The Logic of Theory Assessment*

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Abstract

This paper starts by indicating the analysis of Hempel's conditions of adequacy for any relation of confirmation (Hempel 1945) as presented in Huber (submitted). There I argue contra Carnap (1962, §87) that Hempel felt the need for two concepts of confirmation: one aiming at plausible theories and another aiming at informative theories. However, he also realized that these two concepts are conflicting, and he gave up the concept of confirmation aiming at informative theories.

The main part of the paper consists in working out the claim that one can have Hempel's cake and eat it too – in the sense that there is a logic of theory assessment that takes into account both of the two conflicting aspects of plausibility and informativeness. According to the semantics of this logic, α is an acceptable theory for evidence β if and only if α is both sufficiently plausible given β and sufficiently informative about β. This is spelt out in terms of ranking functions (Spohn 1988) and shown to represent the syntactically specified notion of an assessment relation.

The paper then compares these acceptability relations to explanatory and confirmatory consequence relations (Flach 2000) as well as to non-monotonic consequence relations (Kraus & Lehmann & Magidor 1990). It concludes by relating the plausibility-informativeness approach to Carnap's positive relevance account, thereby shedding new light on Carnap's analysis as well as solving another problem of confirmation theory.

1 Hempel's Logic of Confirmation

In his (1945) Hempel presents the following conditions of adequacy for any relation of confirmation $\mid\!\sim\ \subseteq \mathcal{L} \times \mathcal{L}$ on some language $\mathcal{L}$ (I have added the name for 3.1), where $\vdash$ is the classical consequence relation and '$A \vdash B$' is short for '$\{A\} \vdash B$'. For any sentences $E, H, H' \in \mathcal{L}$,

1. Entailment Condition: If $E \vdash H$, then $E \mid\!\sim H$.
2. Consequence Condition: If $\{H \in \mathcal{L} : E \mid\!\sim H\} \vdash H'$, then $E \mid\!\sim H'$.
   2.1 Special Consequence Cond.: If $E \mid\!\sim H$ and $H \vdash H'$, then $E \mid\!\sim H'$.
3. Consistency Condition: $\{E\} \cup \{H \in \mathcal{L} : E \mid\!\sim H\} \nvdash \bot$.
   3.1 Special Cons. C.: If $E \nvdash \bot$, $E \mid\!\sim H$, and $H \vdash \neg H'$, then $E \not\mid\!\sim H'$.
4. Converse Consequence Condition: If $E \mid\!\sim H$ and $H' \vdash H$, then $E \mid\!\sim H'$.

Condition 2 entails condition 2.1; similarly for 3. Hempel then shows (Hempel 1945, 104) that the conjunction of 1, 2, and 4 entails his triviality result that any two sentences confirm each other. This is clear since the conjunction of 1 and 4 implies this: By the Entailment Condition, $E \mid\!\sim E \vee H$; as $H \vdash E \vee H$, the Converse Consequence Condition yields $E \mid\!\sim H$ for any sentences $E, H \in \mathcal{L}$.

Since Hempel's negative result there has hardly been any progress in developing a logic of confirmation. The exceptions I know of and to be discussed later are Flach (2000)[1], Milne (2000), and Zwirn & Zwirn (1996). One reason for this seems to be that up to now the predominant view on Hempel's conditions is the analysis Carnap gave in §87 of his (1962).

Carnap's analysis can be summarized as follows. In presenting his first three conditions of adequacy Hempel was mixing up two distinct concepts of confirmation, viz. (i) the concept of incremental confirmation according to which E confirms H iff $\Pr(H \mid E) > \Pr(H)$, and (ii) the concept of absolute confirmation according to which E confirms H iff $\Pr(H \mid E) > r$. The special versions of Hempel's second and third condition, 2.1 and 3.1, respectively, hold true for the second explicandum (for $r \geq .5$), but they do not hold true for the first explicandum. On the other hand, Hempel's first condition holds true for the first explicandum, but it does so only in a *qualified* form (Carnap 1962, 473) – namely only if E is not assigned probability 0, and H is not assigned probability 1.

[1] I owe this reference to Hykel Hosni.

This, however, means that, according to Carnap's analysis, Hempel first had in mind the explicandum of incremental confirmation for the Entailment Condition; then he had in mind the explicandum of absolute confirmation for the Special Consequence and the Special Consistency Conditions 2.1 and 3.1, respectively; and then, when Hempel presented the Converse Consequence Condition, he got completely confused and had in mind still another explicandum or concept of confirmation (neither the first nor the second explicandum satisfies the Converse Consequence Condition). Apart from not being very charitable, Carnap's reading of Hempel also leaves open the question what the third explicandum might have been.

The following two notions of the plausibility-informativeness theory (Huber to appear b) will prove useful. A relation $|\!\sim\ \subseteq \mathcal{L} \times \mathcal{L}$ is an *informativeness relation* on $\mathcal{L}$ iff

If $E \mid\!\sim H$ and $H' \vdash H$, then $E \mid\!\sim H'$.

$|\!\sim$ is a *plausibility relation* on $\mathcal{L}$ iff

If $E \mid\!\sim H$ and $H \vdash H'$, then $E \mid\!\sim H'$.

The idea is that a sentence is the more informative, the more possibilities it excludes. Hence, the logically stronger a sentence, the more informative it is. On the other hand, a sentence is more plausible the more possibilities it includes. Hence, the logically weaker a sentence, the more plausible it is. The qualitative counterparts of these two comparative principles are the defining clauses above: If H is informative relative to E, then so is any logically stronger sentence H'. Similarly, if H is plausible relative to E, then so is any logically weaker sentence H'.

The two main approaches to confirmation that have been put forth in the last century are qualitative *Hypothetico-Deductivism* HD and quantitative probabilistic *Inductive Logic* IL. According to HD, E HD-confirms H iff H logically implies E (in some suitable way that depends on the version of HD under consideration). According to IL, E absolutely IL-confirms H to degree r iff $\Pr(H \mid E) = r$. The natural *qualitative* counterpart of this quantitative notion is that E *absolutely IL-confirms* H iff $\Pr(H \mid E) > r$ for some $r \in [.5, 1)$ (this is Carnap's second explicandum).

As noted above, this is not the way Carnap defined qualitative IL-confirmation in chapter VII of his (1962). There he required E to raise the probability of H, $\Pr(H \mid E) > \Pr(H)$, in order for E to qualitatively IL-confirm H. Nevertheless, the above is the natural qualitative counterpart of the degree of absolute confirmation. The reason is that later on the difference between $\Pr(H \mid E)$ and $\Pr(H)$ –

however it is measured (Fitelson 1999) – was taken as the degree of *incremental confirmation*, and Carnap's proposal is the natural qualitative counterpart of this notion of incremental confirmation.

HD and IL explicate *conflicting* concepts of confirmation. HD-confirmation *increases*, whereas absolute IL-confirmation *decreases* with the logical strength of the theory to be assessed. More precisely, if E HD-confirms H and H' logically implies H, then E HD-confirms H'. So HD-confirmation is an informativeness relation. On the other hand, if E absolutely IL-confirms H (to some degree) and H logically implies H', then E absolutely IL-confirms H' (to at least the same degree). Hence absolute IL-confirmation is a plausibility relation.

The epistemic values behind these two concepts are *informativeness* on the one hand and *truth* or *plausibility* on the other hand. First, we want to know what is going on "out there", and hence we aim at true theories – more precisely, at theories that are true in the world we are in. Second, we want to know as much as possible about what is going on out there, and so we aim at informative theories – more precisely, at theories that inform us about the world we are in. But usually we do not know which world we are in. All we have are some data. So we base our evaluation of the theory we are concerned with on the plausibility that theory is true in the actual world given that the actual world makes the data true and on how much the theory informs us about the actual world given that the actual world makes the data true.

Turning back to Hempel's conditions, note first that Carnap's second explicandum satisfies the Entailment Condition *without* the second qualification: If E logically implies H, then $\Pr(H \mid E) = 1 > r$ for any $r \in [0, 1)$, provided E does not have probability 0. So the following more charitable reading of Hempel seems plausible: When presenting his first three conditions, Hempel had in mind Carnap's second explicandum, the concept of absolute confirmation, or more generally, a plausibility relation. But then, when discussing the Converse Consequence Condition, Hempel also felt the need for a second concept of confirmation aiming at informative theories. Given that it was the Converse Consequence Condition that Hempel gave up in his (1945), the present analysis makes perfect sense of his argumentation: Though he felt the need for two concepts of confirmation, Hempel also realized that these two concepts were *conflicting* – this is the content of his triviality result – and so he abandoned informativeness in favour of plausibility.

2 Assessing Theories

However, in a sense one can have Hempel's cake and eat it too: There is a logic of confirmation or theory assessment that takes into account both of these two conflicting concepts. Roughly speaking, HD says that a good theory is informative, whereas IL says that a good theory is plausible or true. The driving force behind Hempel's conditions is the insight that *a good theory is both true and informative*. Hence, in assessing a given theory by the available data one should account for these two conflicting aspects.

What one does according to the plausibility-informativeness theory (Huber to appear b) is to evaluate how much theory H informs us about some piece of evidence E given a body of background information B and to evaluate how plausible H is in view of E and B. Then one combines these two values to get the overall assessment value of H in the light of E and B. Informativeness about the data is measured by a strength indicator, and plausibility given the data is measured by a truth indicator.

Definition 1 *A possibly partial function $f : \mathcal{L} \times \mathcal{L} \times \mathcal{L} \to \Re$ is a* truth indicator *on $\mathcal{L}$ iff for all $\langle H, E, B\rangle$, $\langle H', E, B\rangle \in Dom_f$:*

$$B, E \vdash H \to H' \quad \Rightarrow \quad f(H, E, B) \leq f(H', E, B).$$

f is a strength indicator *on $\mathcal{L}$ iff for all $\langle H, E, B\rangle$, $\langle H', E, B\rangle \in Dom_f$:*

$$B, \neg E \vdash H \to H' \quad \Rightarrow \quad f(H', E, B) \leq f(H, E, B).$$

An assessment function measuring the overall epistemic value of theory H in light of evidence E and background information B should not be both a strength indicator and a truth indicator. Any such function is constant. This observation – call it *the singularity of simultaneously indicating strength and truth* – is the quantitative counterpart of Hempel's triviality result. Instead, an assessment function should weigh between these two conflicting aspects in such a way that any surplus in informativeness leads to a greater overall value when the difference in plausibility becomes small enough.

Definition 2 *Let s and t be a strength and a truth indicator on $\mathcal{L}$, respectively. A possibly partial function $f : \mathcal{L} \times \mathcal{L} \times \mathcal{L} \to \Re$ is an s, t* assessment function *iff there is a possibly partial function $g : \Re \times \Re \times X \to \Re$ such that (i) $\langle H, E, B\rangle \in Dom_f$ and $f(H, E, B) = g(s(H, E, B), t(H, E, B), x)$ for all $\langle H, E, B\rangle \in Dom_s \cap Dom_t$, and (ii)*

1. *Continuity: Any surplus in informativeness succeeds, if the difference in plausibility is small enough.*

$$\forall \varepsilon > 0 \quad \exists \delta_\varepsilon > 0 \quad \forall s_1, s_2 \in R_s \quad \forall t_1, t_2 \in R_t \quad \forall x \in X : \\ s_1 > s_2 + \varepsilon \quad \& \quad t_1 > t_2 - \delta_\varepsilon \quad \Rightarrow \quad g(s_1, t_1, x) > g(s_2, t_2, x) .$$

2. *Demarcation:* $\quad \forall x \in X : \quad g(s_{\max}, t_{\min}, x) = g(s_{\min}, t_{\max}, x) = 0.$

If $s(\bot, E, B)$ and $s(\top, E, B)$ are defined, they are the maximal and minimal values of s, $s_{\max}$ and $s_{\min}$, respectively. R_s is the range of s. Similarly for t. $f(H, E, B)$ is a function of, among others, $s(H, E, B)$ and $t(H, E, B)$. I will sometimes write '$f(H, E, B)$', and other times '$g(s_1, t_1)$', dropping the additional argument place, and other times '$f(s_1, t_1)$', treating f as $g(s, t)$.

This is the general plausibility-informativeness theory. Particular accounts arise by inserting particular strength indicators and truth indicators. Here I will focus on the rank-theoretic version and the logic this gives rise to. As ranking theory is closely related to, but much less well-known than probability theory, it is helpful to briefly look at the Bayesian version.

