The incompleteness of extensional object languages of physics and time reversal. Part 2.

PART 2: Intensional semantics for physics object languages and the deduction of time reversal transformations in physics.

Andrew Holster. Original 2003. Reposted unchanged 2023.

[This paper continues from Part 1 (Holster 2003(C)). Reference is made to Eq. (1)-(7) from there, which are repeated for convenience in Appendix 4 here.] It was shown in Part 1 that it is impossible to construct a general compositional operator to represent the *T* transformation in physics if we have only an extensional interpretation of propositions. It may be wondered if this is a realistic goal anyway – after all, a number of leading writers on time reversal have noted that no systematic way of defining *T* for theories generally in physics is known[1]; they do not appear to think any fully systematic definition is possible; and they appear satisfied to continue with various *ad hoc* interpretations of *T*. But I will give a different answer to this question.

To do this, I introduce a simple extension of the object language, to include a representation of *contingency,* through a basic kind of intensional logic, using *intensionalisation on worlds.* I also observe that, given we have a decisive logical interpretation of a fundamental theory, then the *T* operator is defined analytically through this interpretation, and does not require further *ad hoc* or empirical considerations. The problem is defining the *interpretation of theories*, not defining the *T* operator. I also demonstrate (see Appendix 2) that if the original object language is compositional to start with, then it *must* be possible to define a general compositional *T* operator. I also reiterate that the problems of defining a time reversal operator have led to practical problems, which undermine the reliability of the analysis of time symmetry in applied physics.[2]

7. Intensional *versus* extensional semantics for physics.

A system of formal semantics for a language can be thought of as a specification of a *meaning function*, which maps each well-formed *term* of the language, *A,* to some kind of objects, ***A***. We take *A* to be the name for the *symbol "**A**"*, and we can write schematically:

$$Meaning(A) = \boldsymbol{A}$$

An explicit specification of a *Meaning* function is called *formal semantics* or *objectual semantics* when we specify a direct mapping from *terms* or symbols of a language, to *objects of reference*[3]. The specification of a system of meanings is called an *interpretation of the*

[1] This point and related problems about the definition of time symmetry concepts are discussed in the references to de Beauregard, Davies, Earman, Hutchison, Liu, Penrose, Sachs, Watanabe, Zeh.

[2] These points are pursued in detail in Holster 2003 (A) and (B); see also references to Watanabe, de Beauregard, Callender and Healy. This paper deals with the underlying logical problem that has led to these problems in the applied analysis, not the analysis itself.

[3] The concept of formal semantics was first clearly explicated by Frege. The most popular developments are based on the work of Montague: see references to van Benthem, Janssen, Partee for useful summaries. An alternative development originated with Pavel Tichy (1971, 72, etc), and has been pursued by Materna, Duzi,

language. To give it, we start with some *fundamental interpretations,* of fundamental terms as referring to certain kinds of fundamental objects (we have to start somewhere); we then impose rules for the construction of the meanings of *complex terms* from their syntactic constructions in terms of fundamental component terms, and the meanings of the fundamental terms. We will see the power of this later in the principle of *compositionality,* which says that the meaning of a complex term is determined by the meanings of its component terms, and the manner in which they are combined. But first we specify how this kind of semantics works for physics.

Note that the term 'objects' is used in a wide sense here: in physics, 'basic objects' may be individual particles, points of space, moments of time, space-time manifolds, masses, charges, and so on; but the general class of objects used to interpret a theory of physics includes all kinds of functions or logical constructions that may be defined from these 'basic objects'.

In theoretical physics, the *objects* used to interpret a theory are usually thought to be *real* physical things and their properties. However, this is only obviously the case when we give the abstract theories empirical or experimental applications. To begin with, in pure theoretical physics, we do not have to think of theoretical terms as making any direct reference to real physical things at all. Instead, the basic reference is to a *mathematical (or abstract) model.*

The theoretical interpretation is normally introduced as an explicit *extensional, set-theoretic interpretation*, taking the language terms to refer to entities from an abstract model. We may call these the classes of *theoretical* particles, *theoretical* positions, *theoretical* moments of time, *theoretical* masses, etc. The mathematical structures involved in these classes are assumed to be well-defined.[4]

The theory is subsequently interpreted *empirically* because these theoretical entities and constructions are intended to be applied to give descriptions of real physical things, which we identify as 'physical particles', 'physical space', 'physical time', etc; but we need not assume that any given mathematical theory is necessarily descriptive of real or physical objects when we initially define it as a mathematical structure.

It is primarily the level of the theoretical interpretation that we will be concerned with. The problem of empirical application is separate. The key point is that when we do add an empirical interpretation, there must already be an apparatus in the theoretical language to represent *contingency*. This apparatus will be represented by an intensional semantics for the object language. But first we introduce the extensional interpretation.

A basic *extensional interpretation* for classical mechanics may be sketched as follows. First, for the main basic terms:

- We take the term 't' as a variable ranging over a basic class **T** of moments, and 't_0', 't_1', *etc,* as constants referring to moments.[5]

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and others in the 'Transparent Intensional Logic' (TIL) program: see TIL website for more detail. But the controversies between different approaches to intensional logic are not relevant to the main issues in this paper.

[4] See Kobayashi and Nomizu, 1963, and Spivak, 1979, for the most explicit kind of extensionalist interpretation typically developed in modern physics.

[5] Note that 'Δt' refers to an interval of time, and is vectorial, and the variable t can also be treated as vectorial. Note also that there is a common ambiguity between treating t, i, etc, as variables or constants. Strictly, if these are defined as variables, they should be treated uniformly as such. But when we instantiate t at a particular moment of time (or i at a particular particle, or m at a mass) it is common to intuitively 'exchange its meaning'

- We take '*i'* to signify a variable ranging over a basic class **I** of individual particles, and 'i_1', 'i_2', *etc,* as constants referring to particles.
- We take '***X', 'Y'***, to signify constant point-vectors, or points from a basic class $\mathbf{R^3}$ of positions.
- We take '*m'* to signify a variable ranging over a basic class **M** of masses, and 'm_1', 'm_2', *etc,* as constants referring to particle masses.

We also have two special terms:
- We take '***r****(i,t)'* to denote the point-vector on the trajectory of a particle *i* at a time *t.*
- We take '*m(i,t)'* to denote the mass of a particle *i* at a time *t.*[6]

Of course, **T**, $\mathbf{R^3}$ and **M** are really structured entities, rather than just classes: e.g. **T** has the structure of a linear continuum, $\mathbf{R^3}$ has the Euclidean manifold structure of a 3-dimensional vector space, **M** has the structure of a ray. These structures are evident through the existence of functions giving *time intervals* (i.e. distances between points of time), *vectors* and *lengths* (i.e. relations and distances between points of space), or *mass additions*. We simply take the normal interpretations of these structures for granted here: they are exhaustively discussed in foundational studies. We assume that a good interpretation of this kind has been supplied already – our problem will be to expand this interpretation to an *intensional* one.

We call such classes the *base sets of the ontology of the theory.* There is a definite class of such base sets for a well-defined theory, and other theoretical entities (e.g. tensors; fields; etc) are constructed from these sets (or more accurately, these fundamental structures).

Specifying the *base of the ontology* is the first part of the interpretation. The second part identifies the kinds of *facts* that the theory recognizes. In classical physics, this is essentially through the interpretation of the special constants that refer to *particle trajectory functions, particle masses,* and so forth.

Thus, the special fundamental term: "***r****(.,.)"* is introduced in classical mechanics to represent *the trajectories of particles.* This term is interpreted as referring to *a function from particles, i, and moments of time, t,* to *positions in space,* ***X****.* A second special function is "*m(.,.)",*interpreted as referring to *a function from particles, i, and moments of time, t,* to *masses.* The basic kinds of facts represented in classical mechanics are facts about *positions and masses of particles at moments of time.*

These special functions allow us to state *propositions of the theory.* Thus we may write: ***r****(i,t)* = ***X***¸ for some *i, t,* and (specific position) ***X***, to state that *the particle i has position* ***X*** *at time t.* Having the trajectory function, ***r****(.,.),* we can also define additional mathematical operators on them, such as differential operators like: *d****r****(i,t)/dt* (velocity), in the usual ways. We also frequently use terms like: $\boldsymbol{r}_i(.)$ to represent the specific trajectory of a constant particle *i.*

The laws of a theory are generalised propositions, such as Eq. (4) (see Appendix 4), which state identities between various mathematical operations on trajectory functions, mass

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as a variable for a new meaning, where it is treated as a constant, taken to refer to a specific moment. This lets us avoid using so many subscripted terms for constants. This practice is so deeply embedded that I will not try to deal with it here.

[6] This is a generalization of the ordinary symbolism: in classical mechanics, masses are taken to be constant and we only need to identify the constant mass, m_i, of a particle, *i.* But the generalization is required for a logical view, and immediately comes into play in relativistic physics.

functions, etc. These laws involve one further kind of entity, *physical constants,* such as the gravitational constant, G, which I comment on later.

We formally specify the notion of *facts* through the fundamental semantic notion of *worlds.* A *world, W,* is a *complete class of facts*. In natural language, we are used to allowing any kind of jumble of facts to represent a (logically possible) 'complete truth about the world', but in fundamental physics, the ontology of the theory specifies an extremely limited class of possible *types of facts.* In a basic classical theory, the complete class of facts to represent a world may be represented as a collection, W:

Definition of the logical form of a simple classical world.

$$W = \{(i,\boldsymbol{r},t,m)\text{: particle } i \text{ has position } \boldsymbol{r} \text{ and mass } m \text{ at time } t \text{ in } W\}$$

Comments on the generality of this kind of scheme are made in Section 13 below[7], but for the moment we will just work through this simple example. This specification is a necessary part of the logic of the theory. Logically possible worlds of such a theory are defined as classes of facts of this form. Such a class has a strictly limited logical structure. It is this specification of the 'fundamental logical form' of worlds that gives *fundamental theories of physics* such powerful content. They specify the 'logical space' of the world before they even start to propose specific contingent laws or propositions about the actual world. The progress of fundamental physics, from this point of view, lies in altering the idea of what the fundamental underlying logical space of physical possibilities may be.

It may be noted that this is a distinctly *metaphysical* idea: the notion that there is a fundamental 'logical space' for the world (or indeed, that *there is a single well-defined world at all*) is a metaphysical idea, and is not proved by direct empirical observation. For instance, an obvious objection to this idea is: *what if there is no ultimate level of fundamental facts underlying the real world at all?* What if, as we continue doing fundamental physics, we keep finding that there are deeper and deeper levels of more fundamental composition of the actual physical world, with no ultimate end? Now this is a real possibility: but the specification of the semantics for a hypothetical theory, like classical mechanics, does not depend on its metaphysical assumptions being *correct.* It is rather an explication of how concepts of the theory are *intended to be interpreted.* And the concept of classical mechanics *as a fundamental theory* is intended to be interpreted as a specification of a certain kind of fundamental logical structure of the world.

