The Nature of a Constant of Nature: the Case of G

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Abstract

Physics presents us with a symphony of natural constants: G, $\hbar$, c, etc. Up to this point, constants have received comparatively little philosophical attention. In this paper I provide an account of dimensionful constants, in particular the gravitational constant. I propose that they represent inter-quantity structure in the form of relations between quantities with different dimensions. I use this account of G to settle a debate over whether mass scalings are symmetries of Newtonian Gravitation. I argue that they are not, but only if we interpret mass anti-quidditistically. This is analogous to anti-haecceitism in the presence of spacetime symmetries.

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1 Introduction

What are constants of nature? The laws of physics contain terms, such as c or G, that are known as physical constants. But what do these terms refer to?

The question is impossible to answer in its full generality, for there are many different types of constants.[1] I will consider one constant in particular:

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[1] For an attempt at categorisation, see Lévy-Leblond (2019).

the gravitational constant, denoted G. This constant has three distinctive features. First, it is not associated to any particular kind of particle, contrary to the unit charge e of an electron. Secondly, it is a *fundamental* constant, unlike the Rydberg constant R_∞ which can be derived from quantum mechanics. Finally, it is a dimensionful constant, distinct from the fine structure constant α. Despite these features, G is not *sui generis*. The Planck constant, for instance, is also universal, dimensionful and not associated to any particular kind of object. I therefore suspect that my analysis in this paper also applies to $\hbar$ and similar constants.

Apart from the fact that constants occur in most laws of physics, the nature of G in particular is significant because it is decisive in a recent debate over whether *mass scalings* are symmetries of Newtonian mechanics.[2] The tentative answer that has been reached in this discussion is that if one scales G with all particle masses, then this results in an empirically equivalent possibility. This could potentially lead to the sort of underdetermination familiar from spacetime symmetries, such as the Leibniz shift.[3] The remaining disagreement concerns whether a world in which G has a different value is *physically* possible. If not, then the purported underdetermination is not of any worrisome kind. Some assertions on this topic have been made, but I believe that the question cannot be settled without an account of the gravitational constant.

The aim of this paper is two-fold. First, it provides a definitive answer to the question of whether mass scalings are symmetry-like transformations in Newtonian mechanics. Based on the account of the gravitational constant I will provide, the answer is: yes, if quidditism is true; no, if quidditism is false. I will argue that the doctrine of quidditism saddles Newtonian mechanics with redundant structure, so mass scalings ultimately do not pose an underdetermination problem. Again, these conclusions mirror debates about spacetime symmetries: the conclusion I advance is analogous to anti-haecceitistic (or 'sophisticated') substantivalism.[4]

Secondly, my account of the gravitational constant is of independent interest. I hope it will provide the first step for a comprehensive study of

[2] See Baker (2020); Dasgupta (2013, 2020); Jacobs (2021); Martens (2019,?); Roberts (2016).

[3] I hesitate to call transformations that scale G 'symmetries', since this is inconsistent with standard use of that term in physics. Cf. section 8.

[4] The idea that one can apply sophistication to internal symmetries was first advanced by Dewar (2019). Wolff (2020) has developed a 'substantivalist' position for mass similar to the one I present here.

constants of nature.[5] I will develop an account on which constants are *inter-quantity connections*. One can think of constants as 'exchange rates' between the values of different quantities. For example, G has units of $L^3M^{-1}T^{-2}$. The role of G is thus to determine what length value is 'equal' to a pair of values for mass and duration. The structure of space-time and the structure of mass value space are thus intimately linked. The natural conclusion is that in order to account for gravity, one should move from separate space-time and mass structures to a joint space-time-mass structure.

The proposal presupposes a realist account of quantities: the determinate values of a determinable quantity such as mass are universals that bear second-order relations to each other. I will not defend this form of realism here; for a defence, see Mundy (1987) or Eddon (2013).

2 The Structure of Newtonian Mechanics

In the tradition of the semantic view of theories, I will present Newtonian Gravitation (NG) as a class of models. The *kinematically possible models* (KPMs) of a theory are models that contain the correct sort of mathematical objects to formulate the theory's dynamics. The *dynamically possible models* (DPMs) of a theory are those KPM that in addition satisfy the theory's equations of motion.

Here is a first proposal for the structure of NG, which will require significant revision later on. The KPMs of NG are of the form:

$$\text{NG 1: } \langle D, \mathbb{E}, \mathbb{T}, x_i(t), m_i, \mathbb{R}^+ \rangle$$

where D is a bare set of particles; $\mathbb{E}$ is a three-dimensional Euclidean affine space; $\mathbb{T}$ is a one-dimensional Euclidean affine space; $x : D \times \mathbb{T} \to \mathbb{E}$ is a smooth function which represents the particles' trajectories; and $m : D \to \mathbb{R}^+$ is a function into the positive real numbers which represents the particles' masses (in some unit of mass). Since $\mathbb{E}$ is an affine space, for any pair of points $x, y \in \mathbb{E}$ there is a unique vector from x to y denoted $y-x$; and likewise for $\mathbb{T}$. I will denote the associated vector spaces $\mathbb{V}_E$ and $\mathbb{V}_T$ respectively. Moreover, both vector spaces come attached with a positive real-valued norm $|.| : \mathbb{V} \to \mathbb{R}^+$ that represents the magnitude of each vector (in some unit of length or time).

Notice that it is perhaps more common to set NG on a Newtonian space-time $\langle M, t_{ab}, h^{ab}, \nabla, \sigma^a \rangle$, where M is a differentiable manifold and t, h, ∇

[5] Constants have received little philosophical attention. For an exception, see Johnson (1997) and references therein.

and σ are certain geometric objects. But the idea that NG is more appropriately set on a Euclidean affine space is familiar from Stachel (1993); see also Saunders (2013) and Dewar (2015). I will not discuss this issue further.

The DPMs of NG are those KPMs that satisfy the following equation of motion:

$$\ddot{x}_i(t) = \sum_{j \neq i} G \frac{m_j}{|\mathbf{r}_{ij}(t)|^3} \mathbf{r}_{ij}(t) \tag{1}$$

where $\mathbf{r}_{ij}(t)$ is the unique vector between $x_i(t)$ and $x_j(t)$, $|\mathbf{r}_{ij}(t)|$ is the real-valued norm of that vector, and m_j is the mass value of j. The value of G is an experimentally determined real number.[6]

The left-hand side of this equation deserves some comment. For a given particle i, $x_i(t)$ is a function $\mathbb{T} \to \mathbb{E}$. The velocity of particle i at time t is a measure of how much distance i covers (and in which direction) over an infinitesimal period of time $\mathbf{t}$ from t. In other words, velocity is the *directional derivative* of $x_i(t)$:

$$\nabla_{\mathbf{t}} x_i(t) \coloneqq \lim_{\epsilon \to 0} \frac{x_i(t + \epsilon \mathbf{t}) - x_i(t)}{\epsilon} \tag{2}$$

where ϵ is a real number.[7] This means that the velocity of i at t is represented by a function from $\mathbb{V}_T$ into $\mathbb{V}_E$.