2.1 Assessing Theories, Bayes Style

In the Bayesian paradigm of subjective probabilities we get for every probability $\Pr$ on a language $\mathcal{L}$ the strength indicator $i = \Pr(\neg H \mid \neg E \wedge B)$ and the truth indicator $p = \Pr(H \mid E \wedge B)$. For instance, the Joyce-Christensen measure of incremental confirmation

$$s = \Pr(H \mid E \wedge B) - \Pr(H \mid \neg E \wedge B) = i + p - 1$$

(Joyce 1999, Christensen 1999) is an i, p assessment function. It can be rewritten as the *expected informativeness* of H relative to E and B,

$$s = i \cdot \Pr(H \mid E \wedge B) - i \cdot \Pr(\neg H \mid E \wedge B) .$$

For regular $\Pr$ one can show that s as well as all other i, p assessment functions lead to the most informative among all true theories in almost every world when presented data that separate the set of all models. For more on confirmation theory from the plausibility-informativeness point of view see (Huber to appear a).

2.2 Assessing Theories, Spohn Style

The Spohnian paradigm of ranking functions (Spohn 1988) is in many respects like an order-of-magnitude reverse of subjective probability theory. Ranks represent grades of disbelief. Whereas a high probability indicates a high degree of belief, a high rank indicates a high grade of disbelief. A function κ from a non-empty set of possibilities W into the set of natural numbers extended by ∞, $N \cup \{\infty\}$, is a *pointwise ranking function* on W iff $\kappa(\omega) = 0$ for at least one $\omega \in W$. A pointwise ranking function κ is extended to a function ϱ_κ on a field of propositions $\mathcal{A}$ over W by defining for each $A \in \mathcal{A}$,

$$\varrho_\kappa(A) = \begin{cases} \min\{\kappa(\omega) : \omega \in A\}, & \text{if} \quad A \neq \emptyset, \\ \infty, & \text{if} \quad A = \emptyset. \end{cases}$$

Unlike probabilities, Spohnian ranking functions are only indirectly – via pointwise ranking functions on the underlying set of possibilities W – defined on a field of propositions $\mathcal{A}$ over W. In Huber (to appear c) I have defined (*finitely minimitive*) *ranking functions* as functions ϱ from a field $\mathcal{A}$ over a set of possibilities W into the set of natural numbers extended by ∞ such that for all $A, B \in \mathcal{A}$:

1. $\varrho(\emptyset) = \infty$
2. $\varrho(W) = 0$
3. $\varrho(A \cup B) = \min\{\kappa(A), \kappa(B)\}$

If $\mathcal{A}$ is a σ-field / complete field, ϱ is a σ-*minimitive* / *completely minimitive* ranking function iff, in addition to 1-3, we have for every countable / possibly uncountable $\mathcal{B} \subseteq \mathcal{A}$:

$$\varrho\left(\bigcup \mathcal{B}\right) = \min\{\varrho(B) : B \in \mathcal{B}\}$$

A ranking function ϱ on a field $\mathcal{A}$ is *regular* iff $\varrho(A) < \varrho(\emptyset)$ for every non-empty $A \in \mathcal{A}$. It is a *pre-ranking* iff $\varrho(\bigcup \mathcal{B}) = \min\{\kappa(A) : A \in \mathcal{B}\}$ for every countable $\mathcal{B} \subseteq \mathcal{A}$ such that $\bigcup \mathcal{B} \in \mathcal{A}$. The conditional ranking function $\varrho(\cdot \mid \cdot) : \mathcal{A} \times \mathcal{A} \to N \cup \{\infty\}$ based on the ranking function $\varrho : \mathcal{A} \to N \cup \{\infty\}$ is defined such that for all $A, B \in \mathcal{A}$:

4. $\varrho(B \mid A) = \varrho(B \cap A) - \varrho(A) \quad (= 0 \quad \text{if} \quad \varrho(A) = \infty)$

This differs from Huber (to appear c), where the above equation is restricted to non-empty B and it is stipulated that $\varrho(\emptyset \mid A) = \infty$ for every $A \in \mathcal{A}$. The latter stipulation guarantees that $\varrho(\cdot \mid A)$ is a ranking function for every $A \in \mathcal{A}$. The present definition renders the formulation of assessment models simpler. *Rankings* $\varrho : \mathcal{L} \to N \cup \{\infty\}$ on languages $\mathcal{L}$ are defined such that for all $\alpha, \beta \in \mathcal{L}$:

0. $\alpha \dashv\vdash \beta \quad \Rightarrow \quad \varrho(\alpha) = \varrho(\beta)$
1. $\vdash \alpha \quad \Rightarrow \quad \varrho(\alpha) = 0$
2. $\alpha \vdash \bot \quad \Rightarrow \quad \varrho(\alpha) = \infty$
3. $\varrho(\alpha \vee \beta) = \min\{\varrho(\alpha), \varrho(\beta)\}$
4. $\varrho(\beta \mid \alpha) = \varrho(\alpha \wedge \beta) - \varrho(\alpha) \quad (= 0 \quad \text{if} \quad \varrho(\alpha) = \infty)$

$\vdash$ is the classical consequence relation. ϱ is called *regular* iff $\kappa(\alpha) < \kappa(\bot)$ for every consistent $\alpha \in \mathcal{L}$.

If ϱ_κ is induced by a pointwise ranking function κ, then ϱ_κ is a completely minimitive ranking function (the converse is not true). The triple $\mathbf{A} = \langle W, \mathcal{A}, \varrho\rangle$ with W a set of possibilities, $\mathcal{A}$ a field over W, and ϱ a ranking function on $\mathcal{A}$ is called a *ranking space*. $\mathbf{A}$ is called *regular* iff ϱ is regular.

Observation 1 *For any ranking space* $\mathbf{A} = \langle W, \mathcal{A}, \varrho\rangle$ *and all* $A, B \in \mathcal{A}$*:*

1. $\min\{\varrho(A), \varrho(\overline{A})\} = 0$

2. $A \subseteq B \quad \Rightarrow \quad \varrho(B) \leq \varrho(A)$

A proposition $A \in \mathcal{A}$ is *believed* in ϱ iff $\varrho(\overline{A}) > 0$. ϱ's belief set $Bel_\varrho = \{A \in \mathcal{A} : \varrho(\overline{A}) > 0\}$ is consistent and deductively closed in the finite / countable / complete sense whenever ϱ is finitely / σ- / completely minimitive. A set $Bel \subseteq \mathcal{A}$ is consistent in the finite / countable / complete sense iff $\bigcap \mathcal{B} \neq \emptyset$ for every finite / countable / possibly uncountable $\mathcal{B} \subseteq Bel$. Bel is deductively closed in the finite / countable / complete sense iff for every $A \in \mathcal{A}$: $A \in Bel$ whenever $\bigcap \mathcal{B} \subseteq A$ for some finite / countable / possibly uncountable $\mathcal{B} \subseteq Bel$.

One advantage of ranking theory vis-á-vis probability theory is that it easily admits of qualitative notions as, for instance, belief. This is one reason why the logic of theory assessment – which is based on the qualitative notion of acceptability – is spelt out in terms of ranking functions rather than probability measures. Another reason is to illustrate the claim that the plausibility-informativeness theory is general or paradigm independent.

In order to get the rank-theoretic version of the plausibility-informativeness theory we only have to specify a rank-theoretic strength indicator and a rank-theoretic truth indicator. This is easily achieved. For any ranking space $\langle W, \mathcal{A}, \varrho \rangle$ the plausibility rank of H relative to E and B is given by

$$\varrho\left(\overline{H} \mid E \cap B\right) - \varrho\left(H \mid E \cap B\right) \begin{matrix} > \\ = \\ < \end{matrix} 0 \Leftrightarrow \varrho\left(H \mid E \cap B\right) \begin{matrix} < \\ = \\ > \end{matrix} \varrho\left(\overline{H} \mid E \cap B\right).$$

(Remember: *Lower* ranks indicate *lower* grades of *dis*belief.) Similarly, the informativeness rank of H relative to E and B is given by

$$\varrho\left(H \mid \overline{E} \cap B\right) - \varrho\left(\overline{H} \mid \overline{E} \cap B\right) \begin{matrix} > \\ = \\ < \end{matrix} 0 \Leftrightarrow \varrho\left(\overline{H} \mid \overline{E} \cap B\right) \begin{matrix} < \\ = \\ > \end{matrix} \varrho\left(H \mid \overline{E} \cap B\right).$$

How to measure informativeness and plausibility in ranking terms and how to combine these two values is not the task of the present paper. Here we are interested in the qualitative counterpart of the quantitative assessment value, which is the notion of an *acceptable theory given the data*. ‘Accept’ is not used in the sense of believing or holding to be true. Rather, the proposed attitude towards theories is similar to the attitude one has towards bottles of wine. One has a certain amount of money and one would like to buy a good bottle of wine. On the one hand, one wants to spend as little money as possible (one’s theory should be as informative as possible). On the other hand, one wants to drink reasonably good wine (one’s theory should be sufficiently plausible). Sometimes one need not care much about money, and the main focus is on the quality of the wine – as when one is concerned with several alternative theories all sufficiently informative to answer one’s questions, and one wants to choose the most plausible one. At other times money does matter, for one cannot spend more than one has. Likewise, in many situations very plausible theories won’t do, because they are too uninformative to be of any use.

Just as this picture of the trade-off between price and quality does not tell one when a bottle of wine is worth its price and when one should buy which bottle of wine (except when one gets a bottle of good wine for free), the plausibility-informativeness theory does not tell one when one should adopt or stick to a theory (except when a theory is sufficiently informative to answer one’s questions and known to be true). Instead, a theory which is acceptable given the data is a *possible* candidate to stick with.

Neglecting the background information B, it is tempting to say that H is an acceptable theory for evidence E iff the overall assessment value of H relative to E is greater than that of its complement $\overline{H}$ relative to E. This, however, has the consequence that the notion of acceptability depends on the way one combines plausibility and informativeness. One may, for instance, simply take the sum $s + t - 1$, or else one may judge informativeness measured by s more important than plausibility measured by t and stick with $s + t^x - 1$, for some $x > 1$. The only clear case in which H is acceptable given E is when H is at least as plausible given E as its complement $\overline{H}$, and H informs more about E than does $\overline{H}$; or else, H is more plausible given E than $\overline{H}$, and H informs at least as much about E as does $\overline{H}$. This will be our definition of acceptability.

3 The Logic of Theory Assessment

3.1 Assessment Models

Let us do some stage setting. A *language* $\mathcal{L}$ is a countable set of closed well-formed formulas that contains $\bot$ and is closed under the propositional connectives $\neg$ and $\wedge$ ($\vee$, $\rightarrow$, $\leftrightarrow$ are defined as usual). A language is not required to be closed under the quantifiers. $Mod_{\mathcal{L}}$ is the set of all models for $\mathcal{L}$. If $\mathcal{L}$ is a propositional language over the set of propositional variables PV, $Mod_{\mathcal{L}}$ is the set of all truth value assignments $\omega : PV \rightarrow \{0, 1\}$. If $\mathcal{L}$ is a first-order language, $Mod_{\mathcal{L}}$ is the set of all pairs $\langle D, \varphi \rangle$ with D a non-empty set and φ an interpretation function. φ assigns every k-ary predicate symbol 'P' a subset $\varphi\left(\text{'}P\text{'}\right) \subseteq D^k$ ($\varphi\left(\text{'}p\text{'}\right) \in \{0, 1\}$ for propositional variables 'p' conceived of as 0-ary predicate symbols), and every k-ary function symbol 'f' a function $\varphi\left(\text{'}f\text{'}\right) : D^k \rightarrow D$ ($\varphi\left(\text{'}a\text{'}\right) \in D$ for individual constants 'a' conceived of as 0-ary function symbols). $\vdash\ \subseteq \wp\left(\mathcal{L}\right) \times \mathcal{L}$ is the classical consequence relation on $\mathcal{L}$. '$\alpha \dashv\vdash \beta$' is short for '$\alpha \vdash \beta$ and $\beta \vdash \alpha$', and '$\alpha \vdash \beta$' is short for '$\{\alpha\} \vdash \beta$'. $\models\ \subseteq Mod \times \mathcal{L}$ is the classical satisfaction relation, and for $\alpha \in \mathcal{L}$, $Mod\left(\alpha\right) = \{\omega \in Mod_{\mathcal{L}} : \omega \models \alpha\}$. $\models$ is compact – a set of wffs is satisfiable iff all its finite subsets are – and such that $\omega \models \alpha$ iff $\omega \not\models \neg\alpha$ and $Mod\left(\alpha \wedge \beta\right) = Mod\left(\alpha\right) \cap Mod\left(\beta\right)$. If every $\omega \in Mod_{\mathcal{L}}$ that satisfies all wffs $\alpha \in \Gamma$ also satisfies β, we write '$\Gamma \models \beta$'. '$\alpha \models \beta$' is short for '$\{\alpha\} \models \beta$', and '$\models \alpha$' is short for '$\emptyset \models \alpha$'.