We can also note that, although this interpretation may seem to make the theory involve 'metaphysical' assumptions, the resulting theory is not *non-empirical,* because the structure of fundamental facts implied by the theory may be discovered to be too simple, for instance, to represent the nature of facts in the actual world that we discover by experience. This is the case with classical mechanics: the particular logical structure specified by classical mechanics is just too simple to represent the causal connections we discover empirically, and we are forced to reject the metaphysical basis of this theory when we move on to quantum mechanics or relativity theory. But I will not pursue the discussion of the epistemology of physics here: the aim is merely to explicate the semantics, and to do this, we simply interpret the metaphysical *assumptions* behind the construction of the theory as accurately as we can. If we get them wrong, the reader can object that we have not represented the *theory*

[7] In particular, the application of this kind of semantics to quantum theory is problematic, because of deeper problems in interpreting quantum theory, such as the notion of a complete quantum *world*; see section 13.

accurately; the objection that we have misrepresented the true nature of the *world* accurately (or that the theory we consider is empirically wrong) is beside the point at this stage.

At any rate, we can now state the difference between an *extensional* and an *intensional interpretation.* This is primarily revealed through the interpretation of *statements representing propositions,* and is reflected directly in the interpretation of the contingent terms, such as the trajectory functions and mass functions.

- An *extensional interpretation* takes a proposition, *L*, to be a specific *truth value.* Equally, an extensional interpretation takes ***r****(.,.),* to be a specific *trajectory function,* i.e. a specific function from *(i,t)*-couples to positions.
- An *intensional interpretation* takes a proposition to *take truth values at worlds*, and thus be a *mapping from worlds to truth values.* Equally, an intensional interpretation takes the trajectory function be a mapping *from worlds to specific trajectory functions in worlds.*

The *intensional semantic* approach is far more natural: the extensional approach arose originally in the context of mathematical theories, where propositions are either necessarily true or false; they have the same truth values at all worlds, so we do not have to take the variation between worlds into account at all[8]. But contingent propositions are naturally identified as taking different truth values at different worlds, and are formally identified as *mappings from worlds to truth-values*, or more simply, as the classes of possible worlds where they are true. (Some versions of intensional logic, e.g. Tichy's *TIL*, take propositions as mappings from worlds and *times* to truth values, so that propositions can *change* their truth values with time. This is convincing for natural language; but we can avoid the complication of involving *times* here because we essentially only consider either universal 'laws', which are stated to hold at all times, or else propositions specified at definite times, and these never change their truth values.)

The key to intensional semantics lies in the *explicit representation* of this. It is essential for the approach here that the concept of *worlds* is precisely defined, so that the world variables are precisely defined, and we can quantify over them properly. We have done this above (defined worlds for the present simple theory of classical mechanics), and we can now add an *explicit representation of world references* to the theoretical object language. I will now propose a natural interpretation of this.

To explicitly represent an *intensional semantics for trajectory functions,* we simply add an extra *world-argument to the trajectory terms*, and write an expanded trajectory function. I will capitalize ***R****(.,.,.)* to distinguish the intensional term from the ordinary ***r****(.,.)*. To make the connection between the two formalisms, we impose definitions like this:

$$\boldsymbol{r}(i,t)\ [\textit{obtained in World } W] = \boldsymbol{R}(i,t,W)$$

$$\boldsymbol{r}(i,.)\ [\textit{obtained in World } W] = \boldsymbol{R}(i,.,W)$$

$$\boldsymbol{r}(.,.)\ [\textit{obtained in World } W] = \boldsymbol{R}(.,.,W)$$

[8] This is made particularly clear in Section 2 of Tichy (1988 *unpublished*).

The conditional phrase "*obtained in world W"* is not explicitly represented in ordinary physics formalism itself: it is, however, constantly referred to in the informal reasoning that occurs in the meta-language of physics text-books. It is typically evident in phrases introducing and generalising mathematical arguments in physics, which take forms like:

> "*Let us suppose that* **r***(.,.) is a certain trajectory function* [for a world W] *satisfying the axioms of the theory. Then…* <There follows some mathematical derivation in the object language, producing a derived theorem>. *This obtains for any trajectory function* [for a world W] *that satisfies the theory, and hence represents a general theorem."*

The present aim is to incorporate this level of reasoning formally into the object language. The following schemas indicate how to extend the object language of ordinary physics to make such world references explicitly evident.

An intensional translation for basic terms of physics.

Classical Trajectories

Extensional System:	***r****(i,t) [in World W]*	***gives[9]: spatial point-vector***
Intensional System:	***R****(i,t,W)*	***gives: spatial point-vector***

Mass Functions

Extensional System:	*m(i,t) [in World W]*	***gives: mass***
Intensional System:	*M(i,t,W)*	***gives: mass***

Scalar Fields (e.g. potential fields; quantum wave functions):

Extensional System:	$\Psi(i,t,\mathbf{r})$ *[in W]*	***gives: scalar value (real or complex)***
Intensional System:	$\Psi(i,t,\mathbf{r},W)$	***gives: scalar value (real or complex)***

Simple Differential Operators:

Extensional System:	$\frac{d}{dt}((\lambda t)\, f(t))$ *[f in W]*	$\frac{\partial}{\partial \mathbf{r}}((\lambda \mathbf{r})\phi(\mathbf{r}))$ *[ϕ in W]*	***gives: function***
Intensional System:	$\frac{d}{dt}((\lambda t)\, f(t,W))$	$\frac{\partial}{\partial \mathbf{r}}((\lambda \mathbf{r})\phi(\mathbf{r},W))$	***gives: function***

Note that the *differential operators themselves* are *world independent mappings*, taking functions to their derivative functions. Hence, the *differential operators themselves have no world variables;* only the functions being differentiated require world variables.

Universal Physical Constants:

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[9] I.e. this gives a spatial vector when we take a valuation of the variables *i*, *t*, and *W*.

Extensional System:	c	G	h	*(World independent constants)*
Intensional System:	$c(W) = c$	$G(W) = G$	$h(W) = h$	*(World independent constants)*

The interpretation of the differential operators and the physical constants will require special comment, because there are substantial peculiarities when analysed carefully. However, first we turn to applying this system to solving our problem about the time reversal transformations, and consider what happens to the representation of propositions.

8. Intensional propositions in physics.

We take propositions in general to be represented by *statements, L.* We can now represent the difference between *propositions in intension,* and *the extensional values of propositions at specific worlds,* and *at the actual world.*

Propositions as Intensions

Extensional system: <not represented>
Intensional System: $L(.) = (\lambda W)L(W)$ **gives:** *Mapping from Worlds to Truth values*

(The λ-operation abstracts W from $L(W)$, to form a function: see Appendix 1.)

Extensions of Propositions

Extensional System:	L =	*L [at world W]*	**gives:**	*Truth value*
Intensional System:	L_W =	$L(W)$	***gives:***	*Truth value*

Actual Extensions (truth) of Propositions

Extensional System: *L is actually true* = *L [In the actual world, @]* ***gives:*** *Truth value*
Intensional System: $L_@$ = $L(@)$ ***gives:*** *Truth value*

The first point to note is that the value of a term: *L(W),* for any specified proposition, *L(.)* and world *W, is determined analytically or logically,* because *L* is defined as a class of mappings, or a class of worlds, and *L* is true or false of *W* by their definitions. But of course, the 'actual truth' of a proposition is generally contingent, not analytic. This *contingency* is represented by the value of: *L(@),* i.e. the value of *L(.) at the actual world, @*. This is because @ is not defined analytically: rather, it *takes a specific world as its value*, but the value of @ is only determined by determining contingent facts about the actual world. The same goes for ***R****(i,t,W)* and ***R****(i,t,@).* (See Appendix 3).

Of course, we can never fully determine the value of @ as a unique world in practice: rather, we can only partially determine its content, by determining the truth or falsity of various propositions: *L(@).* This requires that we have some way of empirically determining the truth or falsity of certain propositions. But we need not analyse the epistemology of this in any detail at this point: we merely assume that some such determinations *can be done.*

In fact, the definition of the empirical actual world is a subject of philosophical dispute, and at least four different ways have put forward to define it. The simplest way, adopted here, is just that, *within the theoretical ontology,* @ must refer a particular, constant world.

At any rate, for our purposes, @ is assumed to take a unique value as if it is a primitive constant, *within the theory*. That is: the theory presumes that there is a unique world, called @, and we treat this as a constant. But until we connect the theoretical ontology with an empirical interpretation, no empirical interpretation of @ is possible.

This is the basic idea of intensional propositions: we now turn to see how propositions as intensions are constructed in detail from the intensional representation of the fundamental terms of our object language. We reconsider the earlier statements, (4), (5), and *L*, from previous sections, and substitute the intensional version of terms. First we begin with (4) which is the simplest.

(4)(.) $$(\lambda W)(\forall i,t)[m(i,t,W)d^2\mathbf{R}(i,t,W)/dt^2 = \mathbf{\Sigma}_{j\neq i} -Gm(i,t,W)m(j,t,W)(\mathbf{R}(i,t,W)-\mathbf{R}(j,t,W))/|\mathbf{R}(i,t,W)-\mathbf{R}(j,t,W)|^3]$$

Here, all terms are directly substituted for their intensional versions, and we obtain a general intensional proposition, as required. When we *state (4)(.) as actually true*, we apply it to the actual world, @, to obtain:

(4)(@) $$(\forall i,t)[m(i,t,@)d^2\mathbf{R}(i,t,@)/dt^2 = \Sigma_{j\neq i} -Gm(i,t,@)m(j,t,@)(\mathbf{R}(i,t,@)-\mathbf{R}(j,t,@))/|\mathbf{R}(i,t,@)-\mathbf{R}(j,t,@)|^3]$$

This represents a truth-value: it is true if *(4)(.)* is true of the actual world, @, and false if not[10].

