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Shaking an invisible hand

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The research program of general equilibrium theory originated with Adam Smith's vision of the emerging world economy. It evolved through a long series of efforts by economists and mathematicians. However, a major challenge arose with the discovery that the shape of Walrasian excess demand functions makes unique stable equilibria the exception rather than the rule. This has led to a highly problematic disconnect between modeling for policy support and the understanding of the world economy that is actually available. A promising route to overcome these difficulties has been opened by recent research on the dynamics of social norms, building on the theory of perturbed dynamical systems. This work could lead to a fruitful new reading of Adam Smith. The marginal analysis of current general equilibrium theory can be successfully applied to processes within a historically established basin of attraction, and it can be complemented by the "inframarginal" study of historical transitions from a given basin of attraction to one of several future possibilities.

1. Adam Smith's metaphor

Less than two decades before the French revolution, Adam Smith – a Scotsman who had travelled France extensively – coined the metaphor of the invisible hand of the market. His basic idea was that attempts by governments to control society as a whole were futile and counterproductive, because it was much better to leave markets work out the directions of economic activities by themselves.

He saw important exceptions where government intervention was appropriate, e.g. to avoid cartels and monopolies or to make sure that people do not get trapped in lives where they can only exercise an extremely narrow range of skills. But as a rule, he argued that if people pursue whatever legally accepted interests they may wish to, competitive markets will lead to a highly desirable outcome that would be impaired by government intervention.

Smith thought that the requirements of production define a natural price for every good and that market prices will lie below or above natural prices if supply exceeds de-

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mand or falls short of it. This difference will then induce investors to change their plans, so as to expand or contract capacity, and then production until supply equals demand.

He also thought that production costs will fall as markets expand, because larger production allows for lower unit costs thanks to greater division of labor. Just as he saw domestic economic interventions of governments as usually counterproductive, he considered restrictions on free trade as harmful because they restrict the size of markets.

More generally, Smith saw the growth potential of actual economies as constrained by the size of the market. He took that size as given by historical circumstances, advocating free trade as a means to increase it. An important implication of this view is that the world economy evolves beyond the control of nation states – a reality and a challenge that has become more and more important since the days of Smith (1776).

2. Walras and the origins of mathematical economics

About a century later, Walras (1874, 1877) argued that Smith's ideas should be elaborated in mathematical terms. He was a French economist who had to work in Lausanne, Switzerland, because his Parisian colleagues found little interest in his ideas. According to his proposal, the basic functioning of markets is to be represented by economic agents trading bundles of goods. There are $j = 1, \dots, n$ possible goods, and their quantities are to be represented by nonnegative real numbers, so a bundle of goods is a point in $\mathbb{R}^n_{\geq 0}$. Each good has a price, and all prices are represented in terms of some good or bundle of goods used as numéraire. A price pattern p then is a list of nonnegative real numbers with $p \in \mathbb{R}^n_{\geq 0}$ and the price of the numéraire equal to one.

There are $i = 1, \dots, m$ agents; they have initial endowments $\omega_i \in \mathbb{R}^n_{\geq 0}$ and can demand any bundle of good in the set $\{x : x \in \mathbb{R}^n_{\geq 0},\ \omega_i \cdot p = x \cdot p\}$, i.e. any bundle of good they could afford by selling their initial endowment. The interests of the agents are represented by utility functions $u_i : \mathbb{R}^n_{\geq 0} \to \mathbb{R}$.

It has become customary to consider functions with $\frac{\partial u_i}{\partial x_j} > 0$ everywhere and with strictly convex indifference curves as paradigmatic examples. With functions of such shape, at each price pattern each agent will select exactly one optimal demand x_i. By considering demand as a function of prices one obtains individual demand functions, by adding those over all agents one obtains an aggregated demand function $f_D : \mathbb{R}^n_{\geq 0} \to \mathbb{R}^n_{\geq 0}$. An aggregated supply function can be construed by simply adding all individual endowments over all agents – this function is constant for all prices. Walras that there is a price pattern at which these functions intersect, so that supply is equal to demand on all markets, and that market dynamics converges towards that situation, known as a general economic equilibrium. Proving or disproving that conjecture turned out to be a major challenge, as mathematicians in those days did not have the instruments to treat problems of this kind.

