Constructing Worlds

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Abstract: You and I can differ in what we say, or believe, even though the things we say, or believe, are logically equivalent. Discussing what is said, or believed, requires notions of content which are finer-grained than sets of (metaphysically or logically) possible worlds. In this paper, I develop the approach to fine-grained content in terms of a space of possible and impossible worlds. I give a method for constructing ersatz worlds based on theory of substantial facts. I show how this theory overcomes an objection to actualist constructions of ersatz worlds and argue that it naturally gives rise to useful notions of fine-grained content.

Keywords: Content, hyperintensionality, impossible worlds, ersatz worlds, modality.

1 Introduction

'Even if your premises were true', sighs Professor Hauteur, 'your conclusion still wouldn't follow'. The cowed presenter, giving her first grad talk, mumbles that the conclusion *would* follow. Someone suggests that we reason it though: *first, assume the premises* ...; someone else suggests that the argument goes through if we first assume a deviant logic. 'But look', puffs an irate Hauteur, 'it's a consequence of your account that ..., but not that ...'.

And so it goes in philosophical debates. We reason counterfactually about our opponent's views; we assume their premises and see what does and doesn't follow; we try out views we do not in fact believe to see where they lead. It's somewhat surprising, therefore, that the most prominent philosophical account of the content of an assertion, on which contents are sets of possible worlds, treats such dialectical moves as being contentless in many cases. Suppose we are debating a philosophical position which is necessarily true if it is in fact true (and necessarily false if it is in fact false), as many philosophical positions are supposed to be. That theory is either true in all such worlds or false in all of them. Then, according to the sets-of-possible-worlds account of content, counterfactual and assumption-based reasoning about the theory is trivial: it's just like reasoning about whether 1 = 1 (in the case of true theories) or about whether 0 = 1 (in the case of false theories). Now, philosophers have been known to talk all kinds of nonsense; but surely it isn't *all* bosh?

Any decent account of the content of our sayings must be able to distinguish between logically (and metaphysically) equivalent assertions: it must deliver *hyperintensional* contents.[1] Two philosophical views might both be necessarily false, yet in asserting (or believing, or assuming) one, a speaker needn't thereby

1. 'Content' has many different (although related) uses in these debates: it may mean *proposition*, *what is said* or *compositional value*. Each of these notions generates a hyperintensional context. As I use the term, these are all types of content.

assert (or believe, or assume) the other. The need for a hyperintensional contents is frequently discussed (and is a theme of many of the papers in this volume). Two main approaches emerge: either we impose additional structure on contents, or we enlarge our set of worlds beyond the possible ones. I'll pursue the latter route.

My aim in this paper is to provide a general method for constructing worlds, possible and impossible, with an actualist's resources (that is, using only those entities which actually exist), in terms of which we can define fine-grained contents. I'll show how to build worlds which represent particulars which don't actually exist and properties which no particular actually possesses. Several notions of content fall out of the resulting space of worlds, which are suited to different philosophical tasks. The remainder of the paper is as follows. In §2, I consider the debate between structural and impossible-worlds approaches to fine-grained contents, and opt for the latter. In §3, I consider whether impossible worlds are of a kind with possible worlds. The construction of worlds from facts appears in §4. I show in §5 how this approach solves a problem for actualists. Finally, in §6, I discuss how useful notions of content arise naturally within the constructed space of worlds.

2 Spaces of Worlds

We want to distinguish between necessarily equivalent contents. We can do so either by taking them to differ in their *structure*, or by taking them to differ in their *membership*. In this section, I'll argue in favour of the latter view, and consider some immediate worries it raises.

First, consider the suggestion that contents are structured entities, rather than 'flat' sets of worlds. A structured account of content is used by Carnap (1947), Lewis (1970), Cresswell (1985), Salmon (1986), Soames (1987) and King (1995; 1996). Typically, the structure imposed on contents mirrors (some of) the syntactic structure of the corresponding sentences. Such structures are taken to be ordered tuples (coded as sets) which 'bottom out' either at sets of possible worlds or at the denotations of the relevant terms. The latter gives a 'Russellian' view of content, on which 'Greg pegged Meg's leg' might be assigned the content (*pegging*, Greg, Meg's leg).

