Aristotle’s intermediates and Xenocrates’ mathematicals

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In **Revue de philosophie ancienne** **Issue 1, January 2022**, pages 79 to 112

ISSN 0771-5420
DOI 10.3917/rpha.401.0079

ARISTOTLE'S INTERMEDIATES AND XENOCRATES' MATHEMATICALS

ABSTRACT

This paper investigates the identity and function of τὰ μεταξύ in Aristotle and the Early Academy by focussing primarily on Aristotle's criticisms of Xenocrates of Chalcedon, the third scholarch of Plato's Academy and Aristotle's direct competitor. It argues that a number of passages in Aristotle's *Metaphysics* (at B 2, M 1-2, and K 12) are chiefly directed at Xenocrates as a proponent of theories of mathematical intermediates, despite the fact that Aristotle does not mention Xenocrates there. Aristotle complains that the advocates for mathematical intermediates produce theories that are ontologically and epistemologically inefficient (related to, but not confined by, the "Uniqueness Problem"); that their so-called "intermediates" feature properties opposed to those of the forms on which those intermediates are thought to depend; and that what is μεταξύ must be between objects of a different genus. In all three cases, Aristotle's criticisms are shown to be reactions to the metaphysics and cosmology of Xenocrates, especially his doctrine of the intermediary demonic isosceles triangle souls. Xenocrates emerges as a prime candidate for Aristotle's critique of Platonist τὰ μεταξύ theories – equal to, and possibly even exceeding, Plato as target.

RÉSUMÉ

Dans cet article, j'examine l'identité et la fonction des "entités intermédiaires" (τὰ μεταξύ) chez Aristote et dans l'Ancienne Académie, en me concentrant surtout sur la critique aristotélicienne de Xénocrate de Chalcédoine, troisième scholarque de l'Académie de Platon et concurrent direct d'Aristote. Je soutiens qu'un certain nombre de passages de la *Métaphysique* d'Aristote (en B 2, M 1-2, et K 12) ont pour cible Xénocrate comme partisan de théories des Intermédiaires mathématiques, en dépit du fait qu'il n'y est pas mentionné. Aristote reproche aux partisans des Intermédiaires d'avancer des théories qui sont inefficaces d'un point de vue ontologique et épistémique (en relation avec le "Problème de l'Unicité", mais pas seulement), d'attribuer à leurs "intermédiaires" des caractéristiques opposées à celles des Formes dont ils sont censés dépendre, et de n'avoir pas vu que ce qui est μεταξύ doit l'être entre des objets de genre différent. Je montre que, sur ces trois points, les critiques d'Aristote sont des réactions à la métaphysique et à la cosmologie de Xénocrate, et notamment à sa doctrine des âmes-démons intermédiaires ayant la forme de triangles isocèles. Ainsi, Xénocrate s'avère être un excellent candidat pour le rôle de cible de la critique aristotélicienne des théories platoniciennes des Intermédiaires – au même titre que, et peut être plus que, Platon.

Introduction[1]

The topic of this paper, namely the identity and function of τὰ μεταξύ in Aristotle and the Early Academy, might be usefully described as a mid-century modern problem. It became a central concern among scholars following the influential studies of Harold Cherniss, which took the field of ancient philosophy by storm especially after the second World War. Cherniss had altered the study of ancient philosophy by subjecting Aristotle's critiques of Presocratic and Academic philosophy to stringent evaluations, revealing the deep bias in Aristot-

[1] This paper was presented remotely at the conference "τὰ μεταξύ - Les Intermédiaires Mathématiques", Archives Henri-Poincaré, Université de Lorraine, and in a hybrid format at the "New Directions in Platonism" Conference, Centro Italiano di Studi sul Platonismo, Università di Torino, in 2021. I want to thank the organizers of these conferences, Lorenzo Corti and Federico Petrucci, for their invitations, as well as the audiences for their suggestions for improvement (especially Ben Morison and Jan Opsomer). Lorenzo Corti and Alain Lernould also helpfully pointed out some oversights on my part, for which I thank them. All translations from the Greek are mine, unless otherwise indicated.

le's accounts[2]. In Europe, scholars such as Anders Wedberg (1955) and Konrad Gaiser (1962) found that there could be an accommodation, of sorts, between Aristotle's accounts of τὰ μεταξύ and Plato's metaphysics, and that Aristotle had special insight into what Plato thought about mathematical objects, which he had so carefully left out of his writings that deal with mathematics (especially *Republic*, *Timaeus*, and *Philebus*). Importantly, the assumption on the part of Wedberg and Gaiser was that when Aristotle referred to τὰ μεταξύ, he was chiefly thinking of Plato, and not of the members of the Academy subsequent to him. Hence, the writings of Plato were to be used to inform Aristotle's otherwise sketchy or oblique accounts of τὰ μεταξύ; and where any gap remained, it could be filled by, on the one hand, 20th-century mathematical theory (Wedberg) or the "unwritten doctrines" of Plato (Gaiser)[3]. By the time Julia Annas's 1975 article on the mathematical intermediates and 1976 translation and commentary on *Metaphysics* M and N had arrived, the question concerning the mathematical intermediates was already sounding a bit old[4], and while she took Gaiser in

[2] H.F. Cherniss, *The Riddle of the Early Academy*, Berkeley, Los Angeles, University of California Press, 1945; H.F. Cherniss, *Aristotle's Criticism of Plato and the Academy*, Baltimore, The Johns Hopkins Press, 1944. He treats intermediate mathematicals at H.F. Cherniss, *Aristotle's Criticism of Plato and the Academy*, *op. cit.*, p. 180-182, and somewhat aporetically at H.F. Cherniss, *The Riddle of the Early Academy*, *op. cit.*, p. 76-77; he additionally denies the identification by later ancient commentators of the intermediate mathematicals and the soul in Appendix IX (H.F. Cherniss, *Aristotle's Criticism of Plato and the Academy*, *op.cit.*, p. 565-580).

[3] A. Wedberg, *Plato's Philosophy of Mathematics*, Stockholm, Almquist & Wiksell, 1955, p. 80-91; K. Gaiser, "Quellenkritische Probleme der indirekten Platonüberlieferung", *Idee und Zahl*. Studien zur platonischen Philosophie. Abhandlungen der Heidelberger Akadeime der Wissenschaften, *Phil.-hist. Kl.*, 2, p. 31-84, repr. in K. Gaiser, *Gesammelte Schriften*, ed. by T.A. Szlezák with K.-H. Stanzel, Sankt Augustin, Academia Verlag, 2004, p. 205-264.

[4] Cf. Annas's (J. Annas, "On the 'Intermediates'", *Archiv für Geschichte der Philosophie* 57, 1975, p. 146-166: p. 146) comment, concerning the status of intermediates in Plato's philosophy: "It may seem unrewarding to ask this question again".

particular to task for denying the plausibility of Xenocrates as the target of Aristotle's attack, her notes on the topic reveal a somewhat desperate appeal to move beyond the debate[5]. Indeed, in 1985, Richard Mohr would go so far as to refer to the pursuit of τὰ μεταξύ in Plato as like a "parlor game" among scholars[6]. Studies of τὰ μεταξύ in Aristotle, and their relationship to Plato and the Platonists, would appear to have reached their acumen.

But this is not the end of the story; for in Italy Margherita Isnardi Parente was pursuing a new agenda, with her editions of Speusippus (1980) and Xenocrates (1982), to fill in the gaps where the ascription to Plato would be question-begging; her studies, however, were furthermore almost totally ignored by influential Anglo-American discussions of Aristotle's critique of Plato and mathematics in the 1990s, such as those of Gail Fine and Edward Hussey[7]. In Lille, Michel Crubellier produced a monumental commentary on Aristotle's *Metaphysics* M and N as a doctoral thesis (1994), but this was never published in a revised or updated version. In the new millennium, John Dillon set out to present a systematic account of the Old Academy (2003), with more careful attention to Isnardi Parente's contributions; but he neglected to discuss τὰ μεταξύ in any

[5] J. Annas, *Aristotle's Metaphysics. Books M and N*, Oxford, Clarendon Press, 1976, p. 209-212.

[6] R.D. Mohr, *God and Forms in Plato*, Leiden, E.J. Brill, 1985, repr. in R.D. Mohr, *God and Forms in Plato. And Other Essays in Plato's Metaphysics*, Las Vegas, Parmenides Publishing, 2005, p. xxv.

[7] Fine (G. Fine, *On Ideas. Aristotle's Criticism of Plato's Theory of Forms*, Oxford, Clarendon Press, 1993, p. 290) refers to Isnardi Parente's article on Aristotle's *On Ideas* (M. Isnardi Parente, "Le *Peri Ideôn* d'Aristote: Platon ou Xénocrate?", *Phronesis* 26, 1981, p. 135-152) but does not seriously consider her argument that Xenocrates is the target of Aristotle's criticisms (and she does not cite Isnardi Parente's editions of Speusippus or Xenocrates at all). Hussey (E. Hussey, "Aristotle on Mathematical Objects", *Apeiron* 24.4, *ΠΕΡΙ ΤΩΝ ΜΑΘΗΜΑΤΩΝ: Peri Tōn Mathēmatōn*, 1991, p. 105-133) does not refer to Isnardi Parente, or to Speusippus, Xenocrates, or any other individual Platonist.

depth[8]. In 2010, Thomas Bénatouïl and Dimitri El Murr published a comprehensive and systematic account of geometry in the Academy, from Plato through to Carneades. While their study provided range and depth in the study of mathematics in the Academy, it did not analyse Aristotle's discussion of τὰ μεταξύ specifically[9]. A 2018 volume of *Plato Journal* featured several articles devoted to the intermediates in Plato, of which the most important for our study is Emily Katz's "The Mixed Mathematical Intermediates"; but while Katz's charitable analysis of Aristotle's criticisms of the Platonists' metaphysical views is valuable, she curiously makes no attempt to differentiate the various Platonists views from one another, or from the view of Plato.[10] Her concern is to salvage Aristotle's argument from the criticisms levelled by, among others, Annas. Finally, in her 2020 doctoral thesis, Giulia De Cesaris steered clear of τὰ μεταξύ, opting to focus on Aristotle's critique of Xenocratean mathematicals as possessing the same "one nature" as other superior objects in the Platonist's ontology and epistemology[11].

