Vagueness and Relative Truth

ANDREA IACONA
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According to a view called *nihilism*, sentences containing vague expressions cannot strictly speaking be true or false, because they lack definite truth conditions. While most theorists of vagueness tend to regard nihilism as a hopeless view, a few isolated attempts have been made to defend it. This paper aims to develop such attempts in a new direction by showing how nihilism, once properly spelled out, can meet three crucial explanatory challenges that respectively concern truth, assertibility, and communication.

1 OVERVIEW

Nihilism rests on the idea that a sentence can be true or false only if it has definite truth conditions. Since vague expressions lack definite extensions, a sentence that contains such expressions — from now on, a *vague sentence* — does not have definite truth conditions, so it cannot be true or false. This idea, which goes back to Frege, has been elaborated and defended in different ways by Ludwig and Ray, Braun and Sider, and Iacona.[1]

One way to express the essence of nihilism in three words is the following: *supervaluationism without supertruth*. As is well known, supervaluationism assumes that the vagueness of an expression consists in its capacity of being made precise in more than one way:

(P) If an expression is vague, it admits different precisifications

For example, 'bald' is vague in that there are different ways of delimiting its extension, each of which may be regarded as part of an admissible precisification of the language. Supervaluationism relies on (P) to provide an analysis of truth:

1. Frege 1903 claims that truth requires precision, as it treats vague epressions as expressions whose definition is incomplete. Ludwig and Ray 2002, Braun and Sider 2007, and Iacona 2010 develop this claim in different ways. The term 'nihilism' occurs in Williamson 1994 and in Braun and Sider 2007.

(S) A sentence is true if and only if it is true in all admissible precisifications.

Supervaluationism thus hinges on a distinction between two semantic levels marked by two kinds of value assignments. At the first level — the *valuation* level — each sentence receives a classical value for each element of a set of indices, namely, admissible precisifications. The notion of truth that pertains to this level may be called *relative truth*. At the second level — the *supervaluation* level — the sentence gets a non-indexed value on the basis of the indexed values it receives at the first level. Truth *simpliciter* is defined at this level as "supertruth", that is, truth in all admissible precisifications.[2]

Nihilism differs from supervaluationism in that it contemplates only one semantic level, the valuation level. This is to say that it grants (P) without retaining (S). Since (S) does not follow from (P), one can endorse (P) as a general hypothesis about vagueness without thereby being committed to the supervaluationist analysis of truth *simpliciter*. According to nihilism, (S) can be rejected on the ground that, as far as vague sentences are concerned, there is no such thing as truth *simpliciter*: the only intelligible notion of truth for vague sentences is relative truth.

The thought that underlies this denial of truth *simpliciter* is that a vague sentence does not express a definite proposition, where 'proposition' refers to a content evaluable as true or false. A vague sentence expresses a plurality of propositions, which amounts to a set of admissible precisifications. Since all the precisifications in the set are admissible, there is no such thing as *the* proposition expressed by the sentence. In other words, there is a sense of 'what is said' in which *nothing is said* by uttering a vague sentence. This is the negative claim to which nihilism owes its name, and which distinguishes it from the main extant theories of vagueness.

As to the name itself, two clarifications may help to avoid potential misunderstandings. First, we saw that nihilism differs from supervaluationism in one respect, namely, that it does not retain (S). In fact nihilism does not even imply that there is something wrong with calling a sentence supertrue when it is is true in all admissible precisifications. The only point it questions is that (S) provides an adequate analysis of truth *simpliciter*. This difference might be regarded as relatively minor or even negligible for some purposes, given that (P) holds in both cases. Accordingly, nihilism could equally be described

2. This is the standard construction of supervaluationism, which derives from Lewis 1970, Fine 1975, Dummett 1978, Kamp 1975.

as a non-standard form of supervaluationism, or as a distinct variation of one and the same basic view.[3]

Second, nihilism is not the same thing as "alethic nihilism", the claim that nothing is true.[4] Relative truth is a perfectly coherent notion of truth which applies non-vacuously to vague sentences. To say that a vague sentence is true relative to some admissible precisifications is to say that some propositions expressed by the sentence are true. So, something *is* true. In other words, even though truth *simpliciter* cannot apply to vague sentences, it can apply to the propositions that precisify those sentences. In what follows, the term 'nihilism' will be used in the non-alethic sense just explained, which is but one of the many uses of this term.

The structure of the paper is as follows. Sections 2 and 3 outline what I take to be the most plausible version of nihilism. Sections 4-8 address three crucial explanatory challenges that nihilism has to face, which concern respectively truth, assertibility, and communication. As will emerge — and this is a key point of the paper — the semantic method that best suits the three explanatory tasks identified is quantitative rather than qualitative.

