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Merging DEL and ETL

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Abstract This paper surveys the interface between the two major logical trends that describe agents' intelligent interaction over time: dynamic epistemic logic (DEL) and epistemic temporal logic (ETL). The initial attempt to "merge" DEL and ETL was made in [12] and followed up by [11] and [29]. The merged framework provides a systematic comparison between these two logical systems and studies new logics of intelligent interaction. This paper presents the main results and the recent developments at the interface between DEL and ETL.

1 Introduction

Two issues are important when describing intelligent interaction. One is how the agents' epistemic states change as a result of informational events. Different informational events change the agents' information differently and this change can be quite subtle. We call this aspect *epistemic dynamics*. The second issue concerns which informational events can take place in the course of the agents' interaction. The information that the agents have and the way that this information changes depend not only on which informational event happens but also on what kind of process governs the agents' interaction. We call this kind of information *protocol information*. One of the goals of this paper is show how both aspects of an interactive situation can be represented in the same formal model.

There are two main logical frameworks that have been developed to represent intelligent interaction: *Dynamic Epistemic Logic* (DEL, see, for example, [23,3,6,9,18]) and *Epistemic Temporal Logic* (ETL, see, for example, [20,34]). Each of these systems is oriented towards *only one* of the above two aspects. In DEL, the epistemic states of the agents is described by an epistemic model and informational change is captured by model transformations induced from event models by a procedure called *product update*. The mechanism of product update provides a systematic method that captures

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the epistemic dynamics of a social situation. However, an underlying assumption in the DEL framework is that any event can happen at any moment. DEL does not provide machinery to describe protocol information. On the other hand, in the ETL framework, the temporal evolution of the agents' epistemic states is represented by sequences of events in *time-branching tree structures*. Such a tree structure represents the protocol information by describing when events can or cannot happen. These events are unanalyzable primitive objects, and so ETL does not provide machinery to systematically represent informational events and their effects. The point here is that DEL provides a *local* perspective by systematically describing informational events and their effects and ETL provides a *global* perspective by explicitly describing the protocol information as branching-time structures.

One natural question is whether we can merge the two perspectives and develop a formal framework that suitably represents *both* epistemic dynamics and protocol information. The initial attempt to deal with this question was made by van Bentham, Gerbrandy and Pacuit in [12] and followed up by van Bentham et al. [11] and Hoshi [29]. The key idea is that successive applications of product update to an initial epistemic model generates an ETL time-branching structure. Given an epistemic model, a set of sequences of event models, called a *protocol*, is assigned to each state of the epistemic model. The protocol assigned to a given state is interpreted as the set of sequences of events that can take place at that state. By applying the product update successively to the epistemic model based on the assigned protocols, ETL tree structures are generated. The generated tree structures represent all possible temporal evolutions of the agents' initial informational states that accord with the protocol information.

There are three perspectives from which we can view the merged framework.

1. Generating ETL tree structures by the mechanisms of DEL provides a concrete way to compare and contrast the two logical frameworks. In several places, the question of how to best compare DEL and ETL has been investigated (e.g., [23, 13, 14, 19]). Using DEL product update to produce ETL models provides a "bridge" between the two logical paradigms allowing us to investigate their precise relationship.
2. The merged framework generalizes models in DEL. As mentioned above, DEL assumes the *universal protocol* where any event can happen at any moment. By introducing protocols, the merged framework has the capability of constraining what events can happen in the course of an interactive situation. So, models in the merged framework drop the assumption of the universal protocol. We can then reinterpret the formal language of DEL over this class of generalized models and investigate new logical systems.
3. Models in the merged framework are ETL models armed with a powerful representational device for describing epistemic dynamics. The models have the time-branching structures of ETL and the event models of DEL together in one framework. As such, they can capture epistemic dynamics and protocol information at the same time. Thus these models are powerful tools for studying concrete scenarios of intelligent interactions.

This paper surveys recent development at the interface between DEL and ETL, mainly focusing on the first two perspectives above (with some discussion of the third perspective). In Section 2, we provide some details about how to merge DEL and ETL. Section 3 then gives a systematic comparison between DEL and ETL. In particular, we discuss the representation theorem that characterizes all ETL models generated from DEL models, which was first proved in [11]. In Section 4, we present some main results of logics over classes of ETL models generated from DEL models. Our main focus will be on the system of *Temporal Public Announcement Logic* (TPAL, [11]), but we will discuss other systems that have been discussed in the literature. In Section 5, we conclude the paper by mentioning other relevant results and recent applications.

2 Generating ETL Models from DEL Models

We start by reviewing the systems of DEL and ETL. Fix a finite set $\mathcal{A}$ of agents and a countable set At of propositional letters.

2.1 Dynamic Epistemic Logic

An *epistemic model* $\mathcal{M}$ is a tuple $\langle W, \sim, V\rangle$, where W is a nonempty set, $\sim: \mathcal{A} \to 2^{W\times W}$, and $V: At \to 2^W$. W is interpreted as a set of possible states. The relation $\sim$ assigns an indistinguishability relation on W for an agent in $\mathcal{A}$. By convention, we will write $w \sim_i v$ for $(w, v) \in \sim(i)$. $V(p)$ represents the set of states where p is true. We denote W, $\sim$ and V by $Dom(\mathcal{M})$, $Rel(\mathcal{M})$, and $Val(\mathcal{M})$ respectively.

The language $\mathcal{L}_{EL}$ of epistemic logic extends that of propositional logic with the knowledge modality operator $[i]$, where $[i]\varphi$ reads as "i knows φ." The semantics of this modality is defined by:

- $\mathcal{M}, w \models [i]\varphi$ iff, for all v such that $w \sim_i v$, $\mathcal{M}, v \models \varphi$.

An *event model* $\mathcal{E}$ is a tuple $\langle E, \to, \mathsf{pre}\rangle$, where E is a nonempty set of events, $\to: \mathcal{A} \to 2^{E\times E}$, and $\mathsf{pre}: E \to \mathcal{L}_{EL}$. E is interpreted as a set of possible events. Given two events, e and f, the intended interpretation of $(e, f) \in \to(i)$ is as "when e happens, an agent i considers it possible that f has happened." When $(e, f) \in\to(i)$, we write $e \to_i f$ by convention. The function pre determines *preconditions* of events. Given $\mathsf{pre}(e) = \varphi$, an event e can happen at a world iff φ is true at the world. Given an event model $\mathcal{E}$, we denote its domain, indistinguishability relation, and precondition function by $Dom(\mathcal{E})$, $\to_{\mathcal{E}}$, and $\mathsf{pre}_{\mathcal{E}}$ respectively. Note that preconditions are restricted to $\mathcal{L}_{EL}$. We will discuss this in Section 5.

