ANNOTATIONS TO THE SPEECH OF THE MUSES (PLATO *REPUBLIC* 546B–C)

Oxyrhynchus Papyrus XV 1808 is a carefully written 2nd–century book roll and the oldest extant copy of *Republic* 546b–d, Plato's fabulously obscure discussion of the cycles behind divine and human engendering.[1] Its marginalia, presumably contemporary but less carefully written, are the only commentary on the passage to survive among papyri. The most valuable bits of information preserved there, however, have so far been inaccessible. [2] This is mainly because the annotator, unlike annotators of any other literary papyri, uses shorthand.[3] Although many of the symbols he employs belong to the system deciphered long ago by Wessely, some appear uniquely here among published shorthand texts. Others, assumed for decades to be tachygraphic signs, turn out not to be shorthand at all. We have been able to correct some readings and add others, with the result that this remarkably erudite set of marginalia is now almost completely intelligible. What follows is a new edition of these notes.

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[1] For the names of ancient authors and works we use the abbreviations adopted for LSJ. Papyrological references are from John F. Oates et al., *Checklist of Editions of Greek, Latin, Demotic, and Coptic Papyri, Ostraca and Tablets*, Oakville, Conn.[5] 2001; *DNP* stands for *Der neue Pauly* (edd. H. Cancik, H. Schneider, Aug. F. v. Pauly), Stuttgart 1996–. Works referred to more than once are cited by author's name, as follows:

J. Adam, ed., *The Republic of Plato* (2nd ed. by D. A. Rees), Cambridge 1965;

N. Blößner, *Musenrede und 'geometrische Zahl': Ein Beispiel platonischer Dialoggestaltung (Politeia VIII, 545c8–547a7)*, Stuttgart 1999;

D.R. Dicks, *Early Greek Astronomy to Aristotle* (London 1970);

W. Burkert, *Lore and Science in Ancient Pythagoreanism* (transl. by Edwin L. Minar Jr.), Cambridge, Mass. 1972;

A. Diès, *Le Nombre de Platon: Essai d'exégèse et d'Histoire*, Paris 1936;

E. Ehrhardt, "The Word of the Muses (Plato, *Rep.* 8.546)", *CQ* 36 (1986) 407–20;

D.H. Fowler, *The Mathematics of Plato's Academy: A New Reconstruction*, Oxford 1987;

K. Gaiser, "Die Rede der Musen über den Grund von Ordnung und Unordnung: Platon, *Politeia* VIII 545D–547A", *Studia Platonica: Festschrift für Hermann Gundert zu seinem 65. Geburtstag am 30. 4. 1974* (Amsterdam 1974);

GH: B.P. Grenfell and Arthur S. Hunt, edd., *P.Oxy.* XV 1808 (Pack[2] 1421) with pl. IV;

Haslam: M.W. Haslam, reed. of *P. Oxy.* 1808 in *CPF: Corpus dei papiri filosofici greci e latini*,Florence 1999, I.1.72;

Heath *Euc*.: Th.L. Heath, ed., *The Thirteen Books of Euclid's Elements*, New York[2] 1956;

Heath, *GM*:: idem, *A History of Greek Mathematic*s, Oxford 1921;

C.A. Huffman, *Philolaus of Croton* (Cambridge 1993);

F. Hultsch, "Drei Exkurse" *ap.* Proclus Diadochus, *In Platonis Rem Publicam Commentarii* (ed. Kroll), Leipzig 1901, II 384–415;

G.S. Kirk, *Heraclitus*: *The Cosmic* Fragments, Cambridge 1962;

W.R. Knorr, *The Evolution of the Euclidean Elements*, Boston 1975,

K. McNamee, *Abbreviations in Greek Literary Papyri and Ostraca*, Chico, California 1981;

H.J.M Milne, *Greek Shorthand Manuals* (London 1934),

K. Reinhardt, *Parmenides und die Geschichte der Griechischen Philosophie,* Bonn 1916;

F. Susemihl, *Aristotelous Politika*, Leipzig 1879;

I. Thomas, *Selections Illustrating the History of Greek Mathematics*(Cambridge, Mass. 1957),

H.Vogt, "Die Entdeckungsgeschichte des Irrationalen nach Plato und anderen Quellen des 4. Jahrhunderts", *Bibliotheca Mathematica* 10 (1909–10) 97–155;

M.L. West, *Early Greek Philosophy and the Orient*, Oxford 1971;

E. Zeller, *Die Philosophie der Griechen in ihre geschichtlichen Entwicklung* (Leipzig[5] 1922).

[2] GH and Haslam are the two previous critical editions of these notes.

[3] Why this scribe and no other ancient annotator used tachygraphic signs is an open question. The very learned content of the notes indicates that he was highly educated. Since at the time the notes were written notarial skills were not yet the passport to bureaucratic success that they later became, and since the writer's knowledge of the system is old–fashioned and imperfect, we may guess that he had learned a few essential signs for use in note–taking, and that he may have written these notes during an oral presentation; K. McNamee, "A Plato Papyrus with Shorthand Marginalia", *GRBS* 42 (2001) 91–117. On Greek shorthand in general see Arthur Mentz, *Ein Schülerheft mit altgriechischer Kurzschrift* (Bayreuth 1940), Milne, Hans C. Teitler, *Notarii und Exceptores* (Amsterdam 1985), K. Wessely, "Ein System altgriechischer Tachygaphie", *Denkschr.Wien.Akad.* (1896) Abh. 4.

The eleven marginal comments in *P. Oxy*. XV 1808 explain the passage in which the muses, invoked by Socrates, reveal that living things are regulated by cycles of generation. Their first observation concerns the gods: "For divine engendering, on the one hand, there is a period that a perfect number encompasses." The first three marginalia in the papyrus deal with this cycle of divine generation. Note 1 describes what is engendered, Note 2 appeals to Heraclitus to state the length of the cycle, and Note 3 gives an interpretation of divine engendering.

The remaining eight notes concern the cycle of human generation, which the muses describe as follows:

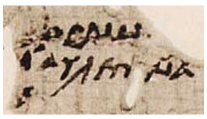
> ἔϲτι δὲ θείῳ μὲν γεννητῷ περίοδοϲ ἣν ἀριθμὸϲ περιλαμβάνει τέλειοϲ, ἀνθρωπείῳ δὲ ἐν ᾧ πρώτῳ αὐξήϲειϲ δυνάμεναί τε καὶ δυναϲτευόμεναι, τρεῖϲ ἀποϲτάϲειϲ, τέτταραϲ δὲ ὅρουϲ λαβοῦϲαι ὁμοιούντων τε καὶ ἀνομοιούντων καὶ αὐξόντων καὶ φθινόντων, πάντα προϲήγορα καὶ ῥητὰ πρὸϲ ἄλληλα ἀπέφηναν·
>
> For human engendering, on the other hand, there is a number, the first in which augmentations—both the ones ruling and the ones ruled, taking three intervals but four terms (of likenings and unlikenings, and of increasings and diminishings)—reveal all things mutually agreeable and rational with respect to each other.

Notes 4 and 5 treat the expression "augmentations—both the ones ruling and the ones ruled." Note 6 comments on the four terms and three intervals. The seventh through tenth notes treat Plato's reference to rational and irrational diagonals, referred to in the next sentence of the passage:

> ὧν ἐπίτριτοϲ πυθμὴν πεμπάδι ϲυζυγεὶϲ δύο ἁρμονίαϲ παρέχεται τρὶϲ αὐξηθείϲ, τὴν μὲν ἴϲην ἰϲάκιϲ, ἑκατὸν τοϲαυτάκιϲ, τὴν δὲ ἰϲομήκη μὲν τῇ, προμήκη δέ, ἑκατὸν μὲν ἀριθμῶν ἀπὸ διαμέτρων ῥητῶν πεμπάδοϲ, δεομένων ἑνὸϲ ἑκάϲτων, ἀρρήτων δέ, δυοῖν, ἑκατὸν δὲ κυβῶν τριάδοϲ.
>
> Of these, a base with a third added, married by the pentad, three times augmented, furnishes two harmonies, the one being equal an equal number of times, one hundred so many times; the other being equal in length to it, but oblong—first, of a hundred of the numbers of rational diagonals of the pentad, each one lacking one, but from irrational, two; and then of one hundred cubes of three.[4]

In Note 11, the annotator identifies one of the harmonies referred to in this sentence and indicates how it should be interpreted. This interpretation anticipates and confirms Konrad Gaiser's suggestion that Plato thinks the harmonies regulate the proper ages for parenthood.

Note 1: i.1–3, treating θείῳ μὲν γεννητῷ περίοδοϲ



i.1	]κοϲμω ̣	[οὐ (τῷ)] κόϲμῳ·
	]μ̣ ʹ αυ⟋επ̣ ̣ ν̣ᵚ α` ̣ ʹ	[τὰ] μ̣(ὲν) ἄϋ(λα) ἐπ̣ά̣νω̣ αϲ̣( )
	]ε̣	[̣ ̣ ̣]ε̣

Palaeographical notes Above and to the left of te note are traces of line ends from the preceding column which suggest there is room for two or three letters at the left of each line i.1]κοϲμω ̣ [̣ ̣] GH :]κοϲμωι (line end) Haslam, but the annotator's omission of iota adscript elsewhere is against this (cf. Notes 3.2, 7.1). After ω, there is an ambiguous trace of ink visible in the published plate. R. A. Coles reports that it disappeared after he unfolded a tiny piece touching the suprascript ω directly below, and that the trace may have belonged to that letter i..2 ⟋, tachygraphic symbol (Menci) επ̣ ̣ ν̣ᵚ α` ̣ ʹ, Coles (*per litt.*) reports, of the third letter: "there is a trace … [which] could be α, if a *minute* one." For alphas of such small size see Note 3, i.7, πλανητ and Note 4, i.9 δυναμεν  νᵚα` ̣ ʹ KM, the second suprascript being the lower part of a curved letter (ε, θ, ο, ϲ) : ν̌ α (line end) GH : ν( )α Haslam (line end)]ε̣ : Haslam ("can see ends of all 3 strokes", Coles) :] α̣ GH

[4] Plato uses πεμπάϲ for the number five. This is usage of the same sort as μονάϲ and δυάϲ for the numbers one and two. Plato's uses of μονάϲ and δυάϲ imply that he thinks that their referents have explanatory and metaphysical heft (*Phd*. 101c, *Phlb.* 15a). Aristotle uses the expressions in his report of Plato's lectures 'On Philosophy' (*de Anima* 404b19–b26).

"] not for the cosmos: . . . , in the first instance, the immaterial . . . above"

Papyrological Commentary

i.1 [οὐ (τῷ)]κόϲμῳ: alternatively, [οὐ τ(ῷ)]κόϲμῳ: Editors have seen κόϲμῳ as a gloss on θείῳ γεννητῷ. In what we think the likeliest reconstruction, however, the annotator instead *distinguishes* the visible cosmos from the objects of divine engendering, rather than likening them. Our restoration in line 1 fits the space available, provided only that we make the plausible assumption that the annotator used shorthand or an abbreviation for τῷ (~, as elsewhere in the notes; less probably τ′ or a similar abbreviation).

i.2 μ(ὲν): the implied antithesis is between divine and human engendering (Procl. *in R.* 15. 4–5, "Therefore he means 'all divine engendering,' after which comes human engendering," πᾶν ἄρα λέγει θεῖον γενητόν, μεθ' ὃ τὸ ἀνθρώπειόν ἐϲτι γενητόν; cf. *ibid.* 14.27–15.1, 17.5–24).

ἄϋ(λα): thus KM. G. Menci, noting the similarity between the shorthand symbol following αυ and Milne no. 36 (λ representing λίαν), first proposed a form of ἄϋλοϲ (*per litt.*); αὐ(τόν?) GH, Haslam.