Such accounts can distinguish between equivalent contents. Suppose the negation sign '¬' denotes a function $[\![\neg]\!]$ and that sentence A expresses the content $[\![A]\!]$.[2] Then $\neg\neg A$ is assigned the content $([\![\neg]\!], ([\![\neg]\!], [\![A]\!]))$, which is distinct from $[\![A]\!]$. This move appeals to the structural differences between the sentences $\neg\neg A$ and A. Yet there are cases to be distinguished which do not differ in their structure. Frege-puzzle cases provide one source of examples: 'Greg is Greg' and 'Greg is Meg' are alike in their syntactic structure and so will be assigned contents alike in their structure. So if Greg *is* Meg, both sentences will be assigned the same content. Yet we can reject, question whether and argue about 'Greg is Meg' without thereby

2. Throughout, logical formulae (including single italicised upper-case letters) come with implicit Quine-quotes, except when following 'that'.

rejecting, questioning whether or arguing about 'Greg is Greg'. So it appears that structure won't get us hyperintensional contents in all the required cases.[3]

In looking for hyperintensional contents, the second option frequently considered is to expand the set of worlds, to include 'impossible' as well as possible worlds. Impossible worlds represent some impossible state of affairs as being the case. There are, however, several different uses of 'impossible world' in the literature. For some authors, they are worlds where some contradiction $A \wedge \neg A$ is true (Lycan 1994; Berto 2010). Typically, on this view, the impossible worlds correspond to the relational models of paraconsistent logic (see, e.g., Priest 1987). For others, impossible worlds are worlds governed by some non-classical (e.g., intuitionistic) logic (Cresswell 1973). An alternative and far less constrained notion is to allow that, if it is impossible that A, then there's a world which represents that A (Nolan 1997; Priest 2005).[4]

Here is a prima facie worry for any decision between these notions of 'impossible world'. If we insist that all worlds must be governed by some logical structure (even if it is non-classical), then we risk conflating contents. Suppose that (as a result of the logical structure we've imposed on worlds) it's the case that, whenever A is true at a world, B is also true at that world. Then $[\![B]\!]$, the set of worlds which represent that B, will include the content of A, $[\![A]\!]$. Yet we may worry that some agent could take an attitude towards B but not to A. Perhaps not: but the worry is there.

Pushed by this worry, we might take the alternative path, and allow worlds not governed by any logical laws (except identity, $A \vdash A$) at all. Such worlds are extremely fine-grained: for any arbitrary set of sentences Γ, we will have a world w which represents that A, for each $A \in \Gamma$, and represents nothing else. But now, we may worry that we have a completely unstructured concept of content about which there's nothing informative we can say. Such contents are as fine-grained as the target sentences themselves and so, the worry goes, we've gained nothing of philosophical or linguistic interest by assigning sets of fine-grained worlds to them.

The first worry seems genuine to me. I've argued elsewhere (Jago 2008; 2009) that there are no valid logical rules (other than identity) such that, if agent a believes (or knows) that A_1 and A_2 and ..., then she must thereby believe (or know) whatever follows from $A_1, A_2, \ldots$ using just those rules. As such, contents suitable for analysing states of belief or knowledge should not be closed under any logical rule (other than identity). So, to model doxastic and epistemic states, we'll need to work with impossible worlds of the most unrestricted variety, which need not be subject to any logical rules. I'm assume a *plenitude principle* for impossible worlds: for any impossible situation which can be meaningfully thought about,

3. Naturally, there are moves a defender of structured contents can make. Dave Ripley discusses (and rejects) the view in more detail elsewhere in this volume, so I'll move on, noting only that I'm a Ripleyist on the shortcomings of structured contents.