We can then define acceleration as the directional derivative of velocity:

$$\nabla_{\mathbf{s}} \nabla_{\mathbf{t}} x_i(t) \coloneqq \lim_{\epsilon \to 0} \frac{\nabla_{\mathbf{t}} x_i(t + \epsilon \mathbf{s}) - \nabla_{\mathbf{t}} x_i(t)}{\epsilon} \tag{3}$$

From this definition it follows that acceleration is a function from $\mathbb{V}_T^2 \to \mathbb{V}_E$. We are normally only interested in the case in which $\mathbf{t} = \mathbf{s}$, so acceleration really is another function from $\mathbb{V}_T \to \mathbb{V}_E$. We can turn this into a vector quantity by choosing the unique unit vector $\hat{\mathbf{t}}$ in the positive time direction as our unit of time. $\mathbb{T}$ has no privileged orientation, so the choice of direction is conventional. Once this choice is made, let $\ddot{x}_i(t) \coloneqq \nabla^2_{\hat{\mathbf{t}}} x_i(t)$. This quantity takes value in $\mathbb{V}_E$.

It hardly seems necessary to point out that (1) is a well-defined equation. On the left-hand side is a displacement vector. The quantities G, m, and

[6] Note that I will not consider alternative formulations of the law of universal gravitation, such as Martens' (2019) Machian comparativism.

[7] This definition departs from Dewar (2021), who instead lets $\dot{x}$ take value in a separate velocity value space. Although either approach works out mathematically, I prefer the parsimony of defining velocity in terms of the vector spaces that are already part of NG's spacetime structure.

$|\mathbf{r}|$ are all real numbers, and $\mathbf{r}$ is a vector; scalar multiplication of vectors is well-defined, so on the right-hand side of (1) is also a displacement vector. The equation of motion compares like with like. However, I will show below that this is an artefact of the redundant real number structures encoded into the theory's kinematics. When this redundant structure is removed, (1) is no longer well-defined. In order to resolve this puzzle, an account of the nature G is required. I will give one that does so in §5.

3 Redundant Real Numbers

The theory of Newtonian Gravitation has three dimensionful quantities: length, time and mass. Each of these quantities has an associated *value space*. Consider, for example, mass value space. This space represents the determinable quantity mass; elements of it represent determinate mass magnitudes. The structure of mass value space encodes the relations between mass magnitudes. For example, the magnitude 'being 10 kg' stands in the relation of 'being twice as much' to the magnitude 'being 5 kg'. Similarly, lengths and durations stand in certain relations to each other.

This raises the question which *particular* relations these quantities stand in to each other. What is the structure of the theory's value spaces? In the above, I assumed that these value spaces are isomorphic to the positive real numbers ($\mathbb{R}^+$). For instance, the mass function m simply assigned each particle a positive real number. This is often tacitly assumed: in foundational treatments of NG considered as a field theory, for instance, the mass density field ρ is defined as a scalar field, i.e. a function from spacetime points into (positive) real numbers (Friedman, 1983; Malament, 2012). But it turns out that this real number structure is too rich for a dimensionful quantity. This will lead to a second, more realistic proposal, namely that value spaces have an *additive extensive structure*.

The problem with $\mathbb{R}^+$ is that it, in effect, endows quantities with a preferred unit. Consider the case of mass, and suppose that, for some particle i and some positive real number x, $m(i) = x$. Since $x \in \mathbb{R}^+$, the mass of i is associated to a particular real number. And since mass value space is meant to represent objective physical structure, this association does not depend on any particular choice of unit. But this association between numbers and masses is spurious. In Martens' (2019) words: there is nothing 'five-ish' about a 5 kg mass. So, mass value space cannot have the structure of the real numbers. Wolff (2020, §8.2.1) presents a similar *reductio*: her conclusion is that when mass value space has no non-trivial automorphisms, then there is a unique homomorphism from that value space into the positive real numbers—contrary to our freedom to choose a mass scale.

Martens (2019, 2518) correctly claims that the received view amounts to a "fail[ure] to distinguish between *physical magnitudes* and the *numerical quantities* used to represent them". Instead, both Martens and Wolff endorse a type of structure that is studied under the guise of 'measurement theory'.[8] These structure are concisely characterised as so-called *principal homogeneous spaces* for the group $\langle \mathrm{R}^+, \times \rangle$ of positive real numbers under multiplication. The principal homogenous space (PHS) of $\mathbb{R}^+$ is a set R^+ over which one has defined a regular action of $\mathbb{R}^+$. The result is a structure that has 'forgotten' the multiplicative identity of $\mathbb{R}^+$, but retains the latter's order and composition structure.

There is a different way of defining these structures that is physically more perspicuous. Known as *additive extensive structures*, they are of the form $\langle M, \leqslant, \circ \rangle$, where M is a set of cardinality $2^{\aleph_0}$, $\leqslant$ imposes a total order on M, and $\circ$ is an associative binary function. When these relations satisfy certain axioms, the resulting structure is equivalent to the PHS defined above.[9] In physical terms, $\leqslant$ is interpreted as the binary relation of one mass being less than or equal to another, and $\circ$ as the tertiary relation of two masses equalling a third. The intuitive thought behind the fact that $\circ$ admits of no inverses or identity is that there are neither negative nor zero masses.

Hölder (1901) proves that one can represent $\langle M, \leqslant, \circ \rangle$ on $\mathbb{R}^+$ in the following sense: there exists a function $f_r : M \to \mathbb{R}^+$ such that (i) $x \leqslant y$ iff $f(x) \leq f(y)$ and (ii) $x \circ y = z$ iff $f(x) + f(y) = f(z)$. Furthermore, this representation is unique up to multiplication by a positive constant α, so that f_r and f'_r both represent the same structure $\langle M, \leqslant, \circ \rangle$ iff there is some $\alpha > 0$ such that $f'_r = \alpha f_r$. This non-uniqueness provides a precise sense in which the former has less structure than the latter. Unlike the real number structure, an additive extensive structure does not define a privileged system of units. The fact that f_r is defined up to multiplication by a positive constant means that any system of units related by such a transformation represents the structure of mass value space equally well.

Therefore, additive extensive structures are apt to represent the value spaces of dimensionful quantities such as mass, length and time. This means that the norm over $\mathbb{V}_E$ is not real-valued; it takes value in a value space isomorphic to the PHS for $\mathbb{R}^+$. The same is true for the norm over $\mathbb{V}_T$. I

[8] The *locus classicus* is Krantz et al. (1971); see Wolff (2020, Ch. 5) for a recent treatment.