Definition 1 (Product Update) The *product update* $\mathcal{M} \otimes \mathcal{E}$ of an epistemic model $\mathcal{M} = (W, \sim, V)$ and an event model $\mathcal{E} = (E, \to, \mathsf{pre})$ is the epistemic model $(W', \sim', V')$ with

1. $W' = \{(w, e) \mid w \in W, e \in E$ and $\mathcal{M}, w \models \mathsf{pre}(e)\}$,
2. $(w, e) \sim'_i (v, f)$ iff $w \sim_i v$ in $\mathcal{M}$ and $e \to_i f$ in $\mathcal{E}$, and
3. $(w, e) \in V'(p)$ iff $w \in V(p)$ for all $p \in \mathsf{At}$.

The language $\mathcal{L}_{DEL}$ extends $\mathcal{L}_{EL}$ with operators $[\mathcal{E}, e]$ for every pointed event model $(\mathcal{E}, e)$ (with $e \in Dom(\mathcal{E})$). The intended interpretation of $[\mathcal{E}, e]$ is "If the event $(\mathcal{E}, e)$ happens, then φ will be true" and truth is defined as follows:

- $\mathcal{M}, w \models [\mathcal{E}, e]$ iff if $\mathcal{M}, w \models \mathsf{pre}(e)$ then $\mathcal{M} \otimes \mathcal{E}, (w, e) \models \varphi$

The dual $\langle\mathcal{E}, e\rangle\varphi$ is defined in the standard way and is intended to mean "The event $(\mathcal{E}, e)$ can happen after which φ will be true."

Using the definitions above, intelligent interaction between agents over time is captured by the successive applications of product update, and the agents' epistemic states at given stages are expressed by the iteration of associated product update and knowledge modalities.

Example 1 (Public Announcement) The simplest kind of event models are *public announcements*. The public announcement $!\varphi$ of a formula φ represents the event in which the true information that φ is *publicly announced*. The public announcement, $!\varphi$, can be thought of as the event model $\mathcal{E}_\varphi = (E, \rightarrow, \mathsf{pre})$, where (i) $E = \{!\varphi\}$, (ii) for each $i \in \mathcal{A}$, $\rightarrow_i = \{(!\varphi, !\varphi)\}$, and (iii) $\mathsf{pre}(!\varphi) = \varphi$. The product update of an epistemic model $\mathcal{M} = (W, \sim, V)$ with an event model $\mathcal{E}_\varphi$ produces the *relativization* of $\mathcal{M}$ containing only the states where φ is true (in $\mathcal{M}$). The system, *Public Announcement Logic* (PAL, e.g. [36,24,5]), deals with this particular kind of event models.

2.2 Epistemic Temporal Logic

Let Σ be a non-empty set and call elements in Σ *events*. A *history* is a finite sequence of events from Σ. We write Σ^* for the set of histories built from elements of Σ. For a history h, we write he for the history h followed by the event e. Given $h, h' \in \Sigma^*$, we write $h \preceq h'$ if h is a *prefix* of h', i.e. there is some $k \in \Sigma^*$ such that $hk = h'$. $H \subseteq \Sigma^*$ is *closed under finite prefix* if, for every $h \in H$ and $h' \preceq h$, $h' \in H$. We denote the empty sequence by λ.

An *ETL model* is a tuple $(\Sigma, H, \sim, V)$ where (i) H does not contain λ and is a subset of Σ^* closed under finite prefix, (ii) $\sim$ is a function from $\mathcal{A}$ to $2^{H \times H}$, and (iii) V is a function from At to 2^H. H represents the temporal structure with $h' = he$ representing the temporal point after the event e has happened at the point h. For each $i \in \mathcal{A}$, the relation $\sim(i)$ (also denoted by $\sim_i$) represents the indistinguishability relation on histories for i. V is a valuation function on H.

Different modal languages describe ETL models (see, for example, [20,26,28]). The minimal language $\mathcal{L}_{ETL}$ of ETL, which we deal with here, extends $\mathcal{L}_{EL}$ with the operator $[e]\varphi$, where $e \in \Sigma$, where $[e]\varphi$ reads as "after the even e happens, φ will be true. The dual $\langle e \rangle$ of $[e]$ is defined in the standard way and $\langle e \rangle\varphi$ reads as "The event e can happen after which φ will be true." It is often natural to extend the language $\mathcal{L}_{ETL}$ with group knowledge operators (e.g., common or distributed knowledge) and more expressive temporal operators (e.g., arbitrary future or past modalities). This may lead to a high complexity of the validity problem (cf. [27,14]). We will discuss those issues in Section 5.

The truth of a formula φ at a history $h \in H$, denoted $\mathcal{H}, h \models \varphi$ is defined inductively. The clause for the temporal operator $[e]$ is:

$$\mathcal{H}, h \models [e]\varphi \text{ iff } he \in H \text{ implies } \mathcal{H}, he \models \varphi$$

2.3 DEL-Generated ETL Models

The basic idea to merge DEL and ETL is that by repeatedly updating an epistemic model with event models, DEL in effect generates ETL models. To generate ETL models from DEL models, we will assign to each world of a given epistemic model a set of sequences of (pointed) event models. Those assigned sets are called *protocols*. Sequences in protocols represent the sequences of events that can take place at a given world. ETL tree structures are generated by applying the product update mechanism successively to the epistemic model based on the assigned protocols. The generated tree structures represent all possible temporal evolutions of agents' initial informational states that accord with protocol information. Below we describe the model construction presented in [11,29].

Let $\mathbb{E}$ be the class of all pointed event models,

$$\mathbb{E} = \{(\mathcal{E}, e) \mid \mathcal{E} \text{ an event model and } e \in D(\mathcal{E})\}.$$

We denote the set of finite sequences of pointed event models by $\mathbb{E}^*$..

Definition 2 (DEL-Protocol) A *DEL-protocol* is a set $\mathsf{P} \subseteq \mathbb{E}^*$ closed under finite prefix. We denote by $\mathit{Ptcl}(\mathbb{E})$ the class of all DEL-protocols, i.e., $\mathit{Ptcl}(\mathbb{E}) = \{\mathsf{P} \mid \mathsf{P} \subseteq \mathbb{E}^* \text{ is closed under finite prefix}\}$.