A ranking space $\langle W, \mathcal{A}, \varrho \rangle$ is a (*rank-theoretic*) *assessment model* for the language $\mathcal{L}$ iff $W = Mod_{\mathcal{L}}$, $\{Mod\left(\alpha\right) \subseteq W : \alpha \in \mathcal{L}\} \subseteq \mathcal{A}$, and $\varrho\left(Mod\left(\alpha\right)\right) < \varrho\left(\emptyset\right)$ for every consistent $\alpha \in \mathcal{L}$. $\langle W, \mathcal{A}, \varrho \rangle$ is a *pointwise* (*rank-theoretic*) *assess-*

ment model for $\mathcal{L}$ iff $\langle W, \mathcal{A}, \varrho \rangle$ is an assessment model for $\mathcal{L}$ and ϱ is induced by a pointwise ranking function κ on W. So every pointwise assessment model is an assessment model.

Every assessment model for $\mathcal{L}$ induces a ranking $\varrho_{\mathcal{L}}$ on $\mathcal{L}$ by defining $\varrho_{\mathcal{L}}(\alpha) = \varrho(Mod(\alpha))$. The *acceptability relation* $|\!\sim_\varrho \subseteq \mathcal{L} \times \mathcal{L}$ of an assessment model $\langle W, \mathcal{A}, \varrho \rangle$ for $\mathcal{L}$ is defined as follows:

$$\begin{aligned} \alpha \mathrel{|\!\sim_\varrho} \beta \;\Leftrightarrow\; & \varrho(\beta \mid \alpha) < \varrho(\neg\beta \mid \alpha) \quad \& \quad \varrho(\neg\beta \mid \neg\alpha) \leq \varrho(\beta \mid \neg\alpha) \\ & \text{or} \\ & \varrho(\beta \mid \alpha) \leq \varrho(\neg\beta \mid \alpha) \quad \& \quad \varrho(\neg\beta \mid \neg\alpha) < \varrho(\beta \mid \neg\alpha) \end{aligned}$$

By the definition of conditional ranking functions (section 2.2) this is equivalent to

$$\begin{aligned} & \varrho(\beta \wedge \alpha) < \varrho(\neg\beta \wedge \alpha) \quad \& \quad \varrho(\neg\beta \wedge \neg\alpha) \leq \varrho(\beta \wedge \neg\alpha) \\ & \text{or} \\ & \varrho(\beta \wedge \alpha) \leq \varrho(\neg\beta \wedge \alpha) \quad \& \quad \varrho(\neg\beta \wedge \neg\alpha) < \varrho(\beta \wedge \neg\alpha) \end{aligned}$$

If one prefers the definition of conditional ranking functions from (Huber to appear c), the second clause is our definition of acceptability relations.

In words: β is an acceptable theory for α iff β is at least as plausible given α as its negation, and β informs more about α than does $\neg\beta$; or β is more plausible given α than its negation, and β informs at least as much about α as does $\neg\beta$.

In the following we employ the Gabbay-Makinson-KLM framework (Gabbay 1985, Makinson 1989, Kraus & Lehmann & Magidor 1990) and present a list of properties such that the acceptability relation $|\!\sim_\varrho$ defined by an assessment model for a language $\mathcal{L}$ satisfies these properties (correctness). Then we show that the converse is also true: For each relation $|\!\sim \subseteq \mathcal{L} \times \mathcal{L}$ on some language $\mathcal{L}$ satisfying these properties there is an assessment model – in fact, a pointwise assessment model – $\langle W, \mathcal{A}, \varrho \rangle$ for $\mathcal{L}$ such that $|\!\sim = |\!\sim_\varrho$ (completeness).

3.2 Assessment Relations

A relation $|\!\sim \subseteq \mathcal{L} \times \mathcal{L}$ is an *assessment relation* on the language $\mathcal{L}$ iff:

A1. $\alpha \mathrel{|\!\sim} \alpha$ — Reflexivity*

A2. $\alpha \mathrel{|\!\sim} \beta, \quad \alpha \dashv\vdash \gamma \quad \Rightarrow \quad \gamma \mathrel{|\!\sim} \beta$ — Left Logical Equivalence*

A3. $\alpha \mid\!\sim \beta, \quad \beta \dashv\vdash \gamma \quad \Rightarrow \quad \alpha \mid\!\sim \gamma$ Right Logical Equivalence*

A4. $\alpha \mid\!\sim \beta \quad \Rightarrow \quad \alpha \mid\!\sim \alpha \wedge \beta$ Weak Composition*

A5. $\alpha \mid\!\sim \beta \quad \Rightarrow \quad \neg\alpha \mid\!\sim \neg\beta$

A6. $\nvdash \alpha \vee \beta \quad \Rightarrow \quad \alpha \vee \beta \mid\!\sim \alpha \quad \text{or} \quad \alpha \vee \beta \mid\!\sim \beta$

A7. $\alpha \vee \beta \mid\!\not\sim \alpha, \quad \nvdash \alpha \vee \beta \quad \Rightarrow \quad \alpha \vee \neg\alpha \mid\!\sim \neg\alpha$

A8. $\alpha \wedge \neg\alpha \mid\!\sim \alpha, \quad \alpha \vee \beta \mid\!\sim \alpha \quad \Rightarrow \quad \alpha \wedge \neg\alpha \mid\!\sim \beta$

A9. $\alpha \mid\!\sim \alpha \wedge \beta, \quad \alpha \mid\!\sim \alpha \vee \beta \quad \Rightarrow \quad \alpha \mid\!\not\sim \neg\beta$

A10. $\alpha \mid\!\not\sim \alpha \wedge \neg\beta, \quad \alpha \mid\!\sim \alpha \vee \beta, \quad \nvdash \alpha, \quad \alpha \nvdash \bot \quad \Rightarrow \quad \alpha \mid\!\sim \beta$

A11. $\alpha \vee \beta \mid\!\sim \alpha, \quad \beta \vee \gamma \mid\!\sim \beta, \quad \nvdash \alpha \vee \gamma \quad \Rightarrow \quad \alpha \vee \gamma \mid\!\sim \alpha$ quasi-Nr 21

A12. $\alpha \vee \beta \mid\!\sim \alpha, \quad \beta \vee \gamma \mid\!\sim \beta, \quad \vdash \alpha \vee \gamma \quad \Rightarrow \quad \alpha \vee \gamma \mid\!\not\sim \neg\alpha$

supplementary-Nr 21

A13. $\alpha_i \vee \alpha_{i+1} \mid\!\sim \alpha_{i+1}, \quad \nvdash \alpha_i \vee \alpha_j \quad \Rightarrow \quad \exists n \forall m \geq n : \alpha_m \vee \alpha_{m+1} \mid\!\sim \alpha_m$

The *-starred principles are among the *core principles* in Zwirn & Zwirn (1996). A5 is different from Milne's Negation Symmetry (Milne 2000). It has to hold of any acceptability relation $\mid\!\sim_\varrho$ given the definition in section 3.1: The plausibility value of β given α is the informativeness value of $\neg\beta$ given $\neg\alpha$, and the informativeness value of β given α is the plausibility value of $\neg\beta$ given $\neg\alpha$. Hence, if the plausibility and the informativeness of β relative to α are both at least as great as that of $\neg\beta$ given α, and one, say plausibility, is strictly greater, then the plausibility and the informativeness of $\neg\beta$ relative to $\neg\alpha$ are both at least as great as that of β given $\neg\alpha$, and the other, informativeness, is strictly greater.

It is helpful to note that for non-tautological $\alpha \vee \beta$, $\alpha \vee \beta \mid\!\sim \alpha$ means that the rank of α is not greater than the rank of β, or equivalently, that the rank of α is not greater than, and hence equal to, the rank of $\alpha \vee \beta$. For tautological $\alpha \vee \beta$, $\alpha \vee \beta \mid\!\sim \alpha$ means that the rank of α is strictly smaller than that of its negation $\neg\alpha$, which holds iff $\neg\alpha$ has a rank greater than 0.

In terms of acceptability A6 says that at least one of α, β is acceptable given non-tautological $\alpha \vee \beta$: Both α and β inform maximally about $\alpha \vee \beta$, and if not α, then at least β must be at least as plausible given $\alpha \vee \beta$ as its negation $\neg\beta$. By the above meaning of $\alpha \vee \beta \mid\!\sim \alpha$ for non-tautological $\alpha \vee \beta$, A6 amounts to the connectedness of the $\leq$-relation between natural numbers: Either the rank of α is not greater than that of β, or the rank of β is not greater than that of α.

The antecedent of A7 simply says that the rank of α is greater than 0. This is also the meaning of the consequent.

By A5 the first antecedent of A8 says that the rank of α is greater than 0. For non-tautological $\alpha \vee \beta$ the second antecedent means that the rank of α is not greater than the rank of β. Hence the consequent that the rank of β is positive. For tautological $\alpha \vee \beta$ the second antecedent means that the rank of $\neg\alpha$ is greater than 0 – which is not possible, because at leat one of $\alpha, \neg\alpha$ must have rank 0.

For tautological α A9 is an instance of the derived rule Selectivity (see below). For non-tautological α the first antecedent means that the rank of $\alpha \wedge \beta$ is not greater than the rank of $\alpha \wedge \neg\beta$. By A5 the second antecedent means that the rank of $\neg\alpha \wedge \neg\beta$ is not greater than the rank of $\neg\alpha \wedge \beta$. Hence $\neg\beta$ is neither more plausible given α than its negation; nor is it more informative about α than its negation. This implies the consequent of A9.

The first and third antecedent of A10 together say that $\alpha \wedge \neg\beta$ has a greater rank than $\alpha \wedge \beta$. The second antecedent implies that the rank of $\neg\alpha \wedge \neg\beta$ is not greater than the rank of $\neg\alpha \wedge \beta$. Therefore β is more plausible given α than $\neg\beta$, and it is at least as informative about α as $\neg\beta$. This implies the consequent. The proof below only requires the weaker version including the fourth antecedent.

quasi-Nr 21 without the restriction $\nvdash \alpha \vee \gamma$ is the derived rule (21) of the system **P** in Kraus & Lehmann & Magidor (1990) (cf. their lemma 22). Together with supplementary-Nr 21 it expresses the transitivity of the $\leq$-relation between natural numbers. If the rank of α is not greater than the rank of β (for non-tautological $\alpha \vee \beta$) or the rank of α is 0 (for tautological $\alpha \vee \beta$), and if the rank of β is not greater than the rank of γ (for non-tautological $\beta \vee \gamma$) or the rank of β, and hence that of α, equals 0, then the rank of α is not greater than that of γ.

A13 says that the set of natural numbers is well-ordered: There is no strictly $<$-decreasing sequence of natural numbers.

Here are some derived rules:

A14. $\alpha \mid\!\sim \beta \quad \Rightarrow \quad \alpha \mid\!\sim \alpha \vee \beta$ — Weak $\vee$-Composition

A15. $\alpha \mid\!\sim \beta \quad \Rightarrow \quad \alpha \not\mid\!\sim \neg\beta$ — Selectivity*

A16. $\alpha \vdash \beta \quad \Rightarrow \quad \alpha \vee \beta \mid\!\sim \beta$

A17. $\alpha \vee \neg\alpha \mid\!\sim \alpha, \quad \alpha \vdash \beta \quad \Rightarrow \quad \alpha \vee \neg\alpha \mid\!\sim \beta$

As to Weak $\vee$-Composition, $\alpha \mid\!\sim \beta$, A5, and Weak Composition first give $\neg\alpha \mid\!\sim \neg\alpha \wedge \neg\beta$ and then $\alpha \mid\!\sim \alpha \vee \beta$. As to Selectivity, $\alpha \mid\!\sim \beta$ and Weak Composition and Weak $\vee$-Composition yield $\alpha \mid\!\sim \alpha \wedge \beta$ and $\alpha \mid\!\sim \alpha \vee \beta$. Apply A9. As to A16, if $\alpha \vdash \beta$, then $\alpha \vee \beta \dashv\vdash \beta$. Apply Reflexivity and Left Logical Equivalence. As to A17, $\alpha \vdash \beta$ yields $\neg\beta \vdash \neg\alpha$, which yields $\neg\alpha \vee \neg\beta \mid\!\sim \neg\beta$ by A16. $\alpha \vee \neg\alpha \mid\!\sim \alpha$, A5, and Left Logical Equivalence yield $\neg\alpha \wedge \neg\neg\alpha \mid\!\sim \neg\alpha$. A8 gives $\neg\alpha \wedge \neg\neg\alpha \mid\!\sim \neg\beta$, and A5, Left Logical Equivalence, and Right Logical Equivalence give $\alpha \vee \neg\alpha \mid\!\sim \beta$.