The situation with (5) is a little more difficult, because it is really represented as *a definition of the term* ***v****(i,t) based on the values of* ***r****(.,.) in the world in question.* We might take the specific world to be either: *(i) the actual world, @,* or alternatively: (ii) *the world W where we evaluate* ***r****(.,.).* To interpret (5) and (6) correctly, we must choose the latter: for when we apply (6) to a world *W*, we do not want (6) to say that the gravitational accelerations correspond to those defined in the *actual world*, but rather, to those in the world *W* where we evaluate the trajectories ***R****(.,.,W).* Thus we can take:

(5)(.) $$(\lambda V)(\forall i,t,W)(\mathbf{v}(i,t,W) = d\mathbf{R}(i,t,W)/dt)$$

(5)(.) is a definition of ***v****(.,.,.)*, and is true in every world *V*; but the values of ***v****(.,.,W)* are still contingent on ***R****(.,.,W) through W*. Note that ***v****(.,.,.)*is quite distinct from the defined operator ***V****[.]*:

$$\forall \mathbf{r}(.,.)[\mathbf{V}(\mathbf{r}(i,t))=_{df} d\mathbf{r}(i,t)/dt]$$

which is just another symbol for the differential operator, not for the *function constructed by the differential operation on* ***r****(.,.).*

Next we interpret *L* defined above. To obtain its intended interpretation, we take it to be a statement that *the mathematically defined trajectory function,* ***f****(t) = exp(t)****w****,* where ***w*** is some constant velocity vector, is the trajectory function for the particle *i*, giving:

L(.) $$(\lambda W)((\forall t)(\mathbf{R}_i(t,W) = \mathbf{f}(t))$$

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[10] We may also allow it to be null, for instance if the differential operation gives no value at a particular point on a trajectory; but we can ignore the question of null values here.

Note that because the term $\boldsymbol{f}(.)$ is defined as a mathematical function, it does not have world variables. $L(.)$ is not necessarily true. When applied to the actual world, it gives the truth value: $(\forall t)(\boldsymbol{R}_i(t,@) = \boldsymbol{f}(t)$. Now $\boldsymbol{f}(.)$ is independently defined by its mathematical definition. This definition is represented in its turn by:

(8)(.) $$(\lambda W)(\forall t)(\boldsymbol{f}(t) = exp(t)\boldsymbol{w})$$

This is just a simple tautology. Given (8)(.) is a tautology, it is not necessary that: $L(@)$, i.e. $(\forall t)(\boldsymbol{R}_i(t,@) = \boldsymbol{f}(t))$ is true: it depends on the value $\boldsymbol{R}_i(t,@)$ at the actual world, and is genuinely contingent.

And finally, we can interpret the troublesome proposition, (6):

(6)(.) $$(\lambda W)(\forall i,t)[m(i,t,W)d\boldsymbol{v}(i,t,W)/dt = \Sigma_{j\neq i} -Gm(i,t,W)m(j,t,W)(\boldsymbol{R}(i,t,W)-\boldsymbol{R}(j,t,W))/|\boldsymbol{R}(i,t,W)-\boldsymbol{R}(j,t,W)|^3]$$

We also need to mention how the logical or defined propositions, like (5)(.) and (8)(.), are distinguished from the contingent propositions like $L(.)$ and (4)(.). To make this distinction, we *add (5)(.) (8)(.) as logical axioms, so that they form part of the general deduction system of the language itself*. Hence when we turn to obtaining derivations, we have as trivial derivations that:

$$\vdash (\lambda V)(\forall i,t,W)(\boldsymbol{v}(i,t,W) = d\boldsymbol{R}(i,t,W)/dt) \quad \text{and:} \quad \vdash (\lambda W)(\forall t)(\boldsymbol{f}(t) = exp(t)\boldsymbol{v})$$

I.e. these are derived from nothing. On the other hand, we deduce *(4)(.) from the axioms of the empirical theory being considering.* If we name the theory by the term *Classical-Gravity*, then we have an ordinary logical deduction:

$$\textit{Classical-Gravity} \vdash (4)(.)$$

If we wish to state that the law (4)(.) is *actually true*, or that the proposition *L*(.) is *actually true*, then we can write these as statements, and propose that when @ is evaluated, we get the values:

(4)(@) is *True*, *or:* *L(@)* is *True*

This shows how a variety of different kinds of statements are interpreted intentionally. We now turn the system for obtaining their time reversals.

9. Definition of time reversal in intensional logic.

Having the resources of an intensional formalism available, we suddenly find that it is easy to define time reversal. First, however, we must add the fundamental definition of the concept of the *time reversal, TW, of a world, W:*

Definition of ***TW*.**
$TW = \{(i,\boldsymbol{r},-t,m)$: particle i has position $\boldsymbol{r}$ and mass m at time t in $W\}$

The time reversal of an intensional proposition is then defined by:

Definition.
A proposition $L^*(.)$ is the time reversal of a proposition $L(.)$ just in case:

$$(\forall W)(L^*(TW) = L(W))$$

Or alternatively, we can just write it as an axiom that:

General Definition of T **acting on** $L(.)$:
$(\forall W)(TL(TW) = L(W))$

Or equivalently:

$(\forall W)(TL(W) = L(T^{-1}W))$

In fact this applies for all general transformations (see below). A useful equivalent form *in the special case of time reversal* is:

Special case for transformations where: $T = T^{-1}$
$(\forall W)(TL(W) = L(TW))$

This follows for time reversal because: $T(TW) = W$. It does not hold generally for transformations, only when $T = T^{-1}$. Then because W is universally quantified (a dummy variable) in the general axiom, and TW has the same range as W, we can replace W with TW there, to obtain: $(\forall W)(TL(TTW) = L(TW))$, which then simplifies to: $(\forall W)(TL(W) = L(TW))$.

A proposition $L(.)$ is *time reversal invariant just in case:* $L(.) = TL(.)$. This means that:

General Definition.
L(.) is T-invariant just in case:

$$(\forall W)(L(TW) = L(W))^{11}$$

Or using the second form for time reversal of L (which assumes that $T = T^{-1}$):

L(.) is time reversal invariant just in case:

$$(\forall W)(TL(W) = L(TW))$$

Obviously this property must hold for all analytically or necessarily true propositions, such as definitions, because these propositions are by definition invariant w.r.t. worlds, so that if

[11] Note that: $(\forall W)(L(TW) = L(W))$ is an extension, not an intensional proposition; it is intensionalised by abstracting: $(\lambda V)(\forall W)(L(TW) = L(W))$, with V a second world variable. But then it is trivial that $(\lambda V)(\forall W)(L(TW) = L(W))$ takes the same value, $(\forall W)(L(TW) = L(W))$, at every world V. With trivial cases like this, we frequently ignore the intensionalisation, and just write: $(\forall W)(L(TW) = L(W))$

$L(W)$ is true, then $L(TW)$ is true. Similarly for all analytically false propositions. But it will no longer hold automatically if $L(.)$ is contingent.

We can also observe that there will now be an automatic procedure to obtain time reversals of propositions: if the proposition is formed from abstraction of W from a complex entity P, we simply replace W throughout P by TW.

$$\text{If } (\lambda W)(P) \text{ is a proposition, then: } (\lambda W)(((\lambda W)(P))(TW)) = T((\lambda W)(P))$$

Hence, when we apply the world W to $T((\lambda W)(P))$, we get: $((\lambda W)(P))(TW)$.

We will now find that the general time reversal operator is represented directly by the distributive operator, $\mathit{Ŧ}$, as defined previously, but extended naturally to include worlds. I.e. for any world W:

$$\mathit{Ŧ}W = TW$$

And with an additional rule for operating on λ-terms themselves:

$$T(\lambda W) = (\lambda W)$$

And we will obtain the result we wanted in the first place: that for any proposition, $L(.)$:

$$TL(.) = \mathit{Ŧ}L(.)$$

Indeed, we find this for any complex term, *X, in the intensional formalism*:

$$TX = \mathit{Ŧ}X$$

This is what needs to be subsequently proved: that extending to an intensional formalism, the *distributive syntactic operator Ŧ successfully defines real time reversal.*

However, before turning to this, we will first check these claims by obtaining the time reversals of the propositions *(4)(.), (5)(.), (8)(.), L(.),* and *(6)(.).*

Summary.

General Logical Truths for Intensional Propositions, $L(.)$
$(\forall W)(L(W) = TL(TW))$
$(\forall W)(L(T^{-1}W) = TL(W))$
$(\forall W)(T^{-1}L(W) = L(TW))$

Special Logical Truth whenever: $T = T^{-1}$
$(\forall W)(L(TW) = TL(W))$
This is also true *whenever L is T-invariant*

Dependant on the nature of $L(.)$

$L(.) = TL(.)$	True just in case *L is T-invariant*
$L(.) = T^{-1}L(.)$	True just in case *L is T-invariant*
$(\forall W)(L(W) = TL(W))$	True just in case *L is T-invariant*
$(\forall W)(L(W) = L(TW))$	True just in case *L is T-invariant*

10. Examples of time reversal in intensional logic.

Time Reversal Invariance of *(4)(.).*
By definition, for any world W: $T(4)(W) = (4)(TW)$. This satisfied by defining:

$T(4)(.)$
$$(\lambda W)(\forall i,t)[m(i,t,TW)d^2\mathbf{R}(i,t,TW)/dt^2 = \mathbf{\Sigma}_{j\neq i} -Gm(i,t,TW)m(j,t,TW)(\mathbf{R}(i,t,TW)-\mathbf{R}(j,t,TW))/|\mathbf{R}(i,t,TW)-\mathbf{R}(j,t,TW)|^3]$$

This follows since if we apply this proposition to the world W, we obviously obtain the same truth-value as the proposition *(4)(.)* applied to TW.

The identity of *(4)(.)* and $T(4)(.)$ (i.e. the well-known time reversal invariance of (4)(.)) is then obtained by noting that:

(i) $m(i,t,TW) = m(i,-t,W)$;
(ii) On the left hand side: $d^2\mathbf{R}(i,t,TW)/dt^2 = d^2\mathbf{R}(i,-t,W)/dt^2$;
(iii) On the right hand side: $\mathbf{R}(i,t,TW)-\mathbf{R}(j,t,TW) = \mathbf{R}(i,-t,W)-\mathbf{R}(j,-t,W)$
(iv) On the right hand side: $|\mathbf{R}(i,t,TW)-\mathbf{R}(j,t,TW)|^3 = |\mathbf{R}(i,-t,W)-\mathbf{R}(j,-t,W)|^3$

Substituting these into $T(4)(.)$, we obtain exactly the equation for *(4)(.)*, but with $-t$ replacing t. But since *(4)(.)* is universally quantified w.r.t. time, it holds equally for times t and $-t$, and hence *(4)(.)* and $T(4)(.)$ are identical. We will examine (ii) more closely below.

Time Reversal Invariance of *(5)(.).*
By definition, for any world V: $T(5)(V) = (5)(TV)$, so:

$T(5)(.)$ $\qquad$ $(\lambda V)(\forall i,t,W)(\mathbf{v}(i,t,TW) = d\mathbf{R}(i,t,TW)/dt)$

If we apply this proposition to any world V, we obtain the same as $(5)(TV)$:

$T(5)(V)$ $\qquad$ $(\forall i,t,W)(\mathbf{v}(i,t,TW) = d\mathbf{R}(i,t,TW)/dt)$

$(5)(TV) \qquad (\forall i,t,W)(\mathbf{v}(i,t,TW) = d\mathbf{R}(i,t,TW)/dt)$

T(5)(.) is the same as *(5)(.)*, being a tautology.