Definition 3 (State-Dependent DEL-Protocol) Let $\mathcal{M}$ be an epistemic model. A *state-dependent DEL-protocol on* $\mathcal{M}$ (*sd*-DEL-protocol) is any function $\mathsf{p} : \mathit{Dom}(\mathcal{M}) \to \mathit{Ptcl}(\mathbb{E})$. When there is no confusion, we will simply say protocols or *sd*-protocols for DEL-protocols or *sd*-DEL-protocols.

Sd-protocols significantly generalize the usual ETL setting where the *protocol* is assumed to be common knowledge among agents (cf. [20,35]). An *sd*-protocol can assign different protocols to different worlds in a given epistemic model. Consequently, what event can happen at a given moment may not even be known by agents. On the other hand, if an *sd*-protocol p assigns the same protocol, say P, to each world of a given epistemic model, then the protocol P will be common knowledge. This is a special kind of *sd*-protocol, which we call *uniform protocols.*

Definition 4 (Uniform Protocol) An *sd*-DEL-protocol p on $\mathcal{M}$ is a *uniform protocol* on $\mathcal{M}$, if, for all $w \in \mathit{Dom}(\mathcal{M})$, $\mathsf{p}(w) = \mathsf{P}$ for some P. Clearly a given DEL-protocol P induces a uniform protocol on any epistemic model. For this reason, when there is no confusion, we drop the specification of epistemic models and call DEL-protocols *uniform protocols.*

State-dependent and uniform protocols are two extreme cases with many interesting intermediate cases, where agents have only partial knowledge of the type of conversation, experimental protocol, or learning process they are in. One natural example is the assumption that all agents individually know the protocol: for each $w, v \in D(\mathcal{M})$, if $w \sim_i v$, then $\mathsf{p}(w) = \mathsf{p}(v)$.

To introduce the method of generating ETL models from DEL models, we need some notation. Let $\sigma = (\mathcal{E}_1, e_1)(\mathcal{E}_2, e_2) \ldots (\mathcal{E}_n, e_n) \in \mathbb{E}^*$. We denote the length of σ by $\mathsf{len}(\sigma)$, i.e. $\mathsf{len}(\sigma) = n$. When $k \leq \mathsf{len}(\sigma)$, we write $\sigma_{(k)}$ for the initial segment of σ of length k, and σ_k for the kth component of σ. When $k > \mathsf{len}(\sigma)$ or $k = 0$, σ_k and $\sigma_{(k)}$ are the empty sequence λ. Also we write σ^L and σ^R for $\mathcal{E}_1 \cdots \mathcal{E}_n$ and $e_1 \cdots e_n$ respectively. For example, $(\sigma^L)_{(3)} = \mathcal{E}_1\mathcal{E}_2\mathcal{E}_3$ and $(\sigma^R)_3 = e_3$. Clearly, $(\cdot)^L, (\cdot)^R$ on the one hand and $(\cdot)_n, (\cdot)_{(n)}$ on the other commute. Thus, we omit parentheses when there is no danger of ambiguity.

2.3.1 ETL Models Generated by Uniform Protocols

We start by constructing an ETL model from a *uniform* DEL-protocol since the definition is more transparent. However, we stress that the following two definitions are special cases of the more general construction of ETL models from state-dependent DEL protocols, which will be given in Definition 7 and Definition 8.

Definition 5 (σ-Generated Epistemic Model) Given an epistemic model $\mathcal{M} = (W, \sim, V)$ and a finite sequence $\sigma \in \mathbb{E}^*$, we define the *σ-generated epistemic model*, $\mathcal{M}^\sigma = (W^\sigma, \sim^\sigma, V^\sigma)$ as $\mathcal{M} \otimes \sigma_1^L \otimes \sigma_2^L \otimes \ldots \otimes \sigma_{\mathsf{len}(\sigma)}^L$.

Definition 6 (ETL Model Generated from a Uniform DEL-Protocol) Let $\mathcal{M}$ be a pointed epistemic model and P a DEL protocol. The *ETL model generated by $\mathcal{M}$ and* P, $\mathsf{Forest}(\mathcal{M}, \mathsf{P})$, is an ETL model $(Dom(\mathcal{M}) \cup \mathbb{E}, H, \sim, V)$ where

- $H = \bigcup_{\sigma \in \mathsf{P}} W^\sigma$,
- for each $i \in \mathcal{A}$, $\sim_i := \bigcup_{\sigma \in \mathsf{P}} \sim_i^\sigma$, and
- for each $p \in \mathsf{At}$, $V(p) := \bigcup_{\sigma \in \mathsf{P}} V^\sigma(p)$

We will omit $Dom(\mathcal{M}) \cup \mathbb{E}$ and write $\mathsf{Forest}(\mathcal{M}, \mathsf{p}) = (H, \sim, V)$, where there is no confusion. We also identify $(w, \sigma_1, \ldots, \sigma_{\mathsf{len}(\sigma)})$ in $\mathcal{M}^\sigma$ with a history $w\sigma$.

$\mathsf{Forest}(\mathcal{M}, \mathsf{P})$ represents all possible evolutions of the system obtained by updating $\mathcal{M}$ with sequences from P. It is straightforward to verify the following proposition.

Proposition 1 *For every epistemic model $\mathcal{M}$ and every uniform DEL-protocol* P*,* $\mathsf{Forest}(\mathcal{M}, \mathsf{P})$ *is an ETL model.*

Example 2 (ETL Models Generated from Uniform Protocols) Let $\mathcal{M}$ be an epistemic model that have three worlds, w, v, u, where p is true only at w and v and q is true only at w. An agent 1 cannot distinguish w and v and an agent 2 cannot distinguish v and u. Let P be a uniform protocol consisting of sequences of public announcements such that $\mathsf{P} = \{!p!q, !\neg p!\neg q\}$. $\mathsf{Forest}(\mathcal{M}, \mathsf{P})$ can be visualized as in Figure 1. At the bottom, we have three nodes (circled for emphasis) corresponding to $\mathcal{M}$. The model evolves as the permitted sequences of announcements in P are applied. Points in the figure are circled to indicate the evolution of the original model into new models.

2.3.2 Construction with State-Dependent Protocols

Now we present the method to generate ETL models from *sd*-DEL-protocols in general. The basic intuition is the same here. We apply product update based on the sequences of pointed event models that appear in protocols. The process was simple for uniform protocols, since the sequence of events that can happen is the same at all states in each epistemic model. For general *sd*-protocols, events that can happen may differ from a state to state. Thus, even if an event $(\mathcal{E}, e)$ is in the protocol at a world w in an epistemic model $\mathcal{M}$, it may not be in the protocol at another world v. In this case, whether or not the precondition of e is true at v, we cannot create the new node $v(\mathcal{E}, e)$. This means that, dealing with general *sd*-protocols, we cannot simply apply sequences of event models allowed in protocols. In applying product update with an event model, we need to exclude the worlds where the event model is not allowed to happen, as well as the world where the precondition is not satisfied. This is taken care of in the following definitions that generalize Definition 5 and 6.