[9] See Dewar (2021, §A) for a clear proof of this equivalence claim.

will let $\mathcal{V}_M$, $\mathcal{V}_L$ and $\mathcal{V}_T$ denote the value spaces for mass, length and time respectively. Note that these value spaces are distinct, albeit isomorphic. There is no canonical isomorphism between them.

4 A Puzzle for Intrinsic NG

With these revisions to the theory in mind, I will rewrite the models of NG as follows:

$$\text{NG 2: } \langle D, \mathbb{E}, \mathbb{T}, \mathcal{V}_M, \mathcal{V}_L, \mathcal{V}_T, x_i(t), m_i \rangle$$

Although the value spaces for length and time are part of the vector space structure over $\mathbb{E}$ and $\mathbb{T}$, I write them out explicitly in order to emphasise that the norm over these vector spaces is not real-valued. Call this formulation *intrinsic NG*.

Let's return to NG's equation of motion:

$$\ddot{x}_i(t) = \sum_{j \neq i} G \frac{m_j}{|\mathbf{r}_{ij}(t)|^3} \mathbf{r}_{ij}(t) \tag{1}$$

When m_j and $|\mathbf{r}_{ij}|$ are interpreted as real numbers, the right-hand side of this equation is a well-defined vector quantity. But in the previous section I argued that these quantities are not real numbers but elements of the PHS for $\mathbb{R}^+$. The product of a vector in $\mathbb{V}_E$ by these elements is ill-defined. In particular, there is no canonical way to associate such elements to a unique real number.

The left-hand side also requires a different interpretation. In section 2, I defined $\ddot{x}_i(t)$ as the second directional derivative of i's trajectory with respect to the unit vector $\hat{\mathbf{t}}$ in the positive time direction. But when vectors in $\mathbb{V}_T$ take value not in $\mathbb{R}^+$ but in a PHS over $\mathbb{R}^+$, there is no unique unit vector even after a temporal orientation is fixed by convention: there is no unique vector $\mathbf{t}$ such that $|\mathbf{t}|$ is canonically mapped onto the multiplicative identity. The best option is therefore to redefine acceleration such that $\ddot{x}_i(t) \coloneqq \nabla^2_{\mathbf{t}} x_i(t)$, where $\mathbf{t}$ is a variable rather than a fixed input. This means that acceleration becomes a function from $\mathbb{V}_T \to \mathbb{V}_E$.

This leads to a puzzle for the intrinsic formulation of NG. While the left-hand side of (1) is a function $\mathbb{V}_T \to \mathbb{V}_E$, the right-hand side is a product of (a) a vector in $\mathbb{V}_E$, (b) the norm of that vector, (c) a mass value, and (d) the constant G. Even if we can make sense of this product, it is unclear what it means to say that these quantities are equal. It seems that (1), interpreted in this way, does *not* compare like with like. This is a fatal problem for intrinsic NG: it leaves the theory's dynamics undefined.

Of course, once we attach *numbers* to these quantities we can equate them with ease. Using Hölder's representation theorem, it is possible to assign positive real numbers to mass values in a way that is faithful to mass value space's internal structure, and similar for length and time. Based on this procedure, one could formulate NG's dynamics as follows: a KPM of NG is a DPM iff for any faithful maps from $\mathcal{V}_M$, $\mathcal{V}_L$ and $\mathcal{V}_T$ onto $\mathbb{R}^+$, equation (1) is satisfied for that model. This avoids the puzzle raised above.

On this interpretation, the fundamental equation of Newtonian Gravitation quantifies over assignments of numbers to physical quantities, or systems of units. Field has criticised such quantification as illegitimate. Numbers are physically inert: the laws are not true in virtue of facts about them. As Field wrote about G:

> The role [G] plays is as an entity extrinsic to the process to be explained, an entity related to the process to be explained only by a function (a rather arbitrarily chosen function at that). Surely then it would be illuminating if we could show that a purely intrinsic explanation of the process was possible, an explanation that did not invoke functions to extrinsic and causally irrelevant entities [i.e. numbers]. (Field, 1980, 44)

Although I will not defend Field's programme of intrinsic physics here, I find these concerns persuasive.[10] The excision of redundant structure from the theory's kinematics is a virtue; this virtue is undone if the same redundancies recur in the dynamics. I will therefore develop an alternative interpretation of (1) on which no quantification over numbers is required. This interpretation requires an account of the role of G, to which I now turn.

5 The Nature of Dimensionful Constants

I have purposefully remained vague about the role of the gravitational constant, G, in Newtonian mechanics. In the first approximation of NG, the gravitational constant was another real number; but from an intrinsic point of view this is clearly unsatisfactory. The theory has reached a breaking point that can only be overcome by considering the dynamical role of G in more detail. That is the aim of this section.

[10] For more explicit discussions, see Sider (2020), Dewar (2021) and Jacobs (2022).

5.1 Example: Hooke's Law

Consider first a simpler example. Hooke's law relates the restoring force of a spring to the displacement of the load away from equilibrium:

$$F_s = -kx \tag{4}$$

where k is a 'stiffness' constant which depends on the material constitution of the spring in question. This constant is quite unlike G in that it is specific to a particular (kind of) object and not part of any fundamental law. But like G it is dimensionful, which will allow us to draw a comparison between them.

Hooke's law relates one quantity—force—to another—displacement. We cannot equate these quantities directly. It makes no sense to say that the force on a spring is equal to a certain amount of displacement, although this is obscured when we express both quantities vectorially. This is in fact the same puzzle as the one discussed in the previous section: when quantities are interpreted 'intrinsically', equations of motion such as (1) and (4) seem ill-defined.

I propose that the role of the constant k is to facilitate comparisons of forces and displacements. The spring constant determines an 'exchange rate' between these quantities: such-and-so displacement is 'worth' so much force. This is the role of dimensionful constants: to convert the value of one type of quantity into the value of another type of quantity. In this way, constants can restore the balance between both sides of the equation.

In more detail, recall that displacement vectors live in a vector space $\mathbb{V}_E$ over $\mathbb{E}$. In my formulation of NG, forces had no existence of their own: by combining Newton's second law with the law of universal gravitation, forces dropped out of the equation. But suppose for the sake of example that forces are real. Then their values would live in a distinct value space $\mathbb{F}$, which is a vector space isomorphic to $\mathbb{V}_E$. Despite the isomorphism, there is no canonical map between $\mathbb{V}_E$ and $\mathbb{F}$. Although one can compare the directions of vectors in either vector space, one cannot compare their magnitudes. There simply is no answer to the question: how much force is equivalent to a certain displacement in the same direction?