ἐπάνω: the realm of the heavens as the locus of the divine engendering suits Plato's account of the heavenly gods in *Timaeus* (39e–40b).

αϲ( ): possibly ἀϲ(ώματα), "incorporeal things"? Cf. sch. *in* Pl. *Resp.* 546a *bis*, where the commentator speaks of the periodic, simultaneous restoration "of all incorporeal as well as all corporeal movements," (πάντων ἀϲωμάτων κινήϲεων καὶ ϲωματικῶν παϲῶν).

i.3]ε: conceivably [ἔλεγ]ε, "he meant," Plato being the implied subject. Since the marginalia were presumably copied, as often, from a separate commentary, this would account for the inflectional shift from dative to accusative in line 2, a phenomenon relatively common in annotations.[5] Similar shifts appear in Notes 4, 5, and 8. A difficulty is the number of letters to be restored. The annotator, however, tends to begin lines successively to the left ('Maas's Law,' visible in operation in Notes 9, 10, 11, and the last three lines of Note 6), and since his epsilon and gamma are narrow, the entire word may have fit in the lacuna. Alternatively, he used shorthand.

The note treats θείῳ γεννητῷ, "divine engendering." There was an ancient dispute about what exactly Plato thought was being engendered. According to Plutarch, it was the cosmos (κόϲμον, Plut. *Moralia* 1017c [*de anima procreatione in Timaeo*]). According to Amelius, a student of Plotinus, it was neither the world (κόϲμον) nor the heavens (οὐρανόν), but everything under heaven (πᾶν τὸ ὑπουράνιον, Procl. *in R.* Kroll II 30.6–21). According to Proclus (in R II 14.8–12; scholia in Pl. R 546b):[6] "By 'divine engendering' Plato means neither the cosmos in its totality, even if it is principally this very thing, nor the heavens alone, nor what is sublunar. Rather, he means all that is eternally in movement and in revolution, whether in the heavens or under the moon: on the one hand, that which is called the corporeal engendering—for no body is self–substantial— on the other hand, the divine, which is always in movement." We have no view about what Plato really meant here, but we think the annotator agrees with Proclus, even though the note's single legible word, 'cosmos', might give the impression he sided with Plutarch. For if we are correct in restoring ἄϋ(λα) and ἀϲ(ώματα) (the latter much less secure), then the note indicates that Plato refers to *multiple* births, which are immaterial and possibly incorporeal. This rules out Plutarch's singular and corporeal cosmos.

Another possibility is that the annotator is claiming, with Amelius, that the divine engendering is all the things under heaven. Against that, we may observe that some things under heaven are material (and corporeal). Moreover, as Proclus observes (in R. II 29–33), , Amelius' interpretation has the implausible consequence that Plato thinks everything in the cosmos is divine. (Proclus calls it 'paradoxical' [30.6].) We may conclude that the view was unusual. In the absence of other evidence, we should presume that the annotator has an ordinary view, not Amelius' extraordinary one.

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[5] K. McNamee, "The Inflection of Marginal Notes in Literary Papyri," *Sigla and Select Marginalia in Greek Literary Papyri* (Brussels 1992) 65–81.

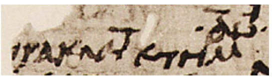
[6] *Scholia Platonica,* ed. W.C. Greene (Haverford, Pennsylvania 1938). The scholia only preserve one other observation on *Resp*. 546b–c and it, too, matches Proclus verbatim (*in R.* II 16.3–8): "It is necessary that we understand the perfect number not only counting upon our fingers (for this is a numbered thing rather than number, and being completed rather than complete, since it is always coming to be). Rather, we should understand the cause of this number, which is on the one hand intelligible and on the other hand that which comprises the defined limit of the whole period of the cosmos."

Our reading stands or falls with the restoration of ἄϋλα, reinforced or not by ἀϲώματα̣. We concede the conjectural nature of the supplements. The terms fit the traces and the space available acceptably, however. ἄϋλοϲ and ἀϲώματοϲ also have a good pedigree in Platonic sources, which routinely use them together in contexts that concern divinity.[7]

According to Proclus (*in R.* II 16.17–22), the objects of divine engendering in *Resp*. 546b include the divine souls which Plato says belong to the heavenly bodies (*Leg*. 899b; cf. *Epin.* 981e–984b). According to a scholium, those incorporeal souls, as well as the souls of all "corporeal movements" (πάντων ἀϲωμάτων κινήϲεων καὶ ϲωματικῶν παϲῶν) are also subject to periodic restoration, κοινῆϲ ϲυναποκαταϲτάϲεωϲ (sch. *Resp*. 546a *bis*). If the annotator has these divine and incorporeal souls in mind, then this first note fits nicely with the third note, which connects the divine number with the orbits of the planets. In addition to the souls of the planets, Plato, Proclus, and the anonymous author of the scholiastic commentary believe in a realm of immaterial and invisible *daimones*.[8] The annotator, we presume, has these in mind as well.

This first note only identifies the entities involved in divine engendering. It does not describe the process of engendering. The annotator's next two notes provide fragments of an account of what Plato thinks happens when 'divine engendering' occurs.

Note 2: i.4, treating ἀριθμὸϲ – – – τέλειοϲ



i.4 ηρακλειτ ετη ·ᾱω̄· μ   Ἡράκλειτ(οϲ)· ἔτη μ(υριὰϲ) ᾱ ω̄

"Heraclitus: 10,800 years"

Papyrological Commentary

Ἡράκλειτ(οϲ): the nominative, the subject of an understood φηϲί or the like, suits the expository style of a commentary better than the genitive proposed by Haslam.

μ(υριὰϲ) ᾱω̄ (·ᾱω̄· μ): "the collocation of figures after ετη is peculiar," GH. The usual abbreviation for myriad is μ(υρι)-ά(ϲ), written μ with α above, but m for μ(υριάϲ) occurs occasionally in subliterary texts and this is how we take it (cf. *P. Lond*. II 265.3, 17 etc.; *P. Mich*. III 145 fr. III vii.2; *P. Michael*. 62 ii.17, iv.18 etc.) Above m in the papyrus are alpha and omega, which the scribe intended to be read together as numerals, for horizontal strokes are written above each one, and there is a dot written, respectively, at the left and the right of the two letters. The dots distinguish, rather than expunge, the text between them, just as the dots on either side of a variant reading draw attention to it (as in *P. Köln* I 12; *P. Oxy*. XIII 1619, XVIII 2180, XXXV 2693, e.g.). Tradition (see below) suggests the scribe intended to indicate one myriad plus 800 years, not 1,800 myriads. It is just possible the annotator's peculiar way of expressing this as a collocation of alpha and omega reflects the influence of the similarly apocalyptic expressions found, for example, in *Rev.* 1.8, which was composed about a century before the papyrus notes were written: ἐγώ εἰμι τὸ ἄλφα καὶ τὸ ὦ, λέγει κύριος ὁ θεός, ὁ ὢν καὶ ὁ ἦν καὶ ὁ ἐρχόμενος, ὁ παντοκράτωρ (cf. 21.6, 22.13). If so, the way the number is written, would point to the perfection of the number 10,800. We owe the suggestion to L. Koenen.

The note associates Plato's perfect number with a tradition of theorizing about 'great years,' a tradition that M. L. West traces back to 'Orpheus', India, and Persia (67 n. 2, 190–92). Its learned precision is rare in papyrus marginalia. The specific attribution to Heraclitus of a great year of 10,800 years also appears, later, in Censorinus (3rd cent. C.E.),[9] according to whom "the orbits of the sun and moon and the five wandering stars make" one such year when "they are brought back together at the same time to the same sign where once they were" (*De die natali* 18.11 [Diels–Kranz A13; Aristotle F 25 R]).

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[7] Plut. 1085c2 (*De communibus notitiis adversus Stoicos*), Iamb. *Myst.* 7.2.14, Procl. *in Ti.* I 32.7; also, with broader connotations, Ph. *Legum allegoriarum* 1.82, 2.80, Iamb. *VP* 29.159, Alex.Aphr. *in Metaph.* 694.40, *de An.* 115.13, ἀπορίαι καὶ λύϲειϲ 39.10, Iamb. *Prot.*118.215, *Comm. Mathematica* 13.67, Procl. *in Ti.* I 396.6, 431.3.

[8] Pl. *Ap.* 27b–28a, *Tim.* 40d–41a, *Pol.* 271d–e, *Lg.* 717b; Procl. *in R.* II 15–16.

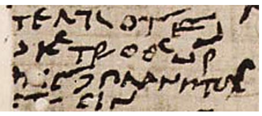
[9] As A. E. Taylor advised Grenville and Hunt (GH 189). Strictly speaking, Censorinus (*DNP* 2.1048 [4]) does not call it a Great Year, but writes that Aristotle calls a certain kind of year *maximum potius quam magnum.* Reinhardt 183 n. 2 conjectures that the reference is to the lost *Protrepticus*.

In the *Timaeus*, Plato refers to a 'perfect year', τέλειοϲ ἐνιαυτήϲ, measured by a "perfect number" (τέλειοϲ ἀριθμός), which "elapses at that moment when the relative speeds of all eight periods [of planetary revolution] have been completed together" (*Tim.* 39d). We infer from Notes 2 and 3 that the annotator and the commentator he quotes equate the perfect number of *Resp.* 546b and the perfect number of *Tim.* 39d, and that they see this number as measuring the length of great year. Neither Plato, nor the annotator, nor any other extant source records how long Plato thinks this period is, however. Perhaps the commentator, lacking direct evidence regarding Plato's opinion, offers the length of Heraclitus' great year as a second–best substitute.

In *Resp*. 546, Plato places considerations about a cosmic cycle of divine engendering next to considerations about a biological life cycle of human engendering. Likewise, Heraclitus' great year seems to be a numerological mix of the astronomical and the human, as Reinhardt (p. 189) shows.[10] He observes that there are 360 periods of 30 years' duration in 10,800 years, and that Heraclitus considered one generation to last for 30 years (Diels–Kranz A19). If Heraclitus assumed, in addition, that one year consists of 360 days, then a day would bear the same relation to year as a human generation, for him, would to a great year (Kirk 302). Aëtius (1[st] cent. C.E.?) gives an alternative length of 18,000 years for Heraclitus' great year, but scholars generally prefer Censorinus' number, partly because of the plausibility of this elucidation by Reinhardt.[11] The Oxyrhynchus annotator's use of 10,800 strengthens Censorinus' claim to accuracy.

In the same passage in which Censorinus assigns a length to Heraclitus' great year, he also attributes to him the idea that the world will be consumed by fire at the end of each great year. That attribution has provoked controversy. Most scholars have denied that Heraclitus believed in such a thing[12] while accepting, however, the period of 10,800 years as originating with Heraclitus and offering various conjectures about what that period really signifies.[13] Note 2 says nothing one way or the other about whether the annotator thinks Heraclitus believed in world conflagration. It simply corroborates, tersely, the report Censorinus gives of Heraclitus' number. The note itself will have been exerpted, however, from a commentary which, in uncut form, must have also discussed the meaning of the number and its significance to Heraclitus and Plato. Note 3 is a slightly longer excerpt from this source–commentary and connects Plato's perfect year with the orbits of the planets. Given the location of the second note between comments on divine generation and on planetary movement, the commentator seems to believe that Heraclitus' great year also has an astronomical significance.