In a way, then, the study of Plato, Xenocrates, and τὰ μεταξύ remains where it was in 1982, when Isnardi Parente's new edition was percolating throughout Europe (excepting the UK), and Annas's trans-

[8] J. Dillon, *The Heirs of Plato. A Study of the Old Academy (347 – 274 BC)*, Oxford, Oxford University Press, 2003, p. 110-112.

[9] Bénatouïl and El Murr (T. Bénatouil, and D. El Murr, "L'Académie et les géomètres: usages et limites de la géométrie de Platon à Carnéade", *Philosophie antique* 10, 2010, p. 41-80) discuss the intermediary status of astronomy, but do not link it to Aristotle's criticisms of the τὰ μεταξύ theorists.

[10] Katz states (E. Katz, "The Mixed Mathematical Intermediates", *Plato Journal* 18, 2018, p. 83-96: p. 93 n. 2): "By 'Platonists' I mean the opponents Aristotle is targeting here [in M.2], namely those who posit forms and intermediates. I consider Aristotle's reasoning independently of the question whether Plato himself posits intermediates". She does not mention Speusippus or Xenocrates, or any other individual members of Plato's Academy, in her article.

[11] G. De Cesaris, *Aristotle's account of Speusippus' and Xenocrates' Metaphysical and Epistemological Theories*, PhD thesis, Durham University, http://etheses.dur.ac.uk/13441/, 2020, Chapter 6.

lation and commentary on M and N was becoming canonical throughout the Anglo-American scholarly world. Forty years is a long enough period of time for a problem to have settled and returned in the history of ancient philosophy. In this paper, I will investigate the problem of Aristotle's characterization of τὰ μεταξύ as contributing to a distinctive Academic theory of being by considering the extent to which Aristotle's comments on it relate to the testimonies regarding Xenocrates of Chalcedon, the third scholarch of the Academy in Athens. In order to do so, I will first start by discussing a paradigmatic example of how Aristotle's presentation of mathematical intermediates presents challenges to the historian of philosophy who would seek to identify the target of his attack, a passage found in *Metaphysics* N 3. Second, I will attempt to hone in on the specific concerns that the theory of mathematical intermediates described by τὰ μεταξύ presents to Aristotle, in consideration of his own philosophical tenets. Third, I will turn to the testimonies of Xenocrates, the second successor to Plato as scholarch of the Academy, in order to show important but understudied correlations between, in particular, Aristotle's criticisms and Xenocrates' theory of geometricals. The goal will be to see how Aristotle's discussion of τὰ μεταξύ informs later accounts of Xenocratean metaphysics. I should like to point out that, due to space constraints, I will not be focussing on indivisible lines in the mathematical ontology of Xenocrates, although there is every reason to think that, if Aristotle was referring to Xenocrates when he attacked theories of τὰ μεταξύ, his criticisms would extend to the latter's indivisible lines as well.

I

As Julia Annas notes, a focal point in the debate concerning Plato, Xenocrates, and τὰ μεταξύ is *Metaphysics* N 3, where it appears that Aristotle wishes to distinguish two figures who advance questionable theories of intermediary mathematical objects[12]. These two figures are

[12] J. Annas, *Aristotle's Metaphysics. Books M and N*, *op. cit.*, p. 210.

known as "those who posit the ideas", or the ideas-positors, and "those who first produced two types of number". I provide the Greek text and translation, with the sections in bold that are included in Isnardi Parente's edition[13] as referring to Xenocrates himself:

> Aristotle, *Metaphysics* N 3, 1090b20-1091a5 = Xenocrates fr. 38 Isnardi Parente² (in bold)
>
> τοῖς δὲ τὰς ἰδέας τιθεμένοις τοῦτο μὲν ἐκφεύγει – **ποιοῦσι γὰρ τὰ μεγέθη ἐκ τῆς ὕλης καὶ ἀριθμοῦ, ἐκ μὲν τῆς δυάδος τὰ μήκη, ἐκ τριάδος δ᾽ ἴσως τὰ ἐπίπεδα, ἐκ δὲ τῆς τετράδος τὰ στερεὰ ἢ καὶ ἐξ ἄλλων ἀριθμῶν·** διαφέρει γὰρ οὐθέν –, ἀλλὰ ταῦτά [25] γε πότερον ἰδέαι ἔσονται, ἢ τίς ὁ τρόπος αὐτῶν, καὶ τί συμβάλλονται τοῖς οὖσιν; οὐθὲν γάρ, ὥσπερ οὐδὲ τὰ μαθηματικά, οὐδὲ ταῦτα συμβάλλεται. ἀλλὰ μὴν οὐδ᾽ ὑπάρχει γε κατ᾽ αὐτῶν οὐθὲν θεώρημα, ἐὰν μή τις βούληται κινεῖν τὰ μαθηματικὰ καὶ ποιεῖν ἰδίας τινὰς δόξας. ἔστι δ᾽ οὐ χαλεπὸν [30] ὁποιασοῦν ὑποθέσεις λαμβάνοντας μακροποιεῖν καὶ συνείρειν. **οὗτοι μὲν οὖν ταύτῃ προσγλιχόμενοι ταῖς ἰδέαις τὰ μαθηματικὰ διαμαρτάνουσιν·** οἱ δὲ πρῶτοι δύο τοὺς ἀριθμοὺς ποιήσαντες, τόν τε τῶν εἰδῶν καὶ τὸν μαθηματικόν, οὔτ᾽ εἰρήκασιν οὔτ᾽ ἔχοιεν ἂν εἰπεῖν πῶς καὶ ἐκ τίνος ἔσται ὁ [35] μαθηματικός. ποιοῦσι γὰρ αὐτὸν μεταξὺ τοῦ εἰδητικοῦ καὶ τοῦ αἰσθητοῦ. εἰ μὲν γὰρ ἐκ τοῦ μεγάλου καὶ μικροῦ, ὁ αὐτὸς ἐκείνῳ ἔσται τῷ τῶν ἰδεῶν (ἐξ ἄλλου δὲ τίνος μικροῦ καὶ μεγάλου; τὰ γὰρ† μεγέθη ποιεῖ.) [1091a] [1] εἰ δ᾽ ἕτερόν τι ἐρεῖ, πλείω τὰ στοιχεῖα ἐρεῖ· καὶ εἰ ἕν τι ἑκατέρου ἡ ἀρχή, κοινόν τι ἐπὶ τούτων ἔσται τὸ ἕν, ζητητέον τε πῶς καὶ ταῦτα πολλὰ τὸ ἓν καὶ ἅμα τὸν ἀριθμὸν γενέσθαι ἄλλως ἢ ἐξ [5] ἑνὸς καὶ δυάδος ἀορίστου ἀδύνατον κατ᾽ ἐκεῖνον.
>
> Those who posit the ideas escape this difficulty; **for they produce magnitudes out of matter and number, lines out of the dyad, and planes surely out of a triad, solids out of the tetrad, or out of other numbers** – it makes no difference. But [25] are these [magnitudes] to be ideas, or what is their mode [of existence]? And what do they

[13] M. Isnardi Parente, *Senocrate e Ermodoro. Testimonianze e frammenti*. Revised edition, éd. T. Dorandi, Napoli, Edizioni della Normale, 2012.

> contribute to reality? Nothing, actually – just as the objects of mathematics contribute nothing, so too these. No, not even any theorem is true in their case, unless someone intends to make the objects of mathematics move or make up certain doctrines of one's own. It is not difficult [30] for them to assume hypotheses of any sort and to string out long-winded stories. **Hence, these thinkers err in this way when they collapse the objects of mathematics into the ideas**.
>
> But those who were the first to produce two types of number – form-number and mathematical number – neither said nor are able to explain how and out of what mathematical number [35] is to exist. For they make it intermediate between form-number and sensible number. Indeed, if it [is to exist] out of the great and the small, it will be the same as the former, form-number. (But from what other great and small? For he produces magnitudes [?].) [1091a1] But if he will call upon another thing, he will be calling upon [too] many elements. And if the first principle is to be a certain 1, unity [lit. "the One"] will be something shared in their case, and we will have to investigate both how the One becomes these many [elements], and [how] at the same time number cannot be generated otherwise than [5] from a 1 and an indefinite 2 – according to him.

I will suspend judgment for the moment about whether the bold sections do indeed refer to the view of Xenocrates. As Annas points out, the context is crucial for determining who is advancing what ontological theory here[14]. Prior to this passage, Aristotle has just attacked Speusippus for believing that only mathematical objects, and not forms, exist; the consequence of this theory, according to Aristotle, is an episodic universe, which Aristotle rejects[15]. Hence,

[14] J. Annas, *Aristotle's Metaphysics. Books M and N*, *op. cit.*, p. 209-210. This is despite the fact that, in her article on the intermediates, she never actually mentions Xenocrates. Indeed, when she concludes "it appears that Aristotle is not talking about the intermediates as these figure add 's', i.e., 'figures' in Plato's dialogues" (J. Annas, "On the 'Intermediates'", art. cit., p. 164), it does not occur to her to consider another Platonist as the target of Aristotle's critique.