2 MEANING AND CONTENT

Let us start from the claim that vague sentences do not express definite propositions. Consider the following sentence:

(1) John is bald

Since 'bald' has no definite extension, (1) has no definite truth conditions, which means that it does not express a definite proposition. So there is a sense in which nothing is said by uttering (1). Although this claim sounds overtly unorthodox, it is far less extreme than it might seem at first sight.

At least two points must be clear. First, to say that vague sentences do not express definite propositions is not quite the same thing as to say that they are meaningless. In order to draw the latter conclusion one would need to assume that sentences have meaning only if they have definite truth conditions. In particular, one would need to assume that a vague predicate such as 'bald' has meaning only if it has a definite extension. Althoug Frege might have entertained this thought, it is not essential for nihilism to endorse it. A more reasonable option for the nihilist is to grant that vague sentences have meaning, and

3. The term 'supervaluationism' might itself be used in a different way. If one were so inclined, one could replace its occurrences in this paper with 'standard supervaluationism'.

4. This how alethic nihilism is understood in Liggins 2024, among other works.

hold that they admit distinct interpretations precisely in virtue of their meaning. Thus one may coherently claim that the meaning of (1) is compatible with a plurality of interpretations.

Second, to say that vague sentences do not express definite propositions is not to deny that speakers have something in mind when they use them. When one utters (1), one associates some mental content to (1): one understands the meaning of the words that occur in (1), has intentions, and exhibits psychological states. The fact is that what one has in mind is not, and does not determine, a unique proposition. That is, when a speaker utters (1), no single specification of the extension of the words in (1) that makes it evaluable as true or false can rightfully be ascribed to the speaker.

The two points just made may be rephrased in terms of the familiar distinction between semantic meaning and speaker's meaning: one thing is what certain words mean, another thing what a speaker means by uttering those words. As is widely recognized, semantic meaning and speaker's meaning can diverge to some extent. But independently of such divergence, neither of these two ways of understanding meaning is ruled out by the claim that vague sentences do not express definite propositions.

In other words, even if it is granted that there is a sense in which nothing is said by uttering (1) — that is, (1) does not express a definite proposition — this does not rule out that there are other senses in which something is said by uttering (1). The expression 'what is said' can be understood in different ways, and the nihilist wants to be nihilist only about one of them.[5]

The foregoing remarks suggest that some terminological choices that have been made in the past to characterize nihilism are far from ideal. Williamson describes nihilism as an extreme view according to which vague predicates are "empty", "semantically defective", and "failed adjectives".[6] But this is misleading. Vague predicates are not "empty": they have meaning, even though they lack definite extensions. And it is seems inappropriate to say that they are "semantically defective" or "failed adjectives". Not having definite extensions should not be regarded as a defect. More generally, one can recognize that the semantics of natural language is not like the semantics of a formal language, where predicates have definite extensions, without thereby taking natural language to be defective. Natural language is good as is, and it would be simply wrongheaded to impose on it the semantic standards of a formal language.[7]

5. This point is made in Iacona 2010, p. 298.

6. Williamson 1994, pp. 168-169. Williamson 1996, p. 173.

7. Here I agree with Braun and Sider 2007, p. 135. According to Sainsbury 1999, for example, vague concepts have no boundaries, so it is pointless to describe them in set-theoretical terms.

The only point that really matters for nihilism concerns the relation between sentences and propositions: to one sentence correspond many propositions instead of one. Arguably, this point is crucial to the understanding of the relation between logic and natural language. Since sentences can be true or false only relative to admissible precisifications — which are suitable sharpenings of their meaning — logic applies to natural language not directly, as it were, but *modulo* such sharpenings.

3 LOGIC

From a logical point of view, the beauty of nihilism lies in its simplicity. Since the only notion of truth contemplated is relative truth, nihilism requires no departure from classical logic. In particular, nihilism does not run into three main problems that have been raised in connection with supervaluationism.

First, supertruth is not disquotational. Suppose that 'p' is true in some admissible precisifications and false in others. For any admissible precisification in which 'p' is true, "p' is true' is not true in that precisification. So the biconditional "p' is true if and only if p' is not true in that precisification, hence it is not supertrue.[8] By contrast, relative truth is disquotational. For every admissible precisification, 'p' is true in that precisification just in case "p' is true' is true in that precisification. So the biconditional "p' is true if and only if p' is always true.[9]

Second, supervaluationism entails that there are true existential generalizations without true instances, and true disjunctions without true disjuncts. This clashes with the intuition that the truth of an existential generalization is grounded in the truth of some of its instances, and that the truth of a disjunction is grounded in the truth of some of its disjuncts.[10] By contrast, relative truth does not have such properties. In every admissible precisification, every true existential generalization has true instances, and every true disjunction has true disjuncts.[11]