Note that Selectivity allows there to be two logically incompatible theories β_1 and β_2 such that both are acceptable given α (cf. Carnap's discussion of Hempel's consistency condition quoted in Huber submitted, section 2).

3.3 A Representation Result

Theorem 1 (Representation Theorem for Assessment Relations) *The acceptability relation $\mid\!\sim_{\varrho}$ induced by an assessment model $\langle W, \mathcal{A}, \varrho\rangle$ for a language $\mathcal{L}$ is an assessment relation on $\mathcal{L}$. For each assessment relation $\mid\!\sim$ on a language $\mathcal{L}$ there is a pointwise assessment model $\langle W, \mathcal{A}, \varrho\rangle$ for $\mathcal{L}$ such that $\mid\!\sim \; = \; \mid\!\sim_{\varrho}$.*

PROOF:
The proof is restricted to the second claim. The plan is as follows: We first define a countable field $\mathcal{A}$ on $Mod_{\mathcal{L}}$. Using only the assessment relation $\mid\!\sim$ on $\mathcal{L}$ we then define a weak order $\preceq$ on $\mathcal{A}$. We go on to show that for each such weak order $\preceq$ on $\mathcal{A}$ there is a regular ranking function ϱ on $\mathcal{A}$ such that ϱ represents $\preceq$, i.e. $A \preceq B$ iff $\varrho(A) \leq \varrho(B)$. This is done by showing that $\preceq$ gives rise to a well-order on the set of equivalence classes $\mathcal{A}/_{\simeq}$, where $\simeq$ is the equivalence relation on $\mathcal{A}$ induced by $\preceq$ ($A \simeq B$ iff $A \preceq B$ and $B \preceq A$). This in turn implies that we can write the elements of $\mathcal{A}/_{\simeq}$ as a sequence. We use the indices of this sequence as the values of ϱ. Finally we show that $\alpha \mid\!\sim \beta$ iff $\varrho_{\mathcal{L}}(\beta \wedge \alpha) \leq \varrho_{\mathcal{L}}(\neg\beta \wedge \alpha)$ and $\varrho_{\mathcal{L}}(\neg\beta \wedge \neg\alpha) \leq \varrho_{\mathcal{L}}(\beta \wedge \neg\alpha)$, where at least one of these inequalities is strict, and $\varrho_{\mathcal{L}}$ is the ranking on $\mathcal{L}$ that is induced by ϱ on $\mathcal{A}$. In fact, ϱ on $\mathcal{A}$ is the pre-ranking induced by $\varrho_{\mathcal{L}}$ on $\mathcal{L}$. The Extension Theorem for Rankings on Languages (Huber to appear c) completes the proof by ensuring that there is a pointwise ranking function κ on $Mod_{\mathcal{L}}$ that induces ϱ.

So suppose $\mid\!\sim \; \subseteq \mathcal{L} \times \mathcal{L}$ is an assessment relation on the language $\mathcal{L}$. Let $\mathcal{A} = \{Mod(\alpha) \subseteq Mod_{\mathcal{L}} : \alpha \in \mathcal{L}\}$. $\mathcal{A}$ is a countable field on $Mod_{\mathcal{L}}$, i.e. a countable set of subsets of $Mod_{\mathcal{L}}$ that contains the empty set and is closed under complementation and finite intersections. The following equivalence will prove useful.

For every ranking space $\langle W, \mathcal{A}, \varrho \rangle$ and all $A, B \in \mathcal{A}$,

$$\varrho(A) \leq \varrho(B) \quad \Leftrightarrow \quad \varrho(A) \leq \varrho\left(\overline{A} \cap B\right). \tag{1}$$

Subproof:
This is easily seen by keeping in mind that

$$\begin{array}{rcl} A \subseteq B & \Rightarrow & \varrho(B) \leq \varrho(A), \\ \varrho(A) & = & \min\left\{\varrho(A \cap B), \varrho\left(A \cap \overline{B}\right)\right\}. \end{array}$$

$\Rightarrow$: $\varrho(A) \leq \varrho(B) \leq \varrho\left(\overline{A} \cap B\right)$.
$\Leftarrow$: If $\varrho(A \cap B) \geq \varrho\left(\overline{A} \cap B\right)$, then $\varrho(B) = \varrho\left(\overline{A} \cap B\right) \geq \varrho(A)$. If $\varrho(A \cap B) < \varrho\left(\overline{A} \cap B\right)$, then $\varrho(B) = \varrho(A \cap B) \geq \varrho(A)$. $\square$

For $A = Mod(\alpha') \in \mathcal{A}$ and $B = Mod(\beta') \in \mathcal{A}$ with $\overline{A} \cap \overline{B} \neq \emptyset$ we define

$$A \preceq B \quad \Leftrightarrow \quad \alpha \vee \beta \mid\!\sim \alpha,$$

for any $\alpha \in [\alpha']$ and any $\beta \in [\beta']$, where $[\gamma] = \{\gamma' \in \mathcal{L} : \gamma \dashv\vdash \gamma'\}$. By Left Logical Equivalence and Right Logical Equivalence it does not matter which representatives $\alpha \in [\alpha']$ and $\beta \in [\beta']$ we choose.

This definition captures the intended meaning, for $\alpha \vee \beta \mid\!\sim_\varrho \alpha$ holds iff

$$\begin{array}{ccc} \varrho(A \cap (A \cup B)) \leq \varrho\left(\overline{A} \cap (A \cup B)\right) & \& & \varrho\left(\overline{A} \cap \overline{A} \cap \overline{B}\right) < \varrho\left(A \cap \overline{A} \cap \overline{B}\right) \\ & \text{or} & \\ \varrho(A \cap (A \cup B)) < \varrho\left(\overline{A} \cap (A \cup B)\right) & \& & \varrho\left(\overline{A} \cap \overline{A} \cap \overline{B}\right) \leq \varrho\left(A \cap \overline{A} \cap \overline{B}\right). \end{array}$$

As $\overline{A} \cap \overline{B} \neq \emptyset$ and ϱ is regular, we get $\varrho\left(\overline{A} \cap \overline{B}\right) < \varrho(\emptyset)$. So the above holds iff

$$\varrho(A) \leq \varrho\left(\overline{A} \cap B\right) \quad \text{or} \quad \varrho(A) < \varrho\left(\overline{A} \cap B\right),$$

i.e. just in case

$$\varrho(A) \leq \varrho(B).$$

For $A, B \in \mathcal{A}$ with $\overline{A} \cap \overline{B} = \emptyset$, equivalence (1) reduces to

$$\varrho(A) \leq \varrho(B) \quad \Leftrightarrow \quad \varrho(A) \leq \varrho\left(\overline{A}\right).$$

As $\varrho(A) \leq \varrho\left(\overline{A}\right)$ iff $\varrho(A) = 0$, we have for $A, B \in \mathcal{A}$ with $\overline{A} \cap \overline{B} = \emptyset$:

$$\varrho(A) \leq \varrho(B) \quad \Leftrightarrow \quad \varrho(A) = 0. \tag{2}$$

For tautological $\alpha \vee \beta$, $\alpha \vee \beta \not\mid\!\sim_{\varrho} \neg\alpha$ holds iff (where $W = Mod_{\mathcal{L}}$, $A = Mod\,(\alpha)$, and $B = Mod\,(\beta)$)

$$\begin{array}{ccc} \varrho\left(W \cap \overline{A}\right) \geq \varrho\left(W \cap A\right) & \text{or} & \varrho\left(\overline{W} \cap A\right) > \varrho\left(\overline{W} \cap \overline{A}\right) \\ & \& & \\ \varrho\left(W \cap \overline{A}\right) > \varrho\left(W \cap A\right) & \text{or} & \varrho\left(\overline{W} \cap A\right) \geq \varrho\left(\overline{W} \cap \overline{A}\right) \end{array}$$

This holds iff $\varrho\left(\overline{A}\right) \geq \varrho\left(A\right)$, which in turn holds iff $\varrho\left(A\right) = 0$. So we define for $A = Mod\,(\alpha') \in \mathcal{A}$ and $B = Mod\,(\beta') \in \mathcal{A}$ with $\overline{A} \cap \overline{B} = \emptyset$:

$$A \preceq B \quad \Leftrightarrow \quad \alpha \vee \beta \not\mid\!\sim \neg\alpha,$$

for any $\alpha \in [\alpha']$ and any $\beta \in [\beta']$. As before, Left Logical Equivalence and Right Logical Equivalence guarantee that it does not matter which representatives $\alpha \in [\alpha']$ and $\beta \in [\beta']$ we choose.

We have to show that $\preceq$ is connected and transitive.

Subproof:
As to Connectedness, suppose $A \not\preceq B$, for some $A = Mod\,(\alpha') \in \mathcal{A}$ and $B = Mod\,(\beta') \in \mathcal{A}$. Assume first $\overline{A} \cap \overline{B} \neq \emptyset$. Then $\not\vdash \alpha \vee \beta$ and $\alpha \vee \beta \not\mid\!\sim \alpha$, for any $\alpha \in [\alpha']$ and any $\beta \in [\beta']$. A6 yields $\alpha \vee \beta \mid\!\sim \beta$. By Left Logical Equivalence, $\beta \vee \alpha \mid\!\sim \beta$, i.e. $B \preceq A$.

Now assume $\overline{A} \cap \overline{B} = \emptyset$. Then $\vdash \alpha \vee \beta$ and $\alpha \vee \beta \mid\!\sim \neg\alpha$, for any $\alpha \in [\alpha']$ and any $\beta \in [\beta']$. By Left Logical Equivalence, it suffices to show that $\alpha \vee \beta \not\mid\!\sim \neg\beta$. Suppose for reductio that $\alpha \vee \beta \mid\!\sim \neg\beta$. As $\neg\beta \vdash \alpha$, A17 yields $\alpha \vee \beta \mid\!\sim \alpha$ – in contradiction to Selectivity. $\square$

Subproof:
As to Transitivity, suppose $A \preceq B$ and $B \preceq C$, for some $A = Mod\,(\alpha') \in \mathcal{A}$, $B = Mod\,(\beta') \in \mathcal{A}$, and $C = Mod\,(\gamma') \in \mathcal{A}$. We have to show that $A \preceq C$. There are four cases:
(i) $\overline{A} \cap \overline{B} \neq \emptyset$ and $\overline{B} \cap \overline{C} \neq \emptyset$: We have

$$\alpha \vee \beta \mid\!\sim \alpha \quad \text{and} \quad \beta \vee \gamma \mid\!\sim \beta,$$

for all $\alpha \in [\alpha']$, $\beta \in [\beta']$, $\gamma \in [\gamma']$. If $\overline{A} \cap \overline{C} \neq \emptyset$, i.e. $\not\vdash \alpha \vee \gamma$, then $\alpha \vee \gamma \mid\!\sim \alpha$ by quasi-Nr 21, and so $A \preceq C$. If $\overline{A} \cap \overline{C} = \emptyset$, i.e. $\vdash \alpha \vee \gamma$, then $\alpha \vee \gamma \not\mid\!\sim \neg\alpha$ by supplementary-Nr 21, and so $A \preceq C$.
(ii) $\overline{A} \cap \overline{B} \neq \emptyset$ and $\overline{B} \cap \overline{C} = \emptyset$: We have

$$\alpha \vee \beta \mid\!\sim \alpha \quad \text{and} \quad \beta \vee \gamma \not\mid\!\sim \neg\beta,$$