Time Reversal Invariance of *(8)(.)*.
By definition, for any world *W: T(8)(W) = (8)(TW)*. This satisfied by defining:

$T(8)(.) \qquad (\lambda W)(\forall t)(\mathbf{f}(t) = exp(t)\mathbf{w})$

Since if we apply this proposition to the world *W*, we obtain the same as *(8)(TW)*. Obviously this is *time reversal invariant*, since:

$$T(8)(W) = (8)(W) = (\forall t)(\mathbf{f}(t) = exp(t)\mathbf{w})$$

Time Reversal Non-Invariance of *L(.)*.
By definition, for any world *W: TL(W) = L(TW)*. This satisfied by defining:

$TL(.) \qquad (\lambda W)((\forall t)(\mathbf{R}_i(t,TW) = \mathbf{f}(t))$

Since if we apply this proposition to the world *W*, we obtain the same as *L(TW)*.
The non-identity of *L(.)* and *TL(.)* is then obtained by noting that:

(i) $\mathbf{R}_i(t,TW) = \mathbf{R}_i(-t,W)$;
(ii) $\mathbf{f}(-t) \neq \mathbf{f}(t)$, for at least one value t;

Substituting (i) into *TL(.)* we get:

$TL(.) \qquad (\lambda W)((\forall t)(\mathbf{R}_i(t,TW) = \mathbf{f}(-t))$

Applying some world, *W*, to *TL(.)*, and instantiating with *t* from (ii) we get:

$$TL(W) \Rightarrow (\mathbf{R}_i(t,W) = \mathbf{f}(-t))$$

But equally:

$$L(W) \Rightarrow (\mathbf{R}_i(t,W) = \mathbf{f}(t))$$

Using (ii): $\qquad TL(W) \Rightarrow \sim L(W)$

Hence, *L(.)* is not time reversal invariant.

Time Reversal Invariance of *(6)(.)*.
By definition, for any world *W: T(6)(W) = (6)(TW)*. This satisfied by defining:

T(6)(.)

$$(\lambda W)(\forall i,t)[m(i,t,TW)d\mathbf{v}(i,t,TW)/dt = \Sigma_{j\neq i} -Gm(i,t,TW)m(j,t,TW)(\mathbf{R}(i,t,TW)-\mathbf{R}(j,t,TW))/|\mathbf{R}(i,t,TW)-\mathbf{R}(j,t,TW)|^3]$$

The correctness of this is shown by applying *T(6)(.)* to the world *W*, to obtain:

T(6)(W)

$(\forall i,t)[m(i,t,TW)d\mathbf{v}(i,t,TW)/dt = \Sigma_{j\neq i}\ \text{-G}m(i,t,TW)m(j,t,TW)(\boldsymbol{R}(i,t,TW)\text{-}\boldsymbol{R}(j,t,TW))/|\boldsymbol{R}(i,t,TW)\text{-}\boldsymbol{R}(j,t,TW)|^3]$

But this obtains just in case *(6)(.)* is true of *TW:*

(6)(TW)

$(\forall i,t)[m(i,t,TW)d\mathbf{v}(i,t,TW)/dt = \Sigma_{j\neq i}\ \text{-G}m(i,t,TW)m(j,t,TW)(\boldsymbol{R}(i,t,TW)\text{-}\boldsymbol{R}(j,t,TW))/|\boldsymbol{R}(i,t,TW)\text{-}\boldsymbol{R}(j,t,TW)|^3]$

We then prove the equivalence of *T(6)(.)* and *(6)(.)* by using the identities (i)-(iv) already observed in the previous discussion of *(6)(.),* and the identity for ***v****(i,t,TW)*:

(i) *m(i,t,TW) = m(i,-t,W);*
(ii) See (v) and (vi) instead;
(iii) On the right hand side: ***R****(i,t,TW)*-***R****(j,t,TW)* = ***R****(i,-t,W)*-***R****(j,-t,W)*
(iv) On the right hand side: $|\boldsymbol{R}(i,t,TW)\text{-}\boldsymbol{R}(j,t,TW)|^3 = |\boldsymbol{R}(i,-t,W)\text{-}\boldsymbol{R}(j,-t,W)|^3$
(v) On the left hand side: ***v****(i,t,TW) =* **-*v****(i,-t,W),* hence:
(vi) *d(**v**(i,t,TW))/dt = d(**v**(i,-t,W))/dt* (reversal of acceleration function).

Substituting these into *T(6)(.),* we obtain:

T(6)(.)

$(\lambda W)(\forall i,t)[m(i,-t,W)d\mathbf{v}(i,-t,W)/dt = \Sigma_{j\neq i}\ \text{-G}m(i,-t,W)m(j,-t,W)(\boldsymbol{R}(i,-t,W)\text{-}\boldsymbol{R}(j,-t,W))/|\boldsymbol{R}(i,-t,W)\text{-}\boldsymbol{R}(j,-t,W)|^3]$

This is the same as *(6)(.),* since these are universally quantified w.r.t. time.

We can now recognize how the problem encountered at the end of Part 1 with (6) is solved by *explicitly recognizing the role of W in the term* ***v****(.,.,.)* as well as the term ***R****(.,.,.)* in *(6).* The physicist's method essentially tries to work by intuitively abstracting on the term ***r****(.,.),* but we only get a systematic method by *abstracting on W.*

Finally, we can note why the transformation of *extensions* always gives the result that they are invariant under *T*. Compare the intensional proposition: *L(.)* with its extension at the actual world: *L(@).* The time reversal of the latter is just: *T(L(@)) = TL(T@).* But by definition, *TL(.)* is true at *T@* just in case *L(.)* is true at @. Thus *T(L(@))* is always the same as *L(@)*, whether *TL(.) = L(.)* or not.

11. Time reversal of the time differential operator.

First consider the time differential operator defined by:

$$(\forall f(.))[\frac{d}{dt}(f(.)) = g(.)]$$

Just in case, for all *t*:

$$g(t) = (\lambda t)(\lim dt \to 0)\frac{f(t+dt) - f(t)}{dt}]$$

And:

$$g(t) = (\lambda t)(\lim dt \to 0)\frac{f(t) - f(t - dt)}{dt}]$$

It is assumed that *dt* limits to 0 from positive values. This double condition ensures that the differential exists with the same value when taken from above and below around *t.* (If we consider the simpler differentials taken just from above, or just from below, we find that they are indeed time asymmetric w.r.t. certain discontinuous functions).

Note that this is generalised over all functions: *f(.).*This differential operator is not world dependant: d[.]/dt: *f(.)*→*g(.),* maps a function*, f(.),* of *t*, to another function, *g*(.*),* of *t.* This mapping is not world dependant. This is because the differential operator is mathematically defined to be *the same operator in every world.*

The fact an operator is *world-invariant* does not by itself mean that it must be invariant under time reversal. But this is a consequence of the specific definition of the time differential operator. The time reversed operator: *T(d[.]/dt)* is defined as follows:

Definition of time reversal of the time differential operator.
If d[.]/dt maps: *f(.)*→*g(.),* then *T(d[.]/dt)* maps: *Tf(.)*→*Tg(.)*

We can show directly that: *T(d[.]/dt) = d[.]/dt* by showing that, for any functions *f(.)* and g*(.),* if: *d[f(.)]/dt = g(.),* then: *d[Tf(.)]/dt = Tg(.).*

Proof. Suppose that: *d[f(.)]/dt = g(.).* By definition: *Tf(.)= T[(λt)(f(t))] = (λt)(Tf(t)) = (λt)(f(-t)),* and: *Tg(.) = T[(λt)(g(t))] = (λt)(Tg(t)) = (λt)(g(-t)).* Hence: *Tf(t) = f(-t),* and: *Tf(t+dt) = f(-t-dt),* and: *Tg(t) = g(-t).* But by definition of the differential operator:

$$g(t) = (\lim dt \to 0)\frac{f(t + dt) - f(t)}{dt}$$

Hence:

$$Tg(t) = g(-t) = (\lim dt \to 0)\frac{f(-t - dt) - f(-t)}{dt}$$

But then by the definition of *Tf(.)* this is equal to the differential of *Tf(.)* (from above) at t:

$$(\lim dt \to 0)\frac{f(-t - dt) - f(-t)}{dt} = (\lim dt \to 0)\frac{Tf(t + dt) - Tf(t)}{dt}$$

(Note that this only exists if the differential of *f(t)* exists at *t* from below, which is why we need the double condition in the definition of the full differentials).
A similar argument applies for the differentials from below. Hence we obtain that:

$$Tg(t) = (\lim dt \to 0)\frac{Tf(t + dt) - Tf(t)}{dt}$$

And similarly:

$$Tg(t) = (\lim dt \to 0)\frac{Tf(t) - Tf(t - dt)}{dt}$$

And so $Tg(.) = d[Tf(.)]/dt$. I.e. $Tg(.)$ is the differential of $Tf(.)$ just in case $g(.)$ is the differential of $f(.)$. Then, by the definition of $Td[.]/dt$, this is the same mapping as $d[.]/dt$. .

Theorem.

$$T(d[.]/dt) = d[.]/dt$$

Physicists assume that time reversal acts by reversing the time differential operator: $T(d/dt)= -d/dt$. But this is false if we take d/dt to represent the differential *operator*. It is only correct for the *differential quantities, dt,* or *1/dt*.

Time reversal of differential quantities.
$T(1/dt)=-1/dt$ and: $T(dt) = -dt$

The apparent anti-intuitiveness of this result is because we know that *the time reversal of the time differential of a function,* $\boldsymbol{R}(i,t,W)$, taken at i, t, W, is not the original *time differential of* $\boldsymbol{R}(i,t,W)$. But this is different matter. It is the fact that: $T[d\boldsymbol{R}(i,t,W)/dt] = d(T\boldsymbol{R}(i,-t,TW))/dt = -d(\boldsymbol{R}(i,t,W))/dt$. Notice also that T is compositional and distributive for this term only if we take $T(d[.]/dt) = d[.]/dt$.