Third, supertruth fails to preserve some classical rules of inference, such as conditional proof, argument by cases, *reductio ad absurdum*, and contraposition. This is due to the definition of validity as necessary preservation of supertruth, *global validity*: an argument is valid if and only if, necessarily, if its premises are true in all admissible pre-

8. Williamson 1994, pp. 162-164, Braun and Sider 2007, p. 147.
9. Braun and Sider 2007, p. 147, Iacona 2010, p. 296.
10. Williamson 1994, pp. 153-154, Braun and Sider 2007, pp. 145-146.
11. Braun and Sider 2007, p. 145.

cisifications, its conclusion is true in all admissible precisifications.[12] Nihilism does not have this problem because the definition of validity that is most plausibly associated with it is *local validity*: an argument is valid if and only if, for every admissible precisification, necessarily, if its premises are true in that precisification, its conclusion is true in that precisification. The second definition, unlike the first, preserves all classical rules of inference.[13]

This last point is directly relevant to the nihilist treatment of the sorites, the paradoxical argument which goes as follows:

(2) 1000 grains make a heap
(3) For every n, if n grains make a heap, $n-1$ grains make a heap

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(4) 0 grains make a heap

This argument is valid in the local sense: in every admissible precisification, if (2) and (3) are true, (4) must be true as well. However, it is unsound in every admissible precisification. Consider any admissible precisification. Since it is admissible, it makes (2) true and (4) false. Moreover, for each of the collections of grains featuring in the series that goes from 1000 to 0 grains, it specifies whether or not that collection is a heap. This means that there is a cut-off point in the series, that is, a number n such that a collection of n grains belongs to the extension of 'heap' while a collection of $n-1$ grains does not belong to it. Therefore, (3) turns out to be false. More generally, the sorites is unsound because its universal premise is false in every admissible precisification.

Nihilism can also explain why (3) seems acceptable. Precisifications are artificial to some extent. As observed in section 2, they are not uniquely determined by the meaning of sentences, and they are not what speakers have in mind when they use sentences. In the case of (3), this emerges clearly if one thinks that any assignment of extension to 'heap' implies a cut-off point. Speakers normally use 'heap' *without* specifying its extension, so they do not draw such a line. Whenever they use 'heap' in an ordinary context to describe a given object, they simply do not consider the possibility that some cut-off point lurks in the vicinity of that object, namely, they tend to assume that relevantly similar objects may equally be described in the same way. This is why there is a natural inclination to reject the following sentence:

(5) There is n such that n grains make a heap but $n-1$ grains do not make a heap

12. Williamson 1994, pp. 146-153.

13. Williamson 1994, pp. 147-148, Keefe 2000, section 3, and Varzi 2007 spell out this distinction. For the adoption of local validity see Braun and Sider 2007, pp. 141-142, Iacona 2010, p. 299.

Since (3) amounts to the negation of (5), one may easily be tempted to accept (3). In other words, (3) draws its intuitive appeal from the fact that our use of 'heap' does not involve specification of a cut-off point.[14]

Note that nihilism, unlike supervaluationism, does not entail that (5) is true. Just as (3) is not false *simpliciter*, (5) is not true *simpliciter*. This relates to the point made above about existential generalizations: nihilism does not hold that (5) is a true existential generalization without true instances. The only sense in which (5) can be true, the relative sense, implies that (5) has one true instance.[15]

4 THREE MAJOR CHALLENGES

As emerges from section 3, nihilism is extremely simple in some crucial respects: it requires no departure from classical logic and yields a straightforward solution to the sorites. The hard question about nihilism is how can it explain language use without assuming that vague sentences express truth-evaluable contents. In particular, three major challenges are to be addressed.

Challenge 1: truth. Speakers normally judge sentences as true or false, and they do so without having in mind precisifications. For example, it is natural to say that (2) is true, or that (4) is false. But according to nihilism this is just loose talk: strictly speaking, (2) cannot be true, and (4) cannot be false. So the challenge is to explain why speakers feel entitled to treat vague sentences as true or false.

Challenge 2: assertibility. Speakers use sentences to make assertions, and it is quite common to judge assertions as correct or incorrect. For example, if one utters (2) it seems that one makes a correct assertion, while the same does not hold for (4). Although there is no universally accepted definition of assertibility, it is usually taken for granted that in order for an assertion to be correct, the speaker must have some justification for thinking that the content asserted is true. But nihilism seems to rule out such an account: not only it implies that the sentence uttered cannot be true, it also denies that there is such a thing as the content asserted. So the challenge is to provide a coherent account of assertibility.