for all $\alpha \in [\alpha']$, $\beta \in [\beta']$, $\gamma \in [\gamma']$. Suppose first $\overline{A} \cap \overline{C} \neq \emptyset$, i.e. $\not\vdash \alpha \vee \gamma$, and assume for reductio that $\alpha \vee \gamma \not\mid\!\sim \alpha$. By A7 $\alpha \vee \neg\alpha \mid\!\sim \neg\alpha$, and so $\alpha \wedge \neg\alpha \mid\!\sim \alpha$ by A5, Left Logical Equivalence, and Right Logical Equivalence. From $\alpha \vee \beta \mid\!\sim \alpha$ and A8 we get $\alpha \wedge \neg\alpha \mid\!\sim \beta$. By assumption we have $\vdash \beta \vee \gamma$. So $\beta \vee \gamma \mid\!\sim \neg\beta$ by A5 and Left Logical Equivalence – a contradiction. Now suppose $\overline{A} \cap \overline{C} = \emptyset$, i.e. $\vdash \alpha \vee \gamma$, and assume for reductio that $\alpha \vee \gamma \mid\!\sim \neg\alpha$. A5, Left Logical Equivalence, and Right Logical Equivalence yield $\alpha \wedge \neg\alpha \mid\!\sim \alpha$. Conclude as before.
(iii) $\overline{A} \cap \overline{B} = \emptyset$ and $\overline{B} \cap \overline{C} \neq \emptyset$: We have

$$\alpha \vee \beta \not\mid\!\sim \neg\alpha \quad \text{and} \quad \beta \vee \gamma \mid\!\sim \beta,$$

for all $\alpha \in [\alpha']$, $\beta \in [\beta']$, $\gamma \in [\gamma']$. Suppose first $\overline{A} \cap \overline{C} \neq \emptyset$, i.e. $\not\vdash \alpha \vee \gamma$, and assume for reductio that $\alpha \vee \gamma \not\mid\!\sim \alpha$. A7 gives us $\alpha \vee \neg\alpha \mid\!\sim \neg\alpha$. By assumption we have $\vdash \alpha \vee \beta$, whence Left Logical Equivalence implies $\alpha \vee \beta \mid\!\sim \neg\alpha$ – a contradiction. Now suppose $\overline{A} \cap \overline{C} = \emptyset$, i.e. $\vdash \alpha \vee \gamma$. Then $\alpha \vee \gamma \not\mid\!\sim \neg\alpha$ by Left Logical Equivalence and the assumptions $\alpha \vee \beta \not\mid\!\sim \neg\alpha$ and $\vdash \alpha \vee \beta$. Hence $A \preceq C$.
(iv) $\overline{A} \cap \overline{B} = \emptyset$ and $\overline{B} \cap \overline{C} = \emptyset$: We have

$$\alpha \vee \beta \not\mid\!\sim \neg\alpha \quad \text{and} \quad \beta \vee \gamma \not\mid\!\sim \neg\beta,$$

for all $\alpha \in [\alpha']$, $\beta \in [\beta']$, $\gamma \in [\gamma']$. Suppose first $\overline{A} \cap \overline{C} \neq \emptyset$, i.e. $\not\vdash \alpha \vee \gamma$, and assume for reductio that $\alpha \vee \gamma \not\mid\!\sim \alpha$. Then $\alpha \vee \beta \mid\!\sim \neg\alpha$ by A7, Left Logical Equivalence, and the assumption $\vdash \alpha \vee \beta$ – a contradiction. Now suppose $\overline{A} \cap \overline{C} = \emptyset$, i.e. $\vdash \alpha \vee \gamma$. Then $\alpha \vee \gamma \not\mid\!\sim \neg\alpha$ by Left Logical Equivalence and the assumptions $\alpha \vee \beta \not\mid\!\sim \neg\alpha$ and $\vdash \alpha \vee \beta$. Hence $A \preceq C$. $\square$

So we have defined a weak order $\preceq \subseteq \mathcal{A} \times \mathcal{A}$ in terms of $\mid\!\sim$. As a consequence, $\simeq \subseteq \mathcal{A} \times \mathcal{A}$, where

$$A \simeq B \quad \Leftrightarrow \quad A \preceq B \quad \& \quad B \preceq A,$$

is an equivalence relation over $\mathcal{A}$, i.e. a reflexive, symmetric, and transitive binary relation over $\mathcal{A}$. Another immediate consequence is that $\prec \subseteq \mathcal{A} \times \mathcal{A}$, where

$$A \prec B \quad \Leftrightarrow \quad A \preceq B \quad \& \quad B \not\preceq A,$$

is asymmetric (if $A \prec B$, then $B \not\prec A$) and transitive. As third corollary we note that $\langle \mathcal{A}/_{\simeq}, \preceq_{\simeq} \rangle$ is a simple order, where for $[C] = \{C' \in \mathcal{A} : C \simeq C'\}$,

$$[A] \preceq_{\simeq} [B] \quad \Leftrightarrow \quad A \preceq B.$$

That $\langle \mathcal{A}/_{\simeq}, \preceq_{\simeq} \rangle$ is a simple order means that $\langle \mathcal{A}/_{\simeq}, \preceq_{\simeq} \rangle$ is a weak order (connected and transitive) that is antisymmetric: If $[A] \preceq_{\simeq} [B]$ and $[B] \preceq_{\simeq} [A]$, then $[A] = [B]$. So the elements $[A]$ of $\mathcal{A}/_{\simeq}$ partition $\mathcal{A}$. We will now show that $\langle \mathcal{A}/_{\simeq}, \preceq_{\simeq} \rangle$ is a well-order, i.e.

1. Reflexivity: $[A] \preceq_{\simeq} [A]$
2. Transitivity: $[A] \preceq_{\simeq} [B] \quad \& \quad [B] \preceq_{\simeq} [C] \quad \Rightarrow \quad [A] \preceq_{\simeq} [C]$
3. Antisymmetry: $[A] \preceq_{\simeq} [B] \quad \& \quad [B] \preceq_{\simeq} [A] \quad \Rightarrow \quad [A] = [B]$
4. Connectedness (Linearity): $[A] \preceq_{\simeq} [B] \quad \text{or} \quad [B] \preceq_{\simeq} [A]$
5. Minimum: $\emptyset \neq M \subseteq \mathcal{A}/_{\simeq} \quad \Rightarrow \quad \exists\, [A] \in M \;\; \forall [B] \in M : [A] \preceq_{\simeq} [B]$

As Reflexivity follows from Connectedness, we only have to show Minimum. It suffices to show that there is no strictly $\prec_{\simeq}$-decreasing sequence $(E_n)_{n \in N}$ of elements $E_n \in \mathcal{A}/_{\simeq}$, where for each $n \in N$ there is an $A \in \mathcal{A}$ such that $E_n = [A]$. Before doing so, note the following useful properties:

$$A \subseteq B \quad \Rightarrow \quad B \preceq A \tag{3}$$

$$A \preceq B \quad \Rightarrow \quad A \simeq A \cup B \tag{4}$$

$$A \prec B, \quad A \prec C \quad \Rightarrow \quad A \prec B \cup C \tag{5}$$

Subproof:
(3) If $A \subseteq B$, for $A = Mod\,(\alpha')\,, B = Mod\,(\beta') \in \mathcal{A}$, then $\alpha \vdash \beta$ for all $\alpha \in [\alpha']$, $\beta \in [\beta']$. By A16 and Left Logical Equivalence, $\beta \vee \alpha \mid\!\sim \beta$. If $\nvdash \beta \vee \alpha$, we have $B \preceq A$. If $\vdash \beta \vee \alpha$, then $\vdash \beta$, and so Reflexivity, Left Logical Equivalence, Right Logical Equivalence, and Selectivity yield $\beta \vee \alpha \not\mid\!\sim \neg\beta$. Hence $B \preceq A$.
(4) Suppose $A \preceq B$, for $A = Mod\,(\alpha')\,, B = Mod\,(\beta') \in \mathcal{A}$. (3) yields $A \cup B \preceq A$. If $\overline{A} \cap \overline{B} \neq \emptyset$, i.e. $\nvdash \alpha \vee \beta$, then $\alpha \vee \beta \mid\!\sim \alpha$, for all $\alpha \in [\alpha']$, $\beta \in [\beta']$. In this case $A \preceq A \cup B$ iff $\alpha \vee \gamma \mid\!\sim \alpha$, for all $\alpha \in [\alpha']$, $\gamma \in [\alpha' \vee \beta']$. But $\alpha \vee \gamma \dashv\vdash \alpha \vee \beta$, for all $\alpha \in [\alpha']$, $\beta \in [\beta]$, $\gamma \in [\alpha' \vee \beta']$, whence the result follows from Left Logical Equivalence.

On the other hand, if $\overline{A} \cap \overline{B} = \emptyset$, then $\vdash \alpha \vee \beta$ and $\alpha \vee \beta \not\mid\!\sim \neg\alpha$, for all $\alpha \in [\alpha']$, $\beta \in [\beta']$. We have to show that $\alpha \vee \gamma \not\mid\!\sim \neg\alpha$, for all $\alpha \in [\alpha']$, $\gamma \in [\alpha' \vee \beta']$. But $\alpha \vee \gamma \dashv\vdash \alpha \vee \beta$, for all $\alpha \in [\alpha']$, $\beta \in [\beta]$, $\gamma \in [\alpha' \vee \beta']$, whence the result follows from Left Logical Equivalence.
(5) follows from (4): By Connectedness $B \preceq C$ or $C \preceq B$. Hence $B \simeq B \cup C$ or $C \simeq B \cup C$. Therefore, by Transitivity, if $A \prec B$ and $A \prec C$, then $A \prec B \cup C$.$\square$

Now suppose there is a strictly $\prec_{\simeq}$-decreasing sequence $(E_n)_{n \in N}$ of equivalence classes $E_n \in \mathcal{A}/_{\simeq}$:

$$\ldots \prec_{\simeq} E_n \prec_{\simeq} \ldots \prec_{\simeq} E_1 \prec_{\simeq} E_0.$$

For each equivalence class E_n there is a representative $A_n \in \mathcal{A}$ and a wff $\alpha'_n \in \mathcal{L}$ such that $E_n = [A_n]$ and $A_n = Mod\,(\alpha'_n)$. So, one level below, we get a strictly $\prec$-decreasing sequence of elements $A_n = Mod\,(\alpha'_n) \in \mathcal{A}$:

$$\ldots \prec A_n \prec \ldots \prec A_1 \prec A_0.$$

Note that for all $i, j \in N$: $\overline{A_i} \cap \overline{A_j} \neq \emptyset$. Suppose not. Then there are $i, j \in N$ such that $\overline{A_i} \subseteq A_j$, and thus $A_{j+1} \prec A_j \preceq \overline{A_i}$ and $A_{i+1} \prec A_i \preceq \overline{A_j}$ by useful property (3). If $i \leq j$, then $A_{j+1} \prec A_j \preceq A_i$ and $A_{j+1} \prec A_j \preceq \overline{A_i}$, whence useful property (5) gives us $A_{j+1} \prec A_i \cup \overline{A_i}$ – in contradiction to $A_i \cup \overline{A_i} \preceq A_{j+1}$, which we get from (3). If $j < i$, then $A_{i+1} \prec A_i \prec A_j$ and $A_{i+1} \prec A_i \preceq \overline{A_j}$, whence (5) gives us $A_{i+1} \prec A_j \cup \overline{A_j}$ – in contradiction to $A_j \cup \overline{A_j} \preceq A_{i+1}$, which we get from (3).

Hence for all $i, j \in N$, all $\alpha_i \in [\alpha'_i]$, and all $\alpha_j \in \left[\alpha'_j\right]$: $\not\vdash \alpha_i \vee \alpha_j$. By the definition of $\preceq$ in terms of $\mid\!\sim$ we have for all $i \in N$, any $\alpha_i \in [\alpha'_i]$, and any $\alpha_{i+1} \in \left[\alpha'_{i+1}\right]$:

$$\not\vdash \alpha_i \vee \alpha_j, \quad \alpha_i \vee \alpha_{i+1} \mid\!\sim \alpha_{i+1}, \quad \text{and} \quad \alpha_i \vee \alpha_{i+1} \mid\!\not\sim \alpha_i.$$

This, however, contradicts A13, according to which there is an $n \in N$ such that for all $m \geq n$, $m \in N$: $\alpha_m \vee \alpha_{m+1} \mid\!\sim \alpha_m$, for all $\alpha_m \in [\alpha'_m]$ and all $\alpha_{m+1} \in \left[\alpha'_{m+1}\right]$.