4. On this latter option, one might allow (as Priest 2005 does) that, for any arbitrary set of propositions, there's a world which represents all and only those propositions as being true. Below, I'll accept a position like this.

there is an impossible world which represents that situation.[5]

The second worry is too embryonic to assess properly. But there is a related worry which can be put more precisely. It seems likely that different notions of content are appropriate for different purposes. As just indicated, epistemic and doxastic contents should be very fine-grained. Compositional content—the values we assign to linguistic units as part of a compositional semantics—should likewise be very fine-grained. We want a belief report '*a* believes that *p*' to be true just in case *a* does believe that *p*; hence the semantic values assigned to *that*-clauses need to be as fine-grained as the contents of doxastic states.

Yet other notions of content benefit from being assigned a coarser grain. We frequently want to keep track of an agent's *rational commitments* during a discourse. It is natural to think of this as the combined content of her assertions, where that notion of content is closed under certain logical principles. For example, if she asserts A and then (without retracting anything) asserts B, she is thereby committed to $A \wedge B$.[6] A related notion concerns what an agent *says* in making an utterance. Plausibly, if she says that $A \wedge B$, she thereby says that $B \wedge A$. if this is right, then a notion of content suitable for unpacking 'what is said' should treat the order in which conjuncts are uttered as irrelevant to what is said is using that conjunction. To capture this notion of content, we require worlds which never distinguish between $A \wedge B$ and $B \wedge A$.

The quick response to these remarks is as follows. If you want just those worlds that obey certain rules, then that's fine: we strike out all the other worlds and work with the resulting restricted space. The worry here, however, is that we have to find out *precisely* which principles the target notion of content should obey, and impose those rules on our total space of worlds. All the work that will then be done by the resulting notion of content is a result of those rules. Our space of worlds itself does no important theoretical work in defining that notion of content.

A preferable situation would be the following. We build a space of very fine-grained worlds, allowing us to deal with the first worry. Yet interesting subspaces of worlds fall naturally out of our construction of worlds. The contents defined on those subspaces then turn out to correspond well to interesting theoretical notions. This situation is preferable because it gives us theoretical tools for reasoning about the target notions, as opposed to having to 'write in' what we've observed about the target notions of content by hand. In the remainder of the paper, I'm going to suggest a way of constructing worlds which places us in this preferable situation. I'll discuss the construction in §4, address some problems it raises in §5 and show how it naturally gives rise to useful notions of content in §6. But first, in §3, I'll consider whether possible and impossible worlds are of a kind with the world we inhabit.

5. For by hypothesis, some non-empty thought-content is about that situation, and that content contains only worlds which represent that situation.

6. Perhaps the logic in question should be a relevant one (Anderson and Belnap 1975). I see no reason to think that, by asserting such-and-such, an agent is thereby rationally committed to theorems of classical logic concerning some completely different subject matter.

3 The Parity Thesis

Debates about the status of worlds (possible and impossible) are often pursued in terms of *concrete* verses *non-concrete* worlds, where 'concrete' things, like you and I, are meant to contrast with 'abstract' things, like numbers and sets. This way of setting up the question is unhelpful. We need impossible worlds according to which Fermat's last theorem is false, and according to which there is a largest prime number. Impossible worlds represent abstract as well as concrete states of affairs. What's really at state is how worlds *represent*.

Lewisian worlds—maximal fusions of spatiotemporally related individuals—represent a situation in which a horse flies by having a flying horse a part. Such worlds walk it like they talk it.[7] I'll call any world which represents in this way a *genuine world*, in contrast with *ersatz worlds*, which represent situations in some other way. Two features of genuine worlds are worth noting. First, they obey the *exportation principle*, that if w represents something as being F, then something is F.[8] For if w represents something as being F, then it contains an F as a part; hence, there exists an F. Second, and as a consequence, genuine worlds commit us to non-actual entities. There are actually no hobbits, but there could have been. So some world represents that there are hobbits. If that world is a genuine world, then it contains hobbits; and so there exist hobbits, simpliciter.