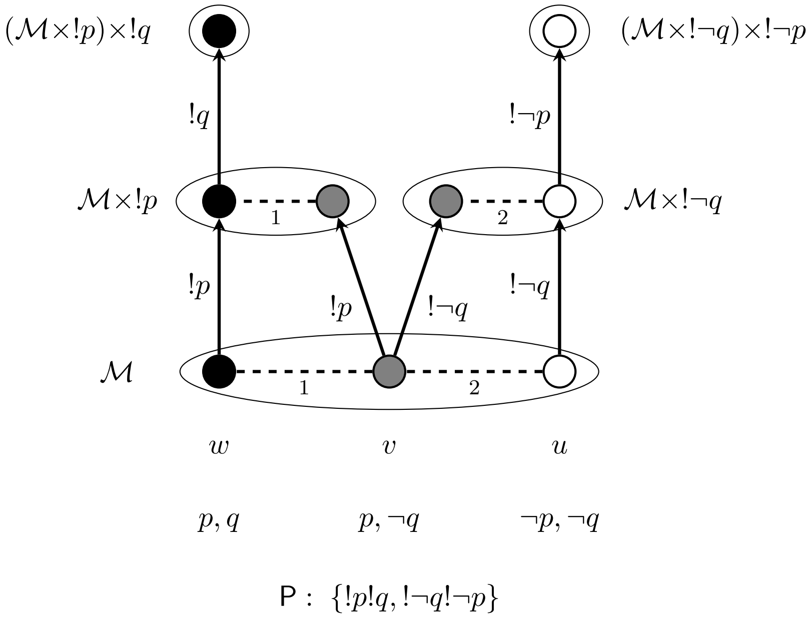

Fig. 1 ETL Models Generated from Uniform Protocols

Definition 7 (σ^L-Generated Model) Let $\mathcal{M} = \langle W, \sim, V \rangle$ be an epistemic model and p, a state-dependant DEL-protocol on $\mathcal{M}$. Given a sequence $\sigma \in \mathbb{E}^*$, the *σ^L-generated model under* p,

$$\mathcal{M}^{\sigma^L,\mathsf{p}} = (W^{\sigma^L,\mathsf{p}}, \sim^{\sigma^L,\mathsf{p}}, V^{\sigma^L,\mathsf{p}}),$$

is defined by induction on the initial segment of σ^L:

- $W^{\sigma^L_{(0)},\mathsf{p}} := W$, for each $i \in \mathcal{A}$, $\sim_i^{\sigma^L_{(0)},\mathsf{p}} := \sim_i$ and $V^{\sigma^L_{(0)},\mathsf{p}} := V$.
- $w\tau \in W^{\sigma^L_{(n+1)},\mathsf{p}}$ iff
  1. $w \in W$,
  2. $\sigma^L_{(n+1)} = \tau^L$,
  3. $w\tau_{(n)} \in W^{\sigma^L_{(n)},\mathsf{p}}$,
  4. $\tau \in \mathsf{p}(w)$, and
  5. $\mathcal{M}^{\sigma^L_{(n)},\mathsf{p}}, w\tau_{(n)} \models \mathsf{pre}_{\tau^L_n}(\tau^R_{n+1})$.
- For each $w\tau, v\tau' \in W^{\sigma^L_{(n+1)},\mathsf{p}}$ $(0 < n < \mathsf{len}(\sigma^L))$, $w\tau \sim_i^{\sigma^L_{(n+1)}} v\tau'$ iff
  1. $w\tau_{(n)} \sim_i^{\sigma^L_{(n)},\mathsf{p}} v\tau'_{(n)}$, and
  2. $(\tau^R_{n+1}, (\tau'_{n+1})^R) \in\to (i)$ in τ^L_{n+1}.
- For each $p \in \mathsf{At}$, $V^{\sigma^L_{(n+1)},\mathsf{p}}(p) = \{w\sigma \in W^{\sigma^L_{(n+1)},\mathsf{p}} \mid w \in V(p)\}$.

Definition 8 (DEL-Generated ETL Model) Let $\mathcal{M} = (W, \sim, V)$ be an epistemic model and p a state-dependent DEL protocol on $\mathcal{M}$. An ETL model $\mathsf{Forest}(\mathcal{M}, \mathsf{p}) = (H, \sim', V')$ is defined as follows:

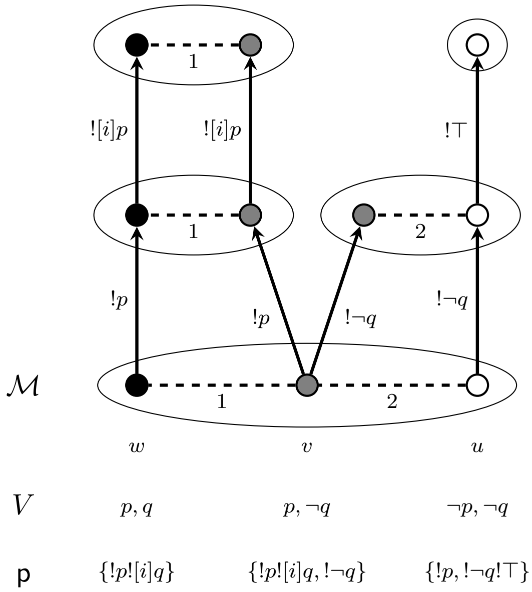

Fig. 2 DEL-Generated ETL Models

- $H =_{\text{def}} \{h \mid \text{there is a } w \in W,\, \sigma \in \bigcup_{w\in W} \mathsf{p}(w) \text{ with } h = w\sigma \in W^{\sigma^L,\mathsf{p}}\}$.
- For all $h, h' \in H$ with $h = w\sigma$ and $h' = v\sigma'$, $h \sim_i h'$ iff$_{\text{def}}$ $w\sigma \sim_i^{\sigma^L,\mathsf{p}} v\sigma'$.
- For each $p \in \mathsf{At}$ and $h = w\sigma \in H$, $h \in V'(p)$ iff$_{\text{def}}$ $h \in V^{\sigma^L,\mathsf{p}}(p)$

