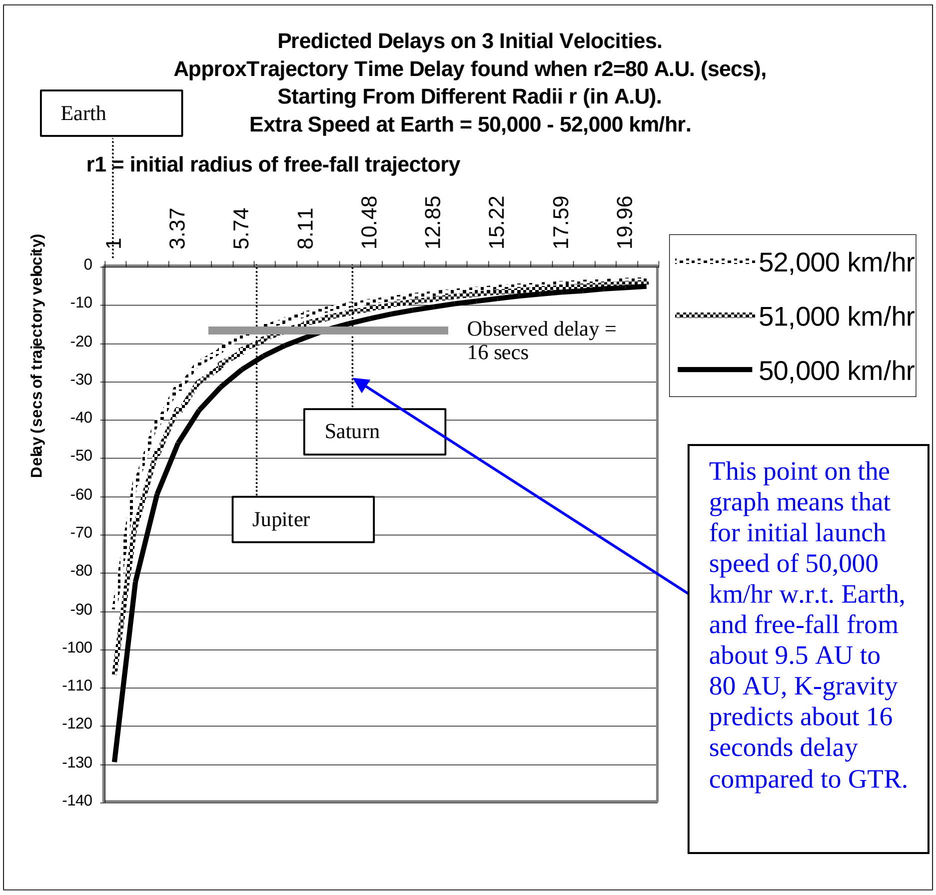

Figure 1. Predicted delays, in seconds of trajectory, for different initial radius of free-fall, and 3 different initial radial velocities.

E.g. this shows that a delay of about 16 seconds would be predicted for a free-fall trajectory starting from radius r_1 at the orbit of Jupiter to 80 AU, assuming an initial launch speed of 44330 m/sec (without significant subsequent radial accelerations from firing rockets).

On the assumption of a free-fall from the orbit of Saturn to 80 A.U., the following delays are estimated for three different initial speeds:

Table 1. Predicted trajectory-time delays in Pioneer spacecraft.

Initial extra velocity at Earth orbit (km/hr)	Initial total velocity (m/sec)	Velocity at 80 A.U. (m/sec)	Predicted delay in journey, from 9.5 A.U. (Saturn) to 80 A.U.
50,000	43775	12338.2	**-16 seconds**
51,000	44052	13290.1	**-13 seconds**
52,000	44330	14183.7	**-11 seconds**

It is also useful to see what happens to the absolute velocities, and Figure 2 below shows the absolute velocity *differences* expected in free-fall from 1 A.U. to 100 A.U.