Challenge 3: communication. Speakers typically use language for the purpose of communication, and most of the time they succeed in doing so. According to a naïve and widely accepted picture of communication, what normally happens when a speaker A communicates with another speaker B is that A utters a sentence that expresses a content X — presumably a content that A takes to be true — and B

14. Ludwig and Ray 2002, p. 432, Iacona 2010, p. 300.
15. See Braun and Sider 2007, p. 145.

grasps X when hearing the sentence uttered by A. This picture, however, is at odds with nihilism, as the latter denies that there is a single truth-evaluable content that A and B share: no unique proposition can be the content that A transmits to B. The challenge, therefore, is to provide an alternative account of communication.

Although some attempts have been made to explain language use in accordance with nihilism, in the works by Ludwig and Ray, Braun and Sider, and Iacona mentioned at the beginning, I think that the three questions just raised have not yet received fully satisfactory answers. The task of the rest of the paper is to show how such answers can be provided through a semantic method — *quantitative supervaluationism* — that resembles supervaluationism in some crucial respects but is quantitative rather than qualitative.[16]

5 QUANTITATIVE SUPERVALUATIONISM

Let L be a language whose alphabet is constituted by a set of sentence letters $p, q, r, \ldots$ and the connectives $\neg, \wedge, \vee$. The formation rules of L are as usual: sentence letters are atomic formulas; if α is a formula, so is $\neg\alpha$; if α and β are formulas, so are $\alpha \wedge \beta$ and $\alpha \vee \beta$.

The semantics of L is defined in two steps, which specify two kinds of value assignments: one pertains to the valuation level, the other pertains to the supervaluation level. While the first step is essentially as in supervaluationism, the second provides a quantitative measure that may be regarded as a generalization of the traditional qualitative measure.

Let us start from the basic ingredients of the semantics.

DEFINITION 1 A *frame* $\mathcal{F}$ for L is a pair $\langle X, E \rangle$, where

- X is a countable set;
- E is a function from X to $\mathcal{P}(\mathcal{P}(X))$ such that, for each $x \in X$, $E(x)$ is countable and $e \in E(x)$ only if $x \in e$.

X is a countable set of *points* understood as contexts, that is, coordinates relative to which sentences are evaluated. E assigns to each point x in X a countable set $E(x)$ of *extensions* of x, that is, of subsets of X which include x. Extensions are understood as precisifications. The distinction between points and extensions can be illustrated geometrically by representing the extensions of a point x as line segments bounded by x: each segment is a set of points that includes x, so the set of segments is centered on x. The countability condition imposed on X and E, which yields a useful technical simplification, is quite

16. Iacona and Iaquinto 2024 outlines quantitative supervaluationism and spells out some of its implications.

reasonable as a constraint on frames. Although uncountable sets of precisifications are perfectly conceivable, it seems that the notion of precisification can fruitfully be applied to paradigmatic examples of vague predicates, such as 'bald' or 'heap', without invoking such sets. Typically, the soritical series involved in those examples are finite.

DEFINITION 2 A model of L is a triple $\langle \mathcal{F}, P, V \rangle$, where $\mathcal{F}$ is a frame $\langle X, E \rangle$ while P and V are functions defined as follows:

- P assigns to each $x \in X$ a countably additive function P_x from $E(x)$ to $[0,1]$ in such a way that $\sum_{e \in E(x)} P_x(e) = 1$;
- V assigns 1 or 0 to each atomic formula of L for each pair x/e, where $e \in E(x)$.

P associates with each point x a *proximity assignment* P_x, which is understood as a measure of admissibility for the elements of $E(x)$. In the literature, admissibility is usually assumed to be a property that either belongs or does not belong to a precisification. But there seems to be nothing conceptually wrong with treating it as a gradable property, leaving room for the possibility that different precisifications have different degrees of admissibility.[17] For our purposes, though, it will suffice to restrict consideration to models in which all admissible precisifications are equally admissible.[18] V is a classical valuation function that assigns values to atomic formulas relative to point/extension pairs. These values are understood as truth values that simple sentences take in contexts relative to precisifications: to say that an atomic formula α has value 1, or 0, relative to x/e is to say that α is true, or false, in the context x relative to the precisification e.

The first step of our semantic construction — the valuation level — can now be completed by defining a function v that assigns values to the formulas of L relative to point/extension pairs:

DEFINITION 3

1 If α is atomic, $v(\alpha, x/e) = V(\alpha, x/e)$;

2 $v(\neg\alpha, x/e) = 1$ iff $v(\alpha, x/e) = 0$;

3 $v(\alpha \wedge \beta, x/e) = 1$ iff $v(\alpha, x/e) = 1$ and $v(\beta, x/e) = 1$;

4 $v(\alpha \vee \beta, x/e) = 1$ iff $v(\alpha, x/e) = 1$ or $(\beta, x/e) = 1$.

A relation of logical consequence is defined accordingly in terms of preservation of the value 1 for all point/extension pairs.