Also we define the class $\mathbb{F}_{sd}$ of DEL-generated ETL models by

$$\mathbb{F}_{sd} = \{\mathsf{Forest}(\mathcal{M},\mathsf{p}) \mid \mathcal{M} \text{ an epistemic model and } \mathsf{p} \text{ an } sd\text{-protocol}\}.$$

Proposition 2 ([11]) *For every epistemic model $\mathcal{M}$ and* sd-*DEL-protocol* p *on* $\mathcal{M}$, $\mathsf{Forest}(\mathcal{M},\mathsf{p})$ *is an ETL model.*

Example 3 (DEL-Generated ETL Model) Let us illustrate the construction by the following example. Take an epistemic model $\mathcal{M}$ given in Example 2. ($\mathcal{M}$ consists of w, v, u, in which p is true only at w, v and q is true only at w.) Let p be an sd-protocol on $\mathcal{M}$ such that $\mathsf{p}(w) = \{!p![i]q\}$, $\mathsf{p}(v) = \{!p![i]q, !\neg q\}$, $\mathsf{p}(u) = \{!p, !\neg q!\top\}$. The ETL model we construct from $\mathcal{M}$ and p can be visualized as in Figure 2.

3 Representation Theorem

DEL-generated ETL models allow us to systematically compare DEL and ETL. The main result here is a *representation theorem* (Theorem 1 [11,29]) characterizing the class of DEL-generated ETL models. The result is an improvement of a previous characterization result found in [5] and provides a precise comparison between the DEL and ETL frameworks. We start by introducing the relevant properties.

Definition 9 (Synchronicity, Perfect Recall, Uniform No Miracles) Let $\mathcal{H} = (\Sigma, H, \sim, V)$ be an ETL model. $\mathcal{H}$ satisfies:

- **Synchronicity** iff for all $h, h' \in \mathsf{H}$, if $h \sim_i h'$ then $\mathsf{len}(h) = \mathsf{len}(h')$ ($\mathsf{len}(h)$ is the number of events in h).
- **Perfect Recall** iff for all $h, h' \in \mathsf{H}$, $e, e' \in \Sigma$ with $he, h'e' \in \mathsf{H}$, if $he \sim_i h'e'$, then $h \sim_i h'$
- **Uniform No Miracles** iff for all $h, h' \in \mathsf{H}$, $e, e' \in \Sigma$ with $he, h'e' \in \mathsf{H}$, if there are $h'', h''' \in \mathsf{H}$ with $h''e, h'''e' \in \mathsf{H}$ such that $h''e \sim_i h'''e'$ and $h \sim_i h'$, then $he \sim_i h'e'$. $\triangleleft$

Another property is needed since we are assuming that product update does not change propositional valuations. An ETL model $\mathcal{H}$ satisfies *propositional stability* provided for all histories h in $\mathcal{H}$, events e with he in $\mathcal{H}$ and all propositional variables p, if p is true at h then p is true at he. This property is not crucial for the result and can be dropped provided we allow product update to change the ground facts as in [10].

Definition 10 (Isomorphism between ETL models) An *isomorphic map* between two ETL models, $\mathcal{H} = (\Sigma, H, \sim, V)$ and $\mathcal{H}' = (\Sigma', H', \sim', V')$, is a one-to-one function f from Σ onto Σ' such that, for every $\sigma_1, \ldots, \sigma_n, \tau_1, \ldots, \tau_m \in \Sigma$, $i \in \mathcal{A}$ and $p \in \mathsf{At}$,

- if $\sigma_1 \ldots \sigma_n \sim_i \tau_1 \ldots \tau_m$, then $f(\sigma_1) \ldots f(\sigma_n) \sim'_i f(\tau_1) \ldots f(\tau_m)$, and
- if $\sigma_1 \ldots \sigma_n \in V(p)$, then $f(\sigma_1) \ldots f(\sigma_n) \in V'(p)$.

If there is an isomorphic map between $\mathcal{H}$ and $\mathcal{H}'$, we say $\mathcal{H} and \mathcal{H}'$ are *isomorphic.*

Theorem 1 ([11, 29]) *An ETL model is isomorphic to some model in* $\mathbb{F}_{sd}$ *iff it satisfies propositional stability, synchronicity, perfect recall, and uniform no miracles.*

The representation theorem can be extended to characterize classes of ETL models generated from other kinds of DEL-protocols. One natural class is the class of ETL models generated from uniform protocols. For this, we need the following properties.

Definition 11 (Epistemic Bisimulation Invariance) Let $\mathcal{H} = (\Sigma, H, \sim, V)$ and $\mathcal{H}' = (\Sigma, H, \sim', V)$ be two ETL models. A relation $Z \subseteq H \times H'$ is an *epistemic bisimulation* provided that, for all $h \in H$ and $h' \in H'$, if hZh', then

(prop) h and h' satisfy the same propositional formulas,
(forth) for every $g \in H$, if $h \sim_i g$ then there exists $g' \in H'$ with $h' \sim_i g'$ and gZg'
(back) for every $g' \in H'$, if $h' \sim'_i g'$ then there exists $g \in H$ with $h \sim_i g$ and gZg'.

If Z is an epistemic bisimulation and hZh' then we say h *and* h' *are epistemically bisimilar*. An ETL model $\mathcal{H}$ satisfies *epistemic bisimulation invariance* iff for all epistemically bisimilar histories $h, h' \in \mathsf{H}$, if $he \in \mathsf{H}$ then $h'e \in \mathsf{H}$.

We also need one technical property. An ETL model *satisfies the finiteness assumption*, if, for each n, the set $\{h \mid he \in \mathsf{H}$ and $\mathsf{len}(h) = n\}$ is finite. For the reason that we need this property, see [11, 29].

Theorem 2 ([11, 29]) *If an ETL model is isomorphic to some model in* $\mathbb{F}_{uni}$ *then it satisfies propositional stability, synchronicity, perfect recall, uniform no miracles, as well as epistemic bisimulation invariance.*

If an ETL model $\mathcal{H}$ *satisfies the finiteness assumption, propositional stability, synchronicity, perfect recall, uniform no miracles, and epistemic bisimulation invariance, then* $\mathcal{H}$ *is isomorphic to some model in* $\mathbb{F}_{uni}$.

We can also consider the class of ETL models consisting of public announcements. Let PAL be the class of all protocols consisting only of public announcements. Recall that $\mathbb{F}(\mathsf{PAL}) = \{\mathsf{Forest}(\mathcal{M}, \mathsf{P}) \mid \mathcal{M}$ an epistemic model and $\mathsf{P} \in \mathsf{PAL}\}$. (This class $\mathbb{F}(\mathsf{PAL})$ is one of the classes that we will present in the next section.) The class is characterized by the following representation theorem.