Having defined how he wanted to analyze the functioning of markets, Walras added production by introducing profit-maximizing firms that can use given technologies to transform input goods into output goods. He also made suggestions for how to include money and credit into the kind of scheme he was proposing, but despite a wealth of proposals it is fair to say that until today it is not really clear how this can be done.

As for economic growth, in a Walrasian perspective it is driven by the desire for greater consumption and constrained by the resources available. Growth potential then depends on the relation between existing resources and outputs available for net investment. However, this relation is not easy to capture, as any measure of economic growth requires prices to compare different goods.

Here, Walras remained ambiguous: sometimes he imagined that in equilibrium goods are traded at prices that are fully consistent with future economic developments (that will affect the amount of inputs and also the technologies available), sometimes he imagined that trade takes place at non-equilibrium prices in such a way as to converge towards equilibrium. As we will see, this ambiguity is still relevant for economic analysis today.

3. Fixed points and equilibria: Brouwer and von Neumann

A few years before World War I, the Dutch mathematician Brouwer proved that any continuous function from a bounded, closed, convex set to the same set has at least one fixed point (Border, 1989, gives an excellent overview of the relevance of fixed points for economics). A simple example of such a set is the unit interval $[0, 1]$ on the real line, and it seems rather obvious that a continuous function f from the unit interval to itself will cross the identity line $y = x$ at lest once, and so has at least one fixed point.

The unit interval is a 1-simplex. More generally, a n-simplex is a construction based on $n+1$ points in $\mathbb{R}^n$ that do not fit $\mathbb{R}^{n-1}$ (the points 0 and 1 on the real line do not fit $\mathbb{R}^0$, i.e. a single point): it is the smallest convex set containing these $n+1$ points (i.e. their convex hull). A simplex is always bounded and closed, and so Brouwer's theorem holds for all of them.

An important property of simplexes is captured by Sperner's lemma, discovered in 1928. For a 1-simplex like the $[0, 1]$ interval, it says that if the endpoints are marked red and blue, and one then adds red and blue points in arbitrary order, at each step one will get either a smaller red-blue subsegment or two additional red-blue subsegments (a simple drawing is sufficient to see why). More generally, if one colors the corners of a n-simplex with n colors and then subdivides it into subsimplexes whose corners are colored with whatever sample from the same n colors, there will always be an odd number of subsimplexes whose corners have n different colors.

Sperner's lemma yields a proof of Brouwer's theorem. Consider a 1-simplex S and a continuous mapping $f : S \to S$. Imagine S as a horizontal line segment, cut it in two halves, call its left endpoint A, its middle point B and its right endpoint C. If

$f(X) = X$, $X \in \{A, B, C\}$, then X is a fixed point and the theorem applies. If none of the three points is a fixed point, color it red if it moves to the right under f and color it blue if it moves to the left. The endpoints are bound to move in different directions, and as a trivial application of Sperner's lemma one of the two subsegments will have endpoints of different colors. Repeat the procedure for that subsegment and so on. Then either the procedure runs into a fixed point after a finite number of steps or it produces a sequence of nested subsegments converging on a single point that will then be a fixed point. An analogous argument holds for arbitrary n-simplexes and then for sets that are topologically equivalent to those, as an arc is to a straight line segment.

In the years before World War II, the Austrian mathematician Karl Menger (son of economist Carl Menger) organized a mathematical colloquium in Vienna that brought together mathematicians, economists and philosophers. In 1937, John von Neumann wrote a paper for the colloquium showing the crucial relevance of Brouwer's theorem for the analysis of general economic equilibria (published in English as von Neumann, 1945).

He designed a model of a growing economy involving n different goods and $k > n$ possible activities. Each activity requires specific quantities, a_{hj}, $h = 1, \ldots, k$, $j = 1, \ldots, n$, of different goods to produce specific quantities b_{hj} of those goods. With n goods there are n prices. By choosing as numéraire a bundle consisting of one unit of each good, the sum of the prices is always equal to one, and the set of possible prices is a $n-1$ simplex, S_p. Each activity can be operated at different activity levels, x_j, including zero for activities not used. By focussing on the case where the sum of activity levels is equal to one, the possible activity levels form a $k - 1$ simplex, S_x.

von Neumann now looks at a general equilibrium as a situation where supply matches demand for each good and where the economy evolves through time without changing the relations between prices. For this purpose, he introduces a function $\phi : S_p \times S_x \to \mathbb{R}$ characterizing any pair of prices and activity levels:

$$\phi(p, x) = \frac{\sum_{h=1}^{k} \sum_{j=1}^{n} \ x_h \, b_{hj} \, p_j}{\sum_{h=1}^{k} \sum_{j=1}^{n} \ x_h \, a_{hj} \, p_j}. \tag{1}$$

He remarks: "A direct interpretation of the function [. . .] would be highly desirable. Its role appears to be similar to that of thermodynamic potentials in phenomenological thermodynamics; it can be surmised that the similarity will persist in its full phenomenological generality (independently of our restrictive idealizations)" (p. 1, loc. cit.).