[15] For a charitable reading of Speusippus' episodicity, now see G. De Cesa-

when he mentions "those who posit the ideas", Aristotle is contrasting these figures with Speusippus, who did not accept the existence of separable transcendent forms/ideas[16]. One might think, then, that "those who posit the ideas" refers to Plato – and indeed, this is what Gaiser thought[17]. But this argument runs into some challenges as well, since formally the text shows that "those who posit the ideas" are contrasted with "those who were the first to produce two types of number, form-number and mathematical number", for two reasons: first, there is a μέν/δέ contrast between these groups – the μέν occurring at the beginning of the passage and being picked up again in the conclusive statement at 1090b31, perhaps to emphasise the point; and secondly, the inclusion of the adjective "first" (πρῶτοι) indicates some sort of priority among the second group. The shift from plural "they" to singular "he" around 1091a1 may be taken to indicate a specification to Plato himself (in the light of this, what appears to be a damaged text in the previous line should not be overlooked)[18]. The production of geometrical objects from numbers associated with the ideal theorists (especially lines from the dyad or 2, planes from the triad or 3, and solids out of the tetrad or 4) is nowhere explicitly or even implicitly suggested by Plato in his dialogues[19]; rather, it

ris, *Aristotle's account of Speusippus' and Xenocrates' Metaphysical and Epistemological Theories*, *op. cit.*, p. 15-25; G. De Cesaris, "The Chicken or the Egg? Aristotle on Speusippus' Reasons to Deny the Principle is (the) Good", *Apeiron* 55.4, 2022; and G. De Cesaris's contribution to this volume.

16 M. Crubellier, *Les livres Mu et Nu de la Métaphysique d'Aristote. traduction et commentaire*, 4 vols., Thèse de doctorat, Université Charles-de-Gaulle Lille III, 1994, p. 497.

17 K. Gaiser, "Quellenkritische Probleme der indirekten Platonüberlieferung", art. cit., p. 214-224.

18 M. Crubellier, *Les livres Mu et Nu de la Métaphysique d'Aristote. traduction et commentaire*, *op. cit.*, p. 504-505.

19 Gaiser (K. Gaiser, "Quellenkritische Probleme der indirekten Platonüberlieferung", art. cit., p. 218-219) links this passage to the pseudo-Platonic *Epinomis* (990e-991a), where planes are produced subsequent to three expan-

looks like an attempt to provide a more elaborate account of the generation of bodies than that presented in Plato's *Timaeus* (48b-53d) through a novel explanation of the production of geometrical entities by the "forms and numbers" (εἴδεσί τε καὶ ἀριθμοῖς) mentioned by Timaeus without further comment at 53b[20]. For Plato leaves open how exactly we get from the ultimate principles – the Great and the Small mentioned by Aristotle[21]? – to the triangles that constitutively inform the geometrical shapes of the elements of the universe[22]. But

sions (τρὶς ηὐξημένους) and thereafter speculates about the relationship between the account of mathematical expansion given there and Xenocrates. Additionally, in the fragment from his work *On Pythagorean Numbers* (fr. 28 Tarán/122 Isnardi Parente ([Iamblichus], *Theologoumena Arithmeticae* 82.10-85.23 De Falco)), Speusippus explicitly speaks of the generation of mathematical objects into magnitudes: "And the same thing occurs too in their [sc. numbers'] generation (ἐν τῇ γενέσει): for the first principle [of generation] into magnitude (εἰς μέγεθος) is a point, the second is a line, the third is a surface and the fourth is a solid figure". For the editions of Speusippus, see L. Tarán, *Speusippus of Athens: a critical study with a collection of the related texts and commentary*, Leiden, Brill, 1981, and M. Isnardi Parente, *Speusippo. Testimonianze e frammenti*, Napoli, Bibliopolis, 1980.

[20] Plato, *Timaeus* 53a7-b5: "Indeed, it is a fact that before this took place [sc. the different kinds of cosmic elements gravitated to their natural places] the four all lacked proportion and measure, and at the same time the ordering of the universe was undertaken, fire, earth and air initially possessed certain traces of what they are now. They are in the condition one would expect thoroughly god-forsaken things to be in. So, finding them in this condition, the first thing the god then did was to give them their distinctive shapes, using forms and numbers".

[21] K. Gaiser, "Quellenkritische Probleme der indirekten Platonüberlieferung", art. cit., p. 221-224.

[22] Plato, *Timaeus* 53c5-d7: "Now everything that has bodily form also has depth. Moreover, it is wholly necessary that the nature of surface comprehend its depth; and the straight [line?] of the face of a surface is composed out of triangles. Every triangle, moreover, originates from two triangles, each of which has one right angle and two acute angles. Of these two triangles, one [the isos-

Aristotle seems to be thinking of these passages when he discusses the views of both the ideas-positers and "those who were first to produce two types of number" – does this mean that Aristotle saw some overlap between their views, and that the latter group forms a subdivision of the former group (as, for example, Crubellier thinks)[23]? If so, what is the difference between positing "magnitude-ideas", as the ideas-positers advance, and the "intermediate number" that is said to be generated by the great and the small? Indeed, would "magnitude-ideas" not be a sort of intermediate object of mathematics?

The point of this brief analysis of this passage is twofold: first, it highlights the very real challenges in assessing Aristotle's accounts of other figures' theories of mathematical intermediates in the *Metaphysics*, especially the challenges in differentiating the views of Plato and Xenocrates; and second, it reminds us of the sad state of our understanding of Aristotle's dialectic with the members of the Early Academy, in part because the activities of the members of the Academy, including Xenocrates, involved engaging with or reacting to what Plato himself wrote in his dialogues. Circularities threaten our investigation from all angles. This means we will have to look elsewhere beyond the writings of Aristotle and Plato to determine with any certainty what sorts of theories Aristotle was criticizing when presenting τὰ μεταξύ.

celes right-angled triangle] has at each of the other two vertices an equal part of a right angle, determined by its division by equal sides; while the other [the scalene right-angled triangle] has unequal parts of a right angle at its other two vertices, determined by the division of the right angle by unequal sides. This, then, we posit as the principle of fire and of the other bodies, *as we pursue our likely account in terms of necessity. Principles yet more ultimate than these are known only to a god, and to any man he may hold dear*" (italics mine for emphasis).

[23] M. Crubellier, *Les livres Mu et Nu de la Métaphysique d'Aristote. traduction et commentaire*, *op. cit.*, p. 497-505.

II

Our investigation now turns to a few of Aristotle's passages that provide a more systematic account of τὰ μεταξύ, in order to flesh out his objections to their theories. This will provide us with a set of attributes that purveyors of τὰ μεταξύ theories are thought to adopt. Additionally, it investigates what Aristotle himself means by the term μεταξύ, and the special way in which his views on this term inform his dialectic with his adversaries in the Academy.

There are two extended treatments of τὰ μεταξύ in Aristotle's *Metaphysics*. The first of these, in B 2, is a slightly more abbreviated version of what appears in M 1-2, which also acts as a sequel of sorts to the earlier passage[24]. I will focus on the passage from B 2, which suffices to introduce us to the main tenets of the theories of mathematical intermediates advanced by Aristotle's adversaries:

> Aristotle, *Metaphysics* B 2, 997b12-998a19
>
> ἔτι δὲ εἴ τις παρὰ τὰ εἴδη καὶ τὰ αἰσθητὰ τὰ μεταξὺ θήσεται, πολλὰς ἀπορίας ἕξει· δῆλον γὰρ ὡς ὁμοίως γραμμαί τε παρά τ᾽ αὐτὰς καὶ [15] τὰς αἰσθητὰς ἔσονται καὶ ἕκαστον τῶν ἄλλων γενῶν· ὥστ᾽ ἐπείπερ ἡ ἀστρολογία μία τούτων ἐστίν, ἔσται τις καὶ οὐρανὸς παρὰ τὸν αἰσθητὸν οὐρανὸν καὶ ἥλιός τε καὶ σελήνη καὶ τἆλλα ὁμοίως τὰ κατὰ τὸν οὐρανόν. καίτοι πῶς δεῖ πιστεῦσαι τούτοις; οὐδὲ γὰρ ἀκίνητον εὔλογον εἶναι, κινούμενον δὲ [20] καὶ παντελῶς ἀδύνατον· ὁμοίως δὲ καὶ περὶ ὧν ἡ ὀπτικὴ πραγματεύεται καὶ ἡ ἐν τοῖς μαθήμασιν ἁρμονική· καὶ γὰρ ταῦτα ἀδύνατον εἶναι παρὰ τὰ αἰσθητὰ διὰ τὰς αὐτὰς αἰτίας· εἰ γὰρ ἔστιν αἰσθητὰ μεταξὺ καὶ αἰσθήσεις, δῆλον ὅτι καὶ ζῷα ἔσονται μεταξὺ αὐτῶν τε καὶ τῶν φθαρτῶν. [25] ἀπορήσειε δ᾽ ἄν τις καὶ περὶ ποῖα τῶν ὄντων δεῖ ζητεῖν ταύτας τὰς ἐπιστήμας. εἰ γὰρ τούτῳ διοίσει τῆς γεωδαισίας ἡ γεωμετρία μόνον, ὅτι ἡ μὲν τούτων ἐστὶν ὧν αἰσθανόμεθα ἡ δ᾽ οὐκ αἰσθητῶν, δῆλον ὅτι καὶ παρ᾽ ἰατρικὴν ἔσται τις ἐπιστήμη καὶ παρ᾽ ἑκάστην τῶν ἄλλων

[24] It is explicitly identified as a such at Aristotle, *Metaphysics* M 2, 1076a37-b5.