17. A proposal along these lines is made in Simons 2010, p. 485.
18. Williams 2011 defines degrees of determinacy in terms of a simple count measure, namely in terms of the number of precisifications that make a sentence true. This is the kind of measure one gets in our framework when one adopts the restriction just considered.

DEFINITION 4 $\alpha_1, \ldots \alpha_n \models_v \beta$ iff for every point/extension pair x/e in every model, if $v(\alpha_1, x/e) = 1, \ldots v(\alpha_n, x/e) = 1$, then $v(\beta, x/e) = 1$.

Leaving aside relativity to point/extension pairs, the relation expressed by the symbol $\models_v$ is nothing but the classical relation of logical consequence for a propositional language.

The second step — the supervaluation level — requires a different function sv that assigns values to the formulas of L relative to points. Assuming that $|\alpha|_x$ is the set of extensions of x such that $v(\alpha, x/e) = 1$, sv is defined as follows:

DEFINITION 5 $sv(\alpha, x) = \sum_{e \in |\alpha|_x} P_x(e)$

The value of α in x is the sum of the proximity values of the extensions of x in which α is true. Since $|\alpha|_x \subseteq E(x)$, and $\sum_{e \in E(x)} P_x(e) = 1$, we get that $sv(\alpha, x) \leq 1$. The limiting case in which $sv(\alpha, x) = 1$ arises when $v(\alpha, x/e) = 1$ for every $e \in E(x)$ such that $P_x(e) > 0$. In particular, tautologies always get value 1, that is, if $\models_v \alpha$, then $sv(\alpha, x) = 1$ for every x. The other limiting case is that in which $sv(\alpha, x) = 0$ because $v(\alpha, x/e) = 0$ for every $e \in E(x)$ such that $P_x(e) > 0$. In particular, contradictions always get value 0, that is, if $\models_v \neg\alpha$, then $sv(\alpha, x) = 0$ for every x. Therefore, for any α and x, $0 \leq sv(\alpha, x) \leq 1$. More precisely, sv turns out to be a probability function, as it satisfies the following constraints, for every x:

Non-negativity: $sv(\alpha, x) \geq 0$;

Normalization: $sv(\alpha, x) = 1$ if α is logically true;

Additivity: $sv(\alpha \vee \beta, x) = sv(\alpha, x) + sv(\beta, x)$ if $\alpha \wedge \beta$ is logically false.

As in the case of the valuation level, one can define a consequence relation that holds at the supervaluation level. Here a plausible option is what Edgington calls the *constraining property*: for any valid argument, any assignment of probability to its premises and conclusion is such that the improbability of the conclusion does not exceed the sum of the improbabilities of the premises.[19] Assuming that u is a function such that $u(\alpha, x) = 1 - sv(\alpha, x)$, this relation, indicated as $\models_{sv}$, is defined as follows:

DEFINITION 6 $\alpha_1, \ldots, \alpha_n \models_{sv} \beta$ iff $u(\alpha_1, x) + \ldots + u(\alpha_n, x) \geq u(\beta, x)$ for every x in every model.

Given Definition 6, a key equivalence result turns out to be provable, that is, $\alpha_1, \ldots, \alpha_n \models_v \beta$ if and only if $\alpha_1, \ldots, \alpha_n \models_{sv} \beta$. This means that the consequence relation $\models_v$ defined at the valuation level and the

19. Edgington 1999, p. 300. This property was first identified in Adams 1966, with a different formulation.

consequence relation $\models_{sv}$ defined at the supervaluation level are extensionally equivalent. In this respect, quantitative supervaluationism is perfectly classical.

The next three sections show how the semantic method just outlined can be employed to address the three challenges presented in section 4. As will be suggested, by relying on this method one can provide coherent accounts of truth, assertibility, and communication that are compatible with nihilism.

6 TRUTH

Let us start with Challenge 1. How can the nihilist explain the fact that speakers commonly ascribe truth to sentences, if there is no such thing as truth *simpliciter*? An interesting suggestion in this respect has been made by Braun and Sider: talk about truth is explainable in terms of *approximate truth*. According to Braun and Sider, speakers typically ignore vagueness, and they can safely do so insofar as the sentences they use are approximately true. Approximate truth, for them, is definable in the same way in which supervaluationism defines supertruth:

> There is typically a cloud of propositions in the neighborhood of a sentence uttered by a vague speaker. Vagueness prevents the speaker from singling out one of these propositions uniquely, but does not banish the cloud. Speaking vaguely (as always), there is a range of legitimate disambiguations for a vague expression. [...] When all the legitimate disambiguations of a sentence are true, call that sentence *approximately true*. An ordinary utterance of 'A man with no hairs on his head is bald' is approximately true, despite failing to be true.[20]

Although Braun and Sider's suggestion is intriguing, their definition of approximate truth is not entirely convincing. It is not obvious that approximate truth is to be characterized in qualitative terms, that is, as a property that either belongs or does not belong to a sentence. Approximation is reasonably understood as a gradable notion, and it is quite natural to think that there are correct comparative judgments concerning closeness to truth. Suppose that John has exactly 10 hairs on his head, while Ron has exactly 100. Then it is plausible to expect that (6) below is closer to truth than (7), even assuming that neither of them is literally true.