In fact, the function has a double economic interpretation. On one hand, it gives a measure of the growth potential implied by any set of activity levels and prices. The potential will be realized if all profits are reinvested. And at given prices, it may or may not be possible to increase economic growth by changing activity levels. On the other hand ϕ gives a measure of the average rate of profits. If some producers make profits at a rate that is higher than average, competitors may be able to undercut them by offering the same product at a lower price.

Against this background, von Neumann looks for pairs of activity levels and prices such that ϕ reaches a maximum with regard to activity levels and a minimum with regard to prices, i.e. a saddle-point of ϕ.[1] For this purpose, one can define a point-to-set mapping Φ by associating to each pair of activity levels and prices a set of prices and activity levels as follows. For any pattern of prices, define the set $A(p)$ of all patterns of activity levels at which ϕ reaches a maximum; for any pattern of activity levels, define the set $B(x)$ of all prices for which it reaches a minimum. Define $\Phi : S_p \times S_x \to \mathcal{P}(S_p \times S_x)$, where $\mathcal{P}$ denotes the power set, and $\Phi(p, x) = A(p) \times B(x)$.

The saddle-point to be found then is a fixed point in the sense that $(p, x) \in \Phi(p, x)$. von Neumann generalized Brouwer's fixed-point theorem so as to be able to tackle this problem and showed that indeed a general equilibrium of this kind can be found. In that equilibrium, the economy grows at a uniform rate $\phi(p, x)$. It is equal to the rate of profit, because net investments in this equilibrium are equal to total profits.

4. Arrow-Debreu and the great existence proof

After World War II, in an important sense science had moved from Europe to America. There, the American Ken Arrow and the French Gérard Debreu developed what was soon to be seen as the core representation of Smith's invisible hand (Arrow & Debreu, 1954). Building on the work of Walras, Brouwer, von Neumann and many others, they were able to prove the existence of a fixed point that can be interpreted as a Walrasian general economic equilibrium.

To capture the basic idea, let there be n goods and represent price patterns as points on the unit sphere in $\mathbb{R}^n$, so that $p \cdot p = 1$ (I follow an exposition due to Saari, 1995). The relation between any two coordinates of such a point then represents the relative prices of the corresponding two goods. As prices are non-negative, only $\mathbb{R}_{\geq 0}$ matters, and the resulting portion of the sphere is topologically equivalent to a $n - 1$-simplex.

There are $i = 1, \ldots, m$ agents, each with her initial endowment ω_i and all confronted with prices p. The set $\{x \in \mathbb{R}^n : \omega_i \cdot p = x \cdot p\}$ of possible demands of agent i is a hyperplane – the budget line in Fig. 1 – including the initial endowment and orthogonal to the price vector, as represented in Figure 1 (it is orthogonal to p because the intercepts must be inversely proportional to the relation between the coordinates of p). Optimal demand x_i is given by the unique tangent point between the budget hyperplane and an indifference curve. Individual excess demand z_i is the difference $x_i - \omega_i$. As all arrows with the same direction and length can be considered representations of the same vector, individual excess demand can be represented by an arrow starting at the endpoint of the price vector and orthogonal to it.

[1] It was through the study of general economic equilibrium that von Neumann came to realize that the saddle-points characterizing equilibria in game theory are related to similar fixed points (see the footnote on p. 5, loc. cit.).