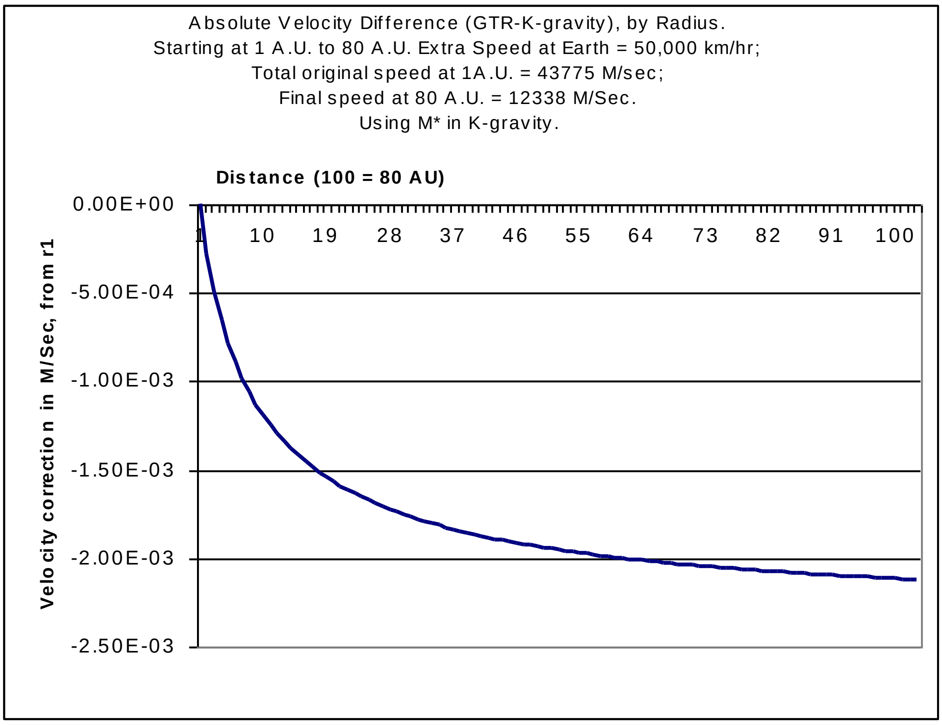

Figure 2. Predicted velocity differences: *v(r) – v*(r)*, for free-fall from 1 A.U. to 100 A.U., for initial radial speed of 50,000 km/s.

Fig. 2 shows that the *absolute speed difference*, *v(r) – v*(r)*, is very small compared to the total speeds. And the effect on total speed falls off rapidly with distance; about half the total speed difference in going from Earth to 80 A.U. is already generated in going just from Earth to the distance of Saturn, at about 9.5 A.U. But the time delay is obtained from integration over the length of the journey, and a long journey magnifies the tiny speed differences into a detectible time delay.

This is shown by comparing the predicted delays shown in Figure 1, for the long free-fall to 80 A.U, with an alternative free-fall from 1 A.U. to Jupiter's orbit, shown in Fig. 3 below.

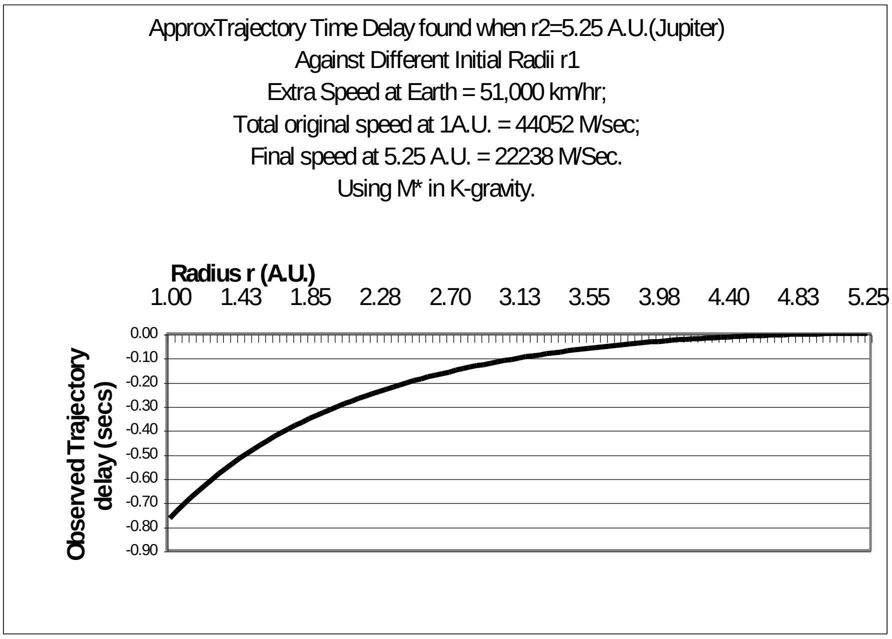

Figure 3. Predicted delays in journey measured from r_1= r to r_2= 5.25 A.U. (Jupiter orbit), for initial radial speed at Earth of 51,000 km/hr.