Proposition 3 (PAL-Generated Models [11]) *An ETL model $(\Sigma, H, \sim, V)$ is isomorphic to some model in $\mathbb{F}(\mathsf{PAL})$ iff it satisfies the minimal properties of Theorem 1, and:*

- *for all $h, h', he, h'e \in H$, if $h \sim_i h'$, then $he \sim_i h'e$ (all events are reflexive)*
- *for all $h, h' \in H$, if $he \sim_i h'e'$, then $e = e'$ (no different events are linked).*

4 Reinterpretation of DEL

The merged models discussed in Section 2.3 generalize standard DEL models. As discussed in the introduction, DEL assumes the *universal protocol* where any event can happen at any moment. However, in the merged framework, protocols restrict the events that can happen at a given moment.

Given this consideration, we can reinterpret the language of DEL over the class of DEL-generated ETL models. The idea is to interpret the event operator $\langle \mathcal{E}, e \rangle$ in DEL as a labeled temporal modality in ETL as follows: Given $\mathsf{Forest}(\mathcal{M}, \mathsf{p}) \in \mathbb{F}_{sd}$ and h in $\mathsf{Forest}(\mathcal{M}, \mathsf{p})$,

$$\mathsf{Forest}(\mathcal{M}, \mathsf{p}), h \models \langle \mathcal{E}, e \rangle \varphi \quad \text{iff} \quad \mathsf{Forest}(\mathcal{M}, \mathsf{p}), h(\mathcal{E}, e) \models \varphi.$$

An easy induction shows that this model transformation preserves truth in the following sense.

Proposition 4 ([11]) *Let $\mathbb{E}^*$ be the DEL-protocol consisting of* all *finite sequences of pointed event models in DEL. Let $\mathcal{M}$ an epistemic model with $w \in Dom(\mathcal{M})$: For any formula $\varphi \in \mathcal{L}_{del}$,*

$$\mathcal{M}, w \models \varphi \textit{ iff } \mathsf{Forest}(\mathcal{M}, \mathbb{E}^*), w \models \varphi.$$

This proposition explains a common intuition about linking DEL to ETL. Also, it makes it explicit that DEL-generated models lift the assumption of the universal protocol.

4.1 Temporal Public Announcement Logic (TPAL)

Given the above reinterpretation of the language of DEL, we can study logics over classes of DEL-generated ETL models. Each set $\mathbf{X}$ of DEL-protocols induces a class $\mathbb{F}(\mathbf{X})$ of DEL-generated ETL models and we can look at the axiomatization of such interesting classes models. For some specific combinations of model classes and logical languages, the answers are already already known. For example, recall $\mathbb{E}^*$ is the set of *all* finite sequences of DEL event models — i.e., the forest of *all possible* DEL event structures. Then $\mathbb{F}(\mathbb{E}^*)$ is the class consisting of all DEL-generated ETL models. Its logic (with respect to the language $\mathcal{L}_{DEL}$) can be axiomatized using the well-known reduction axioms: indeed this is the standard completeness theorem for DEL (cf. [3]).

In this section, we focus on the logic of the class of ETL models generated from protocols consisting of *public announcements*. The logic is called *Temporal Public Announcement Logic* (TPAL) and was first studied in [12] and extended in [11, 29]. Public announcements, as we saw in Section 2, are the simplest kind of event models. Nonetheless, the methods used to study TPAL can be applied to logics over other classes of DEL-generated ETL models, as shown in [32, 29]. We will discuss other logics in Section 5.

Definition 12 (PAL-Protocol) Let PAL be the set of public announcements in $\mathbb{E}$, i.e. $\{!\varphi \mid \varphi \in \mathcal{L}_{EL}\}$. A *PAL-protocol* is a set $\mathsf{P} \subseteq \mathsf{PAL}^*$ closed under finite prefix. We denote the set of PAL-protocols by $\mathit{Ptcl}(\mathsf{PAL})$. A *state-dependent PAL-protocol* (*sd*-PAL-protocol) p on an epistemic model $\mathcal{M}$ is a function that assigns a PAL-protocol to each world in $\mathcal{M}$ a PAL-protocol. We denote the class of *sd*-PAL-protocols by $\mathbb{PAL}$.

Using the above notation, we can denote the classes of ETL models generated from *sd*-PAL-protocols and uniform PAL-protocols by $\mathbb{F}(\mathbb{PAL})$ and $\mathbb{F}(\mathit{Ptcl}(\mathsf{PAL}))$ respectively. We now look at the axiomatization of both $\mathbb{F}(\mathbb{PAL})$ and $\mathbb{F}(\mathit{Ptcl}(\mathsf{PAL}))$.

Restricting attention to public announcements simplifies the definitions for generating ETL models.

Definition 13 (cf. Definition 7) Let $\mathcal{M} = (W, \sim, V)$ be an epistemic model, and p an *sd*-PAL-protocol on $\mathcal{M}$. We define

$$\mathcal{M}^{\sigma,p} = (W^{\sigma,\mathsf{p}}, \sim^{\sigma,\mathsf{p}}, V^{\sigma,\mathsf{p}})$$

by induction on the length of σ:

- $W^{\sigma_0,\mathsf{p}} = W$, for each $i \in \mathcal{A}$, $\sim_i^{\sigma_0,\mathsf{p}} = \sim_i$ and $V^{\sigma_0,\mathsf{p}} = V$.
- $w\sigma_{m+1} \in W^{\sigma_{m+1},\mathsf{p}}$ iff (1) $w \in W$, (2) $\mathcal{M}^{\sigma_m,\mathsf{p}}, w\sigma_m \models \varphi_{m+1}$, and (3) $\sigma_{m+1} \in \mathsf{p}(w)$.
- For each $w\sigma_{m+1}, v\sigma_{m+1} \in W^{\sigma_{m+1},\mathsf{p}}$, $w\sigma_{m+1} \sim_i^{\sigma_{m+1},\mathsf{p}} v\sigma_{m+1}$ iff $w \sim_i v$.
- For each $p \in \mathsf{At}$, $V^{\sigma_{m+1},\mathsf{p}}(p) = \{w\sigma_{m+1} \in W^{\sigma_{m+1},\mathsf{p}} \mid w \in V(p)\}$. $\triangleleft$