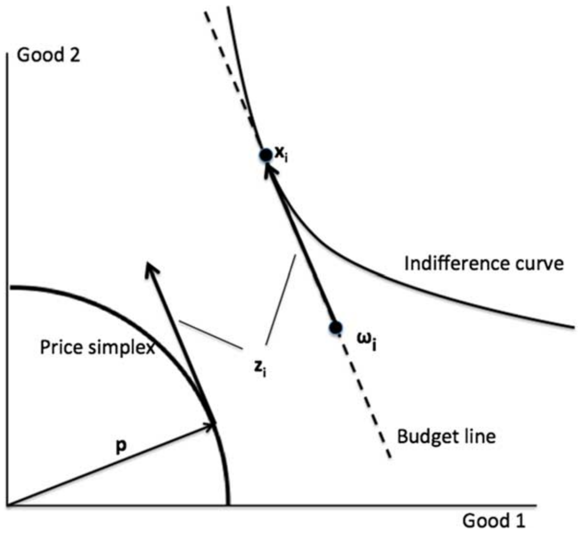

Figure 1. Individual excess demand (based on Saari, 1995).

If in Figure 1 the price vector moves closer to the axis representing good 1, the excess demand vector gets steeper. On the other hand, if the price vector moves close enough to the axis representing good 2, excess demand gets flat and points in the other direction. As the steepness of excess demand can be represented by points on the unit arc used to represent prices, this defines a continuous function on a compact, closed set. By Brouwer's theorem, somewhere in between excess demand will be zero and agent i will see her initial endowment as optimal.

This reasoning carries over to aggregate excess demand: if the price vector gets close enough to one of the axes, the sum of all individual excess demands will yield a vector pointing away from that axis. By Brouwer, then, somewhere on the price simplex lies a point where aggregate excess demand will be zero – a general equilibrium. As long as agents do not all have the same utility function and the same initial endowment, individual excess demands will not be zero in general equilibrium, but they will cancel out in the aggregate.

The argument can be expanded in many ways, introducing firms producing new goods, investment in the course of time, uncertainties about future events, external effects leading to sub-optimal outcomes etc. As for growth potentials, we are back to Walras: very little of interest can be said as long as one does not introduce drastic simplifications like a population of identical agents dealing with a single capital good. In order to assess these simplifications, however, a challenging question must be investigated: how can equilibrium actually be reached?

5. The challenge of multiple equilibria: Sonnenschein-Mantel-Debreu

There is a suggestive image of how markets balance supply and demand: if excess demand is positive, prices increase, lowering demand and increasing supply; if excess demand is negative, an opposite dynamics sets in. This implies

$$\dot{p} = f(\zeta(p)) \tag{2}$$

where $\dot{p}$ is the time derivative of prices, $\zeta : \mathbb{R}^n_{\geq 0} \to \mathbb{R}^n$ the aggregate excess demand function, and $f : \mathbb{R}^n \to \mathbb{R}^n$ some monotonous function with $f(\bar{0}) = \bar{0}$.

In the early 1970s, three economists – the American Hugo Sonnenschein (1973), the Argentinian Rolf Mantel (1974) and the French Gérard Debreu (1974) – showed that as long as the number of agents is not smaller than the number of goods and prices are larger than an arbitrarily small ϵ, the excess demand function can have any shape whatsoever, as long as it satisfies the following three conditions:

- ζ is continuous
- the value of total demand is equal to the value of total supply, i.e. $p \cdot \zeta(p) = 0$
- it depends only on relative prices, i.e. $\zeta(\lambda \cdot p) = \zeta(p)$, $\forall \lambda \in \mathbb{R}^n_{>0}$.

Before looking into why this is so, two remarks are appropriate. First, the Sonnenschein-Mantel-Debreu (SMD) theorem implies that as a rule Walrasian economies have more than one equilibrium. From a purely mathematical point of view, the set of excess demand functions with an infinite number of equilibria is smaller than the one with a finite number of equilibria (in the sense in which the set of rational numbers is smaller than the one of transcendental numbers), but so is the set of functions with exactly one equilibrium. Multiple equilibria are the rule, single equilibria the exception – and even where there is only one equilibrium the shape of the excess demand function may make that equilibrium unstable, driving the economy in an on-going cycle or a chaotic dynamics.

Second, the form of f is quite irrelevant, as it cannot reduce the number of equilibria, and given the variety of possible shapes of ζ, it cannot increase the stability of equilibria either.

To understand why the suggestive image mentioned at the beginning of this section is misleading, consider an arbitrary function $\zeta : \mathbb{R}^n_{>0+\epsilon} \to \mathbb{R}^n$, with ϵ as small as you like, let ζ be twice differentiable and let it satisfy the three conditions mentioned above. *Any* such function is a possible excess demand function.