In this case, for a free-fall journey measured from Earth to Jupiter, the total speed difference between GTR and K-gravity is comparable to that for a journey starting from Saturn's orbit and going to 80 A.U. But the time of the Earth to Jupiter journey is much shorter, and the delay effect is only about 0.8 sec in the trajectory time, or about 6 $x10^{-5}$ seconds delay for round-trip light signals from Earth to the spacecraft. Such a delay is detectible in principle, but unlikely to be noticed in the Pioneer orbits, since there are many confounding factors, including the influence of Earth and Jupiter's planetary gravity, the effect of the solar wind, and any firing of propulsion rockets, which introduce uncertainties into the absolute predicted times, and it may be expected that any observed anomaly in this part of actual Pioneer trajectories would be put down to error. These confounding factors are scarce in the long outer-solar-system journey from Saturn.

8. Circular orbits and light-signal delays.

There are two further features of K-gravity that need to be considered: effects on the radial light signals which provide the measurements of the trajectories of the spacecraft and effects on planetary orbits, which are modeled here by approximating them to circular orbits. A check also needs to be done for effects on the precession of the perihelion of Mercury, but this is not discussed here.

The effect of K-gravity on the light-trip time between the spacecraft and Earth is necessary to establish the differences in the round-trip signaling time, but it is easy to show that this turns out to be tiny ($<10^{-20}$ secs), and may be completely neglected.

I will briefly summarize the effects of K-gravity on circular orbits approximated by planets. We first assume that we have measured the period and radius of the Earth's orbit exactly, and taken the mass of the sun as M so that the acceleration matches with GTR. The relationship for a circular orbit (with non-relativistic speed) is: $v = 2\pi r/T = \sqrt{(MG/r)}$, so T and r determine MG. This is known very precisely. We then measure the period, T, of some other planetary orbit, which is also done very precisely. Using this period T with the GM and the laws of GTR gives us a prediction of this radius, r, of the planetary orbit. We can check GTR by checking this prediction against direct measurements of r.

We then compare with predictions using K-gravity and the modified sun mass M^*. M^* is chosen to make the K-gravity predictions consistent for Earth's orbit. Applying K-gravity to a planetary orbit with a fixed period T using mass M^*, we obtain a different predicted orbit radius, $r^* \neq r$, for the planet. If r^* was detectibly different from r, then the difference between the two theories would be evident in the orbits. The predicted differences are graphed in Fig. 4.

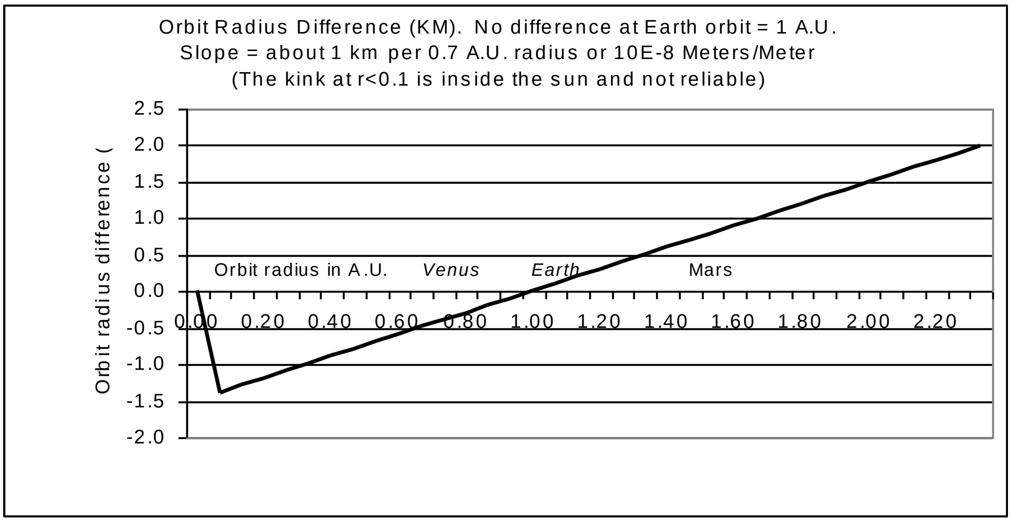

Figure 4. K-gravity modifications of circular orbit radius around the sun, for given rotation period.

- The K-gravity correction for Venus makes its average orbit radius about 0.5 km smaller than expected.
- The correction for Mars makes its average orbit radius almost 1 km larger.
- The correction for Jupiter (not shown in Fig. 4) makes it orbit about 6.4 km larger.

These are the expected effects on the average orbit distances (semimajor axes) for a fixed *period*. But the planetary orbits are of course not exactly circular, and have small perturbations due to other bodies (particularly their satellites), making practical checks of these discrepancies difficult. In practice there are measurement uncertainties in both the periods and orbit distances of the planets, but the periods are easier to measure directly. The question is whether the joint uncertainties are large enough to allow the predicted discrepancies.

Most important however is whether the distance measurements can be used to infer precise distances to the *center of mass.* Measurement uncertainty for average orbit distances for Venus is standardly reported at around 1 km by the late 1990's, and most recently error as small as 0.1 km is reported. The latter is smaller than the predicted discrepancy of 0.5 km. This might indicate a negative empirical result, but only if this represents the measurement uncertainty in the *absolute distance measurement to the center of mass* of Venus. In fact it represents *variations in the center-of-mass distance (from the sun),* but not the precision of the absolute measurement.

It is also not clear whether the reported errors pertain to *direct measurements of distances.* We require *direct measurements* of *the orbit distances throughout the orbits.* We cannot use indirect *inferences* of average orbit distances, based on *the application of GTR* to precise measurements of periods and *MG,* along with direct distance measurements of only the minor and major axes of the orbits, for instance – since K-gravity will slightly distort the non-circular orbits from those predicted by GTR.[10]

I conclude that the current data on planetary orbits appears *prima facie* consistent with K-gravity, but a more careful study is required, and other anomalous effects in the solar system must come to light if the new theory is correct.

9. Experiment.

An experiment could be done to directly test the difference between K-gravity and GTR, by sending a simple space-craft in free-fall from Earth at 1 AU, to a distance of a few AU (Jupiter), avoiding planets, and measuring its trajectory precisely. By choosing the initial speed appropriately, the anomaly predicted by K-gravity can be tested in a journey of about two or three years. The speed of the Pioneer's was not optimal in this respect, and would have only produced an anomaly of about 0.8 seconds in free-fall from 1 AU to about 6 AU; this can be enhanced by choosing an initial speed closer to the solar escape velocity. Nieto and Turyshev [9] have proposed a (more ambitious and general) project to test the origin of the Pioneer anomalies; but testing K-gravity alone would be much quicker and simpler than their proposal.

[10] Also: (c) it is not clear whether a careful comparison of the most recent orbital data has been carefully compared with predictions of GTR anyway.

10. Conclusion.

Solutions of K-gravity for a central mass system are obtained as a modification of the usual Schwarzschild solutions. The application to the solar system requires a reinterpretation of the solar mass, M, (or more exactly, MG), which is currently inferred on the basis of GTR (or just the Newtonian limit), applied to orbital data for Earth. K-gravity is slightly weaker than GTR (or Newtonian) gravity for a given central mass, so we are required to reinterpret the mass of the sun as slightly larger than we do using GTR.

Applying K-gravity to the solar system we recalculate the solar mass M (or the product MG) as about 1.00000004 times the GTR estimation based on the Earth's orbit, and we then use this revised mass in the K-gravity equations.

The overall effect on the trajectory of a spacecraft in free-fall in the outer solar system makes the force of gravity *appear stronger* than predicted by GTR gravity, because using GTR gravity underestimates the solar mass. The predicted anomaly with GTR for the trajectory time of Pioneer 10 in free-fall from the orbit of Saturn at about 9.5 AU to 80 AU is found to be around 10 to 20 seconds (depending on the initial speed). This is a good fit with the observed anomaly, and justifies a more precise study.

K-gravity also predicts slightly different planetary orbits to GTR. The discrepancies are slightly larger than the measurement error reported for the orbit parameters for Venus, which suggests a negative result. However the reported measurement errors do not pertain to *the absolute distance measurement to the center-of-mass*. A more detailed study is required to evaluate K-gravity on these grounds, but it seems unlikely that direct measurements to the center of mass could be done independently of the theoretical assumptions in any case.

K-gravity is indistinguishable from GTR in its effects on light signals in the solar system, and makes no appreciable difference in the expected light-delays for signals between the Pioneers at 80 AU and Earth.

Finally note that while K-gravity differs only very slightly from GTR for weak fields, there are large differences for strong fields, and fundamental theoretical differences between the two theories. It may affect the gravitational dynamics of galaxies, galaxy formation, the interpretation of 'dark matter', 'dark energy', and so on. Observations of the phenomena have anomalies with current theory (primarily

GTR), so radical theory changes remain an open possibility, but this is beyond the scope of this paper.

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