μεταξὺ αὐτῆς τε ἰατρικῆς [30] καὶ τῆσδε τῆς ἰατρικῆς· καίτοι πῶς τοῦτο δυνατόν; καὶ γὰρ ἂν ὑγιείν' ἄττα εἴη παρὰ τὰ αἰσθητὰ καὶ αὐτὸ τὸ ὑγιεινόν. ἅμα δὲ οὐδὲ τοῦτο ἀληθές, ὡς ἡ γεωδαισία τῶν αἰσθητῶν ἐστὶ μεγεθῶν καὶ φθαρτῶν· ἐφθείρετο γὰρ ἂν φθειρομένων. ἀλλὰ μὴν οὐδὲ τῶν αἰσθητῶν ἂν εἴη μεγεθῶν [35] οὐδὲ περὶ τὸν οὐρανὸν ἡ ἀστρολογία τόνδε. [998a] [1] οὔτε γὰρ αἱ αἰσθηταὶ γραμμαὶ τοιαῦταί εἰσιν οἵας λέγει ὁ γεωμέτρης (οὐθὲν γὰρ εὐθὺ τῶν αἰσθητῶν οὕτως οὐδὲ στρογγύλον· ἅπτεται γὰρ τοῦ κανόνος οὐ κατὰ στιγμὴν ὁ κύκλος ἀλλ' ὥσπερ Πρωταγόρας ἔλεγεν ἐλέγχων τοὺς γεωμέτρας), οὔθ' αἱ κινήσεις καὶ [5] ἕλικες τοῦ οὐρανοῦ ὅμοιαι περὶ ὧν ἡ ἀστρολογία ποιεῖται τοὺς λόγους, οὔτε τὰ σημεῖα τοῖς ἄστροις τὴν αὐτὴν ἔχει φύσιν. εἰσὶ δέ τινες οἵ φασιν εἶναι μὲν τὰ μεταξὺ ταῦτα λεγόμενα τῶν τε εἰδῶν καὶ τῶν αἰσθητῶν, οὐ μὴν χωρίς γε τῶν αἰσθητῶν ἀλλ' ἐν τούτοις· οἷς τὰ συμβαίνοντα ἀδύνατα πάντα [10] μὲν πλείονος λόγου διελθεῖν, ἱκανὸν δὲ καὶ τὰ τοιαῦτα θεωρῆσαι. οὔτε γὰρ ἐπὶ τούτων εὔλογον ἔχειν οὕτω μόνον, ἀλλὰ δῆλον ὅτι καὶ τὰ εἴδη ἐνδέχοιτ' ἂν ἐν τοῖς αἰσθητοῖς εἶναι (τοῦ γὰρ αὐτοῦ λόγου ἀμφότερα ταῦτά ἐστιν), ἔτι δὲ δύο στερεὰ ἐν τῷ αὐτῷ ἀναγκαῖον εἶναι τόπῳ, καὶ μὴ εἶναι ἀκίνητα [15] ἐν κινουμένοις γε ὄντα τοῖς αἰσθητοῖς. ὅλως δὲ τίνος ἕνεκ' ἄν τις θείη εἶναι μὲν αὐτά, εἶναι δ' ἐν τοῖς αἰσθητοῖς; ταὐτὰ γὰρ συμβήσεται ἄτοπα τοῖς προειρημένοις· ἔσται γὰρ οὐρανός τις παρὰ τὸν οὐρανόν, πλήν γ' οὐ χωρὶς ἀλλ' ἐν τῷ αὐτῷ τόπῳ· ὅπερ ἐστὶν ἀδυνατώτερον.

And if someone is to posit intermediates besides the forms and sensibles, he will run into a great number of difficulties. For clearly there will be lines [15] besides these [sc. form-lines] and sensible [lines], and similarly with each of the rest of the genera, with the result that, since astronomy is one of these [sciences], there will be a heaven too besides the sensible heaven, and a sun [besides the sensible sun] and moon [besides the sensible moon], and similarly for the rest of the heavenly bodies. And yet how is one to believe these things? For it is not even reasonable to [believe] that [intermediate heaven] is immovable, whereas [20] it is completely impossible for it to be in motion. And similarly too in the case of what optics and, among the mathematical sciences, harmonics, treat; for it is indeed impossible for these [sc. intermediates] to exist besides, for the same reasons. After all, if there will be sensibles and senses intermediate [between form and individual], it is clear that there will be animals intermediate between those

> [that are ideal] and those that perish. [25] Someone might be at a loss about what sorts of objects these [intermediary] sciences ought to pursue. For if geometry is to differ from mensuration in this way alone, that the latter [is a science of] things we perceive, whereas the former [is a science of] things not sensible, clearly there will also be a certain science besides medicine (and besides each of the other sciences) intermediate between medicine-in-itself [30] and individual medicine. Yet how is this possible? For there would be [a class of] objects of health besides sensible objects of health and what is healthy-in-itself. And at the same time, it is not even true that mensuration is [a science of] sensible and perishable magnitudes; for it would perish once they have perished.
>
> Moreover, astronomy cannot be [a science of] sensible magnitudes, nor one concerned with individual heaven. [998a1] For neither are sensible lines such lines as the geometer speaks of (for no sensible [line] is straight or curved in this sense; indeed, the [sensible] circle does not touch a straight edge *at* a point, but <*along* a point>, as Protagoras said when he refuted the geometers), nor are the movements and [5] orbits of heaven similar to those concerning which astronomy produces arguments, nor do [geometrical] points have the same nature as the stars. But there are some who say that so-called "intermediates" exist between forms and sensibles – not actually *apart from* the sensibles, but *in* them. It would be a long discourse [10] to go through all the impossibilities consequent to these [claims], but it is sufficient to consider the following points. It is not reasonable that this should be so in the case of these things [sc. the intermediates] *only*, but clearly *the forms too* should be capable of being *in* the sensibles (for both these are subject to the same argument). Further, it is necessary that two solids be *in* the same place, and that [forms] not be immovable [15], since they are *in* sensible things that are moving. And generally, what is the goal of someone positing that these things [sc. intermediates] exist, and that they exist in sensibles? The same absurdities as those mentioned previously will result. There will be some heaven besides [sensible] heaven – only not apart from it, but in the same place. And this is [even] more impossible.

I will focus on the set of problems raised by Aristotle, because this set of problems helps us to identify what the theory or theories of intermediates he is challenging seem to assume (at least from Aristotle's perspective). First of all, there is a recurrent concern over the universal application of intermediates across all classes of beings, sometimes termed the "Uniqueness Problem"[25]. If, say, we posit intermediate ideal lines in geometry (at or near the fundament of the Platonists' mathematical existence), we will equally be forced to posit intermediates in the other sciences, such as astronomy, with its much-maligned "intermediate heaven" (along with the intermediate sun, and moon). There is an implicit Third Man criticism here as well: what is to stop us from positing *another* intermediate heaven between intermediate heaven and ideal heaven[26]? Additionally, the objects of a science, with the properties that identify and distinguish those objects, are correlate to that science, and hence we would have to posit a separate science whose objects are intermediate astronomical bodies – let us call this science "intermediate astronomy", as contrasted with "ideal astronomy" and "sensible astronomy" (the same goes for medicine and measurement of the earth). At issue here is Aristotle's commitment to differentiation between what he elsewhere, in the *Posterior Analytics* (I 13, 78b32-79a16), calls "superior" and "inferior" sciences (and scholars often refer to as "superordinate" and "subordinate" sciences)[27]. For geometry clearly studies idealised objects whose properties resemble, but are not the same as, the properties of the objects of land mensuration[28] – a key difference is that in mathematical proofs,

[25] As it is by Annas (J. Annas, "On the 'Intermediates'", art. cit., p. 154-155).

[26] This is made more explicit at Aristotle, *Metaphysics* M 2, 1076b11-39.

[27] As noted by Katz (E. Katz, "The Mixed Mathematical Intermediates", art. cit., p. 86).

[28] Or, as Menn states (S. Menn, *The Aim and the Argument of Aristotle's Metaphysics*, Unpublished, obtained via https://www.philosophie.hu-berlin.de/de/lehrbereiche/antike/mitarbeiter/menn/contents, Iγ3: p. 28): "Geometricals thus have a non-separate potential existence dependent on the matter of sensible things, contrasting with the non-separate actual existence of units and numbers, dependent on the forms of sensible things. This quite specially dependent mode of existence is not needed simply to explain how geometry can be a science without separately existing geometricals (since otherwise we

geometry often, but not always, operates with different premises than its subordinate kin, land mensuration, deploys in action[29]. Similarly, for Aristotle, geometry cannot be applied willy-nilly to the study of all space: for example, on an astral map one plots out the entire Milky Way using what is amenable to a geometer or astronomer, namely, a writing or inscribing utensil and a blank template; and while points on a map have practical value to the pilot who is navigating the ship, neither the pilot nor anyone of sense would think that the stars represented there have the same nature as the points! Equally, the astronomer and the navigator might think that they are studying the same objects – the sensible heavenly bodies; but this is not exactly true, as the deployment of the objects in their respective demonstrations reveals that their properties are not exactly the same. Importantly, the science of navigation and the science of astronomy must remain distinct.

A second criticism of Aristotle's opponents who posit intermediate mathematicals relates to the purported mode of being of these objects. According to *some* people who advance a theory of intermediates, which they would appear to "call" (λεγόμενα) τὰ μεταξύ, the intermediates are not separate objects, but rather they inhere in sensible individuals. What these people mean by saying that intermediates are "in" sensibles is not charitably illustrated by Aristotle – neither here, nor in the reprise of this passage in M 2, where Aristotle only compounds the criticisms by noting that an inherence theory of mathematicals not only contradicts the axiom that two solids cannot occupy the

would also need it for units and numbers). Rather, it is needed to explain how geometry can be a science without separately existing geometricals given that geometrical attributes do not apply perfectly to sensible things, and thus to respond to the argument for intermediate geometricals at B#5 997b35-998a4)".

[29] Katz (E. Katz, "The Mixed Mathematical Intermediates", art. cit., p. 91-92) is right in noting that this objection on Aristotle's part does not absolve him from committing the same mistake in his discussion of "mixed" sciences such as optics.

same place, but would lead to a theoretical absurdity based on the assumption that intermediate and sensible objects obtain their properties from the separable forms they imitate (or participate in)[30]. This absurdity lies in the fact that every body would be required to be indivisible: if a point-line-plane-solid progression is assumed, as certain Platonists seem to have held[31], the properties of the point must hold for bodies subsequent to it in generation, and hence bodies would be, like the points that are their ultimate elements, indivisible. But we know that bodies are divisible, and hence the theory is absurd. Additionally, in our passage, if the modality of inherence is the bearer of properties from ideal to intermediary, and from there to sensible, what is to prevent forms from ultimately inhering in sensibles? Indeed, this reveals Aristotle's disdain for the inefficiency of the theory of mathematical intermediates: why do we need mathematical intermediates if their existence is superfluous to a more elegant account of inherence of forms in sensibles?