The pertinent quality for invariance of any operator or function is this: if operator $\boldsymbol{\Theta}$ maps objects: $\mathbf{a} \rightarrow \mathbf{b}$¸ then it is invariant under T just in case it also maps: $T\mathbf{a} \rightarrow T\mathbf{b}$. This means that $\boldsymbol{\Theta} = T\,\boldsymbol{\Theta}$, because by definition, $T\boldsymbol{\Theta}$ maps: $T\mathbf{a} \rightarrow T\mathbf{b}$ just in case $\boldsymbol{\Theta}$ maps $\mathbf{a} \rightarrow \mathbf{b}$. This property is separate from being a *world invariant* mapping. E.g. we can arbitrarily specify a simple function like: [$\boldsymbol{\Theta}(t) = \boldsymbol{X}$ for $t \geq 0$ and: $\boldsymbol{\Theta}(t) = 2\boldsymbol{X}$ for $t<0$], with $\boldsymbol{X}$ some constant. Then: [$T\boldsymbol{\Theta}(t) = \boldsymbol{X}$ for $t \leq 0$ and: $T\boldsymbol{\Theta}(t) = 2\boldsymbol{X}$ for $t>0$], and $\boldsymbol{\Theta}$ is not the same as $T\boldsymbol{\Theta}$, even though $\boldsymbol{\Theta}$ is defined as world invariant.

12. Time reversal of universal physical constants.

The physical constants, like G, c, and h, look simple, but they contain difficulties, because they are really *physical entities*. This is evident when we consider that they map from one king of physical quantity to another. For instance, the *speed of light, c,* when multiplied by an interval of *time,* Δt, gives us a length in *space,* Δr. The gravitational constant, G, and Plank's constant, h, also map from physical quantities to *different* physical quantities. This is reflected in their dimensional analysis. E.g. *c has the physical dimensions:* $c \equiv L/T = Length/Time$. This reflects that it maps from time to space. *G has the physical dimensions:* $G \equiv L^3M^{-1}T^{-2}$. This reflects that it is a more complicated mapping of physical objects. This is also evident from the fact that physical equations must *balance dimensionally*. If we write an equation: $A = B$, that does not balance dimensionally, then it *cannot be physically interpreted,* because the objects interpreted on one side of the equation are *different kinds of objects* to those on the other side.

This potentially makes the interpretation of the transformations on physical constants difficult. They are defined as *world independent quantities,* but this does not necessarily mean that they are invariant under general transformations. It all depends on the objects they map.

However, this problem appears to be simplified by the fact that their mappings are simply from '*lengths'* in space, time, or mass, and not from *vectorial or directional quantities.* For instance, if we defined a *velocity of light,* in a certain direction, as: $\boldsymbol{c} = c\underline{\mathbf{x}}$, where $\underline{\mathbf{x}}$ is a spatial basis vector in a certain direction, and c is just the ordinary speed of light, then we must find that $T\boldsymbol{c} = -\boldsymbol{c}$, as with any ordinary velocity, and equally, $P\boldsymbol{c} = -\boldsymbol{c}$, where P is the usual space reversal (parity) transformation. These reversals follow simply because of the odd power of time and distance in the dimensions of $\boldsymbol{c}$.

However, this does not *appear* to be so with just c: physicists just take: $Tc = c$ and: $Pc = c$. Similarly, if G was replaced with $\boldsymbol{G}$, which mapped in a vectorial fashion w.r.t. space and time, we would find that: $P\boldsymbol{G} = -\boldsymbol{G}$¸ because of the odd power of G in space, although we would still have: $T\boldsymbol{G} = \boldsymbol{G}$¸ since it is even in dimensions of time. Similarly, we would find that: $T\boldsymbol{h} = -\boldsymbol{h}$¸ since the dimensions of h are odd in time.

I will not try to solve this problem here: it requires a deeper examination of the nature of the physical constants as mappings, and a deeper logical treatment of the concept of *dimensional analysis.* What it appears to reflect, however, is that fact that if we P-transform a *length,* r, defined by: $r = |\boldsymbol{r}| = \mathrm{sqrt}(\boldsymbol{r.r})$, then: we obtain: $Pr = r$, even though: $P\boldsymbol{r} = -\boldsymbol{r}$. Similar, if we *T-transform* the *length* of a temporal vector, which we can write as: $|\mathrm{t}| = sqrt(\boldsymbol{t.t})$, we will obtain: $T|t| = -t$. This indicates that we really need to adopt a vectorial representation of time, as we do of space, to make time transformations precise.

13. General time reversal of worlds, atomic facts, and base sets.

The system outlined here only works as a method for defining time reversals in physics if we have a clear definition of *worlds* available to interpret the theory. For this, we have to choose the definition of *atomic facts* that compose worlds in the first place, to define the logical space of the theory. Can this always be done? This is the first major problem. The second major problem is that we have to choose the time reversal transformation on the atomic facts, or on the base sets, to induce the time reversal of worlds. If this is done, then time reversal has a precise definition for the theory: contingent propositions that define the theory (and indeed, all complex entities referred to in the theory) will then have their time reversals determined. I will outline these two problems, without attempting to solve them here.

The first problem is therefore whether there is an *objective interpretation of worlds or atomic facts* for a given theory. This is certainly a problem, because a given 'theory', understood in a broad sense, can often be interpreted, in a precise sense, in many different ways. For instance, we chose atoms like: $(i,\boldsymbol{r},t,m)$ for our simple classical basis. But there are alternative possibilities. For instance, what about: $(\boldsymbol{r},t,m)$, without distinguishing individual particles? The point of this idea is that there is no apparent difference between a world defined from a class of facts like $(i,\boldsymbol{r},t,m)$, and an equivalent world defined from a simple permutation of particles – so is there any need for any absolute identification of particles? It may be seem that without particle identities, we cannot define *trajectories,* or *differentials of trajectories*. But this is not necessarily true. First, the worlds defined by arbitrary classes of $(i,\boldsymbol{r},t,m)$-facts generally *do not display any kinematic properties anyway.* Classical kinematics supposes that only a tiny class of the logically possible $\{(i,\boldsymbol{r},t,m)\}$-worlds are physically possible: those in which particles have continuous trajectories, with properties of being smooth or analytic and differentiable at points. Classical *mechanics* supposes something stronger again: that worlds are determined from mechanical states at moments of time, where the mechanical states are defined by the positions and velocities of all particles, and their masses. A system of classical *dynamics* subsequently imposes a theory of specific forces to implement a relationship between mechanical states and acceleration states, through the additional introduction of particular kinds of charges. Now in a world W with continuous, smooth trajectories, defined by atoms like: $(i,\boldsymbol{r},t,m)$, we can directly define a trajectory function $\boldsymbol{r}(i,t)$ on particles. But do we need i in the *atoms* to individuate *trajectories*? Why not simply take any two points, $(\boldsymbol{r}_1,t_1,m_1)$ and $(\boldsymbol{r}_2,t_2,m_2)$, in world W, to be *on the same trajectory, call it i,* just in case these are smoothly connected by a class of other points in the world W? Then we can *construct trajectories, i,* from the space-time-mass points themselves. Thus we compress the ontology of the theory, and obtain a different type of logical space for classical physics.

For a second example with a rich history of controversy, we might construct a *relational state* instead of an absolute space-time state. Instead of identifying a world with a class of $(i,\boldsymbol{r},t)$ points for absolute space trajectories, why not use a relational-space ontology, with atomic facts like: $(i,j,\boldsymbol{r}_{ij},\mathrm{t})$, which represents a relational vector between two particles, i and j?

A further point is, why not expand our ontology from just containing one type of atom, like $(i,j,\boldsymbol{r}_{ij},\mathrm{t})$, to containing a number of *different types of atoms,* e.g. $(i,j,\boldsymbol{r}_{ij},\mathrm{t})$ to represent facts about trajectories, and: (i,t,m) to represent facts about masses of particles?

The initial point is about *reformulating the theory in a relational manner,* by representing it logically through distinct types of facts. This is an open possibility, which involves deeper problems beyond the scope of this paper. The further point is true, but does not pose any obvious difficulty: we can certainly split our atomic facts into distinct types.

Sometimes this is unnecessary, because we can combine distinct types into a single type. In this example, we could just use: $(i,j,\boldsymbol{r}_{ij},\mathrm{t},m_i,m_j)$ as a single type of atom. It may be objected that then we can take two facts: $(i,j,\boldsymbol{r}_{ij},\mathrm{t},m_i,m_j)$, $(i,j,\boldsymbol{r}_{ij},\mathrm{t},m'_i,m'_j)$, in the same world, and give i and j two different masses simultaneously. But we can do this anyway: we can just take: (i,m_i) and (i,m'_i) in the same world. Equally, we can take: $(i,\boldsymbol{r},t,m)$, and $(i,\boldsymbol{r}',t,m)$ in the same world, and give a particle i two distinct positions at the same time. This is not a problem, because this is merely the definition of *logically possible worlds.* The central part of the classical theory – the kinematic and mechanical laws – subsequently rules out such worlds as physically or nomically possible. But these are contingent laws, not logical ones. It is common for physicists to propose the constrained *kinematic space* as if it is the *logical space* for the theory, but this is a mistake: there should be a *representation of non-kinematic worlds* possible within the theory[12].

What should be distinguished as impossible, though, is the idea of introducing certain logical redundancies into the atoms. The obvious way to do this is by using atoms like: $(i,\boldsymbol{r},d\boldsymbol{r}/dt,t,m)$, where we define both trajectory positions and velocities within atoms. But this leads immediately to logical impossibilities, because *velocity properties, $d\boldsymbol{r}/dt$, are defined from the trajectory properties.* If we have a trajectory defined by a class: $\{(\boldsymbol{r},t)\}$, then any velocity properties, $d\boldsymbol{r}/dt$, are thereby determined *logically* by these points already. The essential feature of the logical atoms is that they are *logically* independent of each other: but

[12] If we really want to logically rule out having one particle at two distinct points at the same time, then we should drop the reference to particles in the atoms, and write: $(\boldsymbol{r},t,m)$.

a class of atoms of the form: (i,$\boldsymbol{r}$,$d\boldsymbol{r}/dt$,t,m) are not logically independent, and we cannot take the power set of this class to define logically possible worlds.

However, there are other cases where we want to use two different types of atoms: specifically, when we incorporate *both particles and fields.* E.g. suppose we want to introduce electric fields as well as charged point particles. Then we can have two types of atoms: (i,$\boldsymbol{r}$,t,m,q) for point-particles, and in addition, ($\boldsymbol{r}$,t,$\boldsymbol{E}$), for a global electric field over all space. Or alternatively, we might want to introduce electric fields associated with specific particles: (i,$\boldsymbol{r}$,t,$\boldsymbol{E}$). Now we cannot combine these into one kind of atomic fact of the simple form: (i,$\boldsymbol{r}$,t,m,q,$\boldsymbol{E}$), because we do not have an appropriate distinction between $\boldsymbol{r}$ for the particle position, and $\boldsymbol{r}$ for the fields point. But we could always find another way: e.g.: (i,$\boldsymbol{r}$,t,m,q,$\boldsymbol{r}'$,$\boldsymbol{E}$), where $\boldsymbol{r}'$ specifies the field points or $\boldsymbol{E}$. But this is rather unnatural and immensely redundant: it is better to split the atomic facts into two types, for two distinct types of entities. There is no specific problem with doing this, however, from the point of view of the theory of propositions or time reversal sketched above.