As a well-order $\mathbf{A} = \langle \mathcal{A}/_{\simeq}, \preceq_{\simeq} \rangle$ has an order type ord $\mathbf{A} = \nu$. A basic fact about well-orders says that every well-ordered set of type $\nu \neq 0$ is isomorphic to the set of all ordinal numbers μ with $0 \leq \mu < \nu$ (ordered according to their magnitude). As $\mathcal{A}$ contains only countably many elements, the order type of $\mathbf{A}$ cannot be greater than the first limit ordinal ω. Hence we can write the elements of $\mathcal{A}/_{\simeq}$ as a sequence

$$E_0, E_1, \ldots, E_n, \ldots, \quad n < \nu = \text{ord}\,\mathbf{A} \leq \omega, \quad \mathbf{A} = \langle \mathcal{A}/_{\simeq}, \preceq_{\simeq} \rangle,$$

i.e.

$$[A_0] \prec [A_1] \prec \ldots \prec [A_n] \prec \ldots$$

Given this we define for every non-empty $A = Mod\,(\alpha') \in \mathcal{A}$: $\varrho\,(A) = n$, where $A \in E_n = [A_n]$. For $\emptyset \in \mathcal{A}$ we stipulate $\varrho\,(\emptyset) = \infty\,(= \omega)$. In this way every

$Mod(\alpha) \in \mathcal{A}$ gets its rank $\varrho(Mod(\alpha))$, and we only have to show that ϱ is a regular ranking function. This is easily done by using the useful properties.

By (3) $Mod_{\mathcal{L}} \preceq A$ for every $A \in \mathcal{A}$. Hence $\varrho(Mod_{\mathcal{L}}) = 0$. Furthermore, $\varrho(A) < \varrho(\emptyset)$ for every non-empty $A \in \mathcal{A}$. By Connectedness, $A \preceq B$ or $B \preceq A$ for all $A, B \in \mathcal{A}$. In the first case (4) yields $A \simeq A \cup B$; in the second case (4) yields $B \simeq A \cup B$. Hence $\varrho(A \cup B) = \min\{\varrho(A), \varrho(B)\}$.

ϱ on $\mathcal{A}$ induces a ranking $\varrho_{\mathcal{L}}$ on $\mathcal{L}$ by defining $\varrho_{\mathcal{L}}(\alpha) = \varrho(Mod(\alpha))$ for all $\alpha \in \mathcal{L}$. We have to show that

$$\alpha \mid\sim \beta \;\Leftrightarrow\; \varrho_{\mathcal{L}}(\beta \wedge \alpha) < \varrho_{\mathcal{L}}(\neg\beta \wedge \alpha) \quad \& \quad \varrho_{\mathcal{L}}(\neg\beta \wedge \neg\alpha) \leq \varrho_{\mathcal{L}}(\beta \wedge \neg\alpha)$$
$$\text{or}$$
$$\varrho_{\mathcal{L}}(\beta \wedge \alpha) \leq \varrho_{\mathcal{L}}(\neg\beta \wedge \alpha) \quad \& \quad \varrho_{\mathcal{L}}(\neg\beta \wedge \neg\alpha) < \varrho_{\mathcal{L}}(\beta \wedge \neg\alpha).$$

Subproof:
$\Rightarrow$: If $\alpha \mid\sim \beta$, then $\alpha \mid\sim \alpha \wedge \beta$ and $\neg\alpha \mid\sim \neg\alpha \wedge \neg\beta$ by Weak Composition and A5. Left Logical Equivalence yields

$$(\alpha \wedge \beta) \vee (\alpha \wedge \neg\beta) \mid\sim \alpha \wedge \beta \quad \text{and} \quad (\neg\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta) \mid\sim \neg\alpha \wedge \neg\beta,$$

which means $A \cap B \preceq A \cap \overline{B}$ and $\overline{A} \cap \overline{B} \preceq \overline{A} \cap B$, for $A = Mod(\alpha)$ and $B = Mod(\beta)$, provided both A and $\overline{A}$ are not empty.

If $A = \emptyset$, i.e. $\vdash \neg\alpha$, then $\neg\alpha \mid\sim \neg\beta$. Left Logical Equivalence then gives us $\beta \vee \neg\beta \mid\sim \neg\beta$, which means $\overline{A} \cap B = B \not\preceq \overline{B} = \overline{A} \cap \overline{B}$. Hence $\varrho(\overline{A} \cap \overline{B}) < \varrho(\overline{A} \cap B)$. As $A = A \cap B = A \cap \overline{B}$, we have $A \cap B \preceq A \cap \overline{B}$, and so $\varrho(A \cap B) \leq \varrho(A \cap \overline{B})$. A similar argument applies in case $\overline{A} = \emptyset$. So assume both A and $\overline{A}$ are not empty. Then

$$\varrho(A \cap B) \leq \varrho(A \cap \overline{B}) \quad \& \quad \varrho(\overline{A} \cap \overline{B}) \leq \varrho(\overline{A} \cap B).$$

It remains to be shown that at least one of these inequalities is strict. The assumption $\alpha \mid\sim \beta$ and Right Logical Equivalence yield $\alpha \mid\sim \neg\neg\beta$. By A9

$$\alpha \not\mid\sim \alpha \wedge \neg\beta \quad \text{or} \quad \alpha \not\mid\sim \alpha \vee \neg\beta.$$

Left Logical Equivalence, A5, and Right Logical Equivalence give us

$$(\alpha \wedge \beta) \vee (\alpha \wedge \neg\beta) \not\mid\sim \alpha \wedge \neg\beta \quad \text{or} \quad (\neg\alpha \wedge \beta) \vee (\neg\alpha \wedge \neg\beta) \not\mid\sim \neg\alpha \wedge \beta.$$

In the first case we get $A \cap \overline{B} \not\preceq A \cap B$, which means $\varrho(A \cap B) < \varrho(A \cap \overline{B})$. In the second case we get $\overline{A} \cap B \not\preceq \overline{A} \cap \overline{B}$, which means $\varrho(\overline{A} \cap \overline{B}) < \varrho(\overline{A} \cap B)$.

$\Leftarrow$: By the definition of ϱ in terms of $\preceq$ we have $A \cap \overline{B} \not\preceq A \cap B$ and $\overline{A} \cap \overline{B} \preceq \overline{A} \cap B$ for $A = Mod(\alpha)$ and $B = Mod(\beta)$ – or the other way round, in which case a similar argument applies. $A \neq \emptyset$, since $\emptyset \preceq \emptyset$. If $\overline{A} = \emptyset$, then $\overline{B} \not\preceq B$, and so $\neg\beta \vee \beta \mid\sim \neg\neg\beta$ by the definition of $\preceq$ in terms of $\mid\sim$. Left Logical Equivalence and Right Logical Equivalence yield $\alpha \mid\sim \beta$. So suppose both A and $\overline{A}$ are not empty. Then we have $\nvdash \alpha$, $\alpha \nvdash \bot$, and, by the definition of $\preceq$ in terms of $\mid\sim$,

$$(\alpha \wedge \beta) \vee (\alpha \wedge \neg\beta) \not\mid\sim \alpha \wedge \neg\beta \quad \text{and} \quad (\neg\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta) \mid\sim \neg\alpha \wedge \neg\beta.$$

Left Logical Equivalence, A5, and Right Logical Equivalence give us

$$\alpha \not\mid\sim \alpha \wedge \neg\beta, \quad \alpha \mid\sim \alpha \vee \beta, \quad \nvdash \alpha, \quad \alpha \nvdash \bot,$$

and A10 yields $\alpha \mid\sim \beta$. □

By the Extension Theorem for Rankings on Languages (Huber to appear c) there exists a unique minimal pointwise ranking function κ on $Mod_{\mathcal{L}}$ such that

$$\varrho(Mod(\alpha)) = \varrho_{\mathcal{L}}(\alpha) = \min\{\kappa(\omega) : \omega \in Mod(\alpha)\}$$

for all consistent $\alpha \in \mathcal{L}$. □

4 Comparisons and Further (Non-) Principles

The papers developing a logic of confirmation I have come across are Flach (2000), Milne (2000), and Zwirn & Zwirn (1996). Zwirn & Zwirn (1996) argue that there is no unified logic of confirmation taking into account all of the partly conflicting aspects of confirmation. Flach (2000) argues that there are two logics of "induction", as he calls it, viz. confirmatory and explicatory induction (corresponding to Hempel's conditions 1-3 and 4, respectively). Milne (2000) argues that there is a logic of confirmation – namely the logic of positive probabilistic relevance – but that it does not deserve to be called a logic.

We have already seen some of the principles of Zwirn & Zwirn (1996). Below the present approach is compared to Flach's explanatory and confirmatory consequence relations and the nonmonotonic consequence relations of Kraus & Lehmann & Magidor (1990). Before doing so let us consider the remaining principles of Zwirn & Zwirn (1996) and a few further ones. The following are admissible:

A18. $\alpha \nvdash \bot \quad \Rightarrow \quad \alpha \not\mid\!\sim \alpha \wedge \neg\alpha$ Consistency*

A19. $\nvdash \alpha \quad \Rightarrow \quad \alpha \not\mid\!\sim \alpha \vee \neg\alpha$ Informativeness

A20. $\alpha \mid\!\sim \alpha \rightarrow \beta \quad \Rightarrow \quad \alpha \mid\!\sim \beta$ Ampliativity I

A21. $\alpha \vee \neg\alpha \mid\!\sim \alpha \quad \Rightarrow \quad \alpha \vee \beta \mid\!\sim \alpha$

A22. $\alpha \mid\!\sim \beta, \quad \alpha \mid\!\sim \gamma \quad \Rightarrow \quad \alpha \mid\!\sim \beta \wedge \gamma$ or $\alpha \mid\!\sim \beta \vee \gamma$

quasi-Composition

A23. $\alpha\vee\beta\vee\gamma \mid\!\sim \beta\vee\gamma, \quad \nvdash \alpha\vee\beta, \quad \nvdash \alpha\vee\gamma \quad \Rightarrow \quad \alpha\vee\beta \mid\!\sim \beta$ or $\alpha\vee\gamma \mid\!\sim \gamma$

As indicated by the *-star, Consistency is one of the core principles of Zwirn & Zwirn (1996) – as is Z-Selectivity, viz. Selectivity restricted to consistent α on the left hand side (Z-Selectivity is, of course, also admissible). Ampliativity I is one direction of Ampliativity (Zwirn & Zwirn 1996, 201). Among the principles of Zwirn & Zwirn (1996) not discussed below are the following inadmissible ones (I use roman numerals for non-principles):

i. $\alpha \mid\!\sim \alpha \wedge \beta \quad \Rightarrow \quad \alpha \mid\!\sim \beta$ Weak Consequence

ii. $\alpha \mid\!\sim \beta \quad \Rightarrow \quad \alpha \mid\!\sim \alpha \rightarrow \beta$ Ampliativity II

Ampliativity II is a special case of

iii. $\alpha \mid\!\sim \beta, \quad \alpha \vdash \beta \leftrightarrow \gamma \quad \Rightarrow \quad \alpha \mid\!\sim \gamma$ Levi Principle

The Levi Principle requires, among other things, that all verified theories are treated the same. It is clear that this does not hold for acceptability, because not all verified theories are as uninformative as tautological theories. Given Carnap's discussion of Hempel's Special Consistency Condition 3.1 (quoted in Huber submitted, section 2), it is particularly interesting to observe that

iv. $\alpha \mid\!\sim \beta, \quad \beta \vdash \neg\gamma \quad \Rightarrow \quad \alpha \not\mid\!\sim \gamma$ Strong Selectivity

is not admissible.

4.1 Explanatory and Confirmatory Consequence Relations

According to (Flach 2000, 167ff) any inductive consequence relation satisfies Left Logical Equivalence, Right Logical Equivalence, Verification, Left Reflexivity, Right Reflexivity, Right Extension, and Falsification (this is indicated by the superscript 'I'). F-Consistency (called Consistency by Flach 2000, 168) is equivalent to Falsification, given Left Logical Equivalence (Flach 2000, Lemma 1). Hence it is also satisfied by any inductive consequence relation (the additional superscript 'd' indicates that it is a derived principle).