Note 3: i.5–8, treating ἀριθμὸϲ... τέλειοϲ



i.5]ʹ τ ε λ ┐ ╲ ο τ ε ʹ ι — [ἀρ](ιθμὸν) τέλ(ει)(ον) ὅτ(ι) ἐ(π)ιῴ
]ω̣ κ ‹ τρ ο θ^{c} ωρ — [δ]ὠ̣(ν) κ(ατὰ) τρ(οπὰϲ) ὁ θ(εὸ)ϲ ὥρ(αϲ)
]θ̣η̣ϲʹ ↗ πλανηταϲ — [. .]θ̣η̣ϲ(ηϲ) (τοὺϲ) πλάνηταϲ
8]ϲτ⸗ϲιν — [ἀ(πο)κ(αθ)ί]ϲτ(η)ϲιν

Palaeographical Notes m[2] KM : m[3]? GH, Haslam i.5]ʹ, an abbreviation mark KM; Coles recognizes the trace of a suprascript (separate from the tail of ρ in ηρακλει^{τ} in the line above) ι, "A short vertical stroke,which might be read as ι, remains unaccounted for" GH, followed by Haslam; ε(π)ι KM i.6:]ω̣ KM. Perhaps slightly suprascript, to indicate abbreviation. Coles is reluctant to read ω ("an odd shape") ⸗ Coles i.7: θ̣η̣ϲʹ Coles : οτι GH, Haslam (ο̣τ̣ι̣) πλανηταϲ, –αϲ cursively written, abuts initial ϲ in the main text ┐, ╲, ‹, ↗, ⸗: standard tachygraphic signs

"(He calls the) [number] perfect because the god, having kept watch over the turnings, ?once a season has […–ed], restores the planets."

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[10] Gaiser 73 observes that the tendency of the pre–Socratics to mix biological periods and cosmic periods resurfaces in the speech of the Muses.

[11] Kirk 300, West 156–57. Aëtius: *DNP* 1.208–209 (2).

[12] Ch.H. Kahn, *The Art and Thought of Heraclitus* (Cambridge 1979) 134–53 is a convincing exception.

[13] Reinhardt 183–201, Kirk 301–03, West 156–58

Papyrological Commentary

1.5–8. Thus GH, Haslam restore τέλ(ει)(οϲ) ὅτ(ι) ἐ(ν) | [αὐτ]ῷ κ(ατὰ) τρ(οπὴν) ὁ θ(εὸ)ϲ ὥρ(ικεν) | ὅτι ϲ(ύν)π(ανταϲ) πλανητὰϲ | [ἀποκ(αθ)ί]ϲ τηϲιν. Alternatives proposed are ὁ τ(έλειοϲ)? Haslam, ἐ(ν)ι[αυτ]ῷ? GH, ἐ(ν) ῷ? Taylor *ap*. GH.

Initial words, at least in i.6–8, were probably more or less alined, although each may have begun progressively further to the left (see the papyrological commentary on Note 1). Very little is missing at the beginning of each line: the position of the other notes in the intercolumnium and of the line ends of the main text suggest 1 to 3 letters have been lost at the start of each line, but the exact number depends on how far each successive line drifted leftward and whether the main text protruded into the margin.

i.5 [ἀρ](ιθμὸν): slightly ecthetic, as in Notes 5 and 8.

τέλ(ει)(ον): the inflection is assured by the shorthand symbol for (ον) (Menci *per litt.*).

i.5–6 ἐ(π)ι|[δῴ(ν): thus KM, ἐ(ν) [ἀυτ]ῷ or ἐ(ν)ι[αυτ]ῷ Haslam and GH, citing Taylor's ἐ(ν) ῷ. It is difficult to make sense of the traces. Our suggestion takes into account that ε′ is the usual abbreviation for ἐ(πί) and that abbreviations of ἐν are unknown in literary papyri. A difficulty with ἐ(π)ι|[δ]ῴ(ν) is the absence of any really clear sign of abbreviation following the supposed omega. Actual abbreviation marks or suprascript letters were not always used, however, and the formulation ε′ιδω (if this is what was written and not ε′ιδ^{ω}) may have been sufficiently clear for the annotator, who was his own intended reader (McNamee xii).

i.6–7 ὥρ(αϲ) [. .]θηϲ(ηϲ)·ὥρ(αϲ: thus KM; ὥρ(ικεν) GH, Haslam. A form of ὥρα is a simpler restoration and more suitable in a comment dealing with the turnings of the planets and, and it accords well with that moment in the great year of the *Timaeus* when god restores them to their initial positions. Given that what follows, the two words presumably belong to a genitive absolute, possibly ὡρ πλ′]θηϲ(ηϲ), for ὥρ(αϲ) πλ(ηϲ)] θηϲηϲ (with iotocistic spelling for πλ(ηϲ)]θείϲ(ηϲ), "when the time has been fulfvilled."

i.8 [ἀ(πο)κ(αθ)ί]ϲτ(η)ϲιν: the supplement, preferred by previous editors, is slightly longer than the supplements we propose in ii.5–7, but we are encouraged to accept it for palaeographical and philological reasons. Iota will have occupied little space. Also, the scribe is likely to have started this last line further to the left than those that preceded, perhaps even intruding it between the lines of the column of text to which the note refers. He may also have compressed his handwriting, which varies considerably in size. From the Platonic viewpoint, ἀποκαθίϲτηϲιν is the word expected. It is the appropriate technical term for the process of planetary restoration, and it has good support in Platonic sources. Periodic collective restoration (συναποκατάϲταϲιϲ) of the movements in the cosmos is mentioned in a scholium on *Resp*. 546a as well as in mathematical and Pythagorean contexts dealing with the return to a starting–point of revolving spheres or souls.[14] Shorter forms from the same root, for example from καθίϲτημι or ἵϲτημι, are not viable options. Plato, Proclus, and others concerned with the subject do not use them in discussing the restoration of periods or of planets.

The annotator's 'turnings' are the turnings of the planets, each of which completes its orbit at a different moment: (ἄϲτρων) ὅϲα δι' οὐρανοῦ πορευόμενα ἔϲχεν τροπάϲ, *Tim*. 39d. 'God' is the demiurge of the *Timaeus*, whom Plato sometimes in fact calls god (*Tim.* 38c, e.g.). He creates the heavenly bodies, whose turnings through the heavens constitute time.

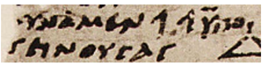
In the discussion of Note 1, we argued that the annotator believes that the objects of divine engendering are the souls of the heavenly bodies and the *daimones* associated with them. The second note suggests that the annotator believes that the 'perfect number' of *Resp.* 546b measures the 'perfect year' of *Tim.* 39d, and Note 3's reference to a period of time determined by the orbits of the heavenly bodies confirms the suggestion. We do not imagine, however, that the commentator believes that Plato thinks the souls of the planets are born and die in the course of a 10,800–year cycle. Rather, they are continuously restored at the conclusion of their cycles. In the *Timaeus*, the demiurge guarantees this immortality by telling the planetary divinities and "the ones who present themselves only to the extent that they are willing" (41a) that "only one who is evil would consent to the undoing of what has been well fitted together and is in fine condition" (41b).

What sort of engendering, then, does the commentator suppose the cosmic divinities are subject to every great year? He tells us what Plato does not: that upon the completion of that period the demiurge

14 κοινῆϲ ϲυναποκαταϲτάϲεωϲ, sch. *in* Pl. *Resp.* 546a. See also ὁ χρόνοϲ ὁ ἐμφανὴϲ... ἀποκαταϲτατικόϲ, Procl. *in R.* II 17.17–19; cf. *ibid.* II 16.12–14, 18.15–17, Theo Sm. 172.115–22 (of the regular revolution of the sun), Iamb. *Theologoumena arithmeticae* 52.15 (of metempsychosis).

intervenes to *re–establish* the cycle of the planets.[15] This might seem to be more an act of preservation than of engendering, but a doctrine that Diotima offers in the *Symposium* suggests a way to understand it. According to Diotima, human and animal engenderings are attempts by mortals to approximate immortality and changelessness (*Symp.* 206e–207d). Plato presumably believes that divine engendering approximates immortality better than human engendering does. We should not be surprised,therefore, if divine engendering is a kind of preservation, rather than a process of bringing something wholly new into existence.

Notes 4 and 5: i.9–10 Treating δυνάμεναί

	i.9	δυναμεν11ϋπο τεινουcαc	δυναμέν(αc)· (τὰc) ὑπο– τεινούcαc

"'the ones ruling': the hypotenuses."

i.10–11 Treating δυναcτευόμεναι

	i.11	1αλλ1πλε̣υρ1 ορθην \ βαcιν	(τὰc) ἄλλ(αc) πλευ̣ρ(ὰc)· ὀρθὴν (καὶ) βάcιν

"The other sides: the perpendicular and base."

Papyrological Commentary

1(–αc), (ταc) and \ (καὶ): the shorthand signs are well attested with these meanings. Presumably the scribe used these accusative forms in place of Plato's nominatives because this was how he found them, in his source–commentary, in statements such as: "N. says the δυναμέναc are hypotenuses."

With Note 4, the annotator moves on to Plato's discussion of human engendering. The rest of the notes explain steps in the calculation of the two harmonies mentioned in *Resp.* 546c. The last note concludes the final step of the calculation of one of the harmonies and comments on its significance. The corresponding conclusions for the second harmony are easy to supply.

Proclus gives the fullest account of Plato's calculation, as well as the fullest compilation of ancient explanations of its significance. We understand the commentator on whom the annotator drew to agree with Proclus about the way the calculation develops. There are striking parallels of language and content between parts of almost all the notes here and parts of Proclus' commentary (exceptions are notes 2 and 8.) Most importantly, however, both authorities agree that 2,700 and 4,800 should be added together to yield the second harmony, 7,500.[16] We shall argue in our discussion of note 11 that the annotator's source disagrees with Proclus about the meaning of this result, however. Agreement about the calculation but disagreement about its interpretation should not surprise us, though. Proclus records no disagreements about what the harmonies are, and he preserves a variety of incompatible accounts of what they signify.

Proclus and the annotator also share, we note, a certain remarkable omission. Neither preserves any information actually identifying Plato's "whole geometrical number that controls better and worse births" (*Resp.* 546c)—the overall number, that is, of human engendering. Instead, they both focus on the calculation of his two harmonies.

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[15] If the commentator is right, then Plato's picture is a bit like Newton's suggestion that God may occasionally intervene to guarantee the stability of the orbits of the planets, since there are irregularities "which may have risen from the mutual Actions of Comets and Planets upon one another, and which will be apt to increase, till this System wants a Reformation," Isaac Newton, *Opticks* (New York 1952, based on the 4th ed. London, 1730) 402.

[16] According to one modern commentator, by contrast, "'If there is anything clear about the number it surely is' that Plato's rectangular harmony is 2700 x 4800, and not 2700 + 4800," Arthur Gordon Laird, *Plato's Geometrical Number and the Comment of Proclus* (Madison 1918) 24, quoting Adam.

Various scholars in the last 125 years have defended either Proclus' interpretation of Plato's calculation or interpretations like it.[17] In forthcoming work, we plan to join that tradition and to defend Proclus' account of Plato's calculation and the commentator's account of its significance. Here, we stick to explaining the annotations in light of Proclus and avoid the question of whether the annotator reads Plato correctly.

As we see it, the annotator's source and Proclus both believe Plato is describing the construction of a three–dimensional solid. Notes 4 through 6 describe the construction of one side of that solid, a large triangle. Proclus explicitly describes the construction of that triangle, and all sympathetic expositors of his interpretation have followed him in doing so. So far as we know, we are the first to consider Proclus' description of the construction of this triangle as the first stage of the construction of a three–dimensional wedge. We shall defend this interpretation of Plato's intentions in a subsequent paper. For present purposes, our interpretation enables us to make sense of the annotator's meaning in Note 6, where he writes that four terms make up a boundary (ὅριον).