At any rate, this outlines the main problem in the first place: is there any *objectively correct interpretation of the logical space for a theory?* This is a deep problem in the foundations of physics. The popular answer is probably that there are always different possible interpretations of the logical space, but these converge on ‘isomorphic’ theories at the level of the stronger empirical consequences. If there is an unsolvable problem in this respect, then it is a problem for interpreting a theory generally, and there may well be *no single theory* in the end. If the choice of logical interpretation affects the time reversal properties, then the theory is not objectively determined in a unique manner. This means, however, that the problem lies in specifying the theory, or its interpretation: it does not mean that the concept of time reversal for a fully specified theory is undefined.

But this leads us to the second major problem: suppose that we have chosen a well-defined logical interpretation. Let us suppose that this is represented by a definite choice of logical atoms, written more generally as: (t, q_1, q_2, ...q_n), where the q_i’s are general variables, interpreted over classes of base sets. The main question is then: *is there an objective interpretation of what the time reversal of these variables should be?*

The problem arises if we have a variable, q_i, which is itself time-dependant, or has an intrinsic construction that relates to time. There are two good examples: first, the magnetic field, **B**¸ is normally thought to reverse on time reversal; so if we have magnetic fields in our atoms, shouldn’t we reverse them on time reversal? But then, how do we decide that this is the appropriate choice? Or take the extended EM theory, with magnetic monopoles, represented by magnetic charges q*. Shouldn’t we reverse these charges? A second example involves the simple quantum mechanical wave function, **Ψ**. If the values of this are complex scalars, z, then on time reversal we normally take z*, i.e. the complex conjugates, rather than just z. But how do we know to do this?

The usual reason for choosing these special transformations is because they ‘work’ in the context of the dynamics laws of these theories: i.e. they render the theories time reversal invariant. Choosing: $T\Psi(\boldsymbol{r},t) = \Psi(\boldsymbol{r},-t)^*$ renders the Schrodinger equation reversible. Choosing $T\mathbf{B}(\boldsymbol{r},t) = -\mathbf{B}(\boldsymbol{r},-t)$ renders the Maxwell equations reversible. But it must be questioned whether this is necessary or proper. My opinion is very briefly that:

(i) If we formulate the logical atoms for simple QM in the form: $(i,\boldsymbol{r},t,z)$, with z being complex values, and take the time reversal of this atom to be: $(i,\boldsymbol{r},-t,z)$, then we get a perfectly sensible concept of time reversal – but it gives an irreversible theory of QM. Alternatively, if we wish to take the reversal, as usual, to be: $(i,\boldsymbol{r},-t,z^*)$, then why? My own opinion is that this is *not viable* – given that we use atoms of the form: $(i,\boldsymbol{r},t,z)$, then time reversal *must be taken as* $(i,\boldsymbol{r},-t,z)$. The conventional result is obtained instead by *reformulating the atoms:* e.g. let us take the atoms as: $(i,\boldsymbol{r},t,\{z,z^*\})$, giving a kind of 'dual' representation of the wave-function. Then the time reversal is: $(i,\boldsymbol{r},-t,\{z,z^*\})$, and, given the appropriate laws, etc, the theory turns out to be reversible after all. But this points to the fact that the reversible interpretation of QM is a *logically different theory to the simpler, irreversible theory.*

But there is a deeper problem of whether any logical representation of QM in terms of 'well-defined worlds' is adequate to represent the broader probabilistic laws of QM. As a result, the usual theory of 'semantics' for QM is usually given in two parts: the first part gives the theoretical model of wave functions for systems of particles, the second gives the 'measurement theory', in terms of 'observables' obtained from the wave functions on measurement[13]. But this introduces the deeply distressing problems of interpreting QM, including the problem of whether QM is *interpretable as a fundamental theory,* which cannot be discussed here.

(ii) If we formulate the logical atoms for EM in the form of charge distributions: $(i,\boldsymbol{r},t,m,q)$, with q being electric charges, and we *construct the* $\boldsymbol{E}$ *and* $\boldsymbol{B}$ *fields from these according to Maxwell's equations,* then we obtain a reversible theory after all by taking: $T(i,\boldsymbol{r},t,m,q) = (i,\boldsymbol{r},-t,m,q)$, because all the **E** and **B** fields are generated from charge distributions, and not introduced as fundamental entities. This is really what we want – the magnetic fields reversed under T because they are generated by the motions of charges, and it is these that are reversed. But if we take the extended theory with magnetic monopoles, and use atoms like: $(i,\boldsymbol{r},t,m,q,q^*)$, with q^* being magnetic charges, we should take the time reversal as: $T(i,\boldsymbol{r},t,m,q,q^*) = (i,\boldsymbol{r},-t,m,q,q^*)$, I think we find that the theory is indeed asymmetric.

(iii) A final point involves the time reversal of probability laws, such as we find in QM and classical thermodynamics. The conventional view is that the time reversal of a law like: $prob(s_2(t+\Delta t|s_1(t)) = p$ is a law: $prob(Ts_1(t+\Delta t|Ts_2(t)) = p$. But I have argued at length in other places that this is a mistake, and the genuine time reversal is: $prob(Ts_2(t-\Delta t|Ts_1(t)) = p$. This result is confirmed by taking a careful semantic analysis as well, but this topic is beyond the scope of the present paper.

These examples indicate real problems about time reversals of well-known theories. The semantic approach outlined here traces these problems back to the logical representation of the theories. There are serious difficulties in deciding what the representations should be. But these difficulties are not solved by adopting the physicist's rough and ready *ad hoc* interpretations of time reversal for specific theories to suit their desires to view all theories as time reversible. Instead, the problems are deeply tied up with issues of interpretation, and this level of analysis needs to be addressed. These difficulties should not be hidden, nor should 'solutions' be adopted merely on the grounds of pragmatic convenience. Most important, the logical representation of theories reflects the scope for potential *developments of theories:* if

[13] See Jauch (1967), Cohen (1989) for examples of the usual approach, and Fine (1986) for a discussion of some key interpretational issues.

our current theories were perfectly adequate, the problem would not be so important, but the deeper problem in physics is to develop better theories, and this requires awareness and reevaluation of the deeper theoretical structures, not merely pragmatic treatments of the current, imperfect theories.

Appendix 1: Semantic concepts: compositionality, abstraction, and general transformations.

A fundamental concept of semantics is *the meaning of terms.* A formal system of objectual semantics formally specifies the content of the *meaning function* for a language, by assigning objects as the meanings of all its terms. We will use bold symbols like "**A", "a",** etc, for terms of the language, and italicized versions of the same letters: "*A", "a",* etc, as terms referring to the symbols themselves, thus introducing the *meta-language* in which we can refer to compositions in the language. Thus, we can write: *A* = **"A"**, *B* = **"B"**, and for the conjunction of two complex terms: *AB* = "**A**"^"**B**" = "**AB**".

Complex terms are assumed to be well-defined by recursive constructions over classes of primitive terms. Any complex formulae or sentence of the language is assumed to have a fundamental form: "$\mathbf{a_1 a_2 \ldots a_n}$", represented by: $a_1 a_2 \ldots a_n$, with bracketing of terms as appropriate.[14] We will generally use $a_1 a_2 \ldots a_n$, etc, for strings of primitive terms, and *A*, *B*, etc, for general terms. We will use the intuitive and commonsense understanding of bracketing, with juxtaposition of terms representing functions and terms representing the arguments with arguments inside the brackets as normal, so that, for instance, we write: *r(t)* rather than: *r(.)t*, to represent the complex term: ***r(t)***.

The definition of *meaning for a language **L*** can then be given by specifying a function, *Meaning:*

$$Meaning(A) = \boldsymbol{A}$$

where the function *Meaning* is defined over all terms ***A*** of the language. Thus, *Meaning* is a mapping *from terms of the language (which are types of symbols)*, to *objects to which the terms of the language refer (which are things)*.

The main formal property we require the meaning function to satisfy in an adequate formalized language is called *compositionality:*

Compositionality.
The meaning of a compound expression is a function of the meanings of its parts and of the syntactic rule by which they are combined.[15]

[14] This formulation for a meta-language only applies exactly with Polish notation, where *functional application of one term to another is reflected syntactically by juxtaposition.* In ordinary physics and mathematics, we find it much clearer to use bracketing notations to indicate functional composition.

[15] This is the formulation of Theo Janssen and Barbara Partee, "Compositionality", *Handbook of Logic and Language*¸ 1997, p.462. Pavel Tichy (1978) calls this the *Frege-Church principle.* But Tichy's theory of meaning does not make meaning a function of the *syntactic rules* of combination: rather, the syntactic rules of combination themselves reflect another level of *semantics,* which Tichy calls *constructions*. This theory has been notably developed by Pavel Materna (1998), and others in the TIL project: see website reference.

We can express this through a general axiom-scheme governing *Meaning*:[16]

$$Meaning(a_1 a_2 \ldots a_n) = \mathbf{a_1 a_2 \ldots a_n}$$

Or more simply:

$$Meaning(AB) = \mathbf{AB}$$

This works because complex terms can always be completely broken down into their basic parts by successively breaking each complex term into a pair of complex or basic parts. Compositionality means that the meaning of a compound expression:

$$A = a_1 a_2 \ldots a_n$$

is determined by:

$$\begin{aligned} \mathbf{A} &= Meaning(A) \\ &= Meaning\ (a_1 a_2 \ldots a_n) \\ &= (Meaning\ a_1)(Meaning\ a_2)\ldots(\ Meaning\ a_n) \\ &= \mathbf{a_1 a_2 \ldots a_n} \end{aligned}$$

Thus we see that this give the (denotation) *Meaning* function a special kind of *distributive property* to begin with.

The Abstraction Operator.

"Where b is a variable of type β and A a formulae of type α, λbA is a formulae of type $\alpha\beta$ denoting the function which assigns to an arbitrary entity b of type β the value taken by A when b takes the value b." (Tichy, 1971, p.285).