Hence, according to Aristotle, the main problems that arise out of theory that embraces intermediate mathematicals are: (a) the theory is ontologically and epistemologically inefficient; (b) it cannot sufficiently explain the presence of properties in sensibles contrary to the properties in forms (if one assumes separate forms); (c) and it confuses the sciences, which ought to be distinguished as according to their respective objects[32]. We also gather somewhat more indirectly from the passage in B 2 and its sequel in M 1-2[33] that the proponents of such theories may have advanced their theory on the basis of a point-line-

[30] Aristotle, *Metaphysics* M 2, 1076a37-b39.

[31] See n. 19 above.

[32] Hence, my assessment of the objections to the intermediates elaborates and goes beyond the three objections Aristotle levels against champions of the theory of intermediates (J. Annas, "On the 'Intermediates'", art. cit., p. 152-154), namely (a) that the Platonists fail to give an account of the existence and status of intermediates; (b) that they multiply first principles; and (c) the so-called "Uniqueness Problem" (mentioned above).

[33] Aristotle, *Metaphysics* M 2, 1076a37-b39.

plane-solid generation of magnitudes; may have attempted to distinguish objects at each of these ontological levels according to their unique properties; may have sought to bring the sciences together, rather than to strictly differentiate them by their respective objects; may have actually advanced notions of intermediate heavenly bodies, and even an intermediate heaven; and may have had special interest in geometricals as the intermediate objects par excellence. It is worth noting what is missing: no mention or reference in this passage to form-numbers, which were of special concern in N 3. In fact, whilst the passage from N 3 could be at least possibly reconciled with what Plato presented in his dialogues, nothing to my eye from B 2 shows strong connections to the *Timaeus*, *Republic*, *Philebus*, or any other writings of Plato[34].

Before moving on to investigate the relationship between N 3 and the testimonies concerning the philosophy of Xenocrates in the third part of this paper, it still remains to explain what, exactly, Aristotle means by the term μεταξύ. Now Bonitz's *Index Aristotelicus* lists around 70 usages of this term, although many of them are more conventional than the technical usage given here[35]. The single passage, however, that does the best to describe what μεταξύ means in the context of what I discussed earlier is from *Metaphysics* K 12, where Aristotle aims to theoretically inform his discussion of place by explaining what me means by the words he uses. The underlined sections are directly related to his idea of μεταξύ:

> Aristotle, *Metaphysics* K 12, 1068b26-1069a14
>
> ἅμα κατὰ τόπον ὅσα ἐν ἑνὶ τόπῳ πρώτῳ, καὶ χωρὶς ὅσα ἐν ἄλλῳ· ἅπτεσθαι δὲ ὧν τὰ ἄκρα ἅμα· <u>μεταξὺ δ᾽ εἰς ὃ πέφυκε πρότερον</u>

[34] Indeed, it is not mentioned by Gaiser in his survey of passages that relate Plato's theory of the principles (K. Gaiser, "Quellenkritische Probleme der indirekten Platonüberlieferung", art. cit.).

[35] H. Bonitz, *Index Aristotelicus*, Secunda Editio, Graz, Akademische Druck – U. Verlagsanstalt, 1870, p. 460-462.

ἀφικνεῖσθαι τὸ μεταβάλλον ἢ εἰς ὃ ἔσχατον μεταβάλλει κατὰ φύσιν τὸ συνεχῶς μεταβάλλον. [30] ἐναντίον κατὰ τόπον τὸ κατ᾽ εὐθεῖαν ἀπέχον πλεῖστον· ἐξῆς δὲ οὗ μετὰ τὴν ἀρχὴν ὄντος, θέσει ἢ εἴδει ἢ ἄλλως πως ἀφορισθέντος, μηθὲν μεταξύ ἐστι τῶν ἐν ταὐτῷ γένει καὶ οὗ ἐφεξῆς ἐστίν, οἷον γραμμαὶ γραμμῆς ἢ μονάδες μονάδος ἢ οἰκίας οἰκία (ἄλλο δ᾽ οὐθὲν κωλύει μεταξὺ [35] εἶναι). τὸ γὰρ ἑξῆς τινὸς ἐφεξῆς καὶ ὕστερόν τι· οὐ γὰρ τὸ ἓν ἑξῆς τῶν δύο οὐδ᾽ ἡ νουμηνία τῆς δευτέρας. [1069a] [1] ἐχόμενον δὲ ὃ ἂν ἑξῆς ὂν ἅπτηται. ἐπεὶ δὲ πᾶσα μεταβολὴ ἐν τοῖς ἀντικειμένοις, ταῦτα δὲ τὰ ἐναντία καὶ ἀντίφασις, ἀντιφάσεως δ᾽ οὐδὲν ἀνὰ μέσον, δῆλον ὡς ἐν τοῖς ἐναντίοις τὸ [5] μεταξύ. τὸ δὲ συνεχὲς ὅπερ ἐχόμενόν τι. λέγω δὲ συνεχὲς ὅταν ταὐτὸ γένηται καὶ ἓν τὸ ἑκατέρου πέρας οἷς ἅπτονται καὶ συνέχονται, ὥστε δῆλον ὅτι τὸ συνεχὲς ἐν τούτοις ἐξ ὧν ἕν τι πέφυκε γίγνεσθαι κατὰ τὴν σύναψιν. καὶ ὅτι πρῶτον τὸ ἐφεξῆς, δῆλον (τὸ γὰρ ἐφεξῆς οὐχ ἅπτεται, [10] τοῦτο δ᾽ ἐφεξῆς· καὶ εἰ συνεχές, ἅπτεται, εἰ δ᾽ ἅπτεται, οὔπω συνεχές· ἐν οἷς δὲ μὴ ἔστιν ἁφή, οὐκ ἔστι σύμφυσις ἐν τούτοις)· ὥστ᾽ οὐκ ἔστι στιγμὴ μονάδι ταὐτόν· ταῖς μὲν γὰρ ὑπάρχει τὸ ἅπτεσθαι, ταῖς δ᾽ οὔ, ἀλλὰ τὸ ἐφεξῆς· καὶ τῶν μὲν μεταξύ τι τῶν δ᾽ οὔ.

Things which are in a single primary place are "together" in place, and "separate" when they are in different places. Things whose extremes are together "touch". That at which the changing thing, when it changes continuously according to nature, naturally arrives before it arrives at the extreme into which it is changing, is "intermediate". [30] That which is most distant in a straight line is "contrary" in place. When something is after the beginning, it is "successive" – how it is so being determined by position or form or some other way – and there is nothing intermediate between things that are in the same genus and that with which they are continuous, e.g. lines and a line, units and a unit, or houses and a house (but there is nothing to prevent *something else* being intermediate [35]). For that which is successive to something is something continuous with and posterior to it. Indeed, 1 is not successive to 2, nor is the new moon to the second day of the month. [1069a1] "Contiguous" is the sort of thing that, being successive, touches upon it. And since all change [occurs] among opposites, and these are contraries or in contradiction, and there is no middle term [5] for things in contradiction, clearly the "intermediate" [occurs] in opposites. The "continuous" is a species of the "contiguous". I say "conti-

> nuous" when the respective extremes [of things], by which they touch and are kept together, become one and the same; hence clearly the "continuous" [occurs] in those things whose nature it is to become a single thing with contiguity. And clearly "successive" is primary (for what is successive does not [always] touch, [10] but that which touches is successive; and if it is continuous, it touches; but if it touches, it is not always continuous; and in things where there is no touching, there is no organic unity). Hence, a point is not the same as a unit; for touching applies to points, but not to units, which [have only] succession. And there is an intermediary between points, but not between units.

The passage in K 12 helps to inform Aristotle's criticisms of the theorists of intermediate mathematicals, because it explains what is meant by "intermediary" within Aristotle's own strictures. First of all, for Aristotle, μεταξύ implies continuous change in the direction of an extreme or a limit – what lies in between the beginning and the extreme is the μεταξύ. Secondly, change occurs either among contraries or contradictories, and since contradictories cannot admit of anything in the middle, μεταξύ only occurs among contraries or opposites. Finally, since μεταξύ occurs among things that are contrary or opposed, it cannot occur between units – it is not the case that 2 units is "opposed" or "contrary" to 4 units, and hence 3 units is not to be considered a μεταξύ between them. But a μεταξύ can occur between two points; one of the reasons for this is because two points are connected by contact, which facilitates change in one direction or another. It is not clear from this passage what, if anything, is expected to be intermediate between two points: Aristotle stipulates that there cannot be a μεταξύ between things that are continuous and in the same genus, e.g., no line between lines, and no unit between units, which means that what is μεταξύ between two points cannot be another point. Indeed, in the *Physics* (V 3, 227a31), Aristotle asserts that what lies between two points is a line, which is of a different genus than a point[36]. These sti-

[36] At *Metaphysics* I 7, 1057a18-33, Aristotle argues in reference to strings on a lyre and colours that intermediates themselves must be in the same genus,

pulations help us to grasp in what special way Aristotle wishes to use μεταξύ, when he attacks those who posited theories of intermediate mathematicals. Intermediate heaven cannot be posited between two other kinds of heaven – ideal heaven and sensible heaven – since an intermediate cannot be posited between two things of the same genus as it. Moreover, if an intermediate is advanced between, say, heaven and earth, heaven and earth must be considered as contraries *qua* place, and whatever would be in between them would have to be different from them; a far more sensible candidate than another heaven or another earth, or worse a heavenly-earth, would be something like atmosphere, or the meterological sphere.