Consider prices on the unit sphere as before, and for each agent pick a function $V_i : \mathbb{R}^n_{>0+\epsilon} \to \mathbb{R}$. Each V_i shall satisfy a set of differentiability conditions that can easily be met and are economically reasonable (see theorem 6 in Chiappori & Ekeland, 2004). In particular, for each p the convex hull of the derivatives of V_i, $i = 1, \ldots, m$ shall include $\bar{0} \in \mathbb{R}^n$.

In economic terms, the functions V_i are indirect utility functions, giving the maximum utility achievable by an agent at any given set of prices. The condition on the convex hull of the derivatives means that a small change of prices can increase the excess demand of at least one agent while decreasing it for at least one other agent. Such divergent effects result from the combination of substitution and wealth effects: if oil gets more expensive, European agents may reduce their oil consumption, while agents in Saudi Arabia may increase theirs because their income is increasing.

For any set of functions V_i fulfilling these conditions, it is always possible to write the function ζ as a weighted average of the derivatives of the functions V_i, where the weights $\mu_i(p)$ change with p:

$$\zeta = \sum_{i=1}^{m} \mu_i \cdot D_p V_i. \tag{3}$$

Here, $D_p V_i$ is the derivative of V_i, and $\mu_i \cdot D_p V_i$ is the excess demand of agent i. Finally, individual utility functions can be constructed from the functions V_i and μ_i.

The SMD theorem has three major implications:

- any explanation of prices in terms of supply and demand must be supplemented by an explanation of equilibrium selection that cannot again rely on supply and demand;
- even if a satisfactory explanation of equilibrium selection should be available, the stability of any given equilibrium needs an explanation that involves more than the difference between supply and demand;
- economic analyses that consider only one possible equilibrium can only be valid under special circumstances whose definition depends on the previous two points.

These implications are directly relevant for the assessment of growth potentials: in a Walrasian setting, multiple equilibria are likely to imply multiple growth potentials.

6. Gintis, Bouchaud and the dynamics of social norms

The SMD theorem triggered a large literature (see Rizvi, 2006, for an overview), but the problems it raised remain unresolved. Economic models used for policy advice simply ignore them – not a very satisfactory approach, of course, but a strategy to keep going.

A promising breakthrough, however, has been achieved by Gintis (2007). He starts with the observation that goods traded at uniform prices are rarely observed: even the price of a cup of coffee varies across coffee-shops. Moreover, no economic agent can perceive more than a sample of all the transactions taking place in the economy as a whole. Gintis then simulates a Walrasian economy with production, but without growth, in which randomly matched agents engage in transactions on the basis of rules of thumb about what prices they consider appropriate – in line with the terminology of game theory, I call these rules price strategies.

Agents modify these strategies in two ways: first, they observe random samples of all transactions and try to adjust their strategies to what they see, and second from time to

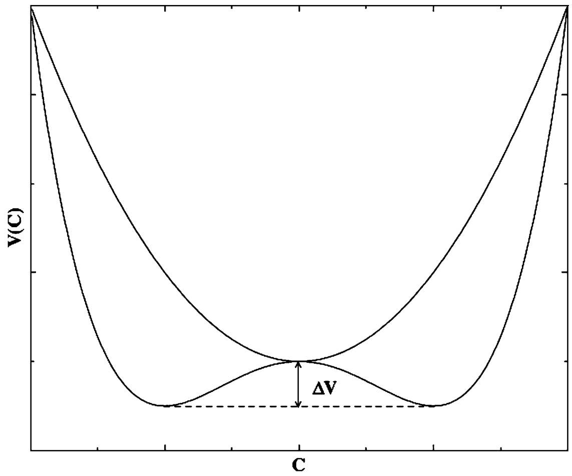

Figure 2. Potential functions for financial markets with heterogeneous trading strategies (C is a parameter driving market dynamics via expectations. Fig. 1 in Wyart & Bouchaud, 2007).

time they modify strategies at random, perhaps to experiment or simply by mistake. He then shows that in many cases a population starting with widely differing price strategies converges rather quickly on a situation where these differences fluctuate within a narrow environment of some equilibrium. This kind of price dynamics can be investigated with techniques that have been developed to study the dynamics of social norms.

That the dynamics of social norms can be helpful in studying markets with multiple equilibria is confirmed by work on financial markets by Wyart & Bouchaud (2007). Figure 2 represents three possible regimes of a financial market where strategic and non-strategic traders operate in the face of uncertain price dynamics and some exogenous signal unrelated to technologies, preferences, or endowments. Wyart and Bouchaud show how a potential function can be constructed that captures the multi-equilibrium dynamics of such markets. The key feature of the system is the possibility of different norms or conventions shaping the trading behavior.