A2.^{I} $\alpha \mid\sim \beta, \quad \alpha \dashv\vdash \gamma \quad \Rightarrow \quad \gamma \mid\sim \beta$ Left Logical Equivalence*

A3.^{I} $\alpha \mid\sim \beta, \quad \beta \dashv\vdash \gamma \quad \Rightarrow \quad \alpha \mid\sim \gamma$ Right Logical Equivalence*

A24.^{I} $\alpha \mid\sim \beta, \quad \alpha \wedge \beta \vdash \gamma \quad \Rightarrow \quad \alpha \wedge \gamma \mid\sim \beta$ Verification

A25.^{I} $\alpha \mid\sim \beta \quad \Rightarrow \quad \alpha \mid\sim \alpha$ Left Reflexivity

A26.^{I} $\alpha \mid\sim \beta \quad \Rightarrow \quad \beta \mid\sim \beta$ Right Reflexivity

A27.^{I} $\alpha \mid\sim \beta, \quad \alpha \wedge \beta \vdash \gamma \quad \Rightarrow \quad \alpha \mid\sim \beta \wedge \gamma$ Right Extension

v.^{I} $\alpha \mid\sim \beta, \quad \alpha \wedge \beta \vdash \gamma \quad \Rightarrow \quad \alpha \wedge \neg\gamma \not\mid\sim \beta$ Falsification

vi.$^{I-d}$ $\beta \vdash \neg\alpha \quad \Rightarrow \quad \alpha \not\mid\sim \beta$ F-Consistency

These principles hold for acceptability relations, if Falsification and F-Consistency are weakended to quasi-Falsification and quasi-F-Consistency, respectively.

A28.$^{I-d}$ $\alpha \mid\sim \beta, \quad \alpha \wedge \beta \vdash \gamma, \quad \alpha \nvdash \gamma \quad \Rightarrow \quad \alpha \wedge \neg\gamma \not\mid\sim \beta$ quasi-Falsification

A29.$^{I-d}$ $\beta \vdash \neg\alpha, \quad \nvdash \neg\alpha \quad \Rightarrow \quad \alpha \not\mid\sim \beta$ quasi-F-Consistency

Left Reflexivity and Right Reflexivity are unconditionally satisfied by acceptability relations. In Flach (2000) the antecedents ensure that α and β are consistent.

Among inductive consequence relations Flach distinguishes between consequence relations for explanatory induction and for confirmatory induction. Explanatory induction $\mid\sim$ is semantically characterised by defining $\alpha \mid\sim_W \beta$ iff (i) there is an $\omega \in W$ such that $\omega \models \beta$, and (ii) for all $\omega \in W$: $\omega \models \beta \rightarrow \alpha$, where W is a subset of the set of all models $Mod_{\mathcal{L}}$ for the propositional language $\mathcal{L}$ and $\models \subseteq Mod_{\mathcal{L}} \times \mathcal{L}$ is a compact satisfaction relation.

Explanatory induction thus focuses more or less exclusively (apart from demanding β to be W-consistent) on the logical strength of β. It satisfies all principles for inductive consequence relations and is syntactically characterised (indicated by the superscript 'E') by Explanatory Reflexivity, Left Consistency, Admissible Right Strengthening, Cautious Monotonicity (called Incrementality by Flach 2000, 172), Predictive Convergence, and Conditionalisation. In addition, it satisfies Admissible Converse Entailment, Consistent Right Strengthening, and Convergence.

A30.^{E} $\alpha \mid\!\sim \alpha, \quad \neg\beta \not\mid\!\sim \alpha \quad \Rightarrow \quad \beta \mid\!\sim \beta$ Explanatory Reflexivity

A31.^{E} $\alpha \mid\!\sim \beta \quad \Rightarrow \quad \neg\alpha \not\mid\!\sim \beta$ Left Consistency

vii.^{E} $\alpha \mid\!\sim \beta, \quad \gamma \mid\!\sim \gamma, \quad \gamma \vdash \beta \quad \Rightarrow \quad \alpha \mid\!\sim \gamma$ Admissible Right Strengthening

viii.^{E} $\alpha \mid\!\sim \gamma, \quad \beta \mid\!\sim \gamma \quad \Rightarrow \quad \alpha \wedge \beta \mid\!\sim \gamma$ Cautious Monotonicity

vix.^{E} $\alpha \wedge \gamma \vdash \beta, \quad \alpha \mid\!\sim \gamma \quad \Rightarrow \quad \beta \mid\!\sim \gamma$ Predictive Convergence

x.^{E} $\alpha \mid\!\sim \beta \wedge \gamma \quad \Rightarrow \quad \beta \rightarrow \alpha \mid\!\sim \gamma$ Conditionalisation

xi.$^{E-d}$ $\beta \mid\!\sim \beta, \quad \beta \vdash \alpha \quad \Rightarrow \quad \alpha \mid\!\sim \beta$ Admissible Converse Entailment

xii.$^{E-d}$ $\alpha \mid\!\sim \gamma, \quad \neg\beta \not\mid\!\sim \gamma \quad \Rightarrow \quad \alpha \mid\!\sim \beta \wedge \gamma$ Consistent Right Strengthening

xiii.$^{E-d}$ $\alpha \vdash \beta, \quad \alpha \mid\!\sim \gamma \quad \Rightarrow \quad \beta \mid\!\sim \gamma$ Convergence

Acceptability relations satisfy Explanatory Reflexivity and Left Consistency, but they violate Admissible Right Strengthening, Cautious Monotonicity, Predictive Convergence, Conditionalisation, Admissible Converse Entailment, Consistent Right Strengthening, and Convergence.

Another class of inductive consequence relations is given by what Flach calls confirmatory induction. These are semantically characterised with the help of *confirmatory structures* $W = \langle S, [\cdot], \|\cdot\| \rangle$, where S is a set of semantic objects, and $[\cdot]$ and $\|\cdot\|$ are functions from the propositional language $\mathcal{L}$ into the powerset of S. $W = \langle S, [\cdot], \|\cdot\| \rangle$ is *simple* just in case for all $\alpha, \beta \in \mathcal{L}$: $[\alpha] \subseteq \|\alpha\|$, $\|\alpha \wedge \beta\| = \|\alpha\| \cap \|\beta\|$, $\|\neg\alpha\| = S \setminus \|\alpha\|$, and $\|\alpha\| = S$ iff $\models \alpha$. Given a confirmatory structure W, the *closed confirmatory consequence relation* $\mid\!\sim_W$ defined by W is the usual KLM consequence relation with the additional requirement that α be consistent in the sense of $[\cdot]$, i.e. $\alpha \mid\!\sim_W \beta$ iff $\emptyset \neq [\alpha] \subseteq \|\beta\|$.

Closed confirmatory induction thus focuses more or less exclusively (apart from demanding α to be $[\cdot]$-consistent) on the logical weakness of β. Simple confirmatory consequence relations are syntactically characterised (indicated by the superscript 'SC') by Selectivity (called Right Consistency by Flach 2000, 179), Right And (called And in Kraus & Lehmann & Magidor 1990, 179, and called Composition in Zwirn & Zwirn 1996, 201), and Cut (called Predictive Right Weakening by Flach 2000, 178). In addition, simple confirmatory consequence relations satisfy Right Weakening (called Consequence in Zwirn & Zwirn 1996, 201) and its instance Admissible Entailment.

A15.^{SC} $\alpha \mid\sim \beta \quad \Rightarrow \quad \alpha \mid\not\sim \neg\beta$ Selectivity*

xii.^{SC} $\alpha \mid\sim \beta, \quad \alpha \mid\sim \gamma \quad \Rightarrow \quad \alpha \mid\sim \beta \wedge \gamma$ Right And

xiii.^{SC} $\alpha \mid\sim \beta, \quad \alpha \wedge \beta \vdash \gamma \quad \Rightarrow \quad \alpha \mid\sim \gamma$ Cut

xiv.$^{SC-d}$ $\alpha \mid\sim \beta, \quad \beta \vdash \gamma \quad \Rightarrow \quad \alpha \mid\sim \gamma$ Right Weakening (Right Monotonicity)

xv.$^{SC-d}$ $\alpha \mid\sim \alpha, \quad \alpha \vdash \beta \quad \Rightarrow \quad \alpha \mid\sim \beta$ Admissible Entailment

As simple confirmatory consequence relations violate Left Logical Equivalence, Verification, and Right Reflexivity, they are no inductive consequence relations (though they do satisfy Right Logical Equivalence, Falsification, Left Reflexivity, Right Extension, and F-Consistency).

$W = \langle S, l, \prec \rangle$ is a *preferential structure* (Kraus & Lehmann & Magidor 1990) iff l is a function from S into $Mod_{\mathcal{L}}$, and $\prec$ is a strict partial order on S such that for all $\alpha \in \mathcal{L}$ and all $t \in \widehat{\alpha} = \{s \in S : l(s) \models \alpha\}$: t is minimal w.r.t. $\prec$, or there is an $s \in S$ which is minimal in $\widehat{\alpha}$ and such that $s \prec t$. A preferential structure $W = \langle S, l, \prec \rangle$ induces a *preferential confirmatory structure* by defining:

$$\begin{aligned} \|\alpha\| &= \{s \in S : l(s) \models \alpha\} \\ [\alpha] &= \{s \in \|\alpha\| : \forall s' \in S\, (s' < s \rightarrow s' \notin \|\alpha\|)\} \end{aligned}$$

Every preferential confirmatory structure is a simple confirmatory structure. Preferential confirmatory consequence relations, i.e. consequence relations $\mid\sim_W$ with W a preferential confirmatory structure, satisfy all principles for inductive consequence relations. They are syntactically characterised (indicated by the superscript 'PC') by Selectivity, Right And, Cut, and, in addition, Left Logical Equivalence, Confirmatory Reflexivity, Left Or (called Or in Kraus & Lehmann & Magidor 1990, 190), and Strong Verification.

A15.^{PC} $\alpha \mid\sim \beta \quad \Rightarrow \quad \alpha \mid\not\sim \neg\beta$ Selectivity*

xii.^{PC} $\alpha \mid\sim \beta, \quad \alpha \mid\sim \gamma \quad \Rightarrow \quad \alpha \mid\sim \beta \wedge \gamma$ Right And

xiii.^{PC} $\alpha \mid\sim \beta, \quad \alpha \wedge \beta \vdash \gamma \quad \Rightarrow \quad \alpha \mid\sim \gamma$ Cut

A2.^{PC} $\alpha \mid\sim \beta, \quad \alpha \dashv\vdash \gamma \quad \Rightarrow \quad \gamma \mid\sim \beta$ Left Logical Equivalence*

A32.^{PC} $\alpha \mid\sim \alpha, \quad \alpha \mid\not\sim \neg\beta \quad \Rightarrow \quad \beta \mid\sim \beta$ Confirmatory Reflexivity

xvi.^{PC} $\alpha \mid\sim \gamma, \quad \beta \mid\sim \gamma \quad \Rightarrow \quad \alpha \vee \beta \mid\sim \gamma$ Left Or

xvii.^{PC} $\alpha \mid\sim \gamma, \quad \alpha \mid\sim \beta \quad \Rightarrow \quad \alpha \wedge \gamma \mid\sim \beta$ Strong Verification

Acceptability relations satisfy Selectivity, Left Logical Equivalence, and Confirmatory Reflexivity, but they violate Right And, Cut, Right Weakening, Admissible Entailment, Left Or, and Strong Verification.

In opposed to closed confirmatory consequence relations *open confirmatory consequence relations* $\mid\sim_W$, where W is a confirmatory structure, are given by: $\alpha \mid\sim_W \beta$ iff $[\alpha] \cap \|\beta\| \neq \emptyset$. Classical confirmatory structures are simple confirmatory structures with $[\cdot] = \|\cdot\|$. So open classical confirmatory consequence is just classical consistency. It satisfies all principles for inductive consequence relations and is syntactically characterised (indicated by the superscript 'OCC') by Predictive Convergence, Cut, F-Consistency, and Disjunctive Rationality, none of which are satisfied by acceptability relations.

viii.^{OCC} $\alpha \wedge \gamma \vdash \beta, \quad \alpha \mid\sim \gamma \quad \Rightarrow \quad \beta \mid\sim \gamma$ Predictive Convergence

xiii.^{OCC} $\alpha \mid\sim \beta, \quad \alpha \wedge \beta \vdash \gamma \quad \Rightarrow \quad \alpha \mid\sim \gamma$ Cut

xviii.^{OCC} $\beta \vdash \neg\alpha \quad \Rightarrow \quad \alpha \mid\not\sim \beta$ F-Consistency

xix.^{OCC} $\alpha \vee \beta \mid\sim \gamma, \quad \beta \mid\not\sim \gamma \quad \Rightarrow \quad \alpha \mid\sim \gamma$ Disjunctive Rationality

As open classical confirmatory induction satisfies both Predictive Convergence and Cut, it somehow combines aspects of explanatory induction on the one hand and confirmatory induction on the other hand. However, the resulting system is so weak that just about anything goes. After all, only logically incompatible sentences do not confirm each other. In contrast to this the combination of the plausibility and informativeness aspects achieved by acceptability relations is much more stringent: In order for β to be a possible inductive consequence of α, β must be at least as plausible given α as and more informative about α than its negation $\neg\beta$, or β must be more plausible given α than and at least as informative about α as its negation $\neg\beta$.