III

In the final section of this paper, I would like to turn to the testimonies concerning Xenocrates of Chalcedon, whom I will advance as one of the central targets of Aristotle's critiques of the theories of mathematical intermediates associated with τὰ μεταξύ. I will attempt to proceed by way of starting with a witness to Xenocrates' philosophy independent of Aristotle, but contemporaneous with him, namely Theophrastus of Eresus, Aristotle's student and successor in the Lyceum. For it is Theophrastus who provides us a crucial insight into how the members of Plato's Academy differentiated their approaches to, in particular, metaphysics. As is well known, Plato's successors Speusippus and Xenocrates advanced strikingly diverse theories of ontology, espe-

and that they be of the same genus as the contraries from which they are formed. Hence, it might sound as if Aristotle is contradicting himself, in reference to mathematicals. But it is not evident that strings and colours are continuous in the same way lines are. Additionally, he stipulates (*Metaphysics* I 7, 1057b2-4) that intermediates must be composed of their contraries; this might work for a line (i.e., a line, which is intermediary between two points, is composed of the two points); but it will hardly work for heaven (i.e., intermediate heaven cannot be composed of ideal heaven and sensible heaven).

cially with reference to mathematics[37]. Because Speusippus rejected the theory of separable transcendent forms, this gives him a certain incentive to work within the category of mathematicals; and in his fragment *On Pythagorean Numbers*, Speusippus makes his mathematicals do a lot of ontological work that otherwise would be associated with the forms – especially heightening the powers of numbers to confer properties on dependent entities[38]. In this way, Speusippus would not appear to be significantly distanced from his successor in the Academy, Xenocrates. And Speusippus seems to have agreed with Xenocrates in positing two main first principles – the One and the Indefinite Dyad – and in deriving reality in some way from these. But it is at the level of cosmology, and in particular the composition of the universe beyond the principles and their immediate products, that Speusippus and Xenocrates differed immensely, as Theophrastus explains in his *Metaphysics*:

> Theophrastus, *Metaphysics* 6a23-b9 = Xenocrates fr. 20 Isnardi Parente[2]
>
> νῦν δ' οἵ γε πολλοὶ μέχρι τινὸς ἐλθοῦντες καταπαύονται, καθάπερ καὶ οἱ τὸ ἓν καὶ τὴν ἀόριστον δυάδα ποιοῦντες· τοὺς γὰρ ἀριθμοὺς γεννήσαντες καὶ τὰ ἐπίπεδα καὶ τὰ σώματα σχεδὸν τἆλλα παραλείπουσιν πλὴν ὅσον ἐφαπτόμενοι καὶ τοσοῦτο μόνον δηλοῦντες, ὅτι τὰ μὲν ἀπὸ τῆς ἀορίστου δυάδος, οἷον τόπος καὶ κενὸν καὶ ἄπειρον, τὰ δ' ἀπὸ τῶν ἀριθμῶν καὶ τοῦ ἑνός, οἷον ψυχὴ καὶ ἄλλ' ἄττα· χρόνον δ' ἅμα καὶ οὐρανὸν καὶ ἕτερα δὴ πλείω, τοῦ δ' οὐρανοῦ πέρι καὶ τῶν λοιπῶν οὐδεμίαν ἔτι ποιοῦνται μνείαν· ὡσαύτως δ' οἱ περὶ Σπεύσιππον, οὐδὲ τῶν ἄλλων οὐθεὶς πλὴν Ξενοκράτης· οὗτος γὰρ ἅπαντά πως περιτί-

[37] See J. Dillon, *The Heirs of Plato*, *op. cit*., p. 107-111.

[38] In particular, Speusippus (fr. 28 Tarán/122 Isnardi Parente ([Iamblichus], *Theologoumena Arithmeticae* 82.10-85.23 De Falco)) focusses on how the perfection of the decad entails other basic properties of reality as evidenced in its numerical constituents' relations (priority and posteriority, simplicity and complexity, etc.). This is a project for another paper.

θησιν περὶ τὸν κόσμον, ὁμοίως αἰσθητὰ καὶ νοητὰ καὶ μαθηματικὰ καὶ ἔτι δὴ τὰ θεῖα.

But as it is, after arriving at a certain point, the majority [of philosophers] stop completely, just like those who posit the One and the Indefinite Dyad; for after they have generated numbers, planes, and solids, they leave out pretty much everything else – except as far as that they [can] apprehend, and to the extent to which they [can] demonstrate this alone, that some things [are generated] from the Indefinite Dyad, e.g., place, void, and the infinite, whereas others [are generated] from numbers and the One, e.g., soul etc. – and time, together with the heaven and many others indeed. But they make no further mention of the heavens and the rest. And, likewise, Speusippus and his followers [make no further mention of these], [as does] nobody else, except Xenocrates: for this man somehow assigns everything a place in the universe, alike objects of sensation, objects of intellection, mathematical objects, and, furthermore, divine things.

It is clear from this passage that Speusippus and Xenocrates are among those who posited the One and the Indefinite Dyad as principles. We hear that these figures generate numbers, followed by planes, solids, and ultimately heaven, by way of the One and the Indefinite Dyad. They also claim that it is by way of ultimate generation from one or the other principle that certain entities are what they are: the One and numbers somehow generate soul, time, and heaven, whereas the Dyad generates place, void, and the infinite. The implication here, as elsewhere in Theophrastus' accounts, is that the generated entities are what they are, i.e., they have the properties they possess, in virtue of their generation from one or another principle: soul, time, and heaven are all unified and limited, whereas place, void, and infinity are potentially or actually empty, unbounded, or marked by division. Scholars have had reason to think that the material here is referring to Speusippus, but it is more likely to present a general account of the Platonists' principles, which would probably include Xenocrates as well[39]. What Theophrastus adds at the end

[39] See Ph.S. Horky, "Theophrastus on Platonic and 'Pythagorean' Imitation", *Classical Quarterly* 63.2, 2013, p. 686-712: p. 700 n. 56.

of this passage gives us a sense of Xenocrates' central contributions to this activity of accounting for the generation and articulation of the objects of the universe: for whereas Speusippus and other Platonists give up after they generate heaven, Xenocrates goes further, mapping the various classes of objects of cognition onto specific places in the universe. This division is effectively into three classes: sensibles (those closest to us), intelligibles (those furthest from us), and mathematicals (those in between). "Divine" objects are, as I have argued elsewhere[40], most likely to be considered those things that are, on a scale of sorts, "more" or "less" divine: the most divine things are intelligibles, the least sensibles, and those in the middle are mathematicals. For Xenocrates had a special inclination towards tripartitions. This onto-epistemological tripartition is also attested in a number of places, most notably in Asclepius' commentary on Aristotle's *Metaphysics* Z 2:

> Asclepius, *In Arist. Metaph.* 379.17-22 Hayduck = Xenocrates fr. 24 Isnardi Parente[2]
>
> ἐντεῦθεν εἰς Ξενοκράτην ἀποτείνεται, καί φησιν ὅτι τὰ εἴδη τῶν πραγμάτων τοῖς ἀριθμοῖς προσηγόρευεν, ἐπειδή, ὥσπερ οἱ ἀριθμοὶ περιοριστικοί εἰσιν ὧν εἰσιν ἀριθμοί, οὕτω δὴ καὶ τὰ εἴδη περιοριστικὰ τῆς ὕλης ὑπάρχουσιν. εἶτα μετὰ τὰς ἰδέας δευτέρας οὐσίας ὑποτίθεται τὰς διανοητάς, τουτέστι τὰ μαθήματα, γραμμὰς καὶ ἐπίπεδα· τελευταῖα δὲ τὰ φυσικά.
>
> Herein he refers to Xenocrates, and he says that he refers to the forms of objects with numbers, since, just as numbers are what gives definition to the things of which there are numbers, so too the forms are what gives determination to matter. Next after the ideas he posits dianoetic substances as secondary, i.e. mathematic[al]s – "lines and planes"; and last natural objects.

[40] Ph.S. Horky, "Theophrastus on Platonic and 'Pythagorean' Imitation", art. cit., p. 699-701 with n. 57.

Aristotle, *Metaphysics* Z 2, 1028b24-27 = Xenocrates fr. 23 Isnardi Parente[2]

ἔνιοι δὲ τὰ μὲν εἴδη καὶ τοὺς ἀριθμοὺς τὴν αὐτὴν ἔχειν φασὶ φύσιν, τὰ δὲ ἄλλα ἐχόμενα, γραμμὰς καὶ ἐπίπεδα, μέχρι πρὸς τὴν τοῦ οὐρανοῦ οὐσίαν καὶ τὰ αἰσθητά.

But some people say that forms and numbers have the same nature, whereas the other objects, lines and planes, are borne as far as the essence of heaven – and the objects of sense-perception.

As is often the case with Xenocrates, one is required to use the later Peripatetic and Neoplatonist commentators to identify Aristotle's target; for Aristotle seldom mentions Xenocrates by name, instead opting to leave us in the dark. However, careful comparison of Asclepius' commentary with Aristotle's original yields further and surprisingly insightful fruit. First of all, once we know this passage can be identified with Xenocrates, we can see something seldom noted by scholars who have worked on Xenocrates' mapping out of reality: as Aristotle says, while Xenocrates posited forms and numbers as having the same nature – the so-called "form-numbers", for which Xenocrates is famous –, he assumed that geometricals (lines and planes) have a *different* nature which only allows them to rise up as far as heaven; and then, below these fall sensibles. Asclepius interprets this to mean that geometricals belong to the dianoetic sphere of knowledge, and it would appear that he does not consider form-numbers to fall into the class of mathematicals at all. This is important supplementary information, because it clarifies that when Aristotle speaks of τὰ μεταξύ, he does not include form-numbers in this class (and hence the problem of τὰ μεταξύ is not a problem of Xenocratean form-numbers, although he surely had objections to those as well). So when Aristotle speaks of "mathematical intermediates", at least in reference to Xenocrates, he is strictly referring to the objects of geometry, which are, in Platonic terms, associated with *dianoêsis* or "thought"[41].