(6) John is bald

(7) Ron is bald

20. Braun and Sider 2007, p. 135.

Even when considering two adjacent items in a soritical series, comparative judgments of acceptability typically exhibit a detectable asymmetry. Consider the following sentences:

(8) 21 grains make a heap

(9) 20 grains make a heap

Although there is a plausible sense in which (8) is slightly closer to truth than (9), there is no plausible sense in which (9) is closer to truth than (8). This is also shown by the intuitive contrast between the following sentences:

(10) 21 grains make a heap and 20 grains do not make a heap

(11) 20 grains make a heap and 21 grains do not make a heap

(10) seems very far from truth. But no matter how far, (11) seems to be unacceptable in a way in which (10) is not.

Considerations such as these suggest that a qualitative definition of approximate truth, such as that provided by Braun and Sider, is unable to account for some crucial connections between sentences. A quantitative account of approximate truth would thus be a better option for the nihilist. This is precisely the kind of account provided by quantitative supervaluationism.

In section 5, quantitative supervaluationism is presented in purely formal terms, without addressing the question of how the values of the function *sv* are to be understood. Now I will focus on the interpretation of the semantics according to which 1 stands for maximal approximate truth, 0 stands for minimal approximate truth — that is, approximate falsity — and every other number in the interval $[0, 1]$ indicates an intermediate degree of approximate truth.

Once the models of L are so construed, Definitions 3 and 5 acquire the expected reading. Definition 3 specifies truth in a context relative to a precisification, while Definition 5 yields the corresponding quantitative account of approximate truth: being approximately true to a certain degree in a given context amounts to being true in a certain proportion of the precisifications that are admissible in that context. Maximal approximate truth and minimal approximate truth — that is, approximate falsity — correspond to supertruth and superfalsity as defined in traditional supervaluationism.

It is easy to see that, on this account, the intuitive judgments about (6)-(11) reported above are easily explained in terms of Definition 5: the value of (6) turns out to be significantly higher than the value of (7); the value of (8) turns out to be slightly higher than the value of (9), (10) has a very low value, and (11) has value 0.

Now consider Challenge 2: how can the nihilist provide a coherent account of assertibility? As in the case of approximate truth, it is doubtful that a qualitative definition of assertibility can be entirely satisfactory. To see why it suffices to recall the examples discussed above. (6) and (7) are not equally assertible, for (6) is definitely more reasonable than (7). Even when considering two adjacent items in a soritical series, comparative judgments of assertibility typically exhibit a detectable asymmetry: although there is a plausible sense in which (8) is slightly better than (9), there is no plausible sense in which (9) is better than (8). This is also shown by the contrast between (10) and (11). No matter how bad (10) may look, (11) seems to be unassertible in a way in which (10) is not.

Arguably, quantitative supervaluationism can provide the account desired. We have seen how the semantics outlined in section 5 can be used to define approximate truth, which is a non-epistemic property. The same semantics admits an epistemic interpretation, in which the property defined is *credibility*: the credibility of a sentence amounts to the degree of acceptance that one should rationally assign to the sentence. Basically, the definitions provided in section 5 remain the same. But instead of assuming that precisifications are actually admissible, one can assume that they are epistemically admissible, that is, that they are reasonably believed to be compatible with the meaning of the expressions that occur in the sentence. Each proximity assignment delivered by P is understood as a measure of epistemic admissibility. This interpretation is directly relevant to the issue of assertibility, assuming that there is a direct relation between assertibility and credibility: a sentence is assertible to the extent that it is credible.[21]

On this account of assertibility, the intuitive judgments about (6)-(11) reported above are easily explained in terms of Definition 5: the value of (6) turns out to be significantly higher than the value of (7); the value of (8) turns out to be slightly higher than the value of (9), (10) has a very low value, and (11) has value 0.

What is the relation between assertibility and approximate truth? The two interpretations of quantitative supervaluationism just sketched provide two structurally identical but extensionally distinct dimensions of variation. The credibility of a sentence in a given context may differ from its closeness to truth in that context, even though formally speaking the two values are obtained in exactly the same way. In order for the credibility of a sentence in a context x to be identical

21. In Iacona and Iaquinto 2021, a notion of credibility along these lines, just as the corresponding quantitative account of assertibility, is applied to future contingents.

to its closeness to truth in x, there should be perfect match between the precisifications that are epistemically admissible in x and those that are actually admissible in x. This amounts to saying that the proximity assignments that model epistemic admssibility and actual admissibility deliver exactly the same values to the extensions of x. But it cannot be taken for granted that there is such a match. The two interpretations considered, however, can in principle be combined in a two-dimensional account where approximate truth and assertibility are defined as distinct properties on the basis of distinct proximity assignments.