Rescher and Brandom (1980) and Berto (2010) discuss the *parity thesis*, the idea that possible and impossible worlds are of a kind (either both genuine or both ersatz). As Berto notes, the thesis has had many supporters, including Rescher and Brandom, Priest (1997) and Yagisawa (1988). Yagisawa puts forward (what he calls) *extended modal realism*, the thesis that 'there are possible worlds in Lewis' sense and also impossible worlds in an equally realistic sense' (Yagisawa 1988, 176). He argues that 'if modal realism is to be accepted at all, we should not stop with the Lewisian modal realism, but go all the way and accept the extended modal realism' (1988, 203). Yagisawa doesn't say much about the metaphysics of impossible worlds, or how they represent what they do; but it seems clear that he wants to treat impossible worlds as being of a kind with Lewisian worlds—i.e., as *genuine* worlds (in the sense above).

There is a very good reason why we shouldn't take the impossible worlds to be of the genuine variety, which Lewis (1986) himself raises. There's an impossible world according to which there's a round square. But there are no round squares: if there were, then both 'square and not square' would be true of some object, committing us to true contradictions. So impossible worlds can't in general represent an F by containing an F. Yagisawa (1988, 203) appears to bite the bullet here: he accepts that one *can* tell the truth about impossible things by

7. One caveat: according to Lewis (1986), ordinary individuals are world-bound, existing in one world only. So a non-actual Lewisian world cannot represent me as having a tail by containing me, entailed, as a part, for I am a part of no world but the actual one. Rather, a non-actual Lewisian world will represent me as having a tail by containing an entailed counterpart of mine.

8. One could deny exportation by relativising all property possession to worlds. Rather than being round, simpliciter, an object would be round-at-w. I am assuming that ordinary objects have at least some properties intrinsically. In saying that this table is round, I mean that it is round, simpliciter; not that it is round-at-this-world.

contradicting oneself. If so, some contradictions are true (simpliciter; and not merely true-in-an-impossible-world). This is the *dialethist* position Priest (1987).

Even dialethists should reject genuine impossible worlds, for although they accept true contradictions, they want to maintain that many sentences are not true. Assuming genuine impossible worlds, however, we obtain many more true contradictions than most dialethists would be willing to countenance. Take any theorem of real analysis, stated as a universal quantification over real numbers. It is equivalent to some statement that there is no real number n such that ϕn. Since that is a theorem, it is impossible for there to be a real number satisfying ϕ. But then, given the plenitude principle, there is an impossible world according to which there *is* such an n. On the assumption that that world is genuine, n is itself a real number such that ϕn (that is, after all, how genuine worlds represent). So we can use the exportation principle: there is (simpliciter) an n such that ϕn, contradicting our theorem. This falsifies our theorem: it is both true and false. The same goes for any other theorem of real analysis, and for many other domains, too. Surely, even for the dialethist, not every mathematical theorem is both true and false!

Here is a further argument to the same conclusion. Dialethists who hold that not everything is true (which, to my knowledge, is all of them) accept a *paraconsistent* notion of consequence: it is not the case that $A \wedge \neg A$ entails B, for arbitrary A and B. Paraconsistent logic usually defines a special atomic sentence, $\bot$, the *Church false constant*, which entails A for arbitrary A.[9] $\bot$ cannot be true: there can be no x such that $\bot$. So there is a world according to which there is an x such that $\bot$. If that world is genuine, it follows that there is (simpliciter) an x such that $\bot$, and hence that $\bot$ is true (simpliciter). But then, since $\bot$ entails A, it follows that A is true, for arbitrary A: *everything* is true. This is just the conclusion the dialethist wanted to avoid by introducing her paraconsistent logic. So the dialethist must take impossible worlds to be ersatz, not genuine.