[41] For an explanation of this relationship, see N. Denyer, "Sun and Line:

Indeed, an important witness to Xenocrates' construction of reality, whose account corroborates what we have said above, is Sextus Empiricus, who provides a more complete account of how Xenocrates populated the universe with various objects:

> Sextus Empiricus, *Against the Logicians* 1.147-49 = Xenocrates fr. 2 Isnardi Parente[2]
>
> Ξενοκράτης δὲ τρεῖς φησὶν οὐσίας εἶναι, τὴν μὲν αἰσθητὴν τὴν δὲ νοητὴν τὴν δὲ σύνθετον καὶ δοξαστήν, ὧν αἰσθητὴν μὲν εἶναι τὴν ἐντὸς οὐρανοῦ, νοητὴν δὲ <τὴν> πάντων τῶν ἐκτὸς οὐρανοῦ, δοξαστὴν δὲ καὶ σύνθετον τὴν αὐτοῦ τοῦ οὐρανοῦ· ὁρατὴ μὲν γάρ ἐστι τῇ αἰσθήσει, νοητὴ δὲ δι᾽ ἀστρολογίας. τούτων μέντοι τοῦτον ἐχόντων τὸν τρόπον, τῆς μὲν ἐκτὸς οὐρανοῦ καὶ νοητῆς οὐσίας κριτήριον ἀπεφαίνετο τὴν ἐπιστήμην, τῆς δὲ ἐντὸς οὐρανοῦ καὶ αἰσθητῆς τὴν αἴσθησιν, τῆς δὲ μικτῆς τὴν δόξαν· καὶ τούτων κοινῶς τὸ μὲν διὰ τοῦ ἐπιστημονικοῦ λόγου κριτήριον βέβαιόν τε ὑπάρχειν καὶ ἀληθές, τὸ δὲ διὰ τῆς αἰσθήσεως ἀληθὲς μέν, οὐχ οὕτω δὲ ὡς τὸ διὰ τοῦ ἐπιστημονικοῦ λόγου, τὸ δὲ σύνθετον κοινὸν ἀληθοῦς τε καὶ ψευδοῦς ὑπάρχειν· τῆς γὰρ δόξης τὴν μέν τινα ἀληθῆ εἶναι τὴν δὲ ψευδῆ. ὅθεν καὶ τρεῖς μοίρας παραδεδόσθαι, Ἄτροπον μὲν τὴν τῶν νοητῶν, ἀμετάθετον οὖσαν, Κλωθὼ δὲ τὴν τῶν αἰσθητῶν, Λάχεσιν δὲ τὴν τῶν δοξαστῶν.
>
> Xenocrates says that there are three forms of existence: the sensible, the intelligible, and that which is composite and opinable; and of these, the sensible is that which exists below the heaven, the intelligible is that which belongs to all things outside the heaven, and the opinable and composite is that of the heaven itself; for it is visible by sense-perception, but intelligible by means of astronomy. This, then, being the situation, he declared that the criterion of the existence which is outside the heaven and intelligible is scientific knowledge, that of what is

The Role of the Good", dans G.R.F. Ferrari (ed.), *The Cambridge Companion to Plato's Republic*, Cambridge, Cambridge University Press, 2007, p. 284-309: p. 296-302.

> below the heaven and sensible is sense-perception, and the criterion of the mixed existence is opinion. And, generally, of these, the criterion afforded by scientific reasoning is both firm and true, whereas that which is afforded by sense-perception is true [only], but not in the same way as that afforded by scientific reasoning; and the [criterion] that is composite shares in both truth and falsehood – for, of opinion[s], one is true, and another false. Hence, too, [Xenocrates says] that tradition has handed down three Fates: Atropos for intelligibles, since she is unchangeable; Clotho for sensibles; and Lachesis for opinables.

This striking account of Xenocrates' kinds of existence evidences the depth and complexity of his theory; for, if we are to discount what is obviously Sextus' framing device (the discussion of the criterion, for which there is no other evidence to my knowledge for Xenocrates), we get a relatively clear account of the types of knowledge, their respective places, the sciences associated with them, their truth and reliability values, and the cosmic deities (from the myth of Er in Plato's *Republic* X) associated with each[42]. According to Sextus, Xenocrates recognized three οὐσίαι: the sensible, the intelligible, and the composite (σύνθετον), which he further qualifies as being the realm of δόξα or opinion. The intelligible is both firm and true, whereas the sensible is true only (but in some deficient way relative to the intelligible; and it is not firm); and the composite is both true and false, given that it is the realm of opinion, and opinions can be either true or false. Unsurprisingly, and mostly in line with what Aristotle and Asclepius say, the sensible lies below heaven, the intelligible beyond heaven, and the composite in heaven. Each of these οὐσίαι, then, is governed by a Fate: Atropos, who literally cannot be "turned", rules over the intelligibles

[42] This association of the Fates with cognitive activities is also evidenced by Proclus (*Commentary on Plato's Republic* 3.249.22-251.17 Kroll), who associated each respectively with the introductory, intermediary, and final the "activity of intellection" (νοήσεων ἐνέργεια): Lachesis refers to holistic or universal intellection, Atropos to partial intellection, and Clotho to intermediary intellection.

and the superouranial sphere; Lachesis rules over heaven and its composite and doxastic objects; and Clotho over the lower realm of sensibles. If we consider this in relation to the account of the Fates in the Myth of Er (Plato, *Republic* 617b-e), further possibilities emerge: since Atropos sings of and is associated with the future, we can imagine that Xenocrates saw intelligibles as reflective of the inevitability of what was to come; it is not beyond the realm of imagination to see the future as the guarantor of what is both firm and true. Lachesis sings of the past, which implicitly links to memory, and it is by dispensation from her lap that the souls receive the lots that provide their order in terms of selection of the next life, which will be a daemon to the souls; we know that memories can be faulty, and hence the past can reasonably be described as either true or false, at least from an epistemic point of view. Finally, Clotho sings of the present, which is always before our sense-perception and hence, in a certain sense, "true", but in a way deficient compared with the truth that the future always brings. It is the sphere of the "composite" that is thus associated with Xenocrates' mathematical objects – the heavenly place associated with memory, truth and falsity, and the life choice that will become the daemon to a person throughout their next embodied life.

A further testimony from Aetius shows us how Xenocrates bridged his account of the first principles and the material elements; hence, this testimony helps to explain what Theophrastus was saying at *Metaphysics* 6a23-b9, and link it to Sextus' more elaborate account of the three types of essence in *Against the Logicians* 1.147-49:

Aëtius, *Placita* 1.7.30 = Xenocrates, fr. 133 Isnardi Parente[2]

Ξενοκράτης Ἀγαθήνορος Καλχηδόνιος τὴν μονάδα καὶ τὴν δυάδα θεούς, τὴν μὲν ὡς ἄρρενα πατρὸς ἔχουσαν τάξιν ἐν οὐρανῷ βασιλεύουσαν, ἥντινα προσαγορεύει καὶ Ζῆνα καὶ περιττὸν καὶ νοῦν, ὅστις ἐστὶν αὐτῷ πρῶτος θεός· τὴν δ' ὡς θήλειαν, μητρὸς θεῶν δίκην, τῆς ὑπὸ τὸν οὐρανὸν λήξεως ἡγουμένην, ἥτις ἐστὶν αὐτῷ ψυχὴ τοῦ παντός. θεὸν δ' εἶναι καὶ τὸν οὐρανὸν καὶ τοὺς ἀστέρας πυρώδεις ὀλυμπίους θεούς, καὶ ἑτέρους ὑποσελήνους δαίμονας ἀοράτους. ἀρέσκει δὲ καὶ αὐτῷ

> καὶ ἐνδιήκειν τοῖς ὑλικοῖς στοιχείοις. τούτων δὲ τὴν μὲν < . . . > ἀειδῆ προσαγορεύει, τὴν δὲ διὰ ὑγροῦ Ποσειδῶνα, τὴν δὲ διὰ τῆς γῆς φυτοσπόρον Δήμητρα. ταῦτα δὲ χορηγήσας τοῖς Στωικοῖς τὰ πρότερα παρὰ τοῦ Πλάτωνος μεταπέφρακεν.
>
> Xenocrates, son of Agathenor, from Chalcedon, [said that] the Monad and the Dyad were gods: the former insofar as he is male and has the role of father who rules in heaven, whom he calls "Zeus", "Odd", and "Intellect", who is his [sc. Xenocrates'] first god; and the latter insofar as she is female, in the sense of mother of the gods, ruler over the allotment that occurs under heaven, who is his [sc. Xenocrates'] soul of the universe [sc. world-soul]. He regards the heavens also as a god, and the stars as fiery Olympian gods, and others [he believes to be] invisible sublunary daemons. It is also his view that they [sc. the daemons] penetrate the material elements as well. Of these, he calls one <...lacuna...> unseen, the one which permeates the water "Poseidon", and the one which permeates the earth "Demeter Seed-Sower". The origins [of these theories] he adapted from Plato and administered to the Stoics.

This account helps to make some sense of both the horizontal classification of entities according to the two first principles mentioned by Theophrastus, the One and the Indefinite Dyad (which are here called the Monad and the Dyad), and the objects generated by them respectively; and the vertical spatial hierarchy of being provided in Sextus' account. The One has the role of "king" and "father" in heaven, the world-soul where the fiery Olympian gods reside as stars; and he is also called "Zeus", "Odd", and "Intellect". Conversely, the Dyad has the role of "mother" under the heaven, i.e., in the realms of the mathematicals and sensibles. The invisible daemons operate in these two areas, bringing the powers from the heavenly realm down to the material parts of the universe, and some of them are named: Poseidon permeates the water, as does Demeter the earth, whereas a lacuna prevents us from knowing who precisely permeates the atmosphere (good conjectures would be Hera and Hades). By virtue of not being fiery or visible, we can assume Xenocrates did not include these deities among the Olympian gods who reside in heaven – we can only wonder whom Xenocrates included in this group beyond Zeus/the One.