Notes 4 and 5 are best understood in light of Proclus' description of the construction of the large triangle. Together, these notes give a geometrical interpretation of Plato's words δυνάμεναί and δυναϲτευόμεναι. According to the annotations, the words mean, respectively, the 'hypotenuse' and 'the other sides' of a right triangle. Susemihl, Diès, and Ehrhardt arrive at such an understanding of the words on the basis of the following passage on Pythagorean terminology in Alexander of Aphrodisias (late 2nd /early 3rd cent.):[18] ἐπεὶ τοίνυν ἡ ὑποτείνουϲα ἴϲον δύναται ἀμφοτέραιϲ ἅμα, διὰ τοῦτο ἡ μὲν δυναμένη καλεῖται, αἱ δὲ δυναϲτευόμεναι, καὶ ἔϲτι πέντε. Since, therefore, the hypotenuse is equal in power to both the other sides together, for this reason it is called 'the one ruling', and the others are 'the ones controlled,' and it is five.[19]

A similar remark appears in Proclus' report of the views of Dercyllides (1st cent. B.C.E. or 1st cent. C.E.), a philosopher and the author of an 11–book work On Plato's Philosophy. He has special value for this enquiry because of his early date and because his work preserves interpretations from the Old Academy. In Tarrant's view, he "belonged to the same species of mathematicizing Platonist as Theon of Smyrna himself, although he was no doubt more original."[20] Proclus records Dercyllides' description of the 3–4–5 triangle, a triangle with "…the sides surrounding (scil. the right angle) having the first ratio in harmony, and the hypotenuse or 'the ruling one' both."[21]

But what can it mean if one says that the number of human engendering contains augmentations of hypotenuses and other sides? What augmentations of right triangles does the commentator think that Plato has in mind? Notice that Plato places δυνάμεναί τε καὶ δυναϲτευόμεναι in apposition to αὐξήϲειϲ. A reader trying to parse this construction could naturally infer, given the syntax, that the hypotenuses and the other sides are in some sense themselves augmentations, and this is how Proclus and, we believe, the commentator interpreted them. Plato does not imply that the nuptial number contains augmentations of hypotenuses and legs of right triangles. He instead implies that it contains hypotenuses and legs, and that these are themselves augmentations.

The most natural way to grasp Proclus' idea is to think of the various sides of the ultimate triangle Plato has in mind as resulting from successive augmentations of the sides of some original triangle. Ac-

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[17] Susemihl (2.370–78), Zeller (2.1.857n–60n), F. v. Ehrenfels, "Zur Deutung der Platonischen 'Hochzeitszahl'", *AGPh* 44 (11962) 240–44, Gaiser (64–69), and Blößner (78–84).

[18] Susemihl 2.374, Diès 5–6, Ehrhardt 414–15; Alex.Aphr. *in Metaph*. 75.18–76.1 (cf. Iamb. *VP* 130–31). GH 189–90 connect the annotations in *P. Oxy.* XV 1808 to the passage from Alexander.

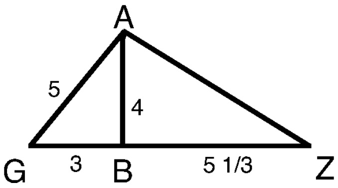
[19] Adam (2.269) remarks on Alexander's words, "Our sympathies being with the hypotenuse, because the odds are against him, we call him 'conqueror,' even though the battle is a drawn one."

[20] H. Tarrant, *Thrasyllan Platonism* (Ithaca 1993) 80–81, and see in general Tarrant 11–13, 72–76 and *DNP* 3.483. Dercyllides' date is the subject of dispute.

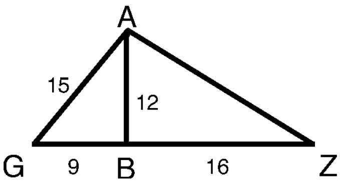
[21] Procl. *in R*. II 25.16–18. In τῆϲ δ' ὑποτεινούϲηϲ ἢ δυναμένηϲ ἀμφοῖν, Proclus' uses the common term ὑποτεινούϲη as an alternative to Pythagorean δυναμένη; see Note 4 and cf. Plato, *Rep.* 546 (quoted at the beginning of this article).

cording to Proclus, the initial triangle is a 3–4–5 triangle (in R. II 40.3). The hypotenuses and other sides generated from this triangle may be called 'augmentations,' in the same sense that we call a facial mole a 'growth.'

Proclus next supplies a series of diagrams that show the sides of right triangles as genuine augmentations (*in R.* II 40.1–24) (Hultsch 407–08). His source, if Hultsch is correct, is Nestorius, also mentioned at *in R.* 64.5–8,[22] where Proclus identifies him as the grandfather of Plutarch, "our guide and our teachers' guide" and credits him with bringing "to light…the secret account about the right triangle." Proclus' diagrams illustrate an interpretation that fits nicely not only with Notes 4 and 5 but also helps us make sense of Notes 6 and 11. Proclus sets out the following procedure:

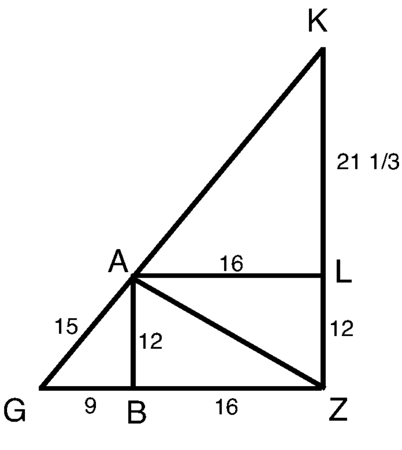


Assume a triangle ABG. AB = 4, BG = 3, AG = 5.
Let a line AZ be drawn to the upright line AG, to which the line BG should be extended.
In the right triangle AZG the segment at the right, AB, will be the altitude.
AB is then the proportional mean between ZB and BG.
Now, the ratio AB: BG, is 4 to 3. It follows tha ratio 4:3 with BA.
Then BZ = 5 1/3.



He continues:
Let us multiply these numbers by 3 to make the fraction disappear.
ZB will then be 16, BA 12, BG 9, and AG 15:

The multiplication by 3 may seem arbitrary, but it is in accordance with a remark Plato makes in book 7 of the *Republic*. Clever calculators—those, that is, who study calculation "for the sake of knowledge rather than selling merchandise"—will not permit fractions. Socrates tells us, "If, in conversation, someone tries to cut up the one, they laugh at him and do not allow it. Rather, if you fragment it, they multiply, taking care lest the one ever seem not to be one but many parts" (*Resp.* 525d–e).[23] When Proclus multiplies by 3, he eliminates fractions. He moves to the next step:



Let us also extend a line ZK perpendicular (to ZG from Z) until it meets K at the extension of GA.
Let us also extend AL parallel to ZG.
Since again, then, ZAK is a right triangle and AL is perpendicular, then AL is the mean proportional between ZL and LK.
So, since ZL measures 12 and AL 16, the line parallel to AL, (BZ), in fact measures 16.
KL is, in turn, in the ratio 4:3 with AL, in the sense that the line KL = 21 1/3, if AL = 16.
He then applies Plato's rule for clever calculators again:
Let us multiply again all these values by 3 because of the fraction 1/3.
Now KL will = 64, LA = 48, LZ = 36, and finally BG = 27.

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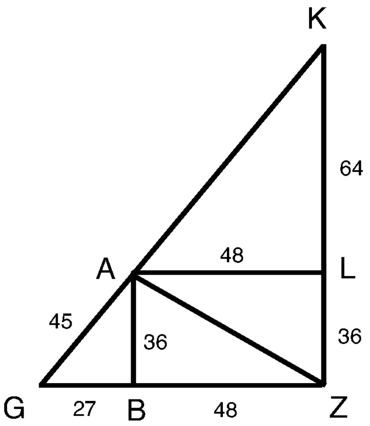
[22] Hultsch 418; a person of the same name also appears at Procl. *in R.* II 324.13 and 325.4, but Kroll thinks he may be different; the latter Nestorius is cited for mystical rather than mathematical talents.

[23] J. Klein, *Greek Mathematical Thought and the Origin of Algebra,* transl. Eva Brann (Cambridge, Mass. 1968) ch. 5 discusses this principle and its consequences.

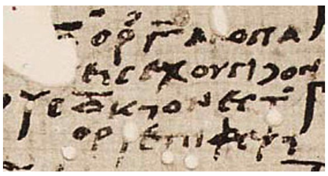
So the final triangle that Proclus has in mind is this:



The constructed triangle KGZ fits in nicely with the rest of Proclus' interpretation. Previously in his discussion, he had demonstrated how to find Plato's two harmonies through arithmetic (in *R.* II 37.4– 20). That procaess involves, as we shall show in our discussion of Note 6, manipulations of the 4:3 ratio to produce the sequence consisting of 27.36, 48, 64. From these Proclus derives the two harmonies he seeks, 10,000 and 7,500. As he concludes his geometrical demonstration (in *R.* II 40.21–24), he links the arithmetical and the geometrical expositions together (cf. Hultsch 408): he observes that the lengths of KL, LA, LZ, and BG are 64, 48, 36, and 27, respectively, and he concludes, "Now these four numbers are precisely the ones which gave rise to the numbers 100 and 75 which form the harmonies10,000 and 7,500."

Let us return to the earlier question: what does the commentator think that Plato means when he speaks of "augmentations of hypotenuses and other sides"? According to the reading suggested by Proclus, he has at least two kinds of augmentation in mind. The first is the construction of larger and larger similar right triangles by appending right triangles to right triangles. The second is the multiplication through by three in order to eliminate fractions.[24] The result is the triangle KGZ.

Note 6: i.13–16 treating τρεῖϲ ἀποϲτάϲειϲ τέτταραϲ δὲ ὅρουϲ λαβοῦϲαι



i.13	δ̄ • ορ° γ̄ αποϲτα	δ̄ ὅρο(ι) γ̄ ἀποϲτά–
	ϲειϲ εχουϲι ↄ ο^ν	ϲειϲ ἔχουϲι· (ἔϲτι) ὄν(τωϲ?)
	δ κ¬ονεϲ τ°	δ κ(ί)ονεϲ τὸ
	ορ|επιφερ⅂	ὅρι(ον) ?ἐπιφέρ(οντεϲ)

Palaeographical Notes i.13 δ • GH : δ̣̄ Haslam Suprascript dot : a similar dot appears in Note 9 i.14 ↄ and i.15 ¬ are standard tachygraphic signs

"Four terms have three intervals. There are actually(?) 4 columns producing the boundary."

Papyrological Commentary

i.13. The dot following the initial delta and the dot in Note 9 are likely to be stray marks. GH suggest the one here is a reference sign corresponding to a similar mark in the lost text.

i.14 (ἔϲτι): Giovanna Menci identifies the symbol ↄ as a quickly formed ⸗, the shorthand sign for εϲτι (Milne no. 72). The singular form of the verb is sometimes used, as here, with plurals (Schwyzer–Debrunner, *Griech. Gramm.*, Hb. d. Altertumsk. II.2.ii 608; *LSJ* s.v. εἰμί A v).

ὄν(τωϲ): ο(ὖ)ν is possible but less likely, as it would be unique. Its usual abbreviation is ο΄.

i.15 κ(ί)ονεϲ: with itacistic ¬ (ει) for ι.

i.16 ὅρι(ον): thus Haslam: ὅρι[ο](ν), i.e., ορι[ο] GH. A tiny lacuna at the top of elongated iota seems too small and wrongly spaced to accommodate omicron. Also, scribes often indicate an abbreviation by elongating the last letter retained (McNamee 119).

?ἐπιφέρ(οντεϲ): tachygraphic ⅂ for εϲ. The scribe provides the inflectional ending to make the syntax clear but suppresses the penultimate syllable, as often happens in shorthand (Milne 6).