There are a number of different theories of lamba-calculus, but the main theory referred to here was developed in the context of recursive function theory by Alonzo Church; see his 1954. The *lamba*-terms are *improper symbols,* because they operate on syntactic items (symbols or terms or formulae) rather than being simple functional terms of the language. Thus, if we have complex symbolic term, call it *A*, which contains occurrences of a term *t*, we can write a new term: $((\lambda t)A)(-t)$, call it *B*. The resulting formulae, $((\lambda t)A)(-t)$, represents a certain object: we can write: $Meaning(((\lambda t)A)(-t))$. The meaning can be obtained through a system of syntactic rules: the formulae is equivalent to the formulae *A* with all the occurrences of *t* uniformly substituted by the term $-t$. But it only operates through basic terms of *A*. For instance, let *A* be the simple term: "$\boldsymbol{t+\Delta}$" (I have now bolded the symbols themselves to distinguish from the names for the symbols). The term: "$((\lambda \boldsymbol{t})(\ \boldsymbol{t+\Delta}))(\boldsymbol{-t})$" is equivalent to: "$\boldsymbol{-t+\Delta}$". Although the $(\lambda \boldsymbol{t})$ acts syntactically, "$(\lambda \boldsymbol{t})(\ \boldsymbol{t+\Delta})$" itself directly represents a function. Thus, when we write: $(\lambda \boldsymbol{t})(\boldsymbol{t+\Delta})$, this has a meaning: it constructs a function which takes moments of time to a new moment. But the principle of compositionality as usually defined does apply directly to the abstraction operator (for denotational meanings at least). I.e. $Meaning(\lambda bA)$ is not equal to:

[16] On the Tichy or Materna theory, the interpretation of meaning offered here is only a *denotational theory,* and the principle of compositionality here is only applicable to denotations, not to full meanings. However, denotation for object languages of theoretical physics is all that we need to consider at this stage.

Meaning(λ)*Meaning*(*b*)*Meaning*(*A*). Instead, *Meaning*(λ*bA*) is obtained by operating on symbolic constructions.

General Transformations.

A general transformation is defined here as a transformation on all the objects referred to in the meaning function for the language, which is generated from an automorphism of basis sets back onto themselves.

An automorphism is a 1-1, invertible, onto, mapping from a class of objects back to itself. Thus for instance the automorphism that *generates the time reversal transformation, **T***, is the mapping: ***T****: t* → *-t*. The general transformation generated by this is obtained (i) taking any object, **A**, (ii) identifying its fundamental *construction* from the elements basis sets, and (iii) applying the mapping ***T*** on the set of times throughout the construction of **A.**

The primary example is the transformation on *worlds*. If we define a world as a class of fundamental atomic facts (reverting to our normal symbolism, rather than bolding all terms): $W = \{(i,\mathbf{r},m,t)\}$, then the the automorphism: ***T****: t*→*-t* may be said to *induce the transformation:* $TW = \{(i,\mathbf{r},m,Tt): (i,\mathbf{r},m,t)\} \varepsilon\, W\}$ on the world *W*.

Other examples of general transformations from physics are:

- The space-reversal transformation, based on: ***P*: r → -r**
- The charge-reversal transformation, based on: ***C:*** *q* → *-q*
- The time translation transformation, based on: ***+T:*** $t \to t+\Delta t$
- The space translation transformation, based on: ***+R*:** $\mathbf{r} \to \mathbf{r}+\Delta\mathbf{r}$
- General Galilean transformations, based on: ***G*: r →*G*(r)**, where ***G*** corresponds to any combination of Galilean space translations, space rotations, and velocity boosts
- General Lorentz transformations, based on: ***L****:* $(\mathbf{r},t) \to \mathbf{L}(\mathbf{r},t)$, where ***L*** corresponds to any combination of space translations, space rotations, and Lorentz velocity boosts

An example of a general transformation from logic is:

- The truth-reversal transformation, based on: ~*: True* → *False*

Appendix 2: Sketch of a more general proof of distributive *T-Operator* in a compositional language.

I will now rapidly sketch how the concepts of general transformations and compositionality combine to generate the result that there must be a general (typed) *distributive syntactic transformation operator*. We want to obtain a lemma:

- *Lemma:* In a *compositional* language, a general (typed) transformation, ***T***, corresponding to an automorphism (or permutation) of the *basic sets* of the ontology back onto themselves, can be represented by a unique (typed) transformation operator-family, *T*, where *T* is also compositional.

- There is a unique operator, $\boldsymbol{T_i}$ **,** for each type of object: $\mathbf{A_i} = Meaning(A_i)$ where A_i is any term of the language; such that:

$$\boldsymbol{T_i}\mathbf{A_i} = Meaning(T_iA_i) = Meaning(T_i)\ Meaning(A_i)$$

For simplicity, we just write ***T*** and *T*, because their types are usually obvious from their arguments. This means that the effect of the general transformation ***T*** can be represented in the language by a (typed hierarchy of) operator term(s), *T*, and the language with *T* is still compositional.

The distributive syntactic property can be seen to reflect a simple feature of homomorphisms, viz. there is a homomorphic mapping from the class of terms to the class of objects, which is the compositional meaning function; the general transformation is by definition an automorphism or isomorphism from the base sets onto themselves; this can be used to induce an isomorphic image of the original language; by definition, the image of the object **A** is the object ***T*A**. This is represented in the image language by the term *T*A.

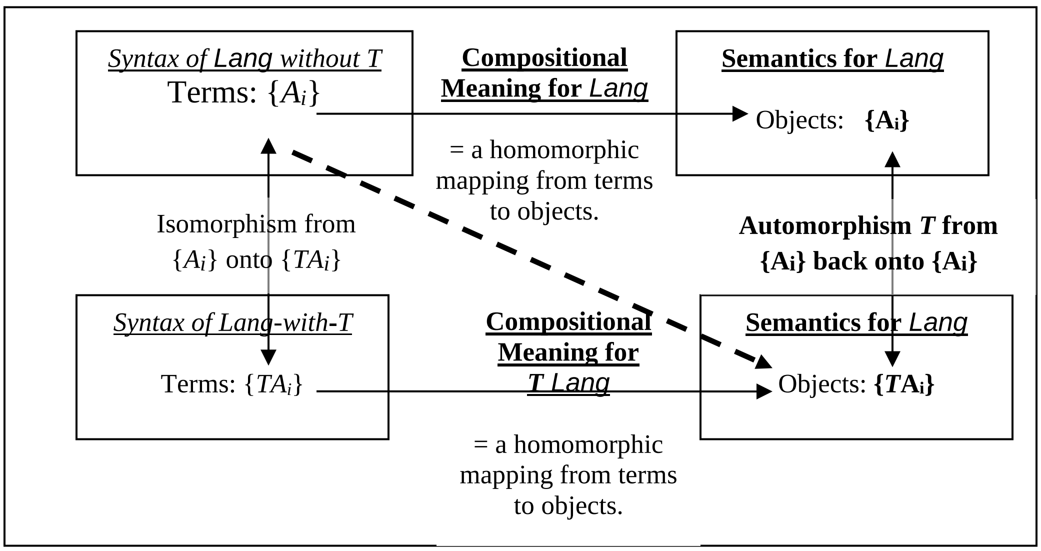

Figure 1. The line – – – – ► could represent an alternative meaning function, which is how coordinate transformations in physics are often introduced.

Let us start with a compositional language, *Lang*, without any *T* terms for transformation operators. Then we must be able to add a class of terms: for every term *A* of the original *Lang*, we just add a new term *TA*. The new, extended language is interpreted by taking the obvious interpretation: if *Meaning(A)* = ***A*,** then: *Meaning(TA)* = ***TA***, where ***TA*** is the transformed object ***A***, obtained from the same construction for ***A***, but substituting all base objects for their ***T****-images* in the construction. This is what leads to compositionality of the extended language with the *TA* terms: the new term *T* distributes through the complex terms *A* because of compositionality of the original *Lang*.

Note that the extended language we form has complex *terms* of the form: *TP* expressing propositions which are images of the corresponding propositions *P* expressed in the language $\{A_i\}$. If *P* = *AB*, then: *TP* = *T(AB)*. We also have the construction: (*TA)(TB)*. The isomorphism guarantees that: (*TA)(TB)* = *T(AB)*. Hence distributivity of *T* at this level.

T is a *distributive semantic operator*, distributing through complex functional constructions of objects; likewise, *T* is a distributive syntactic operator, distributing through complex concatenations of terms. Thus, for a general transformation, $\boldsymbol{T}$, there is a corresponding term T such that:

$$
\begin{aligned}
\boldsymbol{T}(\boldsymbol{BC}) &= \boldsymbol{T}(Meaning(BC)) && \textit{Definition of symbols } B, C, \boldsymbol{B}, \boldsymbol{C}. \\
&= Meaning(T(BC)) && \textit{Assumption that } \boldsymbol{T} \textit{ can be represented by } T \\
&= Meaning((TB)(TC)) && \textit{Assumption that } T \textit{ is distributive} \\
&= Meaning(TB)Meaning(TC)) && \textit{Compositionality} \\
&= \boldsymbol{TBTC} && \textit{Definition of symbols } T, B, C, \boldsymbol{T}, \boldsymbol{B}, \boldsymbol{C}.
\end{aligned}
$$

Rules for the *syntactic* operation of T on terms of the language are defined inductively, in parallel with the semantic operations, from:

(i) basic transformations on the basic terms; e.g. for time reversal:

$$Tt \rightarrow -t;\ TX \rightarrow X;\ etc.$$

(ii) the syntactic distributivity of T across complex terms;

$$T(AB) = (TA)(TB)$$

E.g. for time reversal, if $\boldsymbol{v}_t$ is defined by: $\boldsymbol{v}_t = d/dt(\boldsymbol{r}(t))$, then we can syntactically transform (using complex transformations we have seen earlier):

$$Tv_t \rightarrow T(d/dt(r(t))) \rightarrow Td/dt(Tr(Tt)) \rightarrow d/dt(Tr(-t)) \rightarrow -v_t$$

The inductive step. This is just the first step: we also need to add a second set of terms: $T(TA)$, so we can *reiterate* T generally on terms. This proceeds in a similar way, but there is now a subtlety to obtain distributivity of T, as follows.

We must have for general distributivity of T that a double application of T to a term A: $T(TA) \rightarrow (TT)(TA)$. Since the domain of TA is the same as the domain of A, we thus have that: $T(A) \rightarrow (TT)(A)$, for any A. Hence we require the syntactic transformation rule:

(iii) the syntactic transformation of T to itself;

$$TT \rightarrow T$$

This is a general feature of T. It is self-consistent for reiterated applications, like: $T(T(T(A)))$. We get: $T(T(T(A))) \rightarrow TT(TT(TA))$, and the rule that: $TT \rightarrow T$ is valid. It is obvious that this inductively generalizes to give distributivity for any number of iterations of T.

Note that this is quite distinct from the double application: $T(TA)$. There is a special rule that: $T(T(A)) \rightarrow A$ for transformations where: $\boldsymbol{T} = \boldsymbol{T}^{-1}$. This is the rule that "double-time-reversal operations cancel". The rule (iii) however holds for all general transformations.