4.2 Nonmonotonic Consequence Relations

The following principles from Kraus & Lehmann & Magidor (1990) are satisfied by acceptability relations.

A33. $\alpha \mid\sim \beta \to \gamma, \quad \alpha \mid\sim \beta \quad \Rightarrow \quad \alpha \mid\sim \gamma$ MPC

A34. $\alpha_0 \mid\sim \alpha_1, \quad \ldots, \quad \alpha_{k-1} \mid\sim \alpha_k, \quad \alpha_k \mid\sim \alpha_0 \quad \Rightarrow \quad \alpha_0 \mid\sim \alpha_k$ Loop

A35. $\alpha \wedge \beta \mid\sim \gamma, \quad \alpha \wedge \neg\beta \mid\sim \gamma \quad \Rightarrow \quad \alpha \mid\sim \gamma$ Proof by Cases, D

The following principles are not admissible (xx-xxii are mentioned in both Kraus & Lehmann & Magidor 1990 and in Zwirn & Zwirn 1996). Supraclassicality is again one of the core principles of Zwirn &Zwirn (1996) (hence the *-star), and the numbers refer to the numbering in Kraus & Lehmann & Magidor (1990).

xx. $\alpha \vdash \beta \quad \Rightarrow \quad \alpha \mid\sim \beta$ Entailment, Supraclassicality*

xxi. $\beta \vdash \alpha \quad \Rightarrow \quad \alpha \mid\sim \beta$ Conversion

xxii. $\alpha \mid\sim \beta \quad \Rightarrow \quad \neg\beta \mid\sim \neg\alpha$ Contraposition

xxiii. $\alpha \mid\sim \beta \to \gamma \quad \Rightarrow \quad \alpha \wedge \beta \mid\sim \gamma$ EHD

xxiv. $\alpha \mid\sim \beta, \quad \beta \mid\sim \gamma \quad \Rightarrow \quad \alpha \mid\sim \gamma$ Transitivity

xxv. $\alpha \mid\sim \beta, \quad \beta \mid\sim \alpha, \quad \alpha \mid\sim \gamma \quad \Rightarrow \quad \beta \mid\sim \gamma$ Equivalence

xxvi. $\alpha \wedge \beta \mid\sim \gamma \quad \Rightarrow \quad \alpha \mid\sim \beta \to \gamma$ S

xxvii. $\alpha \mid\sim \beta \quad \Rightarrow \quad \alpha \wedge \gamma \mid\sim \beta$ or $\alpha \wedge \neg\gamma \mid\sim \beta$ Negation Rationality

xxviii. $\alpha \mid\sim \gamma \quad \Rightarrow \quad \alpha \wedge \beta \mid\sim \gamma$ or $\alpha \mid\sim \neg\beta$ Rational Monotonicity

xxix. $\alpha \vee \beta \mid\sim \alpha, \quad \alpha \mid\sim \gamma \quad \Rightarrow \quad \alpha \vee \beta \mid\sim \gamma$ Nr 9

xxx. $\alpha_0 \mid\sim \alpha_1, \quad \ldots, \quad \alpha_k \mid\sim \alpha_{k-1}, \quad \alpha_k \mid\sim \alpha_0 \quad \Rightarrow \quad \alpha_0 \mid\sim \alpha_k$ Nr 15

xxxi. $\alpha \mid\sim \gamma, \quad \beta \mid\sim \delta \quad \Rightarrow \quad \alpha \vee \beta \mid\sim \gamma \vee \delta$ Nr 19

xxxii. $\alpha \vee \gamma \mid\sim \gamma, \quad \alpha \mid\sim \beta \quad \Rightarrow \quad \gamma \mid\sim \alpha \to \beta$ Nr 20

xxxiii. $\alpha \vee \beta \mid\sim \alpha, \quad \beta \vee \gamma \mid\sim \beta \quad \Rightarrow \quad \alpha \vee \gamma \mid\sim \alpha$ Nr 21

xxxiv. $\alpha \vee \beta \mid\!\sim \alpha, \quad \beta \vee \gamma \mid\!\sim \beta \quad \Rightarrow \quad \alpha \mid\!\sim \gamma \to \beta$ Nr 22

The violation of the following principle (called Monotonicity in Kraus & Lehmann & Magidor 1990, 180) means that acceptability relations are not monotonic.

xxxv. $\beta \mid\!\sim \gamma, \quad \alpha \vdash \beta \quad \Rightarrow \quad \alpha \mid\!\sim \gamma$ Left Monotonicity

As already observed in the previous subsection, acceptability relations are *genuinely nonmonotonic* in the sense that they also violate Right Monotonicity.

xv. $\alpha \mid\!\sim \beta, \quad \beta \vdash \gamma \quad \Rightarrow \quad \alpha \mid\!\sim \gamma$ Right Monotonicity, Right Weakening

So not only arbitrary strengthening of the premises, but also arbitrary weakening of the conclusion is not allowed. The reason is this: By arbitrarily weakening the conclusion information is lost – and the less informative conclusion might not be worth taking the risk of being led to a false conclusion.

The logic of theory assessment can also answer the question why everyday reasoning is satisfied with a standard that is weaker than truth-preservation in all possible worlds, and thus runs the risk of being led to a false conclusion. We are willing to take this risk, because we want to arrive at informative conclusions that go beyond the premises. However, as the relation of positive probabilistic relevance, acceptability relations are no proper consequence relations in the sense that their semantics is not in terms of the preservation of a particular property.

5 Carnap's Analysis Revisited

In conclusion let us turn back to Carnap's analysis of Hempel's conditions and his claim that Hempel was mixing up absolute and incremental confirmation. As argued in Huber (submitted, sections 2-4), Carnap's analysis is neither charitable nor illuminating; and the plausibility-informativeness theory provides a more charitable interpretation that is illuminating by accounting for Hempel's triviality result and his rejection of the Converse Consequence Condition. It is nevertheless interesting to consider the relation between Carnap's favoured concept of qualitative confirmation – viz. positive relevance in the sense of a regular probability measure – and our acceptability relations leading to plausible and informative conclusions.

Acceptability relations are unconditionally reflexive, whence any tautology is an acceptable theory for tautological data, and any contradiction is an acceptable theory for contradictory data. In part this is a consequence of stipulating

$\varrho(B \mid A) = 0$ whenever $\varrho(A) = \infty$ and could have been avoided (as in Flach's approach). In contrast to this positive probabilistic or rank-theoretic relevance on a field $\mathcal{A}$ over a set of possibilities W is reflexive except for propositions with extreme probabilities or ranks. The gap can be closed by extending the notion of positive relevance to include all pairs $\langle A, A \rangle$ for $A \in \mathcal{A}$. This means in particular that tautologies are positively relevant for tautologies and contradictions are positively relevant for contradictions. Let us call this broadened notion *extended* positive relevance.

The relation between acceptability and extended positive relevance is still slightly obscured by the fact that acceptability relations so far have been characterised in terms of ranking functions, whereas Carnap's positive relevance account is probabilistic. Given the same framework it is clear that extended positive relevance of α for β is a necessary condition for β to be an acceptable theory for α. More precisely, we have for any probability space $\langle W, \mathcal{A}, \Pr \rangle$ and any $A, B \in \mathcal{A}$ with $\Pr(A) > 0$:

$$\begin{array}{c} \left[\begin{array}{c} \Pr(B \cap A) > \Pr\left(\overline{B} \cap A\right) \\ \& \\ \Pr\left(\overline{B} \cap \overline{A}\right) \geq \Pr\left(B \cap \overline{A}\right) \end{array} \right] \\ \text{or} \\ \left[\begin{array}{c} \Pr(B \cap A) \geq \Pr\left(\overline{B} \cap A\right) \\ \& \\ \Pr\left(\overline{B} \cap \overline{A}\right) > \Pr\left(B \cap \overline{A}\right) \end{array} \right] \end{array} \quad \Rightarrow \quad \Pr(B \mid A) > \Pr(A)$$

Note that the antecedent is implied by the formulation with $\Pr(B \mid A)$ etc. instead of $\Pr(B \cap A)$ etc. Similarly, for any ranking space $\langle W, \mathcal{A}, \varrho \rangle$ and any $A, B \in \mathcal{A}$ with $\varrho(A), \varrho\left(\overline{A}\right) < \infty$:

$$\begin{array}{c} \left[\begin{array}{c} \varrho(B \cap A) < \kappa\left(\overline{B} \cap A\right) \\ \& \\ \varrho\left(\overline{B} \cap \overline{A}\right) \leq \varrho\left(B \cap \overline{A}\right) \end{array} \right] \\ \text{or} \\ \left[\begin{array}{c} \varrho(B \cap A) \leq \varrho\left(\overline{B} \cap A\right) \\ \& \\ \varrho\left(\overline{B} \cap \overline{A}\right) < \varrho\left(B \cap \overline{A}\right) \end{array} \right] \end{array} \quad \Rightarrow \quad \begin{array}{l} \varrho(A \cap B) + \varrho\left(\overline{A} \cap \overline{B}\right) < \\ < \varrho\left(A \cap \overline{B}\right) + \varrho\left(\overline{A} \cap B\right) \end{array}$$

The last clause is the definition of positive rank-theoretic relevance. Indeed,

Observation 2 *The extended positive relevance relation $\mid\!\succ_{\Pr}$ of a probabilistic assessment model $\langle W, \mathcal{A}, \Pr \rangle$ for a language $\mathcal{L}$ satisfies A1-A8, A10-A32, and A34-A35.*

Definition 3 *A probability space $\langle W, \mathcal{A}, \Pr\rangle$ is a probabilistic assessment model for the language $\mathcal{L}$ iff $W = Mod_{\mathcal{L}}$, $\{Mod(\alpha) : \alpha \in \mathcal{L}\} \subseteq \mathcal{A}$, and $\Pr(Mod(\alpha)) > 0$ for every consistent $\alpha \in \mathcal{L}$. The extended positive relevance relation $|\!\succ_{\Pr} \subseteq \mathcal{L} \times \mathcal{L}$ of a probabilistic assessment model $\langle W, \mathcal{A}, \Pr\rangle$ for $\mathcal{L}$ is defined as follows:*

$$|\!\succ_{\Pr} \quad = \quad \perp^{+}_{\Pr} \cup \{\langle \alpha, \beta\rangle \in \mathcal{L} \times \mathcal{L} : \alpha \dashv\vdash \beta\},$$

where $\perp^{+}_{\Pr}$ is the relation of positive relevance on $\mathcal{L}$ in the sense of $\Pr_{\mathcal{L}}$, i.e.

$$\alpha \perp^{+}_{\Pr} \beta \quad \Leftrightarrow \quad \Pr_{\mathcal{L}}(\alpha \wedge \beta) > \Pr_{\mathcal{L}}(\alpha) \cdot \Pr_{\mathcal{L}}(\beta).$$

Observation 3 *The extended positive relevance relation $|\!\succ_{\varrho}$ of a rank-theoretic assessment model $\langle Mod_{\mathcal{L}}, \mathcal{A}, \varrho\rangle$ for a language $\mathcal{L}$ satisfies A1-A8, A10-A32, A34-35, where*

$$|\!\succ_{\varrho} \quad = \quad \perp^{+}_{\varrho} \cup \{\langle \alpha, \beta\rangle \in \mathcal{L} \times \mathcal{L} : \alpha \dashv\vdash \beta\},$$

and $\perp^{+}_{\varrho}$ is the relation of positive relevance on $\mathcal{L}$ in the sense of $\varrho_{\mathcal{L}}$, i.e.

$$\alpha \perp^{+}_{\varrho} \beta \quad \Leftrightarrow \quad \varrho_{\mathcal{L}}(\alpha \wedge \beta) + \varrho_{\mathcal{L}}(\neg\alpha \wedge \neg\beta) < \varrho_{\mathcal{L}}(\alpha \wedge \neg\beta) + \varrho_{\mathcal{L}}(\neg\alpha \wedge \beta).$$

However, as

xxxvii. $\alpha \mid\!\sim \beta \quad \Rightarrow \quad \beta \mid\!\sim \alpha$ — Symmetry

is not satisfied by acceptability relations, the converse is not true. Both probabilistic and rank-theoretic (extended or unextended) positive relevance are symmetric, whereas acceptability relations are not – which, as noted by Christensen (1999, 437f), is as it should be.

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