But the spring constant k provides a link between these value spaces. From the many maps between $\mathbb{F}$ and $\mathbb{V}_E$, it picks out one as privileged. By k's standards, we *can* say whether a vector in $\mathbb{F}$ is equivalent to a vector in $\mathbb{V}_E$. We can construe the spring constant as a function $k : \mathbb{V}_E \to \mathbb{F}$. This means that (4) should really be read as:

$$F_s = -k(x) \tag{5}$$

where x now functions as input for k. On both the left-hand side and the right-hand side of (5) are force-valued quantities: like is compared with like. The puzzle for intrinsic dynamics is solved without recourse to the real numbers.

Since k should ultimately be derived k from more fundamental physics, there is no reason to believe that k represents any *fundamental* relations between displacements and forces. But the same account also applies to more fundamental constants, such as G.

5.2 The Gravitational Constant

The dynamical role of G is to determine the contribution of the gravitational attraction of a massive particle to the acceleration of another particle some distance away from the first. Recall that acceleration is a function from $\mathbb{V}_T \to \mathbb{V}_E$. So, G is a function from mass values $m \in \mathcal{V}_M$ and displacement vectors $\mathbf{v} \in \mathbb{V}_E$ to functions from $\mathbb{V}_T$ to $\mathbb{V}_E$:

$$G : \mathcal{V} \times \mathbb{V}_E \to (\mathbb{V}_T \to \mathbb{V}_E) \tag{6}$$

For a given mass and displacement, G yields an acceleration. The function is surjective but not injective: for every acceleration there is some mass-duration pair which yields that acceleration; but this pair is not unique.[11]

BUt G is not just any such function. It satisfies these four requirements, where λ is a real number:

(i) $G(\lambda m, \mathbf{v})(\mathbf{t}) = \lambda G(m, \mathbf{v})(\mathbf{t})$

(ii) $G(m, \lambda\mathbf{v})(\mathbf{t}) = \lambda^{-2} G(m, \mathbf{v})(\mathbf{t})$

(iii) $G(m, \mathbf{v})(\mathbf{t}) \propto \mathbf{v}$

(iv) $G(m, \mathrm{R}\mathbf{v})(\mathbf{t}) = \mathbf{R}G(m, \mathbf{v})(\mathbf{t})$

These requirements say that (i) G scales proportionally with mass; (ii) G scales inversely proportional to the square of distance; (iii) the acceleration is in the same direction as the displacement; and (iv) rotations have no effect

[11] This proposal bears similarities to Dewar's (2021) account of G as an isomorphism between distinct value spaces, although the details differ. In particular, Dewar posits a separate value space for forces. Again, I prefer the parsimony of an account that only relies on length, time and mass value space.

on the magnitude of the acceleration. These requirements are not a priori principles; they are determined experimentally. It is through careful observation that we know that a test particle accelerates twice as fast towards a body that is twice as massive, and so on.

These constraints jointly determine G up to a scale factor. We can think of this scale factor as a 'choice of unit'—but note that we are not concerned here with numerical scales. This residual freedom in the definition of G will lead to the possibility of different 'values' for G considered in the second half of this paper.

The above only concerns the mathematical representation of G. I do not intend to suggest that the world itself contains a function that causally contributes to the motions of particles. On my view, G is a complex piece of real cross-value space structure between $\mathbb{V}_E$, $\mathbb{V}_T$ and $\mathcal{V}_M$. Unlike k it does not connect just two value spaces, but three. G represents a complex relation that holds between masses, displacements and durations. It is similar to the relations between values of the same quantity, such as mass ratios, in that it is a physical relation between quantity-values; the difference is that it holds between values of different quantities. We can visualise these relations as threads between the elements of these various value spaces. One can follow the thread from one space and arrive at the value of a quantity in a different one. For example, one can follow the thread from a certain mass: this thread has many branches, one for each displacement. Follow one such branch, and one arrives at an acceleration; follow another, and one arrives at a different acceleration. Or start from a displacement, and one is faced with many branches that correspond to different masses. Choose one, and arrive at an acceleration, which may or may not be distinct from before. Thus value spaces are not isolated islands; they are interconnected networks.[12]

6 The Structure of Newtonian Mechanics Revisited

I have argued that the gravitational constant G represents real kinematical structure of Newtonian Gravitation. This kinematical structure connects

[12] This bears similarities to Baker's (2013) 'comparativism with mixed relations'. The main difference is that on Baker's proposal, inter-quantity relations are taken as fundamental, whereas here they are 'induced' by a constant of nature. This may provide a more 'natural' approach to mixed relations than an approach on which they are posited as primitive, but further research is required to evaluate the relative merits of these approaches.

the theory's various value spaces.

Since G represents physical structure, it deserves a place in the theory's models. This yields the third and final version of NG's kinematics:

$$\text{NG 3: } \langle D, \mathbb{E}, \mathbb{T}, \mathcal{V}_M, \mathcal{V}_L, \mathcal{V}_T, x_i(t), m_i, G \rangle$$

It is not always necessary to explicitly list value space structure amongst the theory's posits. I have not explicitly written down the action of $\mathbb{V}_E$ on $\mathbb{E}$, for instance, since this is just part of the structure of $\mathbb{E}$. Yet this is not the case for G, since G links the various value spaces the theory posits; it is not part of any one value space. Therefore, it really is necessary to include G in the theory's models for a full account of the kinematical structure of NG.

This account of the gravitational constant also provides a solution to the puzzle raised in section 2. Recall the puzzle: the left-hand side of (1) is an acceleration—a function from $\mathbb{V}_T$ to $\mathbb{V}_E$—while the right-hand side of (1) is a complicated product. It seems that these are distinct quantities that cannot be compared. But G was the missing piece of the puzzle. Given a mass and a displacement, G outputs an acceleration that we can compare directly to the left-hand side of (1). In other words, (1) should read as follows:

$$\ddot{x}_i(t) = \sum_{j \neq i} G(m_j, \mathbf{r}_{ij}(t)) \tag{7}$$

where as before $\mathbf{r}_{ij}(t) \coloneqq x_j(t) - x_i(t)$.

It may seem odd that this novel equation only involves the displacement vector between x_i and x_j, but not the norm of that vector. But the norm is itself a function from vectors of $\mathbb{V}_E$ into $\mathcal{V}_L$, so G may involve the norm indirectly. This is indeed the case. The occurrence of $|\mathbf{r}|^{-2}$ in the law of universal gravitation tells us that acceleration is inversely proportional to the square of distance: it famously is an inverse-square law. Here, that fact is encoded in condition (ii) on G. Likewise, the occurrence of the unit vector $|\hat{v}_{ij}|$ in the law of universal gravitation tells us that acceleration is in the same direction as displacement. That fact is here encoded in conditions (iii) and (iv). Despite the 'disappearance' of distance, then, (7) has the same empirical content as (1).

This proves that an account of G is not just an optional element of an interpretation of NG. Insofar as we are driven by a desire for intrinsic theories, an account of G is crucial to the enterprise. Once we have an account of G, we can consistently interpret the equation of motion of NG intrinsically.