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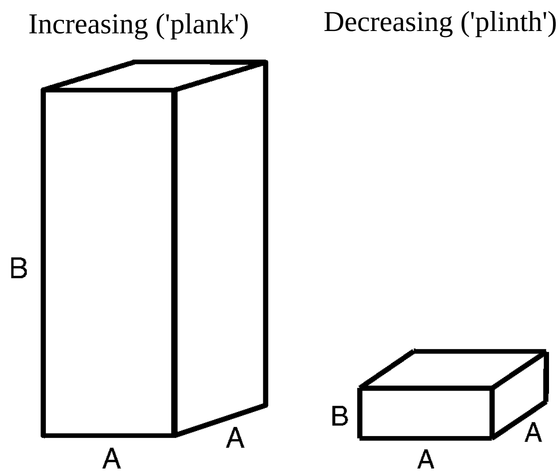
[24] Proclus also reports an alternate geometric procedure, which he attributes to one Paterius (*in R.* II 40.24–41.10). This involves *reducing* the original triangle through subdivision, rather than augmenting it. In the final stage of this calculation, however, however, multiplication is also used in order to eliminate fractions. Proclus prefers the first method (*in R.* II 40.25–27), and so should we, given Plato's explicit stipulation of triple augmentations, not reductions. Nothing significant hangs on the difference between the constructions; each method ends up with the same triangle.

Here our interpretation is fairly conjectural. To explain the note we continue to lean on Proclus but without defending him. At *Resp*. 546b the muses speak of "taking three intervals but four terms of likenings and unlikenings, and of increasings and diminishings." The beginning of the annotator's statement simply metaphrases the first part: "four terms have three intervals" Then he offers an enigmatic clarification: "There are actually(?) 4 columns producing the boundary." Three words in this note, 'terms', 'columns', and 'boundary,' need explication.

ὅροι: 'terms'. The word appears here in its mathematical sense to mean an element in a mathematical proportion (Arist. *EN* 1131b5; Euc. 5 Def. 8), not with the concrete meaning 'boundary stone'. According to Proclus, the first of Plato's four terms is the number 27. When this is augmented three times by four thirds, the resulting sequence is 27, 36, 48, 64 (*in R.* 37.12–13). These four numbers have three intervals among them, since (as Proclus observes in language very close to that of the annotator), "for any four successive terms there are three intervals" (πάντων γὰρ τεττάρων ὅρων ϲυνεχῶν τρεῖϲ εἰϲιν ἀποϲτάϲειϲ, *in R*. II 36.22–23).

κίονεϲ: 'columns'. the second sentence of the note reads, "There are actually(?) four columns producing the boundary." We cannot confidently interpret what 'column' means here. What follows is our best attempt.

Recall, first, that the muses stipulate that the augmentations contained in the nuptial number should produce results that are likening, unlikening, increasing, or decreasing. The ancient mathematicians Nicomachus and Theon and also Proclus agree on the arithmetical meaning of these words.[25] A 'likening' number is a square or cube, while an unlikening number is a number of two or three factors that is neither a square nor a cube. Unlikening numbers include increasing and decreasing numbers. Increasing numbers are those expressible as a^2b, where a and b are natural numbers and b is greater than a. Decreasing numbers can be expressed by a^2b, where a and b are natural numbers, and b is less than a.[26] A decreasing number, furthermore, as Proclus informs us, is a πλινθίϲ, 'plinth', and an increasing number is a δοκίϲ, 'plank'. Geometrically, this can be expressed:



We can apply this terminology to the four terms 27, 36, 48, and 64 as follows: 27 and 64 are cubes, which are 'likening' numbers. 36 (3•3•4) and 48 (3•4•4) are 'unlikening' numbers. 36 additionally is an 'increasing' number, since 3•3•4 can be expressed as a^2b, where b is greater than a. Correspondingly, 48 is a 'decreasing' number, since 3•4•4 is expressible as a^2b where a is greater than b (Procl. *in R*. II 37.10–11). 36 thus is a 'plank' and 48 a 'plinth'.

Where do the annotator's κίονες , 'columns', fit into the picture? Theon's and Proclus' use of architectural diction like 'plinth' and 'plank' for unlikening numbers suggests that in κίονες we are dealing with the same kind of metaphor. Our conjecture is that the commentator uses 'column' to specify the whole genus of arithmetical expressions that comprises 'plinths', 'planks', and, indeed, cubes. κίων, that is, comprehends *all* the species of number mentioned by the muses: likening, unlikening, increasing, and decreasing. If so, then algebraically it is the name for numbers expressible by a^2b, where a can be less than or greater than b, or equal to it. Metaphorically, it is the word for a building element that can be constructed from combinations of cubes, plinths, or planks. The legs of the large triangle discussed in

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[25] Nicom. 2.17.16, Theo Sm. 41, Procl. *in R*. II 36.13–21.

[26] Theon and Proclus do not express themselves algebraically, of course. Cf. A.J. Festugière, *Proclus, commentaire sur la République* (Paris 1970) 143–44 n. 2.

our commentary on notes 4 and 5 turn out to be constructed out of 'columns' in this sense: one composed of the cube 27 and the plank 48, the other of the cube 64 and the plinth 36 (cf. Procl. *in R.* II 40.21–24).

The difficulty with this suggestion is that mathematical writers never use κίων in this sense. Our interpretation can therefore only be probable at best, and we should canvass alternative meanings for the word. One appears in Hero of Alexandria (*fl.* 62 C.E.). He includes κίων in a list of ten kinds of solid and describes it as a column, round or square, which tapers and has a flat top and a flat base (Hero *Stereom.* 1.21 and *passim*). Here is at least an actual geometrical application of the term. Unfortunately it appears to be utterly unhelpful. Because of the tapering of Hero's columns, several parameters determine the shape he envisions. Plato, however, supplies too little information for us to be able to understand his text as describing the construction of four such columns as Hero describes. We set this definition aside.

An astrological reading of κίων seems at first a more attractive alternative than Hero's stereometrical one. Three things favor it. First is the fact that both κίων and δοκίς are well attested as metaphors for types of comet or meteor.[27] Second, and more importantly, the annotator uses κίονεϲ in association with ὅριον, a word sometimes used for one of five subdivisions of a zodiacal sign. ὅριον, moreover, shares both root and meanings with ὅροϲ, which also can designate a 'region' in the sky. Third, Plato uses κίων as a cosmological term in the myth of Er, where he applies it to a column of light (*Resp.* 616b–617b).[28]

Suppose that the κίονεϲ of note 6 are the same sort of thing as the κίων of light in the myth of Er. The proper interpretation of the note, then, would depend on how Plato means for us to understand the column of light. There is an old controversy over the orientation of the column Er sees. On one interpretation, it should be understood as a horizontal band stretching across the sky. Conceivably this is the image the Oxyrhychus commentator has in mind in his reference to 'four columns', since four columns of light, intersecting, could mark out a region of the sky with four sides, each column making up a side. According to the alternate interpretation, Er's column is vertical.[29] If this is what the commentator has in mind, then four such columns might determine an astrological 'region' (ὅριον) by stretching up from the earth to the sphere of the fixed stars, each column determining a corner of the region. An initial problem with either the horizontal or the vertical interpretation, however, is that Er reports only a single column, while the annotator's columns are four. If the annotator's source is talking about cosmological columns like the one in the myth of Er, therefore, clearly at least three of them were not visible to Er. More importantly, we have seen that Proclus uses a phrase very similar to the one that begins note 6. When Proclus says that "for any four successive ὅροι there are three intervals," he means that there are three intervals between any four numerical terms. If we adopt an astrological reading of note 6, however, ὅροι must mean something radically different, since the annotator glosses it with the word κίονεϲ. On such an interpretation, it may mean either 'boundary stone' or 'boundary', but it cannot mean 'term'. Given the close similarity between the marginalia and Proclus' commentary here and elsewhere, however, such a radical divergence seems unlikely. We thus reject the astrological reading of the note.

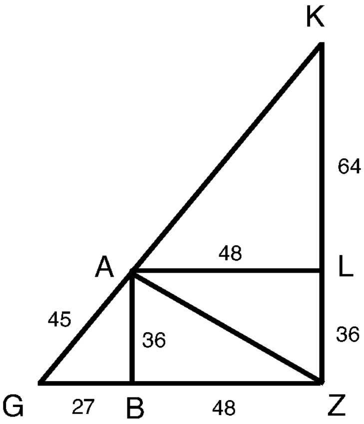
ὅριον : 'boundary'. Four columns, the annotator says, make up a ὅριον, 'boundary.' (LSJ) allows a range of meanings. In the present annotation, its principal meaning, which is the same as the principal

[27] πλινθίς also seems to be a division of the heavens at *Corp.Herm.* 13: ὑπὸ τούτῳ δὲ ἐτάγη ὁ τῶν δαιμόνων χορός, μᾶλλον δὲ χοροί· πολλοὶ γὰρ οὗτοι καὶ ποικίλοι, ὑπὸ τὰς τῶν ἀστέρων πλινθίδας τεταγμένοι (the same statement appears in the *Fragmentum astrologicum* attributed to Hermippus).

[28] Giovanna Menci drew our attention to this fact.

[29] For a judicious, disinterested discussion, see D. R. Dicks, *Early Greek Astronomy to Aristotle* (London 1970) 109–11. John S. Morrison, "Parmenides and Er", *JHS* 75 (1955) 59–68 defends (§4) the first interpretation that the light stretches across the sky. Adam 2.445–47 defends a mixed view, according to which "the light not only passes through the centre of the Universe, but also, since it holds the heavens together . . . round the outer surface of the heavenly sphere. Dicks (236 n.151) observes that the controversy over the orientation of Plato's column extends back to Proclus (*in R.* II 130.4) and Theon (143).

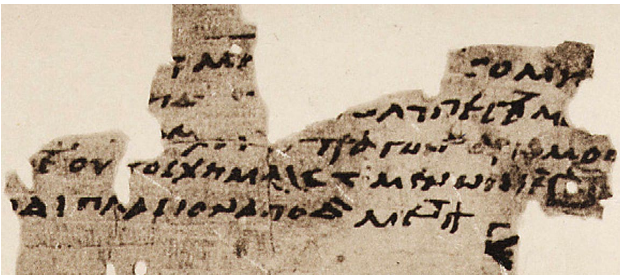
meaning of ὅροϲ, is what we use here, 'boundary', the divergent forms ὅριον and ὅροι ὅριον, aderivative of οροϲ, must signify different things, since they appear so close together in so short a span of text.[30] Although they share a root, that is, they have distinctly different meanings. ὅροι, 'terms', refers to the terms that Proclus derives both arithmetically and geometrically: 27, 36, 48, and 64. ὅριον means 'boundary.' But what is the 'boundary' to which the commentator refers?



We take it to be the largest of the triangles constructed earlier, the one labeled KGZ. It is a boundary because it delimits a side (two sides, strictly speaking) of a certain 3–dimensional wedge. Proclus, in his commentary on Euclid's definition of 'boundary' (called ὅροϲ by Euclid 1 Def. 13), emphasizes the legitimacy of calling the *surface* of a solid a 'boundary'— just as one refers to the perimeter of a plane figure as its boundary (*in Euc*. 136). The Oxyrhynchus annotator's four terms, then, which we take to be the same four numbers recognized by Proclus, produce a 'boundary' of the former sort. For they establish the length of two sides of a right triangle and thus define the area of any solid built upon that surface.