Impossible worlds are ersatz worlds. We can either accept the parity thesis and take all worlds to be ersatz, or deny the parity thesis and take just the possible worlds to be genuine. Berto (2010) takes the latter option. He accepts an ontology of genuine Lewisian possible worlds in addition to ersatz impossible worlds. According to Berto's *hybrid modal realism*, atomic propositions are sets of genuine possible worlds. Ersatz worlds are sets of atomic propositions, i.e. sets of sets of genuine possible worlds.[10] There are two main problems with Berto's hybrid modal realism. First, it conflates intuitively distinct contents. 'Hesperus is F' is verified by exactly the same set of genuine possible worlds as 'Phosphorus is F', and so Berto's account assigns the same content to each sentence. This theory is then no help with certain Frege-problems, which require a notion of doxastic

9. See, e.g., Restall 2004.

10. Berto also appears to take complex propositions to be sets of sets of genuine possible worlds—at least, he takes a conjunctive proposition $\langle A \wedge B \rangle$ of atomic propositions $\langle A \rangle$ and $\langle B \rangle$ to be the set $\{\langle A \rangle, \langle B \rangle\}$ (2010, 482), which is identical to the ersatz world which says that A and that B (but no more). This is clearly not the best way for a hybrid modal realist to define propositions: what set should the disjunction of $\langle A \rangle$ and $\langle B \rangle$ be? Berto would be better off taking complex propositions to be sets of ersatz worlds, i.e. sets of sets of sets of genuine possible worlds.

content on which an agent can believe that Hesperus is F without thereby believing that Phosphorus is F.

Second, there are logical constraints on the way in which Berto's ersatz worlds represent. They cannot represent both that A and that B without thereby representing that $A \wedge B$, for example.[11] Yet this is precisely the kind of fine-grained content we require to make sense of certain beliefs. Subvaluationists about vagueness provide a case in point. Whereas supervaluationists take A to be true iff it is true on all precisifications, subvaluationists take A to be true iff it is true on at least some precisifications (Cobreros 2010). When A is indeterminate, therefore, they believe both A and $\neg A$; but they do not believe $A \wedge \neg A$, for this is true on no precisification.[12] To capture such beliefs, we'll need worlds which are strictly more fine-grained than those provided by Berto's hybrid modal realism.

These two worries are worries for Berto's development of hybrid modal realism, not for hybrid modal realism *per se*. They can be overcome by a hybrid modal realist if they can be overcome by a purely ersatz account of worlds, since a hybrid theorist can adopt all of those ersatz worlds (including ersatz impossible worlds) *in addition to* her genuine possible worlds. Below, I set out a theory of ersatz worlds for which these problems do not arise. There's nothing to stop a hybrid theorist adding those worlds to her account. (It's not as if actualists can take out a patent on certain set-theoretic constructions!) In doing so, she will obtain a theory quite different from Berto's. Propositions (and other notions of content) will be defined in terms of sets of ersatz worlds, which will in turn be constructed set-theoretically from the concrete possible worlds.[13] The concrete worlds provide the concrete building blocks, and they tell us which ersatz worlds count as representing genuine possibilities. But the concrete worlds do not play a direct role in constituting contents.[14]

Even with these concessions granted, I'm going to reject hybrid accounts and accept the parity thesis. On a hybrid theory, it makes sense to talk about a notion of *real possibility*, for which our world-quantifiers range over all the genuine possible worlds, but none of the ersatz ones. This—a non-circular, reductive account of 'real' possibility—is the very point of the hybrid account. But it is a puzzling notion. Logical consequence is suppose to hold between premises and conclusion just in case it is impossible for the premises to all be true but the conclusion false. If we have a single notion of real possibility available to us, we will want to use this to capture real logical entailment. The resulting notion of logical consequence, however, is very different from any of our usual notions, for it makes it a principle of logic that 'Hesperus is F' entails 'Phosphorus is F'.

11. Berto's ersatz worlds may represent that A, that B and that $\neg(A \wedge B)$. But the latter is not the same as *not* representing that $A \wedge B$.

12. As in the supervaluationist case, precisifications are classical models, which verify all tautologies and no contradictions. Whereas supervaluationist disjunctions can be true without either disjunct being true, subvaluationists hold that conjuncts can be true without their conjunction being true.

13. This account would follow the spirit, but not the latter, of Berto's theory.

14. Below, I'll argue that the *actual* world provides sufficient building blocks for all the ersatz worlds we need. If I'm right, merely possible concrete worlds come into the picture only in determining which ersatz worlds represent genuine possibilities. This is already to minimise severely the role of genuine worlds in the theory.