7 Active Mass Symmetries

I have thus established the first aim of this paper: providing an account of the gravitational constant, G. I will now use this account to illumate the recent debate about mass symmetries. I start by defining active mass scalings in this section; in the next section I consider scalings of G.

Unlike passive transformations, which only alter which numbers we use to represent particular masses, an active transformation changes the actual mass values assigned to particles themselves. The effect of an active mass *scaling* is that each particle's mass (as represented on $\mathbb{R}^+$) is multiplied by a constant α.

Take any arbitrary representation f of mass values on $\mathbb{R}^+$ (where f is the 'internal space' analogue of a coordinate chart).[13] We can carry out a passive transformation on this representation that transforms $f(m)$ into $f'(m) = \alpha f(m)$. The effect of this transformation is to assign each mass value m a different real number. In order to turn this passive transformation into an *active* one, we keep fixed the representation relation f and consider an automorphism ϕ_α on $\mathcal{V}_M$ defined such that $\phi_\alpha(m) = (f^{-1} \circ \alpha f)(m)$. If a mass m is associated in some system of units to some number x, then ϕ_α maps m onto the unique mass that is associated to the number αx *in the same system of units.*

The automorphism ϕ_α in turn acts on $m(x)$—the function from particles into $\mathcal{V}_M$—as $\phi_\alpha \circ m(i) \equiv (\phi_\alpha \circ m)(i)$. In this way, an active mass scaling induces a transformation on the models of NG:

$$\langle D, \mathbb{E}, \mathbb{T}, \mathcal{V}_M, x(i,t), m(i)\rangle \overset{\text{Scaling}}{\rightarrow} \langle D, \mathbb{E}, \mathbb{T}, \mathcal{V}_M, x(i,t), (\phi_\alpha \circ m)(i)\rangle \tag{8}$$

This transformation is a symmetry of NG only if the model before the transformation is a solution to NG's equations of motion whenever the same model after the transformation is also a solution.[14]

Are active mass scalings symmetries of Newtonian Gravitation? The obvious answer is 'No'. This follows immediately from NG's equations of

[13] One may wish for a more 'intrinsic' characterisation of active mass scalings, i.e. one that does not proceed via a particular choice of units. This is indeed possible. Hölder (1901) shows that one can associate a unique positive real number to every *pair* of elements of $\mathcal{V}$, which denotes their ratio. In order to multiply an element m_1 by a constant α, then, one maps it to the unique element m_2 such that their ratio $m_2 : m_1$ is equal to α.

[14] This is a necessary but not sufficient condition on symmetries. As a *definition* of symmetries, the condition is far too weak: it would mean that any pair of models are symmetry-related (Belot, 2013). But as a necessary condition it is uncontroversial, and that is all I need to assume here.

motion: the right-hand side of (7) contains a mass quantity and so changes value with mass—in accordance with condition (i) on G—while the left-hand side is independent of mass and so remains the same. Unless $\alpha = 1$, this transformation affects the satisfaction of NG's equation of motion.

Martens (2019) illustrates this with the 'comparativist's bucket', an analogue of Newton's famous bucket experiment based on a thought experiment due to Baker (2020). The escape velocity of a projectile is given by:

$$v_{esc} = \sqrt{\frac{2Gm_2}{r}} \tag{9}$$

where G is the gravitational constant, m_2 is the mass of the body the projectile is escaping from (say, the Earth), and r is the distance between them. Suppose that in one model, the actual velocity of the projectile v is just over v_{esc}—so the projectile escapes. But under an active mass scaling the Earth multiplies in mass by a factor α, hence v_{esc} is multiplied by a factor $\sqrt{\alpha}$. Consequently, v may no longer exceed v_{esc}: the projectile crashes back to Earth. Since the actual trajectory $x_i(t)$ of the projectile is left the same by an active mass scaling, the transformed model is inconsistent with the equations of motion. Therefore, active mass scalings are not symmetries of NG.

8 Scaling G

We have not yet considered the gravitational constant, G. G is a constant with dimensions proportional to M^{-1}. It is commonly thought that if one were to scale G *with* all particle masses, active mass scalings become symmetry-like transformations after all (Roberts, 2016; Wolff, 2020). Wolff, for instance, argues that since we fix the reference of the term 'G' by ostension, it will have a different referent in a mass-scaled world. While that may be true, it is irrelevant to the question of whether G *itself* has a different value in such a world. The inhabitants of a mass-scaled world use the same symbol 'G' to denote a different constant, but this does not mean that the constant *we* denote with 'G' has a different value in that world. I therefore concur with Martens (2019) that there is no reason to believe that one *must* scale G when we scale all particle masses.

It remains possible that one could change the value of G *in addition* to scaling all particle masses. This allows us to define another transformation—call it an *inclusive active mass scaling*—which is a symmetry-like transformation of NG insofar as it seems to relate empirically equivalent physical possibilities. But Martens goes on to make the stronger claim that we cannot change the value of G without moving to a different theory: "It is not at all surprising that if one were allowed to change (the strength of) the laws at will for each possible world one could get any (or at least many) of the evolutions one may have wanted. That is simply not an option within the rules of the game we are playing" (Martens, 2019, 12). Martens claims that changing G is akin to changing the laws, which is against 'the rules of the game'. But who set out these rules? It seems to me that there are different conceptions of what a constant of nature is, that in turn have different consequences for whether a change in G is 'allowed'. In particular, once we reject real number structures as redundant, it becomes untenable to hold that G is a parameter with a fixed numerical value. The cross-value space structure of G is determined by the conditions (i)-(iv) listed above, but these conditions only determine G up to a 'choice of unit'. To demand that G is fully fixed is to introduce an "unnecessary global assumption", in the words of Earman and Norton (1987), akin to hard-coding the particular Minkowski metric $diag(-1, 1, 1, 1)$ into the fabric of relativistic spacetime. It may turn out that different G-functions ultimately represent the same cross-value space structure, as I will argue below. This follows not from any pre-determined rulebook, but from a philosophical analysis of the role of G in Newtonian Gravitation.

It remains possible that Martens simply has a different conception of what counts as 'the' theory of Newtonian Gravitation: perhaps on Martens' conception, the value of G is hard-coded into that theory. In that case, the disagreement is over the precise definition of NG's space of models, rather than over the behaviour of G in particular. Nevertheless, if one adopts a more liberal account of NG, as I have urged above, this has consequences for the possible values of G. In what follows I will assume such a more liberal account.