Here segments of length 27 and 48 and of 36 and 64 make up, respectively, two sides and thus determine a triangle. Thus four terms (ὅροι) produce this triangle, which turns out to be the boundary (ὅριον) of a three–dimensional figure

Note 7: ii.1–5 treating τὴν δὲ ἰϲομήκη μὲν τῇ, προμήκη δέ, ἑκατὸν μέν ἀριθμῶν ἀπὸ διαμέτρων ῥητῶν πεμπάδοϲ, δεομένων ἑνὸϲ ἑκάϲτων, ἀρρήτων δὲ δυοῖν



ii.1]ϲ̣ι̣ μ̅η̅ [. . . . . .]ι̣ϲομη̣^κ [
]η̣ δε (*vac.*) [. .] ̣ω λ⌐πεϲθ̅ μο̣[
] . . [.] . [.] . εχε̣ι̣ τετραγων° αριθμοϲ [
ii.4] . ε̅ ου τ̣ο ϲχημα \ ϲ⌐ μενωνι ε̅[ε̣] . [
]ο διπλαϲιον απο δ` με^τ ⊦ ⊏ [

ii.1]ϲ̣ι̣ μ̅η̅ [. . . . . .] ἰ̣ϲομήκ̣(η) [
?προμήκ]η δέ· . [. .] τῷ̣ λ(εί)πεϲθ(αι) μο̣[νάδι
] . . [.] . [.] . ἔχε̣ι̣ τετράγωνο(ϲ) ἀριθμὸϲ [ἄρρητ(ον) δ(ιά)μετ(ρον), [ὡϲ ἐν
τ]ῷ̣ ἐ(φ)' οὗ τ̣ὸ ϲχῆμα (καὶ) (ἐν) (τῷ) Μένωνι ε̅[ε̣] κ̣[ατασκευάζεται·
ii.4 τ]ὸ διπλάϲιον ἀπὸ δ(ια)μέτ(ρου) γί(νεται) ⊏ [ν̅·

Palaeographical notes ii.1]ϲ̣ι̣ KM :]ι̣ (perhaps ῑ or else]ϲ̣ι̣) Haslam :]̣ι GH a lacuna of perhaps 7 letters, Haslam]ι̣ϲομη̣^κ Haslam:]ι̣ϲομη[κ GH ii.2 ?προμήκ]η̣ GH, Haslam δε̣ (*vac.*) . [KM, δε̣ being followed by unwritten space one letter wide ("certainly no ink visible," Coles) : δε̣[.] . [GH : δε . . [Haslam [. .] . ω Coles, who notes the trace could be the base of τ : [. . τ]ῷ̣ (?) GH : [. . τ]ῷ̣ (?) Haslam ?μο̣[νάδι GH, Haslam ii.3 before εχε̣ι̣, a trace compatible with ρ τ̣ε̣τράγωνο(ϲ) Haslam ἀριθμόϲ KM, Haslam: ἀριθμὸϲ̣ GH ii.4]ῷ̣ (or ⌣) KM :] . Coles :] . GH, Haslam: ε̅ KM : ε′ Coles : ε̄ GH, Haslam \ tachygraphic (καὶ) KM : (ὡϲ)? GH : (ἐϲτιν) Haslam ϲ, ⌐ ,tachygraphic (ἐν) (τῷ) GH : ἐ̣(ν)? τ(ῷ̣) Haslam ε̅[ε̣] . [KM, Coles reports that the topmost mark, if a numeral labelling the figure, lacks a stroke above it; nor is there a stroke above the figure inside the box, unless it is not ε; at the right of the figure, a vertical stroke, with apparently a diagonal rising from its lower extremity, which would suit κ (according to Coles the traces are hardly discernible as ε or ε̄); e.g., κ̣[ατασκευάζεται, KM (cf. Hero [1st cent. C.E.] *Spir*. 1.7, 18, etc.; Eucl. 3.3, 14

30 Their proximity baffled Grenfell and Hunt, who complained, "The introduction of κίονεϲ, as a synonym apparently of ὅροι, is hardly helpful."

etc. *et al.*) : ε ε ε ε GH : ε ε ε Haslam, who notes, however, writing (the square's diagonal?) inside the figure , i.e., MLJ; the diagonals are written with thick strokes ("very crude for a ◇", Coles). The angle formed by the vertical and the upper diagonal is slightly greater that 45 degrees : GH : Haslam KM (a horizontal stroke of ink above the lacuna following the second diagram) : ε (i.e., the measurement of the bottom side of the figure in the line above) GH, Haslam

" … 48 … equal in length … but oblong(?) (vac.) […] by subtracting by 1… a square number has [an irrational diagonal, as in the one (i.e., the square)] upon which the figure also in the Meno, ε ε , *[is* constructed?]…. The double (of the square, i.e.) upon the diagonal[31] is [50]*:* ."

Papyrological Commentary

There is no way to know how many letters are lost at the beginning and the end of each line of the note. It occupies, like many long marginalia, the upper margin. In such notes the scribe's need for writing space determines line length, not the constraints of the intercolumnar space. Restoration is therefore more provisional than usual.

ii.1–2: Three to four millimeters of unwritten papyrus following ?προμήκ]ῃ δὲ in line 2 of the annotation. There are too many gaps in the text to be confident about the sentence structure of what remains, but it seems clear that the unwritten space does not mark a dramatic change of topic. The remains of the note deal entirely with the number 48 and its place in the calculation. We note that the discussion in the note seems to make a fresh start after the gap in writing in line 2. This suggests that ἰ̣ϲομήκ̣(η) and προμήκ]ῃ δὲ̣ may be a lemma introducing the comments in the second part of line 2 and the rest of the note. Plato, however, writes ἰ̣ϲομήκ̣(η) προμήκ]ῃ δὲ̣ consecutively, whereas they are not certainly consecutive here. 48, moreover, which is the result calculated in the process discussed in lines 2–5, actually precedes the supposed lemma. We are therefore inclined not to regard the blank space in line 2 as punctuation separating lemma and discussion.

ii.2 τ̣" λ(εί)πεϲθ(αι) μο̣[νάδι: cf. μονάδι λειπόμενοι μία, Procl. *in R.* II 38.20–21, τὸ μονάδι ἔλαϲϲον...μονάδι ὂν ἔλαϲϲον *in R.* II 38.20–21.

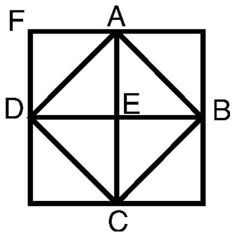
ii.3 ἔχε̣ι̣ τετράγωνο(ϲ) ἀριθμὸϲ [ἄρρητ(ον) δι(ά)μετ(ρον): cf. Procl. *In R.* II 38.12–14, ἐὰν γὰρ πλευρὰν τὸν ε̄ λαβῇϲ τετραγώνου τινόϲ, καὶ ἐν τούτῳ διάμετρον ἄρρητον πάντωϲ διὰ τὸ ῥητὴν εἶναι τὴν πλευράν ‒‒‒. A theoretical alternative, οὐκ (or οὐ γὰρ) ἔχε̣ι̣ τετράγωνο(ϲ) ἀριθμὸϲ ῥητὴν διάμετρον, A square number does not have a rational diagonal" is less attractive. The trace of ink before ¶xe`i` could belong to r, but the rest of the remains do not fit.

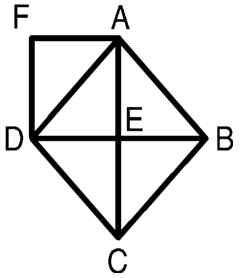
ii.4 ἐν τ]ῷ̣ ἐ(φ)' οὗ τ̣ὸ ϲχῆμα (καὶ) (ἐν) (τῷ) Μένωνι ε ε : the likely antecedent of οὗ is a form of τετράγωνοϲ (L. Koenen). The trace of ink immediately preceding the epsilon of ἐ(φ)' suits the upper tip of the right–hand vertical stroke of omega. Supplements in this and the preceding line are *exempli gratia* and, if correct, may have been distributed differently.

The diagram is difficult to decipher. What is clear is that the numeral epsilon is written at the left, beside a vertical line. The rest is not so clear. Some distance to the right is another, parallel vertical line. Connecting the ends of the two verticals so as to form a rectangular object is a pair of horizontal lines parallel to each other. Where the lines meet at the top right and the bottom left there are right angles. The scribe probably intended the same for the other corners, but where those lines meet, short sloping lines connect the horizontals with the verticals. These presumably (Coles: "surely") reflect the cut of the scribe's pen. Above the figure one might expect a measurement of length, but Coles finds no trace of the characteristic suprascript stroke for indicating numerals. The traces conceivably represent a cursive epsilon with the lower stroke slightly invading the interior of the figure below. Within the box is additional writing, also hard to make out. The strokes bear a resemblance to those above the figure and might also represent ε, but stains in this part of the papyrus complicate the picture. Considerable ink (and staining) remains at the right of the box but is indecipherable. Below the box are traces of a horizontal line. This could either belong to the diagram or be the horizontal stroke designating a numeral in the text in the next line (see below). The smoothest reading of diagram is just that it is a badly executed square, and that is what we have in effect adopted in our reconstruction of the missing text, though without much confidence.

ii.5 τ]ὸ διπλάϲιον ἀπὸ δ(ια)μέτ(ρου) γί(νεται) [ν̄: Cf. ἐὰν γὰρ πλευρὰν τὸν ε̄ λαβῇϲ τετραγώνου τινόϲ, καὶ ἐν τούτῳ διάμετρον ἄρρητον πάντωϲ διὰ τὸ ῥητὴν εἶναι τὴν πλευράν, ἔϲται τὸ μὲν ἀπ' αὐτοῦ κ̄ε, διπλάϲιον δὲ τούτου τὸ τῶν ν̄ χωρίον, οὗ ἦν ἄρρητοϲ ἡ πλευρά. οὐ γὰρ ἔϲτιν ἀριθμὸν λαβεῖν τετράγωνον τετραγώνου διπλάϲιον, Procl. *in R.* II 38.12–17.

[31] Following the advice of Fowler (63–64), we translate διάμετροϲ as 'diagonal' in contexts involving squares.





The second diagram seems to be an attempt to draw the figure Socrates generates in the *Meno* to illustrate the problem of the doubling of the square (*Men.* 84c–85b). Since triangle ADE is half the area of square AEDF and one quarter the area of ABCD, the area of ABCD must be twice the area of AEDF.

In the Greek note the portion of the diagram drawn in solid lines at the top left of the figure is clearly visible in the papyrus. (In the diagram below, this part is represented by lines FD and parts of FA, DA, DE, and DC.) At the bottom left corner, the papyrus that remains may have undergone some kind of damage by abrasion or the loss of its upper layer. Coles reports it is "just possible there has been thin surface loss—the papyrus is stained, as if from ink once there maybe, but such staining occurs elsewhere in blank areas." We think this is what happened. Though it is unusual for ink to come off in this way, the remaining stains are just where they need to be to fit the discussion in the *Meno.*

The alternative is that the annotator's drawing represents only a portion of the figure Socrates describes. Even a partial diagram likethe one on the left would suffice to demonstrate that a square constructed on the diagonal of a square is twice the size of the initial square, since, of course, triangle ADE is still half of square AEDF and one quarter of square ABCD. Such a partial figure would not, however, replicate the one that Socrates draws in *Meno* 84c–85b.

ν̄: we tentatively restore this in the lacuna at the end of the last line of the note. Only the horizontal bar is visible, and previous editors have assumed it belonged to the first diagram (line 4), that it was part of a label for a numeral (presumably ε̄), and that it indicated the length of the lower side of a square. If the figure in line 4 is simply a square, or even if it is a rectangle, it seems to be overlabeled already. This suggests that we should find another explanation for the stroke written below it. If our reconstruction of the second diagram in light of *Meno* 84d–85b is correct, then a little more than half of the figure lies in the lacuna to its right. If the horizontal bar that remains at the right of the lacuna is a numerical label, then the numeral was written immediately to the right of the second figure and was alined with the rest of the text in line 5. The similarities between note 7 and Proclus *In R.* 38.8–15 suggest ν̄, 50.