If the above evidence is to be taken as a genuine representation of Xenocrates' theories of classification of beings and their associated epistemological levels, one can get a better sense of why Aristotle would complain, as he did in N 3, of those form-positers who "make up certain doctrines of [their] own" and "assume hypotheses of any sort and string out long-winded stories". Xenocrates' philosophy looks to have been informed greatly by his creative readings of Plato's imagistic passages, including and especially the Divided Line, the Myth of Er in *Republic* X and the cosmological account of Timaeus in the *Timaeus*, in an attempt to bring together a complete picture of the entire Platonic system. And it makes for fascinating reading, as well as positively exercising the imagination of scholars who wish to find some coherence through a generous principle of charity. Still, as much as this evidence gives us a working sense of what Xenocrates' intermediary mathematicals could have been, and could not have been (i.e., form-numbers), it does not directly respond to Aristotle's specific criticisms of Xenocrates' τὰ μεταξύ; for none of the evidence presented above directly addresses geometricals, which, as far as Xenocrates is concerned, we determined to be the class of objects referred to by Aristotle when he talks about τὰ μεταξύ. Indeed, there is very little evidence (beyond the special case of indivisible lines, which I do not have space to discuss here) of Xenocrates' treatment of geometrical objects. But one passage from Proclus' *Commentary on Plato's Republic* gives us an insight into why Aristotle would be so critical of Xenocrates' τὰ μεταξύ theories. Remarkably, this passage responds to the challenges raised by Plato in *Timaeus* 48b-53d by elaborating a tripartite typology of triangular souls in theological terms:

> Proclus, *Commentary on Plato's Republic* 2.48 Kroll = Xenocrates, fr. 143 Isnardi Parente[2]
>
> [τρισσ]ῶν δὲ ὄντων τῶν τριγώνων τὸ μὲν ἰσόπλευρον, ὡς καὶ Ξενοκράτης ἔλεγεν, ἀνεῖται ταῖς θείαις πάσαις ψυχαῖς ὡς τῷ ἑνὶ κεκρατημέναις· ἡ γὰρ ἰσότης ἑνότης ἐστίν· διὸ καὶ θεῖαι καλοῦνται· τὸ γὰρ ἓν θεότητος ἴδιον. ἐπειδὴ δὲ οὐκ ἦν αὐτὸ ἓν τὸ ἐν ψυχαῖς ἕν, ἀλλὰ μετεχόμενον ὑπὸ τοῦ ἐν αὐταῖς πλήθους, ἰσότης

ἡ ἑνότης γενομένη ταῖς πανταχόθεν ἐκτεθεωμέναις ψυχαῖς καὶ κατὰ πάσας τὰς ζωὰς τὸ πανταχόθεν ἴσον ἀποδέδωκεν τρίγωνον· δι' ὧν καὶ τόδε τὸ πᾶν ἐκθεοῦσιν, τὰς μὲν κινήσεις ταῖς εὐθείαις, τὰς δὲ συνοχὰς τῶν κινήσεων ταῖς γωνίαις. τὸ δὲ ἰσοκελὲς ταῖς μετὰ τὰς θείας ψυχαῖς δαιμονίαις οὔσαις, ἐν αἷς μέσαις οὔσαις ἰσότης τέ ἐστιν καὶ ἀνισοτής, ἕνωσίς τε καὶ ποικιλία δυνάμεων, τῶν βάσεων ἀνομοίων οὐσῶν πρὸς τὰς ἄνωθεν· διότι δὴ καὶ οἱ δαίμονες τοῖς μὲν ἐσχάτοις ἑαυτῶν ἐπάπτονται τῶν χειρόνων, τοῖς δὲ ὑψηλοτέροις τῶν κρεισσόνων, καὶ ἡ ἐπαφὴ τοῖς μὲν ἐξομοιοῖ διὰ τῆς ἰσότητος, τοῖς δὲ συνάπτει διὰ τῆς ἀνισότητος. τὸ δὲ δὴ τρίτον τὸ σκαληνὸν πάντῃ ἀνισούμενον τῶν ψυχῶν ἐστιν ἀνιουσῶν καὶ κατιουσῶν εἰκών, [τοῖς κρείττο]σιν ἀνισουμένων καὶ τοῖς χ[είροσιν]. καὶ γὰρ ἐκείνων ποτὲ μὲν πλέ[ον] ...ται κινούμεναι, ποτὲ δὲ ἔλασσον, [καὶ] τῶν χειρόνων, καὶ τὸ παραδοξότατον, ὅτι καὶ ἑαυτῶν. πάντῃ οὖν αὐταῖς ἀνισουμέναις ἀποδέδοται τὸ πάντῃ ἄνισον.

Of the three types of triangular beings, the equilateral, so Xenocrates claimed, devotes itself to all the divine souls in so far as they are controlled by the One. For equality is unity. Thus they are also called divine, for unity is the property of the divine. But since the one that is in souls is not in-itself, but it participates in the multiplicity that is in them, unity becomes equality in the souls devoted to the divine on all sides and generates in all living beings a triangle equal on all sides. Thereby, they [sc. the wise men of old] also devote the entirety [of the triangle to the divine], both the motions in straight lines and the conjunctions of motions in angles. The isosceles triangle is consecrated to those souls that come after the divine souls and are daemonic. In these souls, which are intermediary (ἐν αἷς μέσαις οὔσαις), are both equality and inequality, unification and diversity of powers, since their bases are unequal to the lines above them. Consequently, then, the daemons attach themselves to the inferior entities at their [lower] extremities, as well as to the superior entities at their higher extremities; and attachment assimilates them to the [superior entities] in virtue of equality, whereas it connects them to the [inferior entities] in virtue of inequality. And, in fact, the third type of souls, the scalene, which is unequal in every way, is the image of those that ascend and descend, the ones which are unequal to the superior and the inferior entities – for, indeed, they [sc. the isosceles souls?], once put into motion, touch

> upon [the superior entities] now more, and now less; and the same with regard to the inferior entities; and, most incredible, the same with regard to one another as well. Thus, to those which are unequal in every way, that which is unequal in all aspects is assigned.

Xenocrates seeks to extend the basic typology of triangles mentioned at *Timaeus* 53c to his theological account of the grades of οὐσίαι, as illustrated by Sextus Empiricus; for Plato only has Timaeus describe the triangles to show how the element fire obtains the properties it has (i.e., it gets them from the properties of the sides and the angles of the triangles that make it up)[43], whereas Xenocrates uses Plato's description to develop a psychic emanation theory that binds the highest and lowest levels of existence. Three types of triangular entities are differentiated: the equilateral, the isosceles, and the scalene. In virtue of falling under the One, the equilateral soul is most divine and most unified, but it is not absolute, like the One above it: because it participates in multiplicity (no doubt through some agency of the Dyad), its unity becomes equality, as all angles and all sides are equal in the equilateral triangle. Second comes the isosceles soul, which features a range of opposites that characterize it, unlike the equilateral: it is both equal and unequal, as well as unified and diverse, as reflected in the qualities its sides and angles take relative to one another. It is described as "daemonic", in the sense that it is divine but not as divine as the equilateral triangle, and it ascends and descends between the superior and inferior entities. The isosceles soul is furthermore described as being "intermediate" (ἐν αἷς μέσαις οὔσαις) between the equilateral and the scalene triangle, in a notable advancement beyond what Timaeus himself says. Moreover, it is the model for the scalene triangle, which is its image, but is marked by inequality in all parts of its existence. Importantly, the intermediary isosceles souls perform actions and are subject to other powers, as they variously "attach themselves" to, i.e. are contiguous with, the superior and inferior entities at both the top and the bottom: they are assimilated by their contact with the equilateral triangles to

[43] Plato, *Timaeus* 53c5-d7. For the text, see n. 22 above.

them, exhibiting the equality that distinguishes them in some parts, whilst also taking on the property of inequality that marks the scalene triangles, as Proclus says, as "now more, now less"[44]. Finally, the intermediate entities are presented as taking on various properties through the language of inherence: equality and inequality, unification and diversification, are expressly described as "in" the isosceles triangles, in virtue of their bases being unequal to the equal lines above them. But the implication is that they have these qualities through participation in and descent from their parent principles, the One and the Indefinite Dyad.

Conclusions

If we return to our summary of Aristotle's criticisms of the theorists of intermediary mathematicals in *Metaphysics* B 2 and M 1-2, we can now see how Xenocrates presents an ideal candidate for these people. Aristotle complained that these people produced theories that are ontologically and epistemologically inefficient, since they tended to populate their world with more and more entities whose job it was to explain how things are, rather than determining the most efficient set of objects to make sense of reality: the list of various beings (the One, the Indefinite Dyad, Heaven/World-Soul, the Olympian gods/stars, the sublunary daemons; the three Fates) nicely fits this criticism. Aristotle furthermore complained that the presence of properties opposed to those of the forms on which the objects were thought to depend in some way was contradictory, and one may consider Xenocrates' ultimate *explanantia* for the propagation of opposing properties, the intermediary demonic isosceles triangle souls, to be ripe for this criticism. Moreover, Xenocrates explicitly asserts that opposing properties of the One and the Dyad inhere in the isosceles triangle souls without fully explaining how this inherence works, something that Aristotle repea-

[44] For the modalities of attachment (ἐπαφή), see Ph.S. Horky, "Theophrastus on Platonic and 'Pythagorean' Imitation", art. cit., p. 698-699.

tedly complains of concerning the purveyors of τὰ μεταξύ theories. In *Metaphysics* K 12, Aristotle explained that what is μεταξύ must be between objects of a different genus, and that it only happens in opposites (of which "divine" and "terrestrial" are not representative); clearly Xenocrates' daemonic isosceles triangle souls fall prey to this criticism as well. Even a cursory examination of the testimonies concerning Xenocrates' philosophy demonstrates that his attempt to fill in the gaps between heaven and earth, as Theophrastus referenced at *Metaphysics* 6a23-b9, leads to the sorts of long-winded stories and speciously made-up doctrines that Aristotle criticized in *Metaphysics* N 3. Hence, we can feel rather certain – without having to consider the supposed Xenocratean doctrine of indivisible lines – that Xenocrates was at least one of the targets, and may have actually been the central target, of Aristotle's attacks against the purveyors of τὰ μεταξύ theories. In a sense, then, we can see that it was Aristotle and Xenocrates who first made the problem of τὰ μεταξύ a mid-century modern problem: the century, however, was the 4th century BCE, and the problem was that the exquisite symmetry thought to underlie the elegant mathematical metaphysics of Xenocrates was actually a façade that covered over a baroque and overstuffed ontology and epistemology – at least according to Aristotle, sadly our most important, but not our most charitable, witness to Xenocrates' philosophy.

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