Definition 14 (cf. Definition 8) Let $\mathcal{M} = (W, \sim, V)$ be an epistemic model and p an *sd*-PAL-protocol on $\mathcal{M}$. A *PAL-generated ETL model* $\mathsf{Forest}(\mathcal{M}, \mathsf{p}) = (H, \sim', V')$ is defined as follows:

- $H =_{\text{def}} \{h \mid h \in W^{\sigma,\mathsf{p}}$ for some $\sigma \in \bigcup_{w \in W} \mathsf{p}(w)\}$.
- For all $h, h' \in H$ with $h = w\sigma$ and $h' = v\sigma$ for some $\sigma \in \bigcup_{w \in W} \mathsf{p}(w)$, $h \sim_i h'$ iff$_{\text{def}}$ $h \sim_i^{\sigma,\mathsf{p}} h'$.
- For each $p \in \mathsf{At}$, $h \in V'(p)$ iff$_{\text{def}}$ $h \in V^{\sigma,\mathsf{p}}(p)$, where $h = w\sigma$ for some $\sigma \in \bigcup_{w \in W} \mathsf{p}(w)$. $\triangleleft$

The language $\mathcal{L}_{TPAL}$ of TPAL extends $\mathcal{L}_{EL}$ with the operator $[!\varphi]$ where $\varphi \in \mathcal{L}_{EL}$. (Note the restriction to $\mathcal{L}_{EL}$. We will discuss this in Section 5). The intended interpretation of $[!\varphi]\psi$ is "After the public announcement that φ, ψ will be true." The dual $\langle !\varphi \rangle$ is defined in the standard way and $\langle !\varphi \rangle \psi$ and means "The public announcement that φ can be made after which ψ will be true." Truth is defined with respect to models in the class $\mathbb{F}(\mathbb{PAL})$: Suppose $\mathcal{H} = \mathsf{Forest}(\mathcal{M}, p) = (H, \sim, V)$, then define

$$\mathcal{H}, h \models \langle !\psi \rangle \varphi \quad \text{iff} \quad h!\psi \in H \text{ and } \mathcal{H}, h!\psi \models \varphi.$$

The usual semantic notions are defined in the standard way.

Now that the public announcement operator is interpreted over the class of ETL models, the following standard PAL validities become invalid (over the class of TPAL models):

(**A**) $\models \langle !p\rangle\langle !q\rangle\varphi \leftrightarrow \langle !(p \wedge q)\rangle\varphi$ (with $p, q \in \mathsf{At}$)
(**B**) $\models \langle !\varphi\rangle \leftrightarrow \varphi$

The validity of **A** in PAL illustrates that sequences of public announcements are identified with some single announcements in PAL. On the other hand, it is invalid in TPAL, since a protocol may not allow the single announcement $!(p \wedge q)$ even when it allows the sequence of announcements $!p!q$. The validity of **B** in PAL reflects the general assumption in DEL that every event can happen if its precondition is true, which equates the announceability of a formula and the truth of it. TPAL removes this assumption and distinguishes announceability and truth (while it still assumes the truthfulness of announcements and validates the left-to-right direction, as we will see shortly in the axiomatization). Because of the invalidity of the principle, the standard reduction axioms of PAL do not hold. Consequently we cannot appeal to the compositional analysis via reduction axioms, as in [3], for the completeness result of TPAL. We need to give a separate completeness argument with the following axiomatization.

Definition 15 (Axiomatization of TPAL) The axiomatization TPAL consists of the following axiom schemes and inference rules.
Axioms

PC Propositional validities
iK $[i](\varphi \to \psi) \to ([i]\varphi \to [i]\psi)$
!K $[!\theta](\varphi \to \psi) \to ([!\theta]\varphi \to [!\theta]\psi)$
R1 $\langle !\theta\rangle p \leftrightarrow \langle !\theta\rangle\top \wedge p$ (with $p \in \mathsf{At}$)
R2 $\langle !\theta\rangle\neg\varphi \leftrightarrow \langle !\theta\rangle\top \wedge \neg\langle !\theta\rangle\varphi$
R3 $\langle !\theta\rangle[i]\varphi \leftrightarrow \langle !\theta\rangle\top \wedge [i](\langle !\theta\rangle\top \to \langle !\theta\rangle\varphi)$
A1 $\langle !\theta\rangle\top \to \theta$

Inference Rules

MP If $\vdash \varphi$ and $\vdash \varphi \to \psi$, then $\vdash \psi$.
iN If $\vdash \varphi$, then $\vdash [i]\varphi$ for any $i \in \mathcal{A}$.
!N If $\vdash \varphi$, then $\vdash [!\theta]\varphi$ for any $!\psi \in \mathsf{PAL}$. $\lhd$

First note that **R1-3** are similar to the reduction axioms in PAL. However they differ from reduction axioms in PAL, since they have formulas of the form $\langle !\theta\rangle\top$ on the right hand side of the equivalences, where the PAL axioms have θ, the precondition of the public announcement $!\theta$. This is exactly because the announceability of φ and the truth of φ are distinguished in TPAL by protocols, as mentioned above. Formulas in TPAL thus do not reduce to formulas in EL.

The completeness proof for TPAL is a variant of the standard Henkin construction. We construct the canonical ETL model from the set of maximal consistent sets in TPAL (*mcs* below). The key idea is that each maximally consistent set contains the information about 'legal' histories of public announcements, since sentences with public announcement operators describe future states. Starting by constructing an epistemic model from the set of maximally consistent sets in the standard way, we can read off the canonical state-dependent PAL protocol. For the details of the proof, see [11,29].

Theorem 3 ([11]) TPAL *is sound and strongly complete with respect to the class of ETL models* $\mathbb{F}(\mathbb{PAL})$.

Also the standard finite model argument applies and TPAL is known to be decidable. However, the exact complexity of the system is still unknown.

Theorem 4 ([29]) *The satisfiability problem for the logic TPAL is decidable.*

4.2 Logics of Other Classes of Protocols

TPAL is the logic over the class of models generated from a particular kind of protocols. Logics over other classes of protocols have also been considered. One natural class to consider is the class $\mathbb{F}_{sd}$ of all DEL-generated ETL models. The axiomatization can be given for the class of models in a similar way and the logic is called *Temporal Dynamic Epistemic Logic* (TDEL) (Hoshi and Yap [32]). The axioms of TDEL look similar to the reduction axioms of DEL, as those of TPAL did to the reduction axioms of PAL, reflecting the fact that the "executability" of events is distinguished from the truth of preconditions of events. For illustration, consider the following axiom in TDEL:

$$\langle \mathcal{E}, e\rangle[i]\varphi \leftrightarrow \langle \mathcal{E}, e\rangle\top \wedge \bigwedge_{\{e\in Dom(\mathcal{E}) \mid (e,f)\in \to_{\mathcal{E}}(i)\}} [i](\langle \mathcal{E}, f\rangle\top \to \langle \mathcal{E}, f\rangle\varphi)$$

If "$\langle \mathcal{E}, e\rangle\top$" is replaced with the precondition of the event e, i.e. "$\mathsf{pre}_{\mathcal{E}}(e)$", we will get the corresponding DEL reduction axiom for the knowledge modality $[i]$. The completeness proof is similar to that of TPAL and a finite model argument can be given to prove the decidability.