8 COMMUNICATION

Finally, consider Challenge 3: how can the nihilist account for communication without assuming that a single truth-evaluable content is shared by two speakers? Imagine that Juan sends the following text to his neighbour Juana:

(12) There is a heap of sand behind your back door

As soon as Juana sees the text, she goes out with a shovel and removes the sand. In this case it is clearly plausible to say that Juan successfully communicates with Juana. However, as noted in section 4, the nihilist cannot appeal to the naïve picture of communication to explain what goes on in their exchange, that is, the explanation cannot be that a single truth-evaluable content is transmitted from Juan to Juana. On the account suggested here, when Juan texts (12) to Juana, there is a cloud of propositions that he associates to (12), and when Juana receives the text, there is a cloud of propositions that she associates to (12). But nothing guarantees that Juan's cloud and Juana's cloud are the same cloud.

In order to account for the exchange between Juan and Juana, the nihilist must adopt some semantic notion that does *not* imply sameness of content. This is quite reasonable after all. Several authors have questioned the naïve picture of communication, arguing that in order to explain communication there is no need to postulate transmission of some unique content. Chomsky famously contended that the pretheoretical notion of content we find in ordinary discourse, as well as the refinements of it provided by contemporary philosophers, is usless from the perspective of an empirical explanation of language use.[22] Marconi argued that all is needed to explain communication is that speakers *converge* on the use of the expressions they employ, namely, that by and large they refer to the same objects by means of

22. Chomsky 1995, p. 49.

those expressions.[23] More recently, Abreu Zavaleta claimed that, when a speaker communicates with another speaker by uttering a sentence, there is no single proposition that the two speakers believe to be the content expressed by the sentence. His point in favour of this claim is that, for nearly every utterance, there are many different propositions that any speaker could easily have believed the utterances to have, none of which is more natural or intrinsically more eligible than the others, so it would be extremely unlikely for two speakers to share exactly the same content.[24]

The account of communication outlined in what follows is in line with the skeptical arguments advanced by the authors just mentioned, and is compatible with some of the positive suggestions they provided for an alternative picture of communication. The distinctive feature of this account is that it is quantitative rather than qualitative. Here is the core idea: when a speaker A utters a sentence 'p' to communicate with another speaker B, there is a cloud of propositions that A associates to 'p', and there is a cloud of propositions that B associates to 'p'; A and B communicate — and B understands A's utterance — to the extent that these two clouds overlap.

This idea can be made precise by using a slightly modified version of the semantics outlined in section 5. Instead of defining the item P in our models as a function that assigns proximity assignments to points, one can define it as a function that assigns proximity assignments to ordered pairs of points and parameters $A, B, \dots$ which represent speakers. Each proximity assignment so obtained is understood as a measure of admissibility for a set of precisifications relative to a speaker: $P_{x,A}$ fixes the precisifications that are admissible in x for A, $P_{x,B}$ fixes the precisifications that are admissible in x for B, and so on. In other words, the interpretation of the pluralist models so defined is like a speaker-relative variant of the epistemic interpretation considered in section 7. For each parameter A — that is, for each speaker A — we call A_x the subset of $E(x)$ to which $P_{x,A}$ assigns a positive value, that is, the set of precisifications that are admissible for A in x.

Using Marconi's terminology, one can define the *convergence* between two speakers A and B in a context x as follows:

DEFINITION 7

$$\mathsf{C}(A, B)_x = \frac{\sum_{e \in A_x \cap B_x} P_{x,A}(e) + \sum_{e \in A_x \cap B_x} P_{x,B}(e)}{\sum_{e \in A_x \cup B_x} P_{x,A}(e) + \sum_{e \in A_x \cup B_x} P_{x,B}(e)}$$

In other words, the convergence between A and B in x is the ratio of the sum of the values assigned by A and B to the precisifications that