There is no need to go as far as Lewis (1969) and argue that conventions are in fact the building blocks of the social universe; but it seems reasonable to pay more attention to conventions, social norms, and the like in the study of economic phenomena.

For this purpose, consider a group of m agents each of which chooses a strategy and each of which has preferences over the possible combinations of strategies. Let $S = (S_1, \ldots, S_m)$ be a tuple of finite sets; an element $s_i \in S_i$ is called a pure strategy, an element $s = (s_1, \ldots, s_m) \in S$ a pure strategy profile. Let $F = (f_1, \ldots, f_m)$ be a tuple of functions $f_i : S \to \mathbb{R}$; a number $f_i(s)$ is a utility index representing the preferences of agent i, it is usually called a payoff. The ordered pair (S, F) is a game.

A mixed strategy of agent i is a probability distribution over S_i with at least two non-zero probabilities. The payoff of a mixed strategy profile for agent i is the weighted average that results by weighting the values of f_i for each element of S with the joint probabilities of all agents.

A Nash equilibrium is a strategy profile whose utility index cannot be increased for any agent by modifying only the strategy of that agent. A coordination game is a game with more than one Nash equilibrium.

To analyze norm dynamics, start with a coordination game where the strategy of each agent is taken from the same set of possible strategies, $S_i = S_j, \forall i, j$ (more complex coordination games can easily be introduced). I call this the small game, because a much larger coordination game based on iterations of the small game will now be constructed as follows.

First, let the agents play the small game at times $t = 1, \ldots, \infty$. Each agent remembers the last τ, $1 < \tau < \infty$ iterations. For initialization, all agents have a memory of τ realizations of the small game. On that basis, each agent chooses a pure small-game strategy that is a best reply to her memory, i.e. a strategy that would deliver an optimal result if she would play it while the other agents would do a replay of the last τ small games she has observed (if there are several best replies, let her choose one by an arbitrary deterministic rule). This defines a large game $G1$ where each agent has only one strategy, namely the best reply function from τ realizations of the small game to the strategy set of the small game. This large game in turn defines a deterministic dynamic system with a finite state space, namely the set of all memories of all agents. Depending on its structure and the initial conditions – i.e. the initial memories of the agents – the system will either reach a fixed state after a finite number of iterations or enter a cycle that it will follow forever. As the small game is a coordination game, the only possible fixed states under a best reply dynamics will be the Nash equilibria of the small game. However, the non-Nash strategy profiles may still result in cycles.

Second, introduce a large population of $M > m$ agents. Their preferences are such that whenever m of them engage in the small game, the set of Nash equilibria of the small game is preserved. At times $t = 1, \ldots, \infty$, a sample of κ agents, $m < \kappa < M$, is selected by some random process, together with a subsample of exactly m agents. The subsample plays the small game and the whole sample observes how it plays out. This defines a large game $G2$ with a much larger state space and a stochastic dynamics. As the state space is still finite, the result is a finite Markov chain. Now, the Nash equilibria of the coordination game provide the basis for the only possible absorbing states of that chain: if all agents have a memory of τ small games played with the same Nash equilibrium, their best replies will reproduce that equilibrium. The non-Nash strategy profiles may or may not lead to stationary distributions, so that the strategy profiles may eventually evolve in a random fashion according to the probabilities defined by such a distribution.

Third and last, let each agent play her best response only with probability $1 - \epsilon$, while playing the remaining strategies with probability $\frac{\epsilon}{\nu - 1}$, where $\nu = |S_i|$. Here, ϵ is a parameter of the large game $G3$. It represents the probability that agents experiment with strategies that they do not consider as optimal at first sight – perhaps by chance, or because they understand that their assessments of optimality might be misleading. ϵ can be generalized to monotonous functions of ϵ that yield different values for different strategies and different agents. $G3$ is a perturbation of $G2$, and can be studied with techniques that build on the theory of perturbations in dynamical systems due to Freidlin & Wentzell (1984). In the study of social norms, these techniques have led to a rich literature (see Young, 2006, for an overview).