Notes 7 through 10 treat the Platonic passage that appears in our heading for Note 7: "the other being equal in length to it, but oblong—first: of a hundred of the numbers of rational diagonals of the pentad, each one lacking one, but from irrational, two.

ii.1 "… 48 …": according to Proclus (*in R.* II 38.12–24) and to almost all scholars since the rediscovery of his commentary on this passage, the rational diagonal of 5 is 49, and the irrational diagonal is 50.[32] Subtracting one from the rational diagonal thus yields 48. Subtracting 2 from the irrational diagonal gives 48 as well.

"Equal in length … but oblong (?)": the annotator quotes Plato's text. On the significance which the commentator perceives in ἰϲομήκη see the discussion, below, of Note 11.

ii.2 "By subtracting by one": there are three possible functions the commentator's words could serve. First, they may simply restate Plato's instruction to subtract one from the rational diagonal of 5. If so, however, the annotator repeats himself in Note 9. Second, they may belong to a general discussion of the Pythagorean algorithm for estimating diagonals, since the square of the estimated measurement is either one less or one greater than the square of the genuine diagonal.**[33]** Any explanation of this algo-

[32] According to Vogt, since the publication of Proclus' commentary in 1901 "kann es wohl nicht mehr zweifelhaft sein" that this interpretation is right. Even so, Ehrhardt (409) and Robert S. Brumbaugh, *Plato's Mathematical Imagination* (Bloomington 1954) 125–27, 134–35 choose to go their own ways.

[33] From the point of view of the history of mathematics (as opposed to the history of numerology), this algorithm is the most important thing in *Resp.* 546. Our knowledge of it comes from Theon and Proclus (Theo Sm. ed. Hiller 42.10–44.17, Procl. *in R.* II 24.16–25). The algorithm begins by roughly approximating the side and the diagonal as each equal to one. In each step after that, one doubles the length of the side and adds it to the diagonal and also adds the length of the diagonal to the side. This produces a second estimate, then, of a side of 2 and a diagonal of 3. A third step (adding 3 to the side, and twice 2 to the diagonal) produces a side of 5 and a diagonal of 7, and this is followed by 12 and 17, 29 and 41, and so on. The accuracy of these estimations is obvious if one squares them and compares the results with what we know, by the theorem in the *Meno*, to be the actual areas of the squares built on the genuine diagonals. that is, for a square with a side of 5, the square of the estimated diagonal is 7^2 or 49, while the area of the square built on the actual diagonal is 5^2 doubled, or 50. For a square with a side of 12, the square of the estimated diagonal is 289, while the square built on the actual diagonal has an

rithm would be fairly lengthy, however. The rest of the annotation seems to follow a different track. Third, the annotator may be noting simply that 50 minus 1 is 49. This is what Proclus does at *in R.* II 38.19–20, where he reports just the bare facts that 50 minus 1 is 49, and that 49 is Plato's rational diagonal of 5. Given the frequent correspondence between Proclus' commentary and the annotations, we think this the likeliest possibility.

ii.3–5 "A square number has [an irrational diagonal, as in the one (i.e., the square)] upon which the figure also in the Meno,  , [is constructed?]…. The double (of the square, i.e.) upon the diagonal[34] is [50]: ": From here onward, note 7 is like a sentence in Proclus' commentary on the *Republic*: For if you take 5 as the side of a square (and in it the diagonal is always irrational since the side is rational) the area from it will 25, and the double of this will be an area of 50, for which the root would be irrational (*in R.* II 38.14–15). Note 7 differs from Proclus' text in that it contains diagrams and a reference to the *Meno.* Since the second diagram and the *Meno* reference would be appropriate in explaining Proclus' thought, they confirm the hypothesis that the annotator's source expressed the same thought. In fact, Proclus expresses this thought somewhat elliptically, and the annotator's reference to the *Meno's* theorem about doubling a square helps unpack his meaning.

The unpacked thought explains why 50 is the irrational diagonal of the pentad. Fifty is called a diagonal because it is a kind of measure of the diagonal of a square with a side of 5, and it is called irrational because that diagonal is incommensurable in length with the side of 5.[35]

How is 50 a measurement of the diagonal of a square with side 5? The Oxyrhynchus annotator addresses this question by referring to the *Meno.* His second drawing reproduces at least part of the diagram that Plato uses to prove a theorem at *Men*. 82b–85b. Socrates proves there (or, rather, has a slave boy recollect) that a square constructed on the diagonal of another square will have twice the area of the initial square. For example, if the area of a square is 25, the area of a second square drawn on that diagonal will be twice 25, or 50.

The areas of a square and a square built on its diagonal stand in a simple 2 to 1 ratio to one another. This fact stands in stark contrast to the incommensurability in length of the side of a square and its diagonal. By the time of Democritus, mathematicians had proven that the side of a square and the diagonal of that square are incommensurable in length,[36] and the incommensurability of those lengths threatened the Pythagorean project of understanding the world as constructed out of numbers.[37] Thinkers sympathetic to this project softened this blow by appealing to the simple 2:1 ratio demonstrated in the *Meno*, thus directing attention to the commensurability of the corresponding *areas* of two such squares as Socrates discusses. Euclid thus defines line segments as 'commensurable in square' when the squares built on them are commensurable with each other (Euc. 10 Def. 2). According to Proclus, Plato and the Pythagoreans were willing to say the same thing (*in R.* II 27.11–28.9; see Knorr, 15, 58 n. 71). In Euclidean terms, then, a side of length 5 is 'commensurable in square' to the diagonal of its square (a square of length √50, we would say), since the area of a square drawn on that diagonal has exactly twice the area of the square of the original square of 5. For our purposes, the important point is that this way of thinking about commensurability only makes sense if one speaks of the area of the square built on the

area of 288. For explanations and discussions, see Thomas 132–38, with notes; Heath *GM* I 307–08, Heath *Euc.* I 398–401, Knorr 32–36, and Fowler 58–60, 100–04.

[34] Following the advice of Fowler (63–64), we translate διάμετρος as 'diagonal' in contexts involving squares.

[35] Two lengths can be said to be 'incommensurable' when no length exists, no matter how small, that goes into each of them a natural number of times without leaving a remainder.

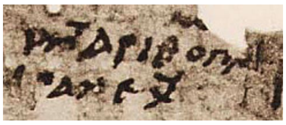
[36] This was about 430 B.C.: Knorr 38. Vogt (144–46) denies that Democritus' work entitled *On Irrational Lines and Solids* deals with the incommensurability of geometrical line segments, but Heath (*GM* 154–57, *Euc.* 1.413) convincingly rebuts his argument. In any case, Vogt allows that the Pythagoreans discover the incommensurability of the diagonal and the side of a square before 410 B.C. (155).

[37] See Knorr 42–49. For skeptical remarks, see Burkert 455–65.

diagonal as if it were the measure of the diagonal itself. Note 7 thus contains the remnants of some such explanation of how Plato can designate 50 as a 'diagonal' of five.[38]

ii.4 "[The square] upon which the figure also in the Meno[is constructed?]:" If the expression 'the figure also in the *Meno*' refers to the second diagram, then the text of the note does not guide us in interpreting the first diagram. Previous editors have taken it as a square with epsilons on each side. A square would be somewhat pointless, however, since nobody needs to be reminded what one looks like. Furthermore, if this is a square, it is badly executed and evidently has more labels than it needs. Nor would there be any explanation for the text at the right of the box, which does not look like a label at all. Unfortunately, we have no better suggestions to explain the traces.

Note 8 ii.6–7 treating ἀριθμῶν ἀπὸ διαμέτρων ῥητῶν



ii.6	ρη^τ αριθ^μ ο πλευ ραν εχ^ω	ῥητ(ὸϲ) ἀριθμ(ὸϲ) ὁ πλευ– ρὰν ἔχω(ν)

"The number having a root is a rational number."

Papyrological Commentary

ii.6: ῥητ(όϲ): thus GH, ῥητ(ῶν) Taylor *ap*. GH, Haslam. The nominative makes the statement grammatically coherent and takes into account that the annotator does not use lemmata.

According to Vogt (Vogt 143), "rational diagonal" and "irrational diagonal" in *Resp.* 546c are the oldest terminology pertaining to incommensurability that Plato preserves. By the 2nd century, this usage has become sufficiently obscure that the annotator needs to write a note to account for why the ordinary and inoffensive number 50 may be classified as an 'irrational' diagonal. Authors of intervening centuries give no help in understanding the terminology. For Euclid, a line segment is rational if it is commensurable with another given line segment, "whether in length and in square or in square only" (*Elements* 10 Def. 4).[39] In Hero, a magnitude is irrational if it is incommensurable with a given magnitude in length (*Definitiones* 136.34). The language employed by each of these authors indicates that neither one would consider the number 50 straightforwardly irrational.

As we have seen in the discussion of the previous note, Plato calls 50 an irrational diagonal because it measures a diagonal that is incommensurable in length with a side of length 5. Fifty is an indirect measurement of the diagonal. It directly measures the area of the square built on that diagonal. According to Proclus and the annotator (see Notes 7 and 9), 49 is the rational diagonal of 5. Forty–nine indirectly measures the rational approximation of the diagonal of a square with a side of length 5. Forty–nine directly measures the area of the square built on that approximation. Forty–nine and 50 are thus not called rational or irrational depending on whether they directly measure areas commensurable with a unit of area. Rather, they are called rational or irrational depending on whether they indirectly measure lengths commensurable with a unit of length. Hence the annotator's explanation of Plato's terminology: a number is called rational if it has roots, that is, roots that are commensurable in length with a unit length.

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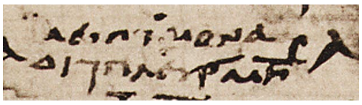
[38]Philo Judaeas tells us that the ancient Jews, following "the very holy instructions of the prophet Moses,… organized a preliminary festival, which the number fifty is assigned to—the holiest and most natural of numbers from the power of the right triangle, which is the beginning of the birth of all things" (*De vita contemplativa* 65). Philo may be injecting an element of Platonism into the Hebrew tradition.

[39] The translation is that of Heath *Euc.* Knorr has useful terminological discussions at 15–16, 20 n. 30.

Note 9 ii.8–9 treating δεομένων ἑνὸϲ ἑκάϲτου



ii.8	λειπἰ˥• μονα	λείπ(ει) μονά–
	δἰ˥ πλευρ´ μ̄η̄	δι (εἰ) πλευρ(ὰ)· μ̄η̄

Palaeographical note λειπἰ˥•, immediately following the tachygraphic symbol is a dot of ink high in the line; probably a stray mark (cf. Note 6)

"It is less by 1 if it has a root: 48".

The annotator paraphrases Plato's instruction to subtract 1 from the rational diagonal of 5 in light of the terminological explanation in Note 8. Since 49 is the rational diagonal of 5, subtracting 1 leaves 48.

Note 10 ii.10–11 treating ἀρρήτων δὲ δυοῖν

ii.10	αρρη^τ ϲ ν̄ ου ∠	ἄρρητ(οϲ) ϲ(ύμπαϲ) ν̄, οὗ (ἡμίϲει)
	ειϲι π^λ	εἰϲὶ πλ(ευραί)

"The total 50 for the half of which roots exist is irrational".

Papyrological Commentary

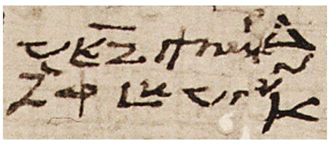
ii.10: ϲ, i.e., ϲ(ύμπαν), "the total" KM (cf. below, Note on *Resp.* 546c) σύμπαϲ δὲ οὗτος ἀριθμὸϲ γεωμετρικός. In *P.Oxy.* XV 1808, GH and Haslam see the sigma as an unattested shorthand sign, perhaps representing (δέ).