23. Marconi 1997, pp. 86-89, 93-97

24. Abreu Zavaleta 2021.

are admissible in x for both A and B to the sum of the values assigned by A and B to the precisifications that are admissible in x for at least one of them. Note that, since $P_{x,A}$ and $P_{x,B}$ are defined in such a way that the total value they assign respectively to the elements of A_x and B_x is 1, we get that $\sum_{e \in A_x \cap B_x} P_{x,A}(e) \leq 1$, $\sum_{e \in A_x \cap B_x} P_{x,B}(e) \leq 1$, and $\sum_{e \in A_x \cup B_x} P_{x,A}(e) = \sum_{e \in A_x \cup B_x} P_{x,B}(e) = 1$. Thus there are two limiting cases. One is perfect convergence in x, that is, $A_x = B_x$, which means that $\sum_{e \in A_x \cap B_x} P_{x,A}(e) = \sum_{e \in A_x \cap B_x} P_{x,B} = 1$, and therefore $\mathsf{C}(A, B)_x = 1$. The other is null convergence in x, that is, $A_x \cap B_x = \emptyset$, which means that $\sum_{e \in A_x \cap B_x} P_{x,A}(e) = \sum_{e \in A_x \cap B_x} P_{x,B}(e) = 0$, and therefore $\mathsf{C}(A, B)_x = 0$. In any other case, $\mathsf{C}(A, B)_x$ has some intermediate value in the interval $[0, 1]$.

The notion of convergence provided by Definition 7 concerns the language as a whole, in that it makes no reference to a specific formula. In order to define a relation of convergence between A and B indexed to a formula α, we need to consider the admissible precisifications in which α is true. Let $A_{x,\alpha} = |\alpha|_x \cap A_x$, the set of precisifications that are admissible for A in x and such that α is true in x. Let $B_{x,\alpha} = |\alpha|_x \cap B_x$, the set of precisifications that are admissible for B in x and such that α is true in x. Then the convergence between A and B in x relative to α is defined as follows:

DEFINITION 8

$$\mathsf{C}(A, B)_{x,\alpha} = \frac{\sum_{e \in A_{x,\alpha} \cap B_{x,\alpha}} P_{x,A}(e) + \sum_{e \in A_{x,\alpha} \cap B_{x,\alpha}} P_{x,B}(e)}{\sum_{e \in A_{x,\alpha} \cup B_{x,\alpha}} P_{x,A}(e) + \sum_{e \in A_{x,\alpha} \cup B_{x,\alpha}} P_{x,B}(e)}$$

Since the value of α relative to each point-extension pair is fixed in the model, so is the same for A and B, $\mathsf{C}(A, B)_{x,\alpha}$ is a function of $\mathsf{C}(A, B)_x$. This means that the convergence between A and B on α depends on their converge on the whole language. As in the case of Definition 7, there are two limiting case. One is perfect convergence in x on α, that is, $A_{x,\alpha} = B_{x,\alpha}$, the other is null convergence in x on α, that is, $A_{x,\alpha} \cap B_{x,\alpha} = \emptyset$. In the first case, $\sum_{e \in A_{x,\alpha} \cap B_{x,\alpha}} P_{x,A}(e) = \sum_{e \in A_{x,\alpha} \cup B_{x,\alpha}} P_{x,A}(e)$, and $\sum_{e \in A_{x,\alpha} \cap B_{x,\alpha}} P_{x,B}(e) = \sum_{e \in A_{x,\alpha} \cup B_{x,\alpha}} P_{x,B}(e)$, therefore $\mathsf{C}(A, B)_{x,\alpha} = 1$. In the second case, $\sum_{e \in A_{x,\alpha} \cap B_{x,\alpha}} P_{x,A}(e) = 0$ and $\sum_{e \in A_{x,\alpha} \cap B_{x,\alpha}} P_{x,B}(e) = 0$, therefore $\mathsf{C}(A, B)_{x,\alpha} = 0$.

The two sets $A_{x,\alpha}$ and $B_{x,\alpha}$ represent the clouds that two speakers A and B respectively associate to a sentence 'p' in a context x, assuming that α is p. So, A and B communicate through 'p' in x to the extent that $\mathsf{C}(A, B)_{x,\alpha}$ is high, that is, to the extent that they converge on the admissible precisifications of 'p'. Similarly, B understands A's utterence to the same extent. When understanding is less than perfect, it is *partial* understanding.[25]

25. Abreu Zavaleta 2023 discusses the notion of partial understanding but suggests a qualitative acconut in terms of content parthood: B partially understands A's

Now we can go back to Juan and Juana. The linguistic exchange between Juan and Juana is successful because their convergence on (12) is sufficiently high. Even if Juan and Juana might disagree, say, on the question whether 4 grains make a heap, by and large they agree on a considerable portion of admissible precisifications of (12). In particular, they agree that the collection of grains behind Juana's back door is a heap. As a result, they understand each other, although only partially.

There is obviously a direct connection between the account of communication just outlined and the treatment of assertibility sketched in section 7. If Definition 5 is suitably indexed in order to account for the plurality of proximity assignments, we get a speaker-relative measure of assertibility that behaves as is plausible to expect in relation to convergence: the more A and B converge on a sentence 'p' in a context x, the more the assertibility values they assign to 'p' in x tend to match.[26]

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utterance when the propositions expressed by A and the propositions B takes the utterance to express have at least one substantive part in common.

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