A key result is that under plausible conditions the large game will be characterized by long stretches where it sticks to one of the Nash equilibria of the small game and short stretches of transition between such small-game-equilibria. Those small-game equilibria that will be played in the large game for long stretches (at least longer than τ) are social norms.

There are two reasons why it seems promising to look at prices from a perspective of norm dynamics. First, the multiplicity of general economic equilibria does lead to a coordination game: in each equilibrium, each agent is in a position that she cannot improve as long as the other agents stick to their strategies, and so the selection among multiple general equilibria is indeed akin to the problem analyzed in studies on norm dynamics. Second, the suggestion that agents operating out of equilibrium trade at different prices is highly plausible both on empirical and on theoretical grounds; at the same time the "law of one price" expresses the fact that heterogeneous prices for homogeneous goods are no Nash equilibria and so will hardly be stable. The simulations by Gintis seem to capture the resulting dynamics quite well, and as Mandel et al. (2009) have shown, they can be generalized to the case of multisectoral growth.

7. Xiaokai Yang: Adam Smith reloaded

Against this background, it is time to read Adam Smith again, with fresh eyes. Yang (2001) has suggested that the kind of marginal analysis that forms the core of current economics captures only a part – if an important one – of Smith's view of the world economy. It misses the "inframarginal" dynamics by which a given economic equilibrium gives way to one of several possible successors.

"We categorize business decisions into two classes: marginal decisions of resource allocation and inframarginal decisions of economic organization. Marginal decisions involve the extent to which resources are allocated to a pre-determined set of activities. Inframarginal decisions are about what activities to engage in" (Cheng & Yang, 2004, p. 138). General equilibrium theory in its present form knows only marginal decisions, and equilibrium selection becomes a mystery.

In the approach to norm dynamics discussed above, inframarginal decisions correspond to the ϵ-events, to those "mutations" that can drive a social system from one basin of attraction to another one. Looking at markets with a perspective of norm dynamics

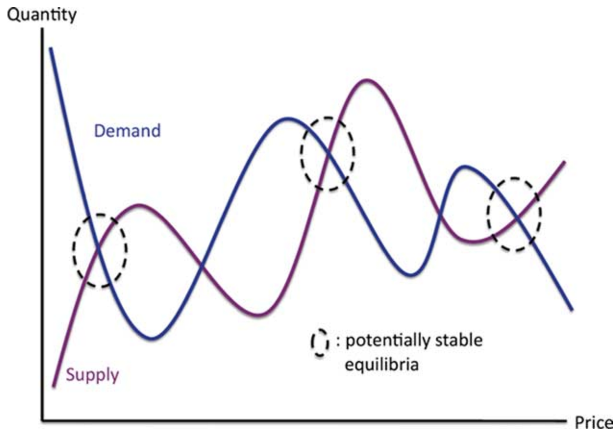

Figure 3. The reality of supply and demand.

leads to the view of supply and demand sketched in Figure 3. Once some equilibrium has been established in the course of history, business decisions are mainly of the marginal kind. They operate in the basin of attraction of that equilibrium, responding to chance events like those leading to a new match between provider and customer of some good or service.

From time to time, however, some set of inframarginal decisions drives the whole system beyond the basin of attraction it was operating in. These decisions may happen spontaneously, or they may respond to exogenous shocks in non-marginal mode (Jaeger et al., 2011, as well as Zhang & Shi, 2011, independently consider possibilities to take advantage of the climate challenge as a possible shock of this kind).

Yang (2001) emphasizes that for Adam Smith the most important driver of such transitions were changes in the division of labor, and suggests that such changes are of the utmost importance in the present world economy. If one focusses less on subdivisions of tasks among unskilled workers and more on the specialization of professional skills that lies at the core of the so-called knowledge economy (Abbot, 1988), this claim has considerable plausibility.

Drawing on models of norm dynamics to address the problem of equilibrium selection in economics offers a remarkable opportunity to move towards a stronger integration of the social sciences. Certainly, the dynamics of professional specialization is closely intertwined with the dynamics of social norms. If Yang's conjecture about the key role of the division of labor for inframarginal business decisions should be corroborated, this might foster stronger synergies between economics and management science. In particular, the resistance of scholars of management to wholeheartedly embrace the optimization perspective underlying most of contemporary economics may be quite intelligible once on accepts the relevance of inframarginal decisions in actual management. Progress in the study of equilibrium selection as an instance of norm dynamics could enhance both the analysis of economic systems and the practice of management that has become a vital part of these systems.

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