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DENNIS E. HESSELING. *Gnomes in the Fog: The Reception of Brouwer's Intuitionism in the 1920s.* Basel, Boston, Berlin: Birkhäuser Verlag, 2003. Pp. xxiii + 448. ISBN 3-7643-6536-6.

Reviewed by LEON HORSTEN*

This book is based on the author's doctoral dissertation. As the subtitle indicates, its main aim is to give a historical and conceptual description of the reception of Brouwer's intuitionism in the 1920s. The first three chapters set the stage. In the first chapter, historical precursors of Brouwer's intuitionism (mainly Kronecker and Poincaré) are discussed. In the second chapter, the genesis and development of Brouwer's foundational view are explained. The third chapter contains an overview of the stances that were taken in the face of the foundational crisis in mathematics in the beginning of the twentieth century. The chapters four and five form the core of the book. In these chapters, a detailed description is given of the discussion in the 1920s of, on the one hand, the relation between mathematical existence and mathematical construction (chapter 4), and of, on the other hand, the principle of excluded third (chapter 5). The final chapter situates the foundational debate about Brouwer's intuitionism in its wider cultural and philosophical context.

The author went through almost all the publications from the 1920s (down to newspaper articles about lectures held at universities) that are somehow concerned with Brouwer's intuitionism and discusses each of them in considerable detail. As a result, one finds a detailed commentary on the reactions of little-known authors such as Barzin and Errera side by side with a commentary on Gödel's contribution to the debate.

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The main theme of the book is the public discussion between intuition-ism and formalism that broke out in the 1920s. It emerges that, whereas by the early 1920s Brouwer's position had taken its more or less definitive shape, Hilbert's formalism only acquired its definitive form in and through the discussion with intuitionism. An interesting finding is also that per-haps the deepest contribution to the debate in the 1920s, by Kolmogorov, went almost completely unnoticed. But the most important and astound-ing discovery of the investigation is the widespread misunderstanding and misrepresentation of Brouwer's views, not only by his critics, but also, albeit to a lesser extent, by his supporters. Even though some participants had a fairly accurate grasp of Brouwer's ideas (Gödel, unsurprisingly, is one of them), misunderstandings about Brouwer's stance were by no means confined to minor figures. Even powerful philosophers of mathematics such as Hermann Weyl misunderstood Brouwer on crucial points.

This is the more regrettable because Brouwer's views had an internal coherence and stability that seems absent in the position of almost all of the participants in the debate. Hesseling points out in a very convincing way that these misunderstandings have particularly affected the case of Brouwer's critics. At almost every juncture where criticism against Brouwer is voiced, Hesseling is able to point out in a few sentences how the criticism is based on a misconception of Brouwer's views.

Nevertheless, the debate in the 1920s did contain germs of potentially fruitful and interesting philosophical discussions. For instance, Menger suggested that constructivists and classical mathematicians disagree about the *meaning* of the term 'existence' (p. 200), thereby raising the question to what extent the question about mathematical existence is an ontological one as opposed to a semantical one, and the even more fundamental question of how one should go about trying to decide between the two. But such islands of lucidity were quickly submerged in confusion.

In the series of misunderstandings, the apostles of Brouwer also played their part. And this is, in my opinion, not always brought out as clearly as it could have been in Hesseling's book. Coming back to the question about matters of meaning *versus* matters of fact, Hesseling cites (p. 202) Heyting's reply to Lévy, who maintained that everybody understands the word 'existence' in its usual meaning:

> Voilà une affirmation bien audacieuse, car dès qu'on sort du domaine de la vie cotidienne, où la signification exacte d'un mot a moins d'importance que son efficacité, pour entrer dans le domaine de la philosophie, le sens du mot 'exister' donne lieu à une controverse des plus profonde[s]; c'est sur ce point que se séparent les grandes systèmes.

Hesseling endorses Heyting's remark (p. 202). But it is not so clear that it is correct. It would be uncontroversial, I suppose, to say that the great

metaphysical systems disagree with respect to the question of what exists. But it is less obvious that the great systems differ with respect to the *meaning* of the word existence. Given the profusion of this kind of questionable argumentative moves from both sides of the fence, it is no wonder that a discouraged Brouwer hardly participated at all in the philosophical debate that his intuitionism had sparked.

The author is up-front about the fact that he sides with the intuitionist's cause (pp. xviii–xix). This has not prevented him from giving by and large an accurate picture of the views and arguments of Brouwer's adversaries. Nevertheless, Hilbert's formalist program was in itself somewhat more subtle and ingenious than the author makes it appear, and was driven by intuitions that turned out to be *partially* correct. The author repeatedly stresses that consistency is one thing, soundness another, and that somehow Hilbert could hardly be brought to acknowledge this (see, *e.g.*, pp. 212–213). This is surely correct, and the distinction between the two was clearly borne out by Gödel's incompleteness theorems. But we now know that the consistency (of ZFC, *e.g.*) implies Π_1-soundness—a fact which holds also by intuitionist lights. So Hilbert was not *completely* wrong. And Brouwer, in maintaining that consistency has nothing whatsoever to do with math-ematical truth (p. 32), is for the same reason not completely correct even by his own standards.

Concerning the influences of the wider context of the debate (Weimar Republic, *Lebensphilosophie*, . . .) the results of Hesseling's investigations are less conclusive. It is somewhat surprising that the author does not in this context explore the link with the construction systems in philosophy (also called 'constitution systems'), that came to the foreground in the work of Mach and culminated in Carnap's 'Der logische Aufbau der Welt' (1928), but which have their roots of course in the work of the modern philosophers such as Locke and Hume. After all, a central claim of Carnap's *Aufbau* is that emipirical concepts (such as 'red') and objects are also constructed out of what is immediately given in our consciousness.

But the misgivings which I have expressed just now are no more than minor quibbles. Hesseling has written an important book. On the concep-tual side, Hesseling does not tell us much that is not already known, but he does clearly show how strong the case for intuitionism really is. The chief importance of Hesseling's book, however, lies in its contribution to the his-tory of the foundational debate and its interpretation. Whoever is engaged in investigations into the foundations of mathematics will want to get the historical facts about the debate between intuitionism and formalism and their conceptual interpretation right. For this, he will find Hesseling's book an invaluable source.