Conceptual truth versus empirical truth

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ABSTRACT

Ontological arguments seek to affirm existence of a priori empirical truths by use of logic. I focus on Gödel's ontological proof. Axioms utilized are definitional by usual standards, which is one reason why controversies surrounding the proof still linger on. I argue that logic without empirical supports can only be used to prove conceptual truths. In order for logic to prove empirical truths, definitions and axioms used must be based on established truths of actual reality. How Thomas Aquinas's criticism of ontological arguments unfolds in context of modern modal higher-order logic is discussed.

KEYWORDS

Gödel's ontological proof, modal higher-order logic, conceptual truth, empirical truth, conceivable property, Thomas Aquinas

1. Definitional nature of Gödel's ontological proof

Consider axioms and definitions in Gödel's ontological proof (Oppy, 2000).

$$\begin{aligned}
\text{Definition 1:}\quad & G(x) \;\Leftrightarrow\; \forall\varphi(P(\varphi) \Rightarrow \varphi(x)) \\
\text{Definition 2:}\quad & \varphi \text{ ess } x \;\Leftrightarrow\; \varphi(x) \wedge \forall\psi\,(\psi(x) \Rightarrow \Box\,\forall y(\varphi(y) \Rightarrow \psi(y))) \\
\text{Definition 3:}\quad & NE(x) \;\Leftrightarrow\; \forall\varphi(\varphi \text{ ess } x \Rightarrow \Box\,\exists y\;\varphi(y)) \\
\text{Axiom 1:}\quad & P(\neg\varphi) \;\Leftrightarrow\; \neg P(\varphi) \\
\text{Axiom 2:}\quad & (P(\varphi) \;\wedge\; \Box\,\forall x(\varphi(x) \Rightarrow \psi(x))) \;\Rightarrow\; P(\psi) \\
\text{Axiom 3:}\quad & P(G) \\
\text{Axiom 4:}\quad & P(\varphi) \;\Rightarrow\; \Box\,P(\varphi) \\
\text{Axiom 5:}\quad & P(NE) \\
\text{Theorem 1:}\quad & P(\varphi) \;\Rightarrow\; \Diamond\,\exists x\;\varphi(x) \\
\text{Corollary 1:}\quad & \Diamond\,\exists x\;G(x) \\
\text{Theorem 2:}\quad & G(x) \;\Rightarrow\; G \text{ ess } x \\
\text{Theorem 3:}\quad & \Box\,\exists x\;G(x)
\end{aligned}$$

A common criticism against the above proof is that some of the axioms are not defensible and need to be modified. (Anderson, 1990; Hájek, 2002) This is relevant if we try to match axioms to some pre-conception one has on God. However, one may not be interested in what properties God should have - one is fine with proving that some form of God - whatever its conception is - exists.

The main argument against 'axiom' detractors of the proof then is this: axioms really just define positiveness P and God-like G in a particular way. One cannot see any axiom that states existence of some object directly, and all the axioms do are defining higher-order predicates. Arguing that these axioms are not definitional goes against how we usually interpret logical applications. As often suggested, it is too much sacrifice to change interpretation of logical applications just to block validity of the ontological proof. Why not just accept the ontological proof?

2. Conceptual truth versus empirical truth

2.1. *Why distinction usually does not matter: higher-order logic makes differences*

In first-order logic, we do not encounter such seemingly tautological but non-trivial logical applications seen in the ontological proof of Gödel. If one wants to prove existence of something in first-order logic, then one has to assert existence of something beforehand. This is why distinguishing conceptual truths from empirical truths has not been that critical for logical applications - logic simply can be said to be about interpreting empirical reality produced by assumptions. Philosophical arguments over whether abstract mathematical objects should be assumed real or not can be made, but as far as applications of logic go, it does not matter we treat mathematical objects as empirically real or only conceptually real.

The above position can no longer be maintained if the Gödel's ontological proof, written in modal higher-order logic, is indeed valid. Even if parody arguments (Nagasawa, 2008; Chambers, 2015) are valid, they do not prove invalidity of the proof.

2.2. *Question of conceivable properties and instantiation of axioms in reality*

The implicit idea behind the ontological proof is that we may define properties and predicates R as we want and leave verification of whether these properties actually exist to checking $\exists Q\ R(Q)$, where Q can be lower-order predicates (properties) or elements (objects).

But let us think of alternative applications of higher-order logic where Gödel's axioms have not been used. Then it is possible that in some of these applications, we get

$$[\Diamond\ \exists x\ \phi(x)] \wedge [\neg\Box\ \exists x\ \phi(x)]$$

which is against the complete modal collapse outcome (Benzmüller and Paleo, 2014) of the Gödel's ontological proof. But this is clearly weird - it was said that the Gödel's ontological proof is tautological! What is going on then?

The answer that can be provided is that logical inference is fundamentally about conceptual truths, not empirical truths. Axioms can be said to be about defining an imaginary world - every axiom, even if it seems innocuous and definitional, is about defining what concepts and properties are instantiated and can be conceived in an imaginary world.

In a world where some properties cannot even be conceived, it would indeed be expected that we would not get the same logical conclusion. All the above modal higher-order logic stories confirm is this point. In a world where Gödel's axioms do not apply, God-like G cannot even be conceived, and one can arrive at different logical conclusions.

It may still then be argued that because we can conceive God-like G and positiveness P, Gödel's ontological proof is valid in our world. Is this really valid?

On this front, one can invoke the point made by Thomas Aquinas against ontological proofs in Summa Theologiae:

> "Granted, however, that someone understands that by this name God is signified this which is said, namely, that than which a greater cannot be thought, nevertheless it does not follow from this that he understands what is signified by the name to exist in reality, but only in the apprehension of the intellect." (Cosgrove, 1974)

That is, Aquinas says that the fact that 'we mentally conceive nature of God' does not mean 'we accurately do conceive nature of God.' Or, 'that we can imagine something' does not mean 'we actually know something.' But how do we verify this? Couldn't it be that 'mentally conceiving nature of God' is sufficient to accurately conceive nature of God, all with its simple appeals? Again, modern modal higher-order logic comes to the front.

It is hard to believe that complete modal collapse predicted by Gödel's ontological proof - which says we do not need modal logic for philosophical purposes - is true. It is much better to accept that 'mentally conceiving' does not equal to 'correctly and actually conceiving.' This, of course, involves philosophical priorities, and it is still possible that for some people, different priorities may result in acceptance of complete modal collapse and empirical relevance of the Gödel's ontological proof. Still, most philosophers would prioritize avoiding modal collapse. (Anderson, 1990; Hájek, 2002)

3. Conclusion

To summarize, whether a particular property is conceivable in a world should affect logical conclusions obtained. Furthermore, that we can mentally conceive some property does not mean that we accurately conceive the property. These statements are supported by what we can get out of modal higher-order logic, given tautological and definitional nature of the

Gödel's ontological proof.

Logical inference thus should be understood as giving out conceptual truths, not empirical truths. Logical inference only gives us empirical truths when underlying assumptions are based on known truths of actual reality, regardless of how innocuous and definitional underlying assumptions and axioms seem to be. Inference from axioms otherwise, however definitional they are, only gives us a picture of an imaginary world.

Conflicts of interest

Authors report no conflict of interest.

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