∠: the standard symbol for forms of ἥμισυς, for the identification of which we thank T. Gagos and Th. Christiansen: οὐ(κ) GH, Haslam.

Continuing the thought of Note 9, the annotator is not showing how to calculate the irrational diagonal of the pentad. Rather, the note states that the result is 50, and proves that, by the definition of 'rational' in Note 8, 50 must be irrational. Proclus again fills in some details (in R. II 38.15–16):διπλάσιον δὲ τούτου τὸ τῶν ν̄ χώριον, οὗ ἦν ἄρρητος ἡ πλευρά. οὐ γὰρ ἔστιν ἀριθμὸν λαβεῖν τετράγωνον τετραγώνου διπλάσιον

The double of this [i.e., of the square of 5] will be an area of 50, of which the root would be irrational. For it is not possible to get as a square number the double of a square." Doubling a square number never yields a square number. Since 25 is a square number, 50 is not. Thus, 50 has no rational roots and, by the definition in note 8, is irrational.

Note 11. ii.12–13 treating ϲύμπαϲ δὲ οὗτοϲ ὁ ἀριθμὸϲ γεωμετρικὸϲ



ii.12	⌣ κ̄ζ ⊢ ημε^ρ	(τῷ) κ̄ζ γί(̄νονται) ἡμέρ(αι)
	Ζ̀ φ̄ ∟κ ⌣ γ^υν	Ζ̀ φ̄, (ἔτη) κ (τῷ) γυν(αικείῳ)

"By 27 the total becomes 7,500 days: 20 years for the female."

Papyrological Commentary

ii.13 (ἔτη) κ : thus KM (without stroke indicating the numeral is); ʹἀκ(ολούθως?) GH, Haslam. This must be the right interpretation of the sign and of the comment. Otherwise, it would be too great a coincidence both that the symbol is the standard sign for 'year' and that 7,500 divided by the number of days in a year is about 20. Assuming a 365–day year, 7,500 days makes 20 years and 200 days.[40]

(τῷ) γυν(αικείῳ): an adjective used as a neuter substantive; alternatively, *scil.* ἀριθμῷ, e.g.

[40] In the middle of the 5th century, Oinopides calculated that a year contains 365 22/59 days (Ael. *VH* 10.7, Censorinus 19.2, Otto Neugebauer, *A History of Ancient Mathematical Astronomy* [Berlin 1975] 2.619). Slightly later, Philolaos less accurately concluded that a year contains 364 1/2 days (Censorinus 18.8, Neugebauer *ibid*., Huffman 276–79). Philolaos may have slanted his numbers for the sake of bringing the numerologically interesting 729 (9 cubed and 27 squared) into his astronomical system (Neugebauer 619, Huffman 279). Plato may have followed him both in being interested in the number 729

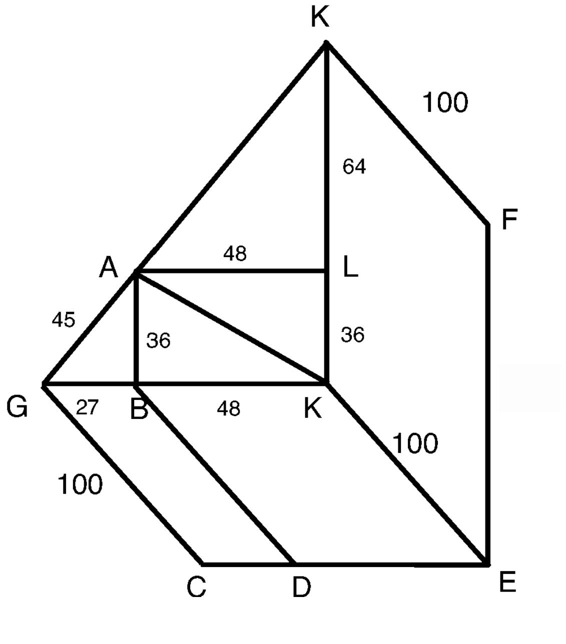
The commentator takes 7,500 to be one of the two harmonies. We shall first show how he reaches it through 27 and 48 and then explain the note's implication that 7,500 represents a length of time measured in days connected with a 'female' number. The fact that commentator uses the number 27 to arrive at 7,500 confirms that he and Proclus are part of a common tradition. Once again, therefore, we rely on Proclus in our explanations.

1. Calculating the Second Harmony

Plato describes "two harmonies, the one being equal an equal number of times, one hundred so many times; the other being equal in length to it, but oblong—first: of a hundred of the numbers of rational diagonals of the pentad, each one lacking one, but from irrational two, and, then, of one hundred cubes of three" (*Resp*. 546c).

The first harmony, according to Plato, is "equal an equal number of times, one hundred so many times." Proclus tells us that this expression refers to 10,000, which is 100 squared (*in R*. II 37.15, 18; Hultsch 406). The second harmony has two constituents. Its first part is "of a hundred of the numbers of rational diagonals of the pentad, each one lacking one, but from irrational, two." Since, as we have seen, Proclus and the annotator's source believe that the rational diagonal of 5 is 49 and the irrational 50, for them, the first constituent of the second harmony is one hundred 48s, or 4,800. The second constituent is "one hundred cubes of three." This is what the annotator's expression 'by 27' must refer to. One hundred 27s make 2,700. Proclus directs us to add the two constituents, 4,800 and 2,700, to produce the second harmony, 7500. This must be what the annotator means by the female number.

So much is arithmetic. In order to understand the significance of the large triangle described in notes 4, 5, and 6 and the sense in which that triangle is a boundary, we need to look at Proclus' geometrical exposition of the calculation. What he describes is Plato's intention to take the large triangle constructed previously from augmented triangles and convert it from plane to solid by giving it a depth of 100, thus:



On this interpretation, Plato's first harmony, 10,000, corresponds to the square ZKFE. "Equal an equal number of times" is the muses' way of calling a number square, and "one hundred so many times" is their way of saying that the corresponding square has a side of 100.

The second harmony is represented by the rectangle GZEC, "equal in length to it [*scil.* the first harmony], but oblong." That is, the lines GZ and EC are the same length as the sides of the square ZKFE, but GZEC is oblong, and not a square. Proclus draws attention to the fact that this rectangle is described indirectly, as the composite of two other rectangles: first BZED, then GBDC (*in R.* II 38.4–39.1). The 'oblong' BZED—the one measuring 4,800—is what Plato refers to in speaking about a figure "of a hundred of the numbers of rational diagonals of the pentad each one lacking one, but from irrational, two"—that is, its dimensions are 48 and 100. GBDC is "one hundred cubes of three"—and soa rectangle measuring 27 by 100. The terms 27, 36, 48, and 64, in short, short, first produce a large triangle with sides of 75 and 100. When this triangle is given a depth of 100, two shapes result. One is an oblong with an area of 7,500, the other a square with an area of 10,000.

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and in believing that a year is 364 1/2 days. 729 is the 'tyrant's number' and at *Resp.* Book 9 (587e–588a), Socrates says that 729 is relevant to days, nights, months, and years (see Adam on this passage at 2.361n). In what follows, we shall assume that there are 365 days in a year, since nothing hangs on being more precise.

Susemihl, Zeller, and Blößner, instead of making Plato's figure three–dimensional, take 7,500 and 10,000 to measure figures that occupy the same plane as the big triangle.[41] Specifically, they understand the square harmony to measure a square built on KZ, while the rectangular harmony measures the rectangle having KZ as one side, and GZ as the other. Our interpretation, on the other hand, fits nicely with Aristotle's enigmatic explanation that when Plato says that a base with a third added, married by the pentad, furnishes two harmonies, "he means when the number of this figure *becomes solid*" (*Pol*. 5.12, 1316a5–6). More importantly for our present purpose, it fits with the annotator's remark that the four terms make up a boundary. We interpret this as the boundary of a three dimensional wedge.

2. 7,500 Days for the Female

Proclus preserves two accounts of what the harmonies signify. According to Dercyllides, the first harmony governs good births, the second bad (*in R.* II 25.20–24). According to Platonist Cronius (late 2nd cent.), the first harmony governs the male, the second the female (Procl. *in R*. II 23.6–8). Given ancient attitudes towards the sexes, it is easy to see how one of these traditions might have arisen from the other. Proclus' sympathies are with Dercyllides: at *in R.* II 174.2–7 he links the square and the oblong described by Plato in *Resp*. 8 with 'the better life' and 'the worse,' respectively. Elsewhere he specifically links Plato's two harmonies to "the more intellectual and the more irrational (*scil*. lives), … those returning to the same point and those coming to be" (*in R.* II 66.22–25).

Now, given the annotator's remark that 7,500 days is "for the female" it is clear he and his source fall, unlike Proclus, into the tradition of Cronius. Plato's harmonies have genders, according to the note. Specifically, the commentator tells us Plato believes the period of time that governs the female engendering is 7,500 days, about twenty years. What is particularly female about 7,500? Burkert rather cautiously remarks about the nuptial number, "a relation with Pythagoreanism is probable." (Burkert 481 n. 76). A good many such connections can be found on the commentator's interpretation of the text. For the sake of focus, we shall defer discussion of most of those antecedents for future work; one of them, however, casts enough light in the present context to be worth mentioning here. According to Aristotle, some members of the Pythagorean school advanced a table of ten pairs of opposites.[42] In this table, the male correlates with the square and the female with the oblong (*Metaph*. 986b22–24). The harmony governing male reproduction measures the area of a square (ZKFE), and the harmony governing female reproduction measures the area of an oblong (GZEC).

Note 11 thus provides a remarkable confirmation and anticipation of Konrad Gaiser's interpretation of the two harmonies. He takes the numbers 7,500 and 10,000 from Proclus, surmises that they measure lengths of time measured in days, and shows that thus understood they correspond closely to the ages that Socrates offers in Book 5 as the best ages for engendering children (Gaiser 56–57, 63–76). There Socrates and Glaucon agree that 20 years is both the age at which a woman's child–bearing should begin and the duration of her proper child–bearing years (*Resp.* 460e). Socrates also gives age limits for when a man ought to be fathering children. The period is supposed to run "from the time that he passes his peak as a runner until he reaches fifty–five," a stretch of time that Socrates says lasts about twenty years (*Resp*. 460d–461a). Gaiser shows that if we assume that Plato's first harmony measures a period of days, we get rough agreement between Socrates' remarks in Book 5 and those of the muses in Book 8 regarding the starting age of fertility, and we get close agreement about the proper age for terminating reproductive activity (Gaiser 71–72):

> Starting age for men: 10,000 days ÷ 365 days/year = 27 years and 145 days
> Ending age for men: 10,000 days x 2 = 54 years and 290 days.

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[41] Susemihl 376–77, Zeller 2.1.860n, Blößner 84. Blößner derives his big triangle from Proclus. Susemihl and Zeller come up with their big triangle in a different way, but it has the same dimensions along the outside.

[42] For a discussion of the table, see Burkert 51–52.

These numbers do not correspond exactly to what Socrates presented in Book 5 as the proper ages according to common consent, but they are very close. The starting age for women is off by 200 days, and the ending age is off by 400 days. The starting age for men is off by 2 years and 145 days, and the ending age is off by 75 days. Since Plato is discussing the proper regulation of births in both passages, however, these numbers are altogether too close for coincidence. It would be an even stranger coincidence if the annotator's source thought that the two harmonies were divisible into genders and measured a period of days, but did not think that these periods corresponded to the periods described in book 5. We thus conclude that the annotator's source interprets Plato's harmonies as Gaiser does.[43]

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