Consequence in this 'logic' depends on the content, rather than merely on the form, of premises and conclusion. This is not a formal logic in the usual sense.

Since I want to preserve genuine connections between (regular notions of) logical consequence and possibility, I'll accept the parity thesis and take all worlds (other than our own) to be ersatz worlds. Since the ontology itself does not distinguish between the possible and the impossible worlds, on this view, we are allowed a degree of pluralism in our use of 'possible', and a corresponding pluralism is our use of 'consequence' (which I take this to be a positive feature of the view: see Restall and Beall 2006).

In this section, I've argued against Yagisawa's extended modal realism and Berto's hybrid modal realism. Whilst an modified hybrid account could overcome the problems I raised for Berto, I accepted the parity thesis. Possible and impossible worlds alike are ersatz worlds. Now for the main feature: in the next section, I give a construction of ersatz worlds, based on a plenitudinous theory of facts.

4 Building Worlds from Facts

In this section, I first introduce a theory of facts.[15] I then use this theory in the construction of possible and impossible ersatz worlds. I'm going to work with an ontology of both positive and negative facts.[16] Negative facts earn their keep in many ways. They help to constitute material objects such as holes and edges; they play a role in causation-by-absence; they can be the objects of a perceptual experience (such as when the batsman sees a gap at extra cover); and they play an indispensable role in a maximalist truthmaking theory, which requires a truthmaker for each truth.[17] I assume, with Skyrms (1981) and Armstrong (2004), that facts are fundamental entities. We continue to individuate facts 'in the vulgar way', as (Skyrms 1981, 200) has it, in terms of particulars-and-properties: we'll talk of the fact *that Greg begged Meg*, for example. But particulars, properties and relations are *abstractions* from facts. Those entities exist in the fullest sense; but their existence is dependent on the existence of certain facts.

In addition to positive and negative atomic facts, I accept conjunctive, disjunctive, existential and universal facts. Conjunctive facts are just the mereological sums of their conjuncts: if facts α and β exist, then so does their mereological sum $\alpha \sqcup \beta$. I'll treat this as a conjunctive fact, which makes true any conjunction $A \wedge B$ where α makes A true and β makes B true. What about disjunctive facts? In the formal theory of abstraction of Jago 2010, we get disjunctive facts for free, as negative conjunctive facts. From a conjunctive fact $[[Fc]^- \sqcup [Gb]^-]^+$, we can abstract to the property $\lambda x[[Fx]^- \sqcup [Gb]^-]^+$. Now suppose a lacks this property:

15. I'll use 'fact' and 'state of affairs' interchangeably. By either term, I mean the substantial, non-linguistic entities in virtue of which truths are true.
16. This ontology is defended at length in Jago and Barker 2010; the technical details are given in detail in Jago 2010.
17. A qualification on the 'indispensability' claim: I'm assuming that a truthmaker for a proposition $\langle A \rangle$ must be such that, necessarily, $\langle A \rangle$ is true if that entity exists. Note that totality facts (Armstrong 2004; 1997) are themselves a kind of negative fact, so their purported viability doesn't detract from the claim.

then the fact $[(\lambda x[[Fx]^- \sqcup [Gb]^-]^+)a]^-$ exists. This fact exists precisely when a is F or b is G: it plays the role of the disjunctive fact *that either a is F or b is G*.[18]

As conjunctive facts are just mereological sums, they obey the semilattice properties of $\sqcup$, that is:

(1) $[\alpha \sqcup \beta]^+ = [\beta \sqcup \alpha]^+$.

(2) $[[\alpha \sqcup \beta]^+ \sqcup \gamma]^+ = [\alpha \sqcup [\gamma \sqcup \beta]^+]^+$.

(3) $[\alpha \sqcup \alpha]^+ = \alpha$.

Parallel principles for disjunctive facts are:[19]

(4) $[\alpha \sqcup \beta]^- = [\beta \sqcup \alpha]^-$.

(5) $[[\alpha \sqcup \beta]^+ \sqcup \gamma]^- = [\alpha \sqcup [\gamma \