Another kind of class of protocols that we can consider are ETL models generated from uniform protocols. We can give an axiomatization for such classes by introducing the existential modality E, where $E\varphi$ mean that "φ is true at some history with the same sequence of announcements". The existential modality is defined as:

$$\mathcal{H}, w\sigma \models E\varphi \;\text{ iff }\; \exists v \in W \;\text{ such that } v\sigma \text{ is in } \mathcal{H} \text{ and } \mathcal{H}, v\sigma \models \varphi.$$

This operator functions as an existential modality at each 'stage' of successive public announcements. With the operator, we can express the uniformity of PAL-protocols by:

Uni $\quad \langle !\theta\rangle\top \to U(\theta \to \langle !\theta\rangle\top)$.

The axiomatization of the class of PAL-generated ETL models can be given by adding **Uni** and the standard axioms for existential modality to TPAL ([11]). A similar method works for TDEL ([29]).

While TPAL looks at protocols generated by protocols consisting of public announcements, we can consider protocols generated by other (fixed) types of DEL event models. For example, an interesting DEL event to consider here is a *semi*-public announcement where a public announcement is made to a subgroup of the entire set of agents. These can also be viewed as public events where only a subgroup of the agents observe a piece of true information while the others do not (though it is public who observes the true fact). Hoshi and Pacuit [31] axiomatize this variant of TPAL relating it to logics of awareness (cf. [21]). For additional results in this direction see [32].

5 Other Results and Relevant Work

Section 2 provides the formal details of a logical framework that *merges* DEL and ETL. The previous sections have emphasized two new perspectives that this framework provides. First, it allows for a systematic comparison between DEL and ETL as witnessed by the representation theorems of [11, 29] characterizing classes of DEL-generated ETL models. Second, reinterpreting the language of DEL over this new class of models calls for new axiomatic systems. We have focused on new completeness results for TPAL and TDEL and discussed other relevant axiomatization in the literature. We conclude this paper by looking at other relevant work.

5.1 Dynamic Doxastic Logic and Doxastic Temporal Logic

The current paper has focused on the interface between DEL and ETL. These systems are designed to represent knowledge and its dynamics in interactive situations. However, in many situations, the agents' information is uncertain or even erroneous, and they may have to make choices based on this "softer" informaiton. A number of logical frameworks have been developed to describes agents *beliefs* over time. Among such systems are *Dynamic Doxastic Logic* (DDL, van Benthem [7], Baltag and Smets [4]) and *Doxastic Temporal Logic* (DTL, cf. Halpern and Friedman [22] and Bonanno [15]). DDL describes beliefs using plausibility orderings on possible states and captures the informational change as model transformations. DTL represents the temporal evolution of the agents' doxastic states by branching-time tree structures with plausibility orderings over the histories. This is analogous to the initial situation with DEL and ETL discussed in the Introduction. An obvious question here is can we *merge* DDL and DTL in the style discussed in this paper? This question has been investigated first in van Benthem and Dégremont [8] and studied further in Dégremont [16]. They have obtained results corresponding to the representation theorem and axiomatization results discussed in Section 3 and 4.

5.2 Extension of Logics

Section 4 looked at the logics of classes of DEL-generated ETL models. Those logical systems can be extended in two directions. One direction is to extend the the language that describes the DEL-generated ETL models. The minimal language of TPAL has the two modalities: the knowledge modality $[i]$ and the public announcement modality $[!\varphi]$. A natural extension of the epistemic part of the language is to consider group knowledge operators such as *common knowledge*. An axiomatization of TPAL and TDEL with common knowledge is presented in [29]. Other extensions focus on the temporal component. An example of an extension in this direction is the addition of a modality that quantifies over *future* events: i.e., "some event can happen after which...", "some sequences of events can happen after which...", and so on. Operators of this kind have been investigated in [2] as extensions of the basic DEL framework. We can also add modal operators that describe what happened in the *past*: "previously,...", "before the event e happens...", and so on. Such past-time operators have been discussed in a variety of contexts [30, 32, 38, 37].

An alternative type of extension generalizes the model construction from Section 2.3. Recall from Section 2.1 that the preconditions in the event models are restricted to *epistemic formulas* (formulas in $\mathcal{L}_{EL}$) and so cannot contain temporal or event modalities. This restriction does not seem substantial in the the standard DEL setting since formulas in DEL are provably equivalent to epistemic formulas via the *reduction axioms*. However, full reduction axioms are not available in TDEL (cf. Section 4) and the frameworks discussed in this paper do not provide a way to express preconditions by formulas containing event operators. This can be an obstacle when using these systems to model concrete scenarios. For instance, in TPAL cannot deal with public announcements expressing what *will* be true in the future. See [29] for an extensive discussion of ways to lift this restriction (cf. also [39]).

5.3 Applications

DEL-generated ETL models are powerful tools for capturing epistemic dynamics and representing protocol information. These models have been used to investigate different aspects of intelligent interaction: for example, [29] has investigated certain philosophical issues, such as Fitch's paradox and the epistemic closure principle. Other authors have found applications in formal learning theory [25] and in game theory [16] (cf. also [33] for a discussion of other applications).

Finally, we remark that the representation of protocol information in the merged framework suggests that there is an interesting logical 'protocol theory' ready to be developed further. Situations of intelligent interaction often involves a variety of restrictions on informational events that can happen. One interesting question is whether or not agents can reach an epistemic state of interest, such as a state of common knowledge, in such restricted situations. Such 'reachability' questions have been discussed by various communities. For example, this is an important question when formally reasoning about *security protocols* (see [33] for some relevant references of applications of DEL and ETL on this topics). It is also one way to think about Aumann's classic "agreeing to disagree" theorem [1] and the resulting literature. Dégremont and Roy [17] have an extensive discussion of these results in the basic DEL framework. These application areas provide fertile ground for concrete scenarios that the logical frameworks discussed in this paper can be used to study.

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