For self-consistency, we see from compositionality applied to the extended language, with T, that:

$$
\begin{aligned}
\boldsymbol{T}(\boldsymbol{BC}) &= meaning(T(BC)) = meaning(T)meaning(BC) \\
&= meaning(T)\boldsymbol{BC}
\end{aligned}
$$

Or:

$$\boldsymbol{T} = meaning(T)$$

This is a consistency condition that we need for compositionality in the fully extended language. When we apply T to itself, in: $TT \rightarrow T$, we then require that: $Meaning(TT) = Meaning(T)Meaning(T) = \boldsymbol{TT}$. To deal with this properly, we need a proper hierarchical type theoretic treatment, where we recognize a hierarchy of different types of $\boldsymbol{T}$. I will not comment on this here, except that to observe that it remains consistent with the meaning of $\boldsymbol{T}$ as a transformation. If we define $\boldsymbol{T_\alpha}$ as the transformation over a specific class of objects, then: $\boldsymbol{T_\alpha : A \rightarrow B = T_\alpha A}$. Then we define a higher-order: $\boldsymbol{T_\beta : T_\alpha \rightarrow T_\gamma}$, for some $\boldsymbol{T_\gamma}$. But since: $\boldsymbol{T_\alpha : A \rightarrow B}$, then by definition: $\boldsymbol{T_\beta(T_\alpha) = T_\gamma : T_\alpha A \rightarrow T_\alpha B = T_\alpha(T_\alpha A)}$. But: $\boldsymbol{T_\alpha : T_\alpha A \rightarrow T_\alpha(T_\alpha A)}$ already. Hence: $\boldsymbol{T_\alpha = T_\gamma}$, i.e.: $\boldsymbol{T_\beta(T_\alpha) = T_\alpha}$.

I conclude that, if the object language for a theory of physics is *compositional*, then we *must be able to* represent a general transformation by general *distributive semantic operator term, T*, corresponding to the transformation $\boldsymbol{T}$, and retain compositionality. The fact that there is no such distributive operator for an extensional language shows that the language is in fact not compositional, and is logically inadequate.

Appendix 3. Possible worlds and the actual world.

The interpretation of the simple term ‘the actual world’ has caused much controversy in natural language semantics and the related discussions of metaphysics underlying empirical languages. However, I am not proposing a natural language theory: I am only proposing a logical interpretation of the *theoretical formalism* of typical theories of physics. It is only when we go on to interpret this *empirically* that we can try to identify the notion of @ as ‘the real actual world’. The view I have adopted here is that: (i) we take @ a constant referring to one possible theoretical world, in the *theoretical ontology*; but: (ii) we do not actually identify @ as any specific world in the theory: we just assume the term is an unknown constant; (iii) @ in the abstract theory does not refer to the ‘empirically real actual world’ at all until the theory is interpreted empirically; and (iv) if the theoretical framework or logical space for the theory itself is wrong, so that *the empirical actual world is not like any world in the theoretical ontology*, then the interpreted theoretical term ‘@’ cannot denote the real actual world at all. (v) If the theoretical framework is compatible with the logical structure of the real world (even if it merely a partial theory), then @ in the theory is interpreted to be ‘the empirical actual world’; but: (v) the stronger contingent propositions of the complete theory may not be true of the real actual world, and thus @ in the theory will lie outside the laws of the contingent laws of the theory.

I will add some comments about the interpretation of natural empirical language. Here, I think we can introduce ‘the actual world’ as a primitive office: we evaluate propositions *at worlds*, but *the actual world* simply maps to a single, specific world. We cannot know what this world is, but we do know some things about it, so we must know its concept. But it is a primitive concept: we cannot define it in any more primitive way.

It is a simple metaphysical thesis that there is only one actual world; and it might be wrong (as the ‘many worlds’ interpretation of quantum theory proposes). We obtain this knowledge from our experience that we live in only one world; we generalise that there are no more actual worlds, because we do not appear to need any more actual worlds to explain all our actual knowledge.

The main alternative view is that ‘the actual world’ taken *at a world W* should be identified with *W*. My argument against this is as follows.

Suppose there is a possible world, *W*, which is *not the actual world*, but is similar. (Remembering that the whole point of allowing possible worlds is so we can talk about possibilities which are *not* actual). Let *P* be a proposition true of *W*, but false of the actual world. Suppose that someone in *W* states that the proposition *P* is actually true, i.e. true of the actual world. Then they are wrong: *P* is not true of the actual world – even though it is true of the world *W* in which the person makes this statement. (A person stating the same proposition in the actual world would clearly be stating something false).

The point is that *the actual world* should not be identified with the world '*within which*' propositions are stated at all. It is a mistake to index the truth of propositions to worlds 'within which' they are stated. Instead: there is an objective class of truths about the actual world. Propositions may be 'stated within possible but non-actual worlds', by 'possible but non-actual people' but the worlds they are 'stated within' is irrelevant to their truth. Otherwise we have to give up the conception that there are 'possible but non-actual worlds' altogether.

The only solution I can see out of this is that we regard the actual world as an office which is satisfied by just one world. *It doesn't vary according to 'where' a proposition is 'stated' or 'evaluated'.* Propositions may be said to be 'evaluated within worlds': but their truth is not determined by the worlds within which they are evaluated. Rather, their truth is evaluated *at worlds*. They are not *actually* true or false according to the worlds 'in which they are stated', or 'in which they are evaluated': they are true or false according to the class of truths that specifies the objective actual world.

The concept of evaluating propositions *'within worlds',* so that 'within a world W, W is regarded as the actual world', leads either to the 'indexical' theories of meaning, where meaning is relative to 'tokens', or to the Lewis view that all worlds are equally real. The objective theory of truth leads to the view that *the actual world is objective,* and *the actual world is not relativised to any kind of evaluation of truth within worlds.*

To reflect this in the theoretical interpretation, @ is not world-dependant: it is a constant. The office of 'the actual world' maps every world to a single world, @.
(Or rather: @ would be a constant if its content didn't change with time.)

The fact that *we don't know what particular world the actual world is* doesn't seem to me to be of any relevance. We take it as a primitive concept and primitive metaphysical intuition that *there is a single and unique actual world.* We can hardly even justify that we do 'know' this. It is just an assumption. But once we make this assumption, there is no way of understanding it in an objective theory of truth except by taking 'the actual world' as an office which specifies a constant world.

A second view, earlier proposed and then rejected by Pavel Tichy, is to take @ as the mapping from worlds to themselves: $@ = (\lambda W)(W)$ (see Tichy, 1975; 1988). This might be taken to represent the idea that the term 'the actual world' is evaluated from 'within' worlds, and from 'within' any world, *W*, the actual world is naturally *evaluated as W* (although this is hardly Tichy's intention). For instance, suppose that we take a world *W* which is a little different to 'our actual world', in which the proposition: "*L(@)*" is evaluated, and suppose that (i) *L(.)* is true of *W*, but (ii) *L(.)* is false of *'our actual world'.* We may then distinguish two different evaluations: (i) *L(@) evaluated within W is true at W*, but (ii) *L(@) evaluated within "our actual world" is false at W.* The immediate problem this seems to raise is that the term @ appears to become infinitely regressive, in the sense that: $L(@) = L((\lambda W)(W)) = L(.)$, and so *L(@)* does not actually specify any truth-value – until it is *evaluated within some world*. Tichy subsequently rejected this idea, and proposed a further theory, which I do not adequately understand.

A third idea is to take @ as an ‘indexical’, so that its meaning is directly ‘indexed’ to *the world in which it is stated.* This is undoubtedly the most popular idea. Thus the meaning of an occurrence of the symbol ‘@’ is said to be ‘*indexed to its own token statement’.* I.e. @ is the world in which the term ‘@’ itself is made. However, despite its superficial appeal, this is not a plausible interpretation, as Tichy has shown: see his 1988. For a start it implies that *the meaning of the term @ alters from token statement to token statement* in a vicious way – and anyway, proponents of this view never seem to formally explicate the notion of ‘indexing to token statements’, they just introduce this notion intuitively as if it makes sense. Tichy’s own earlier theory that: @ = $(\lambda W)(W)$ is much better, but suffers from the defects described above.

A fourth view is the idea championed by David Lewis, usually called ‘modal realism’, which holds that *all possible worlds are (actually) real,* so that *there is no unique actual world.* The reader will have to make up their own mind about this as a metaphysical conjecture: but I think it is safe to say that it is not how the concept of the ‘actual world’ is intended to be understood in ordinary physics, which is what we are analyzing here.

Appendix 4. Statements from *Part 1.*

(1) $\boldsymbol{F}_i = m_i\boldsymbol{a}_i(t) \equiv m_i d^2\boldsymbol{r}_i(t)/dt^2$

(2) $\boldsymbol{F}_{ij} = -\mathrm{G}m_i m_j \boldsymbol{r}_{ij}/|\boldsymbol{r}_{ij}|^3$

(3) $\boldsymbol{F}_i = \Sigma_{j\neq i}\,\boldsymbol{F}_{ij}$

(4) $m_i d^2\boldsymbol{r}_i(t)/dt^2 = \Sigma_{\mathrm{j}\neq\mathrm{i}}\, -\mathrm{G}m_i m_j \boldsymbol{r}_{ij}/|\boldsymbol{r}_{ij}|^3$

(4) $(\forall i,t)[m(i,t)d^2\mathbf{r}(i,t)/dt^2 = \Sigma_{\mathrm{j}\neq\mathrm{i}}\, -\mathrm{G}m(i,t)m(j,t)(\mathbf{r}(i,t)\text{-}\mathbf{r}(j,t))/|\mathbf{r}(i,t)\text{-}\mathbf{r}(j,t)|^3]$

(5) $\mathbf{v}(i,t) =_{df} d\mathbf{r}(i,t)/dt$

(6) $(\forall i,t)[m(i,t)d\mathbf{v}(i,t)/dt = \boldsymbol{\Sigma}_{\mathrm{j}\neq\mathrm{i}}\, -\mathrm{G}m(i,t)m(j,t)(\mathbf{r}(i,t)\text{-}\mathbf{r}(j,t))/|\mathbf{r}(i,t)\text{-}\mathbf{r}(j,t)|^3]$

(7) $(\forall i,t)[m(i,t)d^2\mathbf{r}(i,t)/dt^2 = \boldsymbol{\Sigma}_{\mathrm{j}\neq\mathrm{i}}\, -\mathrm{G}m(i,t)m(j,t)(T\mathbf{r}(i,t)\text{-}T\mathbf{r}(j,t))/|T\mathbf{r}(i,t)\text{-}T\mathbf{r}(j,t)|^3]$

Acknowledgements.

Part of this paper was presented to the *Australasian Association of Logic and Linguistics Conference* at Victoria University, Wellington, December, 2001. Grateful thanks go to Professor Pavel Materna and Dr. Marie Duzi for many helpful comments. All remaining errors are the fault of the author.

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