I will thus formulate an inclusive active mass scaling as follows. For any automorphism ϕ_α of $\mathcal{V}$:

$$\langle D, \mathbb{E}, \mathbb{T}, \mathcal{V}, x(i,t), m(i)\rangle \overset{\text{Inc. Scaling}}{\to} \langle D, \mathbb{E}, \mathbb{T}, \mathcal{V}, x(i,t), (\phi_\alpha \circ m)(i), \phi^*_\alpha G\rangle \quad (10)$$

where ϕ^*_α is the pullback of ϕ_α, defined such that $\phi^*_\alpha G(m, \mathbf{v}) = G(\phi^{-1}_\alpha(m), \mathbf{v})$. The effect of this transformation is to change the cross-value space relations between $\mathcal{V}_M$ and $\mathbb{V}_E$. If m and $\mathbf{v}$ were previously connected to some acceleration $\ddot{x}$, for instance, then after this transformation they are connected to some different acceleration $\alpha\ddot{x}$.

It is easy to see that such a transformation preserves the dynamics: when ϕ acts on both G and m, $\phi^*G(\phi(m(i)), \mathbf{v}) = G(m(i), \mathbf{v})$. Given a particle's mass value and a displacement vector, G yields the same acceleration before and after an inclusive mass scaling. This transformation therefore also preserves escape velocities. Suppose, for instance, that $\phi_\alpha(m) = \alpha m$; then $\phi_\alpha^* G = G/\alpha$. In the formula for escape velocity, these factors of α precisely cancel each other out.

Perhaps it would be a misnomer to call this transformation a *symmetry*.[15] The best definition of symmetries is a matter of current debate, but Earman's (1989) definition of *dynamical symmetries* is a useful first approximation.[16] As the name suggests, dynamical symmetries act on a theory's dynamical objects. In the case of NG, these are the position and mass quantities.[17] Importantly, a dynamical symmetry does not act on the theory's kinematical structure, which here includes G. So, an inclusive mass scaling is not a dynamical symmetry because it does act on G. Earman contrasts dynamical symmetries with *spacetime symmetries*, which leave the dynamical objects alone and instead affect the theory's (fixed) spacetime structure. We can generalise this definition to include non-spatiotemporal kinematic structure.[18] On this definition, *kinematic symmetries* are transformations *only* of the theory's kinematic structure—potentially including G. Again, inclusive mass scalings are not kinematic symmetries, since they transforms both kinematical *and* dynamical structure.

Elsewhere in the literature, the kind of transformation we are interested in is called a *similarity*.[19] This term originates in a passage from Poincaré (2003, originally published in 1908):

> Suppose that in one night all the dimensions of the universe became a thousand times larger. The world will remain *similar* to itself, if we give the word *similitude* the meaning it has in the third book of Euclid. Only, what was formerly a metre long

[15] I thank David Wallace for discussion of this point.

[16] For further debate, see Belot (2013), Dasgupta (2016), Wallace (2022), Read and Møller-Nielsen (2020) and references therein.

[17] It is perhaps controversial to categorise mass as a dynamical object, since masses are fixed across time. But I have in mind a broader notion of dynamics, namely the structure that varies across solutions of the theory. This is opposed to the kinematical structure that remains fixed across solutions. On this definition, masses count as dynamical since the mass of any particle is not fixed across models.

[18] For a similar suggestion, see Hetzroni (2019).

[19] For a recent discussion, see Gryb and Sloan (2021).

> will now measure a kilometre, and what was a millimetre long will become a metre. The bed in which I went to sleep and my body itself will have grown in the same proportion. When I awake in the morning what will be my feeling in face of such an astonishing transformation? Well, I shall not notice anything at all. The most exact measures will be incapable of revealing anything of this tremendous change, since the yard-measures I shall use will have varied in exactly the same proportions as the objects I shall attempt to measure.

The case in which all mass-valued quantities are scaled follows the same pattern as set out by Poincaré. Following Poincaré, I will say that an inclusive mass scaling is a similarity of NG.

Despite the fact that this transformation is a similarity rather than a symmetry, it poses similar metaphysical and epistemological questions. In particular, the models related by an inclusive mass scaling describe empirically equivalent states of affairs. But on certain views, those states are physically distinct. This would lead to a worrisome form of underdetermination: particle masses become undetectable even in principle, since mass measurements have the same outcome in similar worlds despite the fact that particles have different masses in them.[20] The presence of such undetectable quantities is a hallmark of redundant theoretical structure.

The physical distinctness of mass-scaled solutions follows from Martens' (2019) belief that mass values possess primitive identities, or *quiddities*: non-qualitative properties that are uniquely possessed by individual mass magnitudes.[21] The reason Martens postulates quiddities is that the elements of $\mathcal{V}_M$ are *absolutely indiscernible*: for no $m \in M$ does there exist a formula with one free variable that is true of m only.[22] However, Martens claims, the laws treat different masses differently—the more massive an object is,

[20] The argument that variant quantities are undetectable is found in Roberts (2008), Dasgupta (2016) and Martens (2019). Middleton and Murgueitio Ramírez (2020) are sceptical, but see Jacobs (2020) for a response.

[21] It is common to refer to the primitive identities of properties as *quiddities*, but the term is used both for determinables, such as mass or charge, and determinates, such as the property of being five kilograms. I will use the term in the latter sense; cf. Martens (2019, fn. 6).

[22] Martens writes that masses are 'qualitatively identical', but this is false in the sense that elements of m are at least *relatively discernible*, i.e. there are formulas with *more than one* free variable that apply to sets of elements in only one particular order (cf. Saunders (2003)). For example, if $m_1 < m_2$, then m_1 is relatively discernible from m_2 as the smallest of the two. Wolff (2020) uses the more appropriate term 'homogeneous'.

the slower it accelerates in response to gravitational attraction. Therefore, Martens continues, we must introduce quiddities that tell forces how to 'latch onto' masses: they are "required for the forces to be well-defined, in the sense of uniquely matching up instances of initial conditions, including masses, with, say, accelerations" (Martens, 2019, 2519).

When masses are scaled, each particle is assigned a different mass value. When each mass comes attached with a quiddity, this represents a real phyical difference. The presence of qualitatively identical yet physically distinct posibilities is just the sort of problematic underdetermination that is familiar from debates around the structure of spacetime. When a theory's spacetime symmetries match the dynamical symmetries, symmetry-related models seem to represent worlds that differ merely over which spacetime point plays which qualitative role. This is the case, for example, for boost-related models of Newtonian Gravitation set on Galilean spacetime. There, a renouncement of primitive identities as redundant solves the issue: the difference between boost-related models becomes a distinction without a difference. I claim that in the present case, the quiddities of mass values are also redundant. Without quiddities the spectre of underdetermination fades away.

Recall that Martens claims that quiddities are necessary for forces to 'latch on' to the right mass magnitudes. This ignores the gravitational constant. The real way in which forces latch onto masses is via the cross-value space structure represented by G. When we include G within the theory's kinematical structure, quiddities become unnecessary. For example, consider once more the escape velocity of a projectile. Which velocity the projectile requires to escape from a massive body with mass m_2 depends on the acceleration the projectile acquires as a result of the gravitational pull of the massive body. For a displacement $\mathbf{r}$, this acceleration is equivalent to $G(m_2, \mathbf{v})$. We can thus find out the acceleration of the projectile without the quiddity of m_2. The gravitational attraction can simply latch onto masses via G to produce the observed accelerations.

There is a helpful analogy here with Aristotelian spacetime. Aristotelian spacetime results from adding a privileged worldline to a Newtonian spacetime. This additional structure is surplus in Newtonian Gravitation, but would be necessary to express the laws if forces were to depend on absolute positions. It is exactly because translations of all material bodies are not symmetries of such an Aristotelian theory that we would need to introduce a worldline to represent the 'centre' of the universe. Similarly, it is because of the fact that mass scalings are not dynamical symmetries of NG that there must exist some structure that distinguishes the points of mass value space.

Furthermore, translations of both all material bodies *and* the centre-of-the-universe worldline are symmetries of the Aristotelian theory; models related by such translations are isomorphic. Likewise, a scaling of all particle masses *and* G is a similarity of Newtonian Gravitation; models related by such scalings are isomorphic, too. Once one rejects primitive identities, however, these models therefore represent the same possibility. Consider the privileged Aristotelian worldline. If we think of it as a property of a certain special collection of spacetime points, it seems that a translation of the centre of the universe assigns this special property to a different collection of points. The result is an empirically equivalent yet physically distinct state of affairs. It seems more natural to consider the Aristotelian worldline as a property that *individuates* spacetime points: to be *this* spacetime point is just to be the point that is located in a certain way relative to the centre of the universe. This view entails that a shift of all particles *and* spacetime structure has no physical effect: matter remains in the same location with respect to the Aristotelian worldine. Compare this with Stachel's (1993) view of the metric in GR as a dynamical individuating field. Both views follow from *anti-haecceitism*: the view that spacetime points have no primitive identities, but are qualitatively individuated.

The analogue of anti-haecceitism for quantities is *anti-quidditism*: physical magnitudes have no primitive identities, but are qualitatively individuated. I propose anti-quidditism about mass.[23] The idea is that mass magnitudes are identified by their pattern of instantiation. Teller (1991, 393) puts the idea as follows:

> What is it to be the property of having a mass of five grams? Perhaps it is no more and no less than bearing certain mass relations to other masses, or possibly to other exemplified masses. On this account we still take there to be individual mass properties, but we take the principle of individuation of an individual mass to be its mass relations to other masses, so that the mass relations between masses become essential to all of them.

Since the structural relations that the particles' masses stand in to each other and to the elements of $\mathbb{V}_E$ and $\mathbb{V}_T$ (as determined by G) are the same across models related by inclusive scalings, anti-quidditism implies that the

[23] The possibility of anti-quidditism in response to the presence internal symmetries has already been noted by Arntzenius (2012), Dewar (2018), Martens and Read (2020) and Wolff (2020).

same masses are instantiated in these models. The differences between isomorphic models are representationally irrelevant, since they concern quidditistic facts about *which* particular mass values are instantiated. This is also a distinction without a difference.

This conclusion allows us to avoid the spectre of underdetermination that normally accompanies symmetry-like transformations. If inclusive mass scalings merely change the way a possibility is represented, rather than that possibility itself, then there are no empirically equivalent yet physically distinct possibilities to speak of. Therefore, scaling G with all particle masses does not lead to a distinct physical possibility. This conclusion does not follow from any preconceived notion of G as a fixed parameter, but from an account of the metaphysical nature of G in conjunction with anti-quidditism. The position I have presented is therefore structurally similar to sophisticated substantivalism about spacetime (cf. Wolff's (2020) 'sophisticated quantity substantivalism').

9 Constants and Symmetries

I have presented an account of what dimensionful constants represent: cross-value space relations. I have then used this account to argue in favour of anti-quidditism about mass values. But a further question could be raised: why are there constants in the first place? It is not an a priori fact that distinct value spaces should be connected in this way. This is not the type of question that always has an answer. But in this case I believe that there *is* an answer, namely that constants are necessary in order to coordinate the dynamical symmetries of different quantities. I will now elaborate on this idea.

Recall Earman's distinction between dynamical and kinematical symmetries, where I will include the symmetries of non-spatiotemporal kinematical structure in the latter. Earman (1989) famously posited a pair of symmetry principles:

SP1 Every dynamical symmetry is a kinematical symmetry

SP2 Every kinematical symmetry is a dynamical symmetry

The motivation for SP1 is to avoid redundant structure. If SP1 fails, then the theory posits kinematical structure that is dynamically inefficacious. The classical example here is the standard of absolute rest of Newtonian spacetime: since the dynamics of Newtonian Gravitation are boost-invariant, such a standard is superfluous. The motivation for SP2, on the other hand, is to make sure that the theory has enough structure to sustain the dynamics. If SP2 fails, then the dynamics distinguish between elements that are qualitatively the same. An example is Leibnizian spacetime, which lacks a standard of absolute acceleration. Since Newtonian Gravitation is sensitive to absolute accelerations, Leibnizian spacetime does not have enough structure to sustain the theory's dynamics.

Consider now the structure of mass value space, $\mathcal{V}_M$. On the one hand, the mass scalings ϕ_α—applied just to particles—are not dynamical symmetries of NG. On the other, they *are* symmetries of the kinematical structure of $\mathcal{V}_M$, since the relations $\leqslant$ and $\circ$ defined over M are invariant under the ϕ_α transformation. Consider arbitrary m_1, m_2 such that $m_1 \leqslant m_2$. By Hölder's representation theorem, $f(m_1) \leq f(m_2)$ for any representation $f : \mathcal{V} \to \mathbb{R}^+$. Since α is positive, $\alpha f(m_1) \leq \alpha f_r(m_2)$. By another application of the representation theorem, $(f^{-1} \circ \alpha f)(m_1) \leqslant (f^{-1} \circ \alpha f)(m_2)$. Since $\phi_\alpha(m) = (f^{-1} \circ \alpha f)(m)$, this shows that $m_1 \leqslant m_2$ iff $\phi(m_1) \leqslant \phi(m_2)$, hence $\leqslant$ is invariant under the action of ϕ. The proof for $\circ$ is analogous.

There is thus a mismatch between the kinematical and dynamical symmetries of $\mathcal{V}_M$ in violation of SP2. Earman's argument in favour of SP2 (applied to masses) is that were the principle to fail, "the theory would have to contain names, regarded as rigid designators, of [elements] of [mass value space]" (47, 1989). This is exactly what Martens believes: that the laws of NG 'latch onto' masses via their quiddities. The laws must pick out masses independently from their qualitative features; so in this sense the theory 'names' particular mass magnitudes. Earman, however, sees this as a defect: "But such a difference in lawlike behavior is reason to suppose that [m_1] and [m_2] differ in some structural property that grounds the difference in behavior" (47, 1989). In other words, the fact that gravity acts in a certain way on a certain mass should follow from facts about what that mass is like—not just from the irreducible fact that it is *that* mass! This suggests that mass value space has more than an additive extensive structure.

Let us suppose not only that mass scalings are not dynamical symmetries, but that *no* transformation that acts solely on m_i is a symmetry of NG. Put differently, no non-trivial automorphisms of $\mathcal{V}_M$ are dynamical symmetries. It is then tempting to conclude, by SP2, that mass value space itself has no non-trivial automorphisms. This would mean that $\mathcal{V}_M$ has the sort of rigid real number structure discussed in section 3. The real number structure is richer than an additive extensive structure exactly because it has fewer symmetries.[24] Unfortunately, this does not solve the inconsistency with SP1 and SP2. We have already seen that this structure endows mass value space with a privileged unit, contrary to the conventional (and correct) wisdom that the choice of unit is arbitrary. But there is an more general argument against this sort of structure, which turns on the *scale-independence* of Newtonian Gravitation.

Suppose one were to double all particle masses, and all distances between particles, *and* all durations between events. It turns out that this *is* a symmetry of NG. Generally, consider a joint scaling of mass by a factor μ, of length by a factor λ, and of time by a factor τ: this transformation is a dynamical symmetry of NG iff $\lambda^3 = \mu\tau^2$. One can easily see from dimensional analysis that under such a joint scaling, the left-hand side of (7) is multiplied by a factor of $\lambda\tau^{-2}$ whereas the right-hand side is multiplied by a factor of $\mu\lambda^{-2}$. These factors are equal iff $\lambda^3 = \mu\tau^2$. The special case of $\lambda = \tau = \mu$ shows that the theory is invariant under a joint rescaling of all quantities by the same factor, i.e. it is scale-independent.

These transformations are dynamical symmetries: they act on the dynamical objects m_i and $x_i(t)$ (and their derivatives). From SP1, it follows that there are corresponding kinematical symmetries. But these kinematical symmetries are special in that they act simultaneously on quantities with different value spaces. We cannot account for these symmetries with changes to the structure of the theory's value spaces in isolation. If we were to let $\mathcal{V}_M$, $\mathcal{V}_L$ and $\mathcal{V}_T$ individually remain invariant under scale transformations, for instance, then this would entail that scale transformations of mass, length and time are dynamical symmetries of NG even when $\lambda^3 \neq \mu\tau^2$, contrary to the conclusion of section 7 that mass scalings are not dynamical symmetries.

In order to find the kinematical counterpart of NG's scale-independence, a piece of structure that connects the theory's value spaces is required: the graviational constant. On the one hand, G is not invariant under scale transformations of mass, length or time by themselves. This correctly entails that such transformations are not dynamical symmetries of NG. On the other hand, G *is* invariant under any joint scale transformation for which $\lambda^3 = \mu\tau^2$. From (i) and (ii), it follows that $G(\mu m, \lambda\mathbf{v}) = \mu\lambda^{-2}G(m, \mathbf{v})$. But the resultant value of $G(m, \mathbf{v})$ is an acceleration, which scales with $\lambda\tau^{-2}$. The function G is therefore preserved whenever $\mu\lambda^{-2} = \lambda\tau^{-2}$, or equivalently, whenever $\lambda^3 = \mu\tau^2$. While each individual value space is invariant under scale transformations, the connections between these value spaces are

[24] For an account of the relation between symmetries and structure, see Barrett (2020).

only invariant when these transformations are related in this particular way.

Is the occurence of constants a consequence of the dynamics' scale-independence? Or are the dynamics scale-independent because of the occurrence of G in the equations of motion? This question is reminiscent of the debate between the geometrical approach and the dynamical approach to spacetime structure (Brown and Read, 2021). On the former view, the dynamical symmetries of NG are a consequence of the theory's spacetime structure. In the case under discussion, this translates into the view that the scale-independence of NG is a consequence of the occurrence of G in the equation of motion. Suppose that the equations of motion did not contain G; then they would not remain invariant under scale-transformations of the sort discussed above. The latter view reverses the order of explanation: spacetime structure is only a reflection of the theory's dynamical symmetries. On such a view the gravitational constant merely codifies the details of NG's scale-independence. The particular transformation properties of G reflect the fact that these dynamics are invariant whenever $\lambda^3 = \mu\tau^2$. G becomes a 'glorious non-entity', to borrow a phrase from Brown and Pooley (2004). Whether the parallel between the gravitational constant and spacetime structure is sufficiently strong to warrant a similar treatment for both remains to be seen. The benefit of a clear account of constants is that it affords us the means to ask such questions in the first place.

10 Conclusion

I have used the controvery over mass symmetries of Newtonian Gravitation as a case study for an account of the nature of natural constants. My account has yielded an answer to the question whether mass scalings are similarities of Newtonian Gravitation: yes, as long as one endorses anti-quidditism. Although hinted at by Teller (1991), the only contemporary defence of this position is Wolff's (2020). It is therefore a relatively novel position in the debate between absolutists and comparativists, but one that I believe is superior to the alternatives.

Yet the real added value is the account of the gravitational constant. On the view I have presented, G represents cross-value space structure, i.e. relations between the elements of different quantities' value spaces. The value spaces that G connects are mass value space, $\mathcal{V}_M$, displacement space, $\mathbb{V}_E$, and duration space, $\mathbb{V}_T$. Since these value spaces are intricately linked, one cannot consider their structure in isolation. Instead of a theory set on space-time, then, it seems better to conceive of Newtonian Gravitation as a theory set on space-time-mass: each of these spaces is equally indispensible to the theory's dynamics. This parallels the earlier move from space and time considered as separate entities to one joint space-time. The transformation properties of G are closely related to the scale-independence of Newtonian Gravitation, although different accounts of the arrow of explanation are possible.

This account of constants exposes a host of questions for further research. It has focused on G in particular; does it extend to other constants, such as the speed of light, c, or Planck's constant, $\hbar$? The observation that G is dimensionful, universal and not associated to any particular kind of object is highly relevant here. It would seem that a constant such as the mass of an electron, m_e, does not represent cross-value space structure. In addition, it has sometimes been suggested that one can obtain a 'natural' system of units by setting all constants to unity. Can we account for this intuition if we interpret constants as cross-value space relations? On this point, Lesche (2014) seems to suggest that setting a constant to unity amount to *identifying* distinct value spaces. Finally, is it possible to rid physics of dimensionful constants altogether, as has been the desire of physicists from Eddington to Einstein? We have already seen that without constants, physics is not scale-independent. But it is not yet clear whether a well-defined constant-free version of classical mechanics is possible at all.

These and further questions deserve further exploration. I hope to have provided a framework within which to consider them.

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