The incompleteness of extensional object languages of physics and time reversal. Part 1.

Andrew Holster PhD. ATASA030@gmail.com
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1. Introduction.

There are frequent disputes about the *conceptual* implications of theories of physics - yet the principles for settling these are not usually regarded as contentious, because it is generally thought they are settled by careful mathematical or conceptual analysis. I raise a problem about this, and argue that modern physics lacks an adequate proof theory required for many essential results. The primary example of concern here is the analysis of *time reversal transformations of statements or propositions*. The subject of time symmetry and reversibility has been intensively investigated since Boltzmann's work in the late 19th Century, but the literature is still full of controversies even about the correct *conceptual* analysis of many issues. Some leading philosophers of physics have complained for decades that concepts of time directionality remain unclear in physics, and have disputed various claims about the accepted analyses[1].

Two such disputes, concerning the time reversal symmetry of quantum mechanics (QM), are summarized shortly, as examples. I should emphasize that the point is not to try to solve these particular disputes here: detailed discussions are given in Holster 2003(A) and (B), and by writers mentioned in *foot note 1*. I merely summarize 'Orthodox' views on the one hand, and 'Alternative' views on the other, as illustrations of the source of the problem. The peculiarity of these disputes is that they have continued so long without conclusive resolution. Given that they are 'merely' conceptual disputes, about the correct logical analysis of time reversal invariance of QM, why haven't physicists simply given conclusive, deductive, mathematical proofs of the correct results, and settled the issues for good?

The reason I propose here is that there are no conclusive deductive proofs of key results available, *in principle*, because there is no way of constructing an adequate deductive proof theory in the current formalism of physics.

The problem, I hasten to add, is not that there are no proofs possible in principle: only that *the deductive formalism of ordinary physics is currently too weak to provide any proof theory*. This is a strong claim, and this paper (Part 1) is taken up establishing it, before moving on to consider how it may be solved, in Part 2.

> ***Main Claim.*** *Ordinary extensional object languages of physics are too weak to construct adequate syntactically-based system of rules for making formal deductions of the effects of general transformations (such as time reversal) on propositions.*

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[1] E.g. Schrodinger, Watanabe, de Beauregard, Healey, Penrose, and Callender have all argued there are failures in the conceptual analysis of time symmetry of quantum mechanics. These are verified in Holster, 2003(a) and 2003(b).

I go on to argue in Part 2 that the ordinary formalism of physics *can be extended* to allow proofs; but the point is that this involves a radical extension of the formal syntax and semantics. It requires an extension from the *extensional languages* of ordinary physics, to an *intensional language.*

The increased logical power of an intensional formalism is by now well-known to semanticists and logicians generally, following the pioneering work of Montague, Tichý , and many others. It is most obviously required when we need to deal with the logic of 'contingent identities', as opposed to purely logical identities. But the need for such an extension for the object languages of theoretical physics does not appear to have ever been noticed. I would trace this oversight, in first place, to the fact that the object languages of physics appear superficially to be just *mathematical languages*, which require only extensional interpretations; but close analysis shows they are not, because they incorporate contingent identities, and an intensional logic is ultimately necessary to deal with this. A second reason is that intensional logic or semantics is a specialist subject, beyond the orbit of philosophers of science and physicists who examine these problems, and the lessons have not been recognized. The closest approach is through the model theoretic proofs, often employed by John Earman for example; but model theory is quite different to intensional semantics.

The primary example motivating this study is the time reversal transformation, T: $t \rightarrow -t$. But the claim made above holds for any general transformation. A general transformation in this sense is generated by an automorphism on basic sets used in the fundamental models; other examples include time translation, space reflection, space rotation, charge inversion, or general Galilean or Lorentz transformations. These are based on automorphisms on sets of basic or fundamental entities (time, space, charge, etc), and they *induce transformations on higher-order entities*, such as states, processes, laws, and theories. They provide the concepts of general symmetries of theories. However, the focus here is restricted to time reversal transformations, and I begin by summarizing two disputes that illustrate the problems clearly.

We can note, firstly, that orthodox physics does acknowledge a question here already.

Orthodox physics view.

Physicists have well-developed views about the general symmetries of their theories, and usually claim that their fundamental theories are *time reversal invariant* (TRI). To establish this claim about a theory, it is necessary to show the theory is invariant under the transformation *T*, which involves deriving the effect of the transformation on propositions or laws of the theory. Physicists have a system of rules for doing this, described in later sections, which is roughly this:

(i) We first distribute *T* throughout the statement of a law of physics; i.e. we switch everything in the statement to its *T*-image; and

(ii) To complete the job, we have to *interpret* what the time reversals of various *states or properties* mentioned in the statement are. (E.g. the electric field is transformed: $T\mathbf{E} \rightarrow \mathbf{E}$, but the magnetic field is transformed: $T\mathbf{B} \rightarrow \mathbf{-B}$.)

The orthodox view is that the second task, (ii), of *interpreting state reversal*, is a step that cannot be 'deduced' automatically, and must be examined separately within each theory. This is seen as the main source of complication in practical analysis. E.g.

> *Physicists have nevertheless figured out by now how to time reverse most of the physical states, but it is done on a case-by-case basis; no general recipe is found which is independent of particular theories or models. (Liu, 1993, p.622.)*

But once the *T*-images of all the different states have been decided, then it is thought that (i) is a deductive procedure, which conclusively settles the time reversals of statements or propositions. However, I maintain a different position on both these main points.

Alternative View.

(i) First, I maintain that the physicist's usual system for implementing a 'syntactic *T*-operator' does *not* work consistently. In fact, I claim that *it is logically impossible to provide a deductively valid system of general rules adequate for making T-derivations in ordinary object languages of physics.* The reason is that these languages are merely *extensional.* I subsequently argue in Part 2 that if we extend to an intensional formalism, we can indeed construct a *distributive, compositional, syntactic operator* that calculates all the effects of time reversal; and thus we can solve (i).

(ii) I also go on to argue in Part 2 that if we do extend to an intensional language, we are forced to explicitly define the construction of world-variables, and doing this completely determines the effects of *T* on states and such-like. It is then possible to provide fully rigorous deductive proofs about general transformations such as *T*.

2. Two Disputes About Time Reversal in Physics.

Two key disputes, both relating to QM, illustrate these problems.

Problem Example 1. The first problem concerns *the criterion adopted in physics for judging the time reversal invariance of probabilistic theories*. I will refer to the criterion normally adopted as the *Orthodox Criterion for Reversal Symmetry.* It is defined by:

> **Definition.** A probabilistic theory **T** satisfies the *Orthodox Criterion for Reversal Symmetry* just in case:
>
> $$\mathbf{T} \Rightarrow [\ prob(s_2(t+\Delta t)|s_1(t)) = prob(Ts_1(t+\Delta t)|Ts_2(t))]$$

The terms: $s_1(t)$ and: $s_2(t+\Delta t)$ here represent *propositions,* respectively, that a system is found in state s_1 at time t, and in state s_2 at time $t+\Delta t$. Ts_1 and Ts_2 are the time reversed images of s_1 and s_2.

Note that the general logic underlying this kind of criterion is based on two main definitions.

> **Definition 1.** A theory **T** is *time reversal invariant* (TRI) just in case its time-reversed image, *T***T**, is logically equivalent to **T**.

We can write this property as: *T***T** = **T** (or in more standard notation, as: *T***T** ⇔ **T**).

Some theories are represented by a single key equation of motion, and we can examine this equation directly. But complex theories, like quantum mechanics (QM), are represented by classes of laws, rather than single general statements, and we need to derive time reversed images indirectly, by considering the effects of T on the class of laws. E.g. most probabilistic theories entail collections of probabilistic laws, of the general form: $prob(s_2(t+\Delta t)|s_1(t)) = p$, where the value p are given by general functions of the states and Δt. We must then define T**T** indirectly, as follows.

Definition 2. For any theory, **T**¸ and law, *L:*
T**T** entails TL just in case: **T** entails L.

I.e. the time reversed image, T**T**, of a theory **T**, entails precisely the time reversed images of all the logical implications of **T**.

We also assume that double time reversal is the identity, i.e. $TT(L) = L$. This follows from the definition of T as an inversion mapping: $TT: t \rightarrow --t = t$. It follows that: **T** entails TL just in case T**T** entails L. Consequently, we obtain a general logical criterion for time reversal invariance:

General TRI: T**T** = **T** just in case, for all L, if **T** entails L, then **T** entails TL.

This is a general logical result. But to use it, we now have to calculate what the time reversed images of specific *laws* or *propositions* of a theory are. For instance, what is the time reversal of the probability law: $T[prob(s_2(t+\Delta t)|s_1(t)) = p]$?

The Orthodox Criterion arises if we identify this as: $prob(Ts_1(t+\Delta t)|Ts_2(t)) = p$, where: Ts_1 and Ts_2 are the time-reversed images of the states in the original law.[2]

But this is a mistake: the time reversed image of the probability law is not as above, but rather, the (past-directed law): $prob(Ts_2(t-\Delta t)|Ts_1(t)) = p$. This gives a different criterion for TRI:

TRI. A probabilistic theory **T** is time reversal invariant (TRI) just in case:
$$\mathbf{T} \Rightarrow [\, prob(s_2(t+\Delta t)|s_1(t)) = prob(Ts_2(t-\Delta t)|Ts_1(t))]$$

This is logically independent of the Orthodox Criterion (see Holster, 2003(A)). A number of writers over the years, most importantly Watanabe (from 1955), have shown quite decisively that the Orthodox Criterion does not represent time reversal invariance, although they have been largely ignored[3]. This raises the question:

Q.1: *Why have physicists adopted the Orthodox Criterion?*

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[2] Since if $prob(s_2(t+\Delta t)|s_1(t)) = p$ and $prob(Ts_1(t+\Delta t)|Ts_2(t)) = p$ then: $prob(s_2(t+\Delta t)|s_1(t)) = prob(Ts_1(t+\Delta t)|Ts_2(t))$.

[3] See also Healy, Penrose, Callender, and note 10, for similar conclusions.

If physicists do have an adequate syntactic system for deriving time reversal, why haven't they simply applied it to probability laws correctly? The contention here is that *there is no adequate deductive system for obtaining time reversals of statements.*[4]

Problem Example 2. To begin applying either of the Criteria given above, we must decide the effect of time reversal on the *states* mentioned in the laws In QM, the time reversal transformation for states is taken to be:

> **Definition.** The *Orthodox Time Reversal Operator for Quantum States* is represented by the Wigner operator T^*, which is the combination of ordinary time reversal: $T: t \rightarrow -t$, and complex conjugation: $*: (a+ib) \rightarrow (a-ib)$

But this is also problematic: this choice appears to contradict the fact that time reversal in physics is simply *defined* by the general transformation $T: t \rightarrow -t$. Why modify this to T^* especially for QM? This is a more complex issue, involving details of interpretation of QM[5]. But the fact that *there are no valid deductive proofs that T* is the time reversal operator in QM* is broadly acknowledged by writers who support the orthodox choice, such Liu quoted above, who typically say that there can be no *general deductive system* for obtaining time reversed states, and the choice depends instead on special considerations of features of the theory in question – or even on empirical tests.

But if such choices are not *deduced* from general principles, how can they be justified as representing time reversal objectively? For time reversal invariance of a *theory* is surely a logical property of the theory, not an additional empirical result.

Q.2: *Why have physicists adopted T* rather than T for time reversal in QM?*

Failure of Objectivity in the Orthodox Reasoning.

The simple answer to the questions posed above is that the Orthodox choices are adopted *because they represent symmetries that actually hold of QM*. The alternative choices do not represent symmetries of QM, and judged by them, *QM would fail to be TRI*. Physicists often acknowledge that they do not want the latter result: they *want* their 'deepest theories of nature' to be time symmetric.

But I strongly object to the idea that it is a matter of *choice* whether a given theory has this symmetry or not. What is really at stake is *the objectivity of time reversal symmetry*. If the analysis must be cobbled together on a 'case-by-case basis', involving an 'appropriate choice', to deliberately obtain the symmetry, then the usual notion that the symmetry is an *objective, logical property of well-defined laws or theories* goes out the window.

Physicists do offer various 'proofs' that QM is TRI: but these start with the 'orthodox interpretations' of state-reversals or law-reversals. If you question these assumptions, and point out that TRI just means invariance under the general transformation: $T: t \rightarrow -t$, and argue

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[4] Note also that the probability laws themselves contain further embedded propositions, $s_1(t)$ and: $s_2(t+\Delta t)$, and we must be able to obtain their images as well, in the process of obtaining the images of the complete laws.

[5] I discuss this in detail in Holster 2003 (B); and see Costa de Beauregard (1980), and Callender (2000) for similar views.

that the ‘orthodox’ interpretations fail to capture this, the orthodox view is typically to deny that *these choices* can be defined independently of specific theories. It is commonly said that we must examine in detail whether the choices *work* for the theories in question, before adopting them. But then, how is the property of TRI objective if the criteria used to judge it is a matter of *choosing a criteria because it is satisfied by the theory*? Should it not be determined solely and completely by the *transformation, T:* $t \rightarrow -t$, on which it is based? Why should we have to check whether the ‘appropriate’ TRI property is *satisfied* by a theory (like QM), before analyzing the theory for such a property?

In the most important sense, I believe the orthodox approach is simply wrong: *there must be proofs about the correct application of the time reversal transformation to states, processes, propositions, laws, and theories* – given that there is any definite application at all, which is not necessarily true for every theory.

But at the same time, I will hold that the results cannot be deduced in a purely formal way *in the current formalism.* In the rest of Part 1, I demonstrate this claim. It applies to classical theories just as much as QM, and for simplicity, I use a simplified version of classical gravitational theory to illustrate the situation, so that it is clear the problem does not arise from the special complexities of QM.

3. Deductive proofs in physics and mathematics.

I will begin with a brief summary of the usual object-language deductions in physics, which are ordinary logical-mathematical deductions. I then go on to observe that physics differs conceptually from mathematics in the crucial respect that it contains statements of contingent identities.

Textbooks on physics represent theories in precise, formal, mathematical equations, called the *object language*. These are introduced within surrounding text, which is a *meta-language.* For instance, in an introduction to classical physics, we will typically find a passage like:

> “*We suppose that* $\boldsymbol{r}_i(t)$ *represents the trajectory of a particle with a mass m, in three dimensional space, as a function of time. Then Newton’s force law states that:*
>
> (1) $$\boldsymbol{F}_i = m_i\boldsymbol{a}_i(t) \equiv m_i d^2\boldsymbol{r}_i(t)/dt^2$$
>
> *The primary problem in classical physics is then to specify all the forces that exist in nature. Newton’s law for gravitational forces states that any mass particle,* $j \neq i$, *exerts an instantaneous force on the particle i, according to:*
>
> (2) $$\boldsymbol{F}_{ij} = -Gm_im_j\boldsymbol{r}_{ij}/|\boldsymbol{r}_{ij}|^3$$
>
> $\boldsymbol{r}_{ij}$ *is the vector from the particle j to the particle i, defined by:* $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$. *The complete resultant gravitational force on particle i is obtained by adding all the individual components from the different particles j. Thus we may write:*
>
> (3) $$\boldsymbol{F}_i = \Sigma_{j\neq i}\, \boldsymbol{F}_{ij}$$ ”

The meta-language is essentially the open-ended natural language with some specialized terms from mathematics and physics. This language is used for two main purposes:

(i) to introduce the formal interpretation of the object language in terms of intended mathematical models, and to present and conduct much of the mathematical reasoning about the theory;

(ii) to introduce the "empirical interpretation" of the formal theory, by specifying the observational interpretation of terms like $\mathbf{r}_i(t)$, and to explain the empirical evidence provided by experiments and so forth.

The fundamental theories themselves are defined essentially by formal laws or equations. In specialized treatments on the foundations, these are axiomatised, in a similar manner to ordinary mathematical theories (e.g. von Neumann (1932)). The point of such treatments is to show that at the formal level such theories are rigorously defined, and that there are rigorous formal derivations of the consequences of them. Physicists give semi-formal proofs in practice, but this is no blemish as long as rigorous deductive principles or axiomatic proofs are available in principle.

Deductive proofs in physics give general theorems, which are logical consequences of the axioms of a theory. The original laws are typically expressed in a form stating *identities between certain quantities,* as in (1) and (2) and (3) above. The most basic form of deduction allows us to substitute identities: thus we can obtain the specific law for the gravitational effect on a particle *i* from (1), (2) and (3):

$$\textbf{(4)} \qquad m_i d^2\boldsymbol{r}_i(t)/dt^2 = \Sigma_{j\neq i} -Gm_im_j\boldsymbol{r}_{ij}/|\boldsymbol{r}_{ij}|^3$$

More complex deductions involve quantificational logic, where we take a law with general quantifications, propose some specific instantiation by saying: "let us suppose we have a system of particles with such-and-such trajectories", deduce consequences for this particular system, and then generalise the result appropriately to all systems of particles.

This makes us recognize the specific laws are implicitly generalised w.r.t. ordinary variables, such as *time, t* (representing time translation invariance), *space*, (space translation invariance), and *particles* (universality across different systems). And in making deductions, of course we can refer to well-established mathematical identities, such as theorems from the differential calculus.

This is all fine and routine, and it appears at first that the deductive formalism for theoretical physics is really just that for general mathematics. As long as we have given a *formal interpretation* of such a theory, by associating the various terms of a theory with specific intended mathematical entities or models, we can pursue deductions in a purely logical-mathematical fashion, without worrying about the "empirical meaning" of the theory at all. That messy part of interpretation, it seems, arises only at a secondary stage, when we give a specifically *empirical interpretation*.

But we can see there are some features of physical theories not present in mathematical theories:

1. Quantification of laws is often left implicit in physics, but we can add the quantifiers on variables such as time, space, and particles easily enough. However, we are still left with special *constant functions,* in particular, the *particle trajectory function,* $\boldsymbol{r}(.,.)$, which appears in the previous equations. This is a special *contingent function.*

2. Theories of pure mathematics are presented as *universally and necessarily true,* or true in any logically possible world. But theories of physics are *contingent,* and not true in any possible world. As empirical theories, they are proposed to be true of the actual world, or more generally, they are proposed to be true of *physically possible worlds.* But there are many *logically possible worlds* that cannot obey the theory. The need to do physical experiments to confirm such theories reflects their contingent nature. We do not do such experiments to confirm purely mathematical theories.

3. We typically reason in physics by making deductions from general laws to their consequences, using ordinary quantificational logic. But when we analyse general symmetry properties, we need to deduce *properties of the laws themselves.* General transformations, like T, are formally defined by hierarchical families of operators, which act on particular kinds of objects, such as times, states, fields, or tensors of various sorts representing these things. But this hierarchy only includes *first-order objects;* it does not extend to *propositions,* which are of a logically higher-order.

I now examine these points more closely.

4. Logical consequence and formal deduction.

Ordinary variables in physics are quantified, but the special *trajectory functions,* $\boldsymbol{r}_i(t)$, $\boldsymbol{r}_j(t)$, *etc,* are special functions. They map from moments of time, t, to the spatial position of the particles, i, j, *etc,* at t. To make these more explicit we can write them as: $\boldsymbol{r}(i,t)$, $\boldsymbol{r}(j,t)$, etc. Physicists regard $\boldsymbol{r}_i(t)$ as the special constant function arising from the general function: $\boldsymbol{r}(i,.)$ when i is evaluated for a specific particle. $\boldsymbol{r}(.,.)$ is a more general mapping from (*particle, time*)-couples to *positions,* while $\boldsymbol{r}_i(.)$ is a mapping from *times* to *positions,* which is obtained when we put the particle i into the function $\boldsymbol{r}(.,.)$. Similarly, we can regard the mass function, $m(.)$ as mapping from *particles* to *masses,* and m_i is normally a constant value of this function for the particle i. We can expand to the more general form: $m(i,t)$ to allow masses to vary, as in Special Relativity.

With this more explicit notion, we can write the quantification for (3) as:

$$(4) \qquad (\forall i,t)[m(i,t)d^2\boldsymbol{r}(i,t)/dt^2 = \Sigma_{j\neq i} -Gm(i,t)m(j,t)(\boldsymbol{r}(i,t)-\boldsymbol{r}(j,t))/|\boldsymbol{r}(i,t)-\boldsymbol{r}(j,t)|^3]$$

(The summation over j represents a quantification of that variable also). If we interpret this theory for the actual world, then we take $\boldsymbol{r}(i,t)$ to give the actual position of a particle, i, at a time t, in the actual world. I will call $\boldsymbol{r}(.,.)$ a *contingent constant function.*

We should contrast this *contingent* constant function with the constants we meet in purely mathematical theories. For instance, the following is the associative law of the ordinary theory of natural number arithmetic:

$$(\forall i,j,k)[(i+j)+k = i+(j+k)]$$

Here i, j, k are variables over the class of natural numbers, and + is the constant addition function. But the + function is a truly *constant function,* the same in every possible world or model, whereas $\boldsymbol{r}(.,.)$ represents a different function in different possible worlds.

Through the interpretation of ***r**(.,.),* the physical theory is *capable of expressing contingent propositions,* and not merely constrained to *a priori* facts. The specific content of ***r**(.,.)* is not determined by any axioms, nor by the theoretical laws, like (3); rather, it is determined as a description of some *contingent world,* and it varies from world to world. A law like (3) can be true of a class of worlds, and even of the actual world; but *it cannot true of all possible worlds.* Stated as a law of nature, it is intended to be true only of a tiny class of possible worlds.

This makes a difference in the way we present the axioms or laws of the theories. With the mathematical theory, we can state the axioms categorically, as applying to *any world,* and adopt them as necessary or logical truths. With the physical theory, we state the theory in a more restricted sense, as *true of a class of worlds.* When we wish to propose the physical theory as being actually true of the real world, we likewise make an additional claim: that the *actual world* is one of those that belong to this special class. This is what makes it contingent.

In fact, there is one formal representation in physics to indicate the distinction between contingent and necessary statements: the use of a different *identity sign* for definitions (or 'identities'), and for ordinary laws. Identities are often written in a form: $A =_{df} B$, or alternatively: $A \equiv B$. Slightly different rules are used for such 'logical identities' than for ordinary equalities. However, whether these rules are adequate is the problem.

Physicists may wonder whether this makes any real difference. Surely the problem in theoretical physics is just to obtain *logical consequences of theories*, before subsequently examining these consequences empirically, and why should this present a problem? There is a perfectly good way of dealing with the concept of *logical consequence*, which is equally applied to the mathematical and physical theories. We relativise things w.r.t. worlds, or models. We can prove that the law (3) is a *logically consequence of the theory,* represented say by (1) and (2), by proving that *any world (or model) that satisfies (1) and (2) must also satisfy (3).* This shows that (3) is a logical consequence of (1) and (2). In precisely the same way we prove that various arithmetic laws are *logical consequences of the general axioms of arithmetic.* The general concept of logical consequence is essentially the same whether we are dealing with mathematics or physics.

This makes it appear that the difference between contingent and necessary theories or laws does not enter into mathematical physics, because (i) we can establish the logical consequences of a set of theoretical laws by purely mathematical reasoning, and regard this as the basic theoretical development, and (ii) we can treat the problem of *applying* such a theory to the empirical or contingent world as a secondary endeavor. In the latter, of course, we will have to link the empirical interpretation of the theory with the natural language of empirical observations, in which we make claims about what is *actually true* or *actually observed;* but surely, this is a separate issue from the purely theoretical development from an axiomatic statement of the theory?

However, while it may be agreed that the concept of *logical consequence* is the same for both the mathematical and the physical theories, the key question is whether the system of formal representation for pure mathematical theories is necessarily adequate for physics. The point is that logical consequences are *proved* by formal deductions. Logical consequences are regarded as objective or determinate, and derive from the meanings of the propositions. But to prove them, we use formal systems, specified as deductive rules for operating on *syntactic representations.* This is of profound importance in physics as much as mathematics. We require our formal object language of physics to be adequate, in principle, to obtain *rigorous formal deductions of the important theorems.*

By *rigorous formal deductions* we mean deductions based on the use of syntactic rules of inference, which guarantee logical consequences. Logicians represent logical consequence with the special notation:

$P \models Q$ Q is a logical consequence of P.

Logical deducibility is represented by a different symbol to logical consequence:

$P \vdash Q$ Q is deduced from P.

Logical deducibility is only relative to a *system of deductive rules*. Logical consequence is a *semantic notion*, because it involves the relative existence of models for the respective propositions. Deduction is called a *syntactic notion,* because it involves the existence of a formal system of syntactic rules for deriving one statement from another.

Now of course, we know it is unreasonable to ask for a *complete deductive system for physics,* because of Goedel's theorems. But we require a system that is complete enough to obtain major results that we ordinarily claim. For instance, there are well-defined rules for making transformations on tensors; if we lacked some of these rules, the tensor calculus would be inadequate as a *formal calculus.*

This is where I claim there is a problem: the theories of physics do require a distinct proof theory from those of mathematics, because of the fact that their languages have *contingent interpretations,* or contingent constants.

The initial sign of this problem already appears in the use of the distinct *identity symbols,* "=" ('contingent equality') and "$=_{df}$" ('logical identity') in physics, which are not distinguished within pure mathematics. But the problem appears in earnest when we consider the derivation of general transformations of laws or propositions. We wish to know whether various laws are invariant under *T*. To establish time reversal invariance of a law, *L,* we have to deduce what the time reversal of the law is. Given that we have a system for deducing *TL* from *L,* we can then establish whether or not $L = TL$. But the main question is whether there is a fully systematic *deductive* method for deriving *TL* from *L*. We now turn to consider the standard treatment of this.

5. The Physicist's distributive syntactic operator, *Ŧ.*

Physicists refer to a system of syntactic rules for obtaining time reversal transformations, *T*, which I now describe. I will call it the *distributive syntactic transformation,* or the *syntactic operator*. I denote this by the special symbol: *Ŧ,* so we may distinguish it conceptually from the time reversal transformation, *T*. Given a definition of *Ŧ,* we then have to check that when we apply it to laws, *L,* we always obtain: *ŦL = TL,* i.e. that the syntactic operator achieves its goal of generating time reversed images correctly.

Note that, for any object, law, or process, *X*, we assume throughout that *TX* is the *real time reversed image of X,* whereas *ŦX* is defined by the following rules for *transforming the expression for X.*

The Distributive Syntactic Operator, Ŧ.

Suppose we have a statement, L (or indeed, any complex term). To obtain its *syntactic transformation, ŦL,* we substitute every occurrence of any time variable, t, in the statement of L with its reversal, $-t$. We also substitute temporal constants, like Δt or dt, with their time reversals, $-\Delta t$ and $-dt$. We must also substitute other *fundamental variables or constants,* notably fundamental *state variables,* or *property variables*, with their time reversals. That is to say, the operator Ŧ is defined to *distribute through all fundamental terms,* and then to operate on these directly by giving their time reversed images.

In basic classical physics, the fundamental terms include the terms for *positions* or *spatial vectors*, $\boldsymbol{X}$*,* $\boldsymbol{Y}$, *etc*, which are invariant (Ŧ$\boldsymbol{X} = T\boldsymbol{X} = \boldsymbol{X}$); specific *velocities*, $\boldsymbol{v}$*,* which are reversed (Ŧ$\boldsymbol{v} = T\boldsymbol{v} = -\boldsymbol{v}$); specific *momenta*, $\boldsymbol{p}$*,* which are reversed (Ŧ$\boldsymbol{p}=T\boldsymbol{v} = -\boldsymbol{p}$); the time differential operator, d/dt, or $\partial/\partial t$, is reversed (Ŧ$(d/dt) = T(d/dt) = -d/dt$); *masses*, m, are invariant (Ŧ$m = Tm = m$); *particle identities*, i, are invariant (Ŧ$i = Ti = i$); electric charges, q, are invariant (Ŧ$q = Tq = q$), and *dimensional constants, e.g.* G, c, h, are invariant (Ŧ$G = TG = G$, etc).

We also define the operation of Ŧ on fundamental trajectory *functions* like $\boldsymbol{r}(.,.)$ in the same way: Ŧ$\boldsymbol{r}(.,.) = T\boldsymbol{r}(.,.)$, or: Ŧ$\boldsymbol{f}(.) = T\boldsymbol{f}(.)$, for a trajectory function $\boldsymbol{f}$. [6] And similarly for the mass function: Ŧ$m(.,.) = Tm(.,.)$.

All other, non-physical items, e.g. numbers, logical constants, or quantifiers, are invariant under T.

From these basic transformations on the fundamental terms of the theory, *plus* the syntactic rule of distributivity, the Ŧ-transformations are defined for all complex terms. For instance, suppose we have a law with the usual syntactic form of an identity between two complex terms, A and B:

L: $\qquad A = B$

To obtain the transformed law, ŦL*,* the basic idea is that we distribute Ŧ through all the terms of the expression. Thus, first we write ŦL as:

ŦL: $\qquad$ Ŧ$(A = B)$

We then transform this into:

ŦL: $\qquad$ ŦA = ŦB

We then continue to transform through the complex terms that make up A and B, until we get to fundamental terms or entities that have direct Ŧ transformations. This provides a general system for transforming laws or any other complex terms that occur within laws.

[6] We should emphasize, however, that $T\boldsymbol{r}(.,.)$ is a reversed trajectory *function*, and not the same as: $T(\boldsymbol{r}(i,t)) = T(\boldsymbol{r}) = \boldsymbol{r}$, which is just the reversed trajectory *point* on $\boldsymbol{r}(i,t)$ at i, t. Whereas we have: $T(\boldsymbol{r}(i,t)) = \boldsymbol{r}(i,t)$, we do not have: $T\boldsymbol{r}(i,t)) = T\boldsymbol{r}(i,t)$; instead, as we derive below, we have: $T\boldsymbol{r}(i,t)) = \boldsymbol{r}(i,-t)$. The *syntactic rule* however is just that: Ŧ$\boldsymbol{r}(.,.) = T\boldsymbol{r}(.,.)$.

For a simple example, consider directly transforming a specified *trajectory function*, $\boldsymbol{r}_i(.)$, for a particle i, to obtain its time reversed image, $T\boldsymbol{r}_i(.)$. It is perfectly evident to physicists how this must actually be defined:

$$T\boldsymbol{r}_i(t) = \boldsymbol{r}_i(-t)$$

We can verify this rule by reasoning directly about the trajectory function. We assume that $\boldsymbol{r}(i,.)$ is defined as a specific trajectory, mapping times, t, to positions, $\boldsymbol{r}_{i,t} = \boldsymbol{f}(t)$, for some specific function, $\boldsymbol{f}(.)$. For the sake of definiteness, let us suppose that the trajectory function is defined by:

$$\boldsymbol{f}(t) = \exp(t)\boldsymbol{w}$$

where $\boldsymbol{w}$ is a specific velocity in some direction of space, call it $\hat{\mathbf{y}}$. It doesn't matter what we actually choose for $\boldsymbol{f}(.)$, except that it must be dimensionally correct, but it is useful to have a concrete example, and this trajectory is represented in Figure 1.

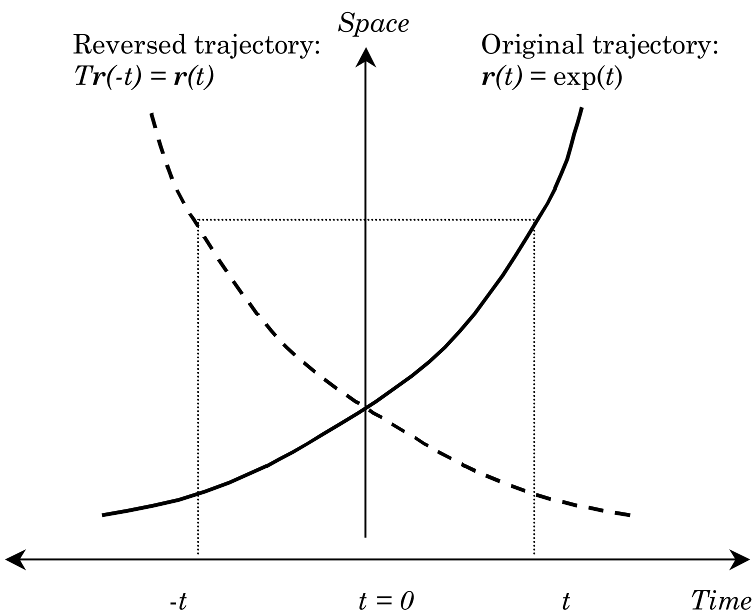

Figure 1. Time reversal of a simple trajectory function: $\boldsymbol{r}(t) = \exp(t)\boldsymbol{w}$.

We can represent $\boldsymbol{r}_i(.)$ schematically as a mapping: $\boldsymbol{r}_i(.)$: $t \rightarrow \boldsymbol{r}_{i,t}$. Or more explicitly, we can represent it set-theoretically as a function represented by a class of ordered couples like:

$$\boldsymbol{r}_i = \{(t,\boldsymbol{r}): \textit{particle } i \textit{ is at position } \boldsymbol{r} \textit{ at time } t\}.$$

Clearly we obtain $T\boldsymbol{r}_i(.)$ as a new function mapping the 'reversed times', $-t$, to the same positions that $\boldsymbol{r}(.)$ maps the original times, t. I.e. in this example:

$$T\boldsymbol{r}_i(-t) = \boldsymbol{f}(t) \quad \text{or equivalently: } T\boldsymbol{r}_i(t) = \boldsymbol{f}(-t)$$

Or more generally:

$$T\boldsymbol{r}_i(-t) = \boldsymbol{r}_i(t) \quad \text{or equivalently: } T\boldsymbol{r}_i(t) = \boldsymbol{r}_i(-t)$$

This is clearly correct: the new reversed trajectory function, $T\boldsymbol{r}_i(.)$, sends the original trajectory ‘backwards’, reflecting the original path in the time axis.

This is also consistent with the direct Ŧ transformation on the complex term: $\boldsymbol{r}(i,t)$, since:

$$Ŧ[\boldsymbol{r}(i,t)] = Ŧ\boldsymbol{r}(Ŧi,Ŧt) = Ŧ\boldsymbol{r}(i,-t) = T\boldsymbol{r}(i,-t) = \boldsymbol{r}(i,t)$$

This is self-consistent with the fact that the value of $\boldsymbol{r}(i,t)$ is a *position*, $\boldsymbol{r}_{i,t}$, which is invariant under Ŧ.

But we now test this on the law *L* used above to define this trajectory, using the syntactic transformation rules for Ŧ described above. First we take the definition of the original trajectory, in quantified form:

$$L \qquad (\forall t)(\boldsymbol{r}_i(t) = \boldsymbol{f}(t))$$

We now obtain the Ŧ transformation on the statement *L* using the syntactic rules:

$$ŦL \qquad Ŧ[(\forall t)(\boldsymbol{r}_i(t) = \boldsymbol{f}(t))]$$

Distributing Ŧ through this statement, we transform to:

$$ŦL \qquad (\forall Ŧt)(Ŧ(\boldsymbol{r}_i(t)) = Ŧ(\boldsymbol{f}(t)))$$

And using the substitutions defined above, we get:

$$ŦL \qquad (\forall -t)(\boldsymbol{r}_i(t) = \boldsymbol{f}(t))$$ [7]

Because of the universal quantifier, *-t* is here a dummy variable, ranging equally over positive and negative values, and we change to the usual form:

$$ŦL \qquad (\forall t)(\boldsymbol{r}_i(t) = \boldsymbol{f}(t))$$

Now something has gone wrong! The transformed law *ŦL* we have calculated is just the original law *L*. But this cannot be *TL*, because in this case, *L* is time asymmetric, and the true time reversed law *TL* must be different to *L*.

We obtain the true *TL* intuitively as follows. Just as the law *L defines the original trajectory*, $\boldsymbol{r}_i(t)$, when we take the time reversal *TL* of *L*, this should *define the reversed trajectory*, $T\boldsymbol{r}_i(t)$. I.e. *TL* must be defined by:

---

[7] We obtain: $Ŧ(\boldsymbol{f}(t)) = Ŧ\boldsymbol{f}(Ŧt) = T\boldsymbol{f}(-t) = \boldsymbol{f}(t)$, since $\boldsymbol{f}(t)$ is a trajectory function, and obeys the same rule as: $Ŧ(\boldsymbol{r}_i(t)) = \boldsymbol{r}_i(t)$.

TL $(\forall t)(\boldsymbol{r}_i(t) = \boldsymbol{f}(-t))$

Or equally, we may reason that the *time reversed proposition, TL,* must be equivalent to defining the reversed trajectory, $T\boldsymbol{r}(.)$, by substituting it in the original equation *L* for the original $\boldsymbol{r}(.)$, i.e. we should obtain:

TL $(\forall t)(T\boldsymbol{r}_i(t) = \boldsymbol{f}(t))$

And this is the same result when we substitute $T\boldsymbol{r}_i(t) = \boldsymbol{r}_i(-t)$.

So we find, in this example, that the syntactic transformation has given the wrong answer for *TL*. What has gone wrong? Is it because we are operating on *a definition* of the contingent trajectory $\boldsymbol{r}(.)$? We will consider the flaw in the syntactic transformation next.

6. The physicist's enhanced method.

Physicists do not really apply the syntactic system defined above to obtain transformations of laws. Instead, they cut the formal derivations short, and reason intuitively, much as we did above. Let us begin with a law like (4) above. We observe that this has the following form: it states an identity *about the trajectory function,* $\boldsymbol{r}(.,.)$. Let us suppose, for simplicity, that *L* has the form:

L $A[\boldsymbol{r}(.,.)] = B[\boldsymbol{r}(.,.)]$

$A[.]$ and $B[.]$ are taken to be two general 'mathematical operators' on the trajectory function.[8] We want to find the time reversal of this law, *TL*. We reason that *if L is true of some trajectory,* $\boldsymbol{r}(.,.)$, then by definition *TL is true of the time reversed trajectory,* $T\boldsymbol{r}(.,.)$. This is undoubtedly the intended meaning of the "time reversed law". This agrees with the notion of *time reversal invariance of a law: L* is time reversal invariant just in case, for any process (trajectory function) that satisfies it, it is satisfied by the time reversed process.

Thus, to obtain *TL*, we need to substitute $T\boldsymbol{r}(.,.)$ for $\boldsymbol{r}(.,.)$ in the original *L*, giving:

TL $A[T\boldsymbol{r}(.,.)] = B[T\boldsymbol{r}(.,.)]$

We define the transformed trajectory functions as above:

$$T\boldsymbol{r}(i,t) = \boldsymbol{r}(i,-t)$$

Then by substitution we can obtain *TL* directly. And this is equivalent to:

TL $A[\text{Ŧ}\boldsymbol{r}(.,.)] = B[\text{Ŧ}\boldsymbol{r}(.,.)]$

---

[8] This is the form of the simple Schrodinger equation for an isolated particle:

$\partial\Psi/\partial t = i\hbar/2m\, \partial^2\Psi/\partial x^2$.

But now the signs of time variables, *t*, *d/dt*, etc, in the terms *A* and *B* have been left unchanged. We have not distributed *T* through the *entire* statement at all: it has been applied only to the *trajectory functions* (the 'contingent parts'). The 'mathematical parts' have been left alone.

With the example of the law *L* of the previous section, for instance, this gives:

$$TL \qquad (\forall t)(\boldsymbol{r}(i,-t) = \boldsymbol{f}(t) = \exp(t)\boldsymbol{v})$$

And this is the correct result.

Alternatively, we can do the converse: we take the 'abstracted law': *A[.] = B[.]* to be a 'law-like property' that is postulated to hold of any contingent trajectory functions, ***r**(.,.),* and we define the *time reversal* of this property as: *TA[.] = TB[.].* We then obtain the transformed law *L* as the statement that this *time reversed property holds of the contingent trajectory functions,* giving:

$$TL \qquad TA[\mathbf{r}(.,.)] = TB[\mathbf{r}(.,.)]$$

To put this into effect, we take the time reversal of the functions that define *A[.]* and *B[.]*, and apply them to the original trajectory function; so once again we only transform 'half' the proposition. To obtain the time reversals of *A[.]* and *B[.]* we no longer have a direct definition (as with *r(.,.)*). Now we use the syntactic transformation *Ŧ* as defined above, and substitute all the individual terms with their time reversals, to obtain:

$$TL \qquad ŦA[\mathbf{r}(.,.)] = ŦB[\mathbf{r}(.,.)]$$

And the application of this rule is well-defined given *Ŧ* is well defined. This is what I will henceforth call *the physicist's method,* or *enhanced method.* In fact, we have defined two distinct procedures, but we will see in the next section that they are logically equivalent.

Now this may seem a good solution: we distinguish the two parts of the statement, separating the 'contingent constants' from the 'logical-mathematical' constants, and treat them separately. But we soon it is not generally adequate either. First, however, I will note a key property that permits it to work in a limited way, and means that it gives the correct answer in simple cases[9].

7. The equivalence of the physicist's two procedures.

An initial question is whether the two different methods defined above ensure the same result. I.e. does taking *TL* as: $A[Ŧ\mathbf{r}(.,.)] = B[Ŧ\mathbf{r}(.,.)]$ always give the same result as taking: *TL* as $ŦA[\mathbf{r}(.,.)] = ŦB[\mathbf{r}(.,.)]$? We can prove this only if we can establish, for a general statement *L*, that:

$$ŦL = L$$

---

[9] For instance, it works for the transformation of the probability law: $prob(s_2(t+\Delta t)|s_1(t)) = p$, introduced in Section 2, to give the result stated there. This provides some justification of the TRI Criterion stated there; but it is hardly conclusive, given argument here, that this method is ultimately inconsistent.

We will see this is indeed true *for any statement L that is interpreted extensionally as a truth-value*. We turn to this next, but first, we observe how the proof follows from this. It allows us to immediately show that the statement: $A[Ŧ\mathbf{r}(.,.)] = B[Ŧ\mathbf{r}(.,.)]$ always has the same truth-value as the statement: $ŦA[\mathbf{r}(.,.)] = ŦB[\mathbf{r}(.,.)]$. First we operate on the first statement with $Ŧ$, which transforms to:

$$
\begin{array}{ll}
 & Ŧ[A[Ŧ\mathbf{r}(.,.)] = B[Ŧ\mathbf{r}(.,.)]] \\
\textit{Apply } Ŧ\textit{:} & ŦA[ŦŦ\mathbf{r}(.,.)] = ŦB[ŦŦ\mathbf{r}(.,.)] \\
\textit{Simplify } ŦŦ\textit{:} & ŦA[\mathbf{r}(.,.)] = ŦB[\mathbf{r}(.,.)]
\end{array}
$$

The last step uses the fact that the double application of $Ŧ$ is the identity, since: $T(TX) = X$, for any X. But then, *if and only if*: $ŦL = L$ for all statements L, we obtain the result that: $A[Ŧ\mathbf{r}(.,.)] = B[Ŧ\mathbf{r}(.,.)]$ is the same as $ŦA[\mathbf{r}(.,.)] = ŦB[\mathbf{r}(.,.)]$, and the two methods are the same. But if we ever find that: $ŦL \neq L$, the two methods will produce different results, since we would have:

$$\big[A[Ŧ\mathbf{r}(.,.)] = B[Ŧ\mathbf{r}(.,.)]\big] = L \neq ŦL = \big[ŦA[\mathbf{r}(.,.)] = ŦB[\mathbf{r}(.,.)]\big]$$

8. Compositionality of Ŧ

We turn now to the critical point, that: $ŦL = L$, *for any statement L that is interpreted extensionally as a truth-value*. This can be shown from the critical property of *compositionality* of the distributive syntactic operator, $Ŧ$.[10] *Compositionality* is the key property to use in these proofs, because it connects the semantic and the syntactic. It is the main formal property we require the meaning function to satisfy in an adequate formalized language. I will only sketch the main idea here.

> **Compositionality.**
> *The meaning of a compound expression is a function of the meanings of its parts and of the syntactic rule by which they are combined.*[11]

We can express this formally by an axiom-scheme governing ('denotational') *Meaning*:

$$Meaning(a_1a_2\ldots a_n) = \mathbf{a_1a_2\ldots a_n} = Meaning(a_1)Meaning(a_2)\ldots Meaning(a_n)$$

Here that italicized terms: "$a_1a_2\ldots a_n$" are used for meta-language *names or variables for the object language symbols* (syntactic items), while bolded terms: "$\mathbf{a_1a_2\ldots a_n}$" are used for the

[10] A full demonstration that $Ŧ$ has this property is only considered in Part 2 of this paper; but it must have this property if it is to be logically adequate. See Janssen (1997) and Tichý (1978) for discussions of compositionality, and an introduction to intensional logic by Partee and Hendricks (1997), or alternative approaches by Tichý , Materna, and the *TIL* website.
[11] This is the formulation of Janssen and Partee, "Compositionality", *Handbook of Logic and Language*¸ 1997, p.462. Tichý (1978) calls this the *Frege-Church principle.* Tichý's theory is unique because it does not make meaning a function of the *syntactic rules* of combination: rather, the syntactic rules of combination themselves reflect another level of semantics, which he defines formally as *constructions*.

object language symbols. If the syntactic transformation $Ƭ$ is *compositional*, and the language is *functional*, then:

$$Meaning(Ƭ(a_1a_2\ldots a_n)) = Meaning(Ƭ)Meaning(a_1a_2\ldots a_n)$$

If $Ƭ$ is *adequate to represent the transformation*, ***T***, then:

$$Meaning(Ƭ(a_1a_2\ldots a_n)) = \boldsymbol{T(a_1a_2\ldots a_n)}$$

If $Ƭ$ is *distributive* then:

$$Meaning(Ƭ(a_1a_2\ldots a_n)) = Meaning(Ƭa_1Ƭa_2\ldots Ƭa_n)$$

Compositionality and distributivity mean that:

$$Meaning(Ƭ(a_1a_2\ldots a_n)) = Meaning(Ƭa_1)\ Meaning(Ƭa_2)\ldots Meaning(Ƭa_n)$$

We noticed previously in Section 3 that applying $Ƭ$ to the particular proposition L considered there does *not* give us the time reversal, TL: instead, it just gives us L again. We need to prove this generally: i.e. if we apply $Ƭ$ systematically to *any law or statement expressing a proposition*, it remains invariant: $ƬL = L$. The crucial property that ensures this is that the language is *extensional*, so that *any law or statement expressing a proposition is formally interpreted as a truth-value.*

Given this, the result follows from compositionality and distributivity of $Ƭ$. Suppose we apply $Ƭ$ to a complex term, X, with component terms $a_1a_2\ldots a_n$. We can obtain the result in two ways: (i) by determining the *value* of X directly, and applying the meaning of $Ƭ$ to the value of X, or (ii) distributing $Ƭ$ through the sub-terms $a_1a_2\ldots a_n$, and obtaining each of these directly: $ƬX = Ƭa_1Ƭa_2\ldots Ƭa_n$.

Compositionality of $Ƭ$ means that we get the same answer either way. We saw this property in the example of the term $\boldsymbol{r}(i,t)$ earlier.

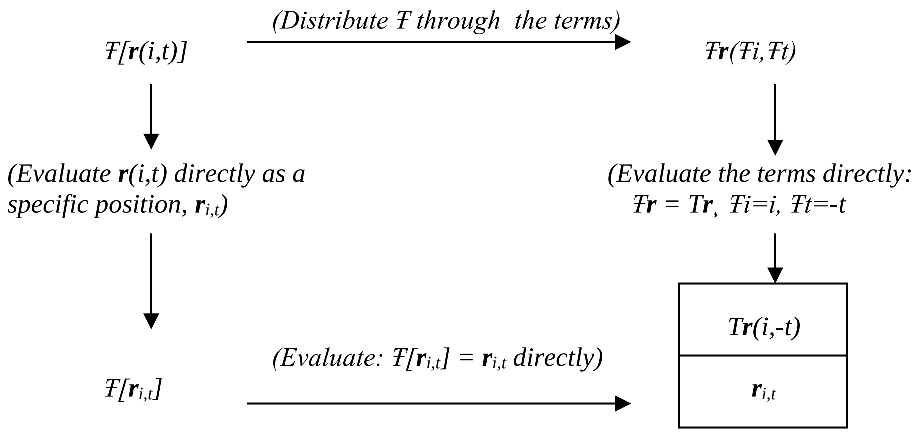

Figure 2. Compositionality diagram for $Ŧ[\mathbf{r}(i,t)]$.

In the bottom right hand corner, we obtain two different results for $Ŧ[\mathbf{r}(i,t)]$. The top result has been obtained by applying the syntactic transformation to the complex term '$\mathbf{r}(i,t)$' first, and then evaluating *Ŧ* applied directly to the fundamental terms. The bottom one has been obtained by evaluating $\mathbf{r}(i,t)$ first, representing it by *a different term*, $\mathbf{r}_{i,t}$, and then evaluating *Ŧ* applied directly to this. *Compositionality of Ŧ* means that these two results must be the same, and we can identify: $T\mathbf{r}(i,-t) = \mathbf{r}_{i,t}, = \mathbf{r}(i,t)$. This works for this case.

But assuming compositionality of *Ŧ* holds generally, it follows that: $ŦL = L$ must hold generally in an extensional interpretation, simply because a statement *L* in an extensional interpretation just denotes *a truth-value.* It takes either the value *True* or the value *False.* (We can ignore *null* values here). These are logical constants, and by definition: *Ŧ(True) = True,* and *Ŧ(False) = False.* Hence, the evaluation of *L* must give the same truth-value as *ŦL.*

This shows why the physicist's two procedures are in fact the same. But now, of course, when, in fact: $TL \neq L$, we must have: $ŦL \neq TL$, and *Ŧ cannot adequately represent T.*

Or alternatively, we could begin by assuming that *Ŧ adequately defines T,* so that: $ŦL = TL$ for all *L*. But then we can infer that *Ŧ* must *fail compositionality, or the language cannot be interpreted extensionally,* because of cases where: $TL \neq L$.

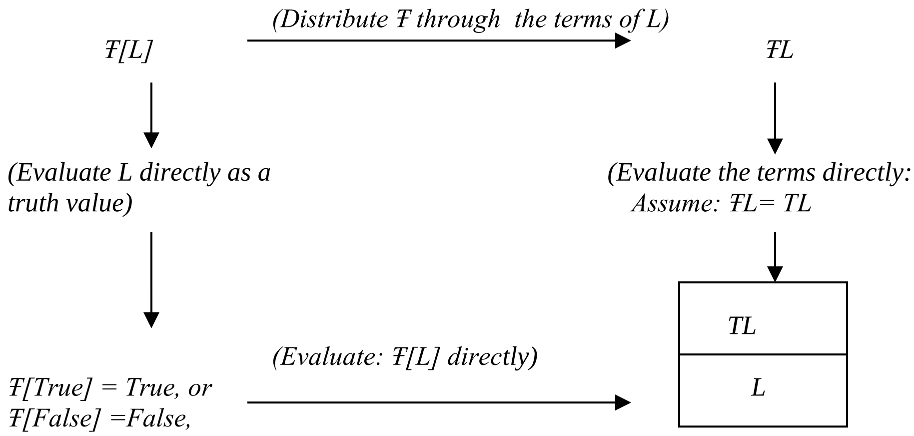

Figure 3. Compositionality diagram for *Ŧ[L]* if: *ŦL= TL*. Compositionality of *Ŧ* fails if *TL* $\neq$ *L*, because we get contradictory answers.

The logic of this is explored in more depth in Part 2 of this paper. But to complete this discussion, I will carry through our earlier example, to illustrate how the physicist's method leads to inconsistencies, and cannot be rescued in any simple manner.

9. The Physicist's Enhanced Method fails for Definitions.

The physicists' 'enhanced method' above is designed for contingent statements: we split the 'contingent functions', ***r****(.,.),* from the mathematical functions that occur in a law, and apply our *Ŧ* uniformly to just one or other. But we can now show that this fails for *definitions*. Consider the following statement of a *definition of the term* ***v****(i,t),* which represents velocities.

(5) $$\mathbf{v}(i,t) =_{df} d\mathbf{r}(i,t)/dt$$

(Universally quantified on *i* and *t*.) Here we have *defined* ***v****(.,.)* as the velocity function corresponding to the trajectory function ***r****(.,.).* What is the time reversal of this? Clearly, we have to require that:

T(5) $$dT\mathbf{r}(i,t)/dt = -\mathbf{v}(i,-t)$$

This is the correct result, because: $T\mathbf{r}(i,t) = \mathbf{r}(i,-t)$, and hence: $dT\mathbf{r}(i,t)/dt = -d\mathbf{r}(i,-t)/dt$. This is easily verified from the example in Figure 1 above: the velocity at a point *t* in the reversed trajectory is in the opposite direction to the velocity at the point *–t* in the original trajectory.

But let us now apply the physicist's method to obtain *T(5).* To do this, we abstract ***r****(.,.)* from the statement *(5)* above, and replace it with *T****r****(.,.).* Thus we would obtain:

*T(5)** $$dT\mathbf{r}(i,t)/dt = \mathbf{v}(i,t)$$

This is clearly wrong: it contradicts *(5)* as a *definition of* ***v****(.,.).*

The physicist's enhanced method does not work for obtaining the time reversals of definitions like *(5)*. Clearly, this has to do with the fact that this is a *definition,* rather than a 'contingent statement' about ***r****(.,.).* In fact, we know that *definitions must be invariant under time reversal generally.* A definition is true in all possible worlds, and is therefore equally true in a world and in its time reversal. This is the case with the correct time reversed image, *T(5)* above, where: *T(5) = (5).* But it is *not* the case with *T(5)** above: $T(5)^* \neq (5)$.

This brings us back to the motivation for introducing the physicist's method in the first place. We found that the distributive syntactic transformation, *Ŧ,* achieves precisely the effect of *leaving all statements invariant*, but this is precisely why it is inadequate for contingent statements, which must be able to alter under time reversal, since $TL \neq L$ in cases of time asymmetric statements. But consequently, it has the wrong effect on *definitions*, which must *not* be able to alter under time reversal.

10. Separate Methods for Analytic and Contingent Statements Fails.

This suggests a further modification to the physicist's method: how about systematically distinguishing *contingent statements* from *definitions* (or analytic statements), and defining time reversal for the former by the physicist's method above, and defining time reversal for the latter by *Ŧ*?

But unfortunately, this does not work either, because *we can combine contingent and analytic statements,* by substituting analytic definitions into contingent statements, and the problem reappears again. For example, consider the following statement, obtained by substituting the definition of ***v****(.,.)* in (5) into the statement (4):

(6) $$(\forall i,t)[m(i,t)d\mathbf{v}(i,t)/dt = \mathbf{\Sigma}_{j\neq i} -\mathrm{G}m(i,t)m(j,t)(\mathbf{r}(i,t)-\mathbf{r}(j,t))/|\mathbf{r}(i,t)-\mathbf{r}(j,t)|^3]$$

This follows analytically from (4). We can easily derive that (4) itself is time reversal invariant. But applying the physicist's method to (6), we obtain:

*T(6)** $$(\forall i,t)[m(i,t)d\mathbf{v}(i,t)/dt = \mathbf{\Sigma}_{j\neq i} -\mathrm{G}m(i,t)m(j,t)(T\mathbf{r}(i,t)-T\mathbf{r}(j,t))/|T\mathbf{r}(i,t)-T\mathbf{r}(j,t)|^3]$$

Then re-substituting from the definition of ***v****(.,.)* in (5), this means that:

(7) $$(\forall i,t)[m(i,t)d^2\mathbf{r}(i,t)/dt^2 = \mathbf{\Sigma}_{j\neq i} -\mathrm{G}m(i,t)m(j,t)(T\mathbf{r}(i,t)-T\mathbf{r}(j,t))/|T\mathbf{r}(i,t)-T\mathbf{r}(j,t)|^3]$$

But this is not the same as (4). It would only be the same if we also substituted *T**r**(.,.)* for ***r****(.,.)* in the right hand term – as the physicist's method applied to (4) directly requires.

I think we can be assured that contradictions will reappear, no matter how we try to further enhance our system, because it is the *extensional interpretation,* or the limitations of having a merely *extensional formalism,* that is ultimately at fault, and this will always confound the problem of making a *compositional operator* to represent deductions of *T*.

11. Conclusion.

The formal interpretation of a theory of physics is distinct from that for a pure *mathematical* theory, and requires additional formal concepts in the object language for rigorous deductions of key results. The problem is because mathematics requires only *extensional* *semantics,* but physics ultimately requires *intensional semantics.* The additional concepts of intensional semantics are not currently represented in physics.

This is reflected in long-standing problems about the time reversal transformations in physics. These problems can be tackled in other ways, by giving semantic proofs, but the situation for physics is not ultimately satisfactory without showing how a better deduction system can be constructed.

Part 2 of this paper will argue that these problems can be resolved by extending to an *intensional* version of physics object languages. This shows that systems for deduction of general transformations are possible. More importantly, the implementation of precise intensional semantics forces us to provide detailed formal interpretations of theories, and I think that this objectively determines the interpretations of time reversal transformations of all entities in the fully interpreted theories, including states, processes, and properties.

The physicist's common view that there is something 'conventional' about *the meaning of time reversal* is consequently rejected. The problem is rather that *there are different possible interpretations of complex theories,* such as QM, and it is not until an interpretation has been adopted that we can objectively determine time reversals. The meaning of time reversal itself is found to be perfectly objective.

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Andrew Holster PhD. ATASA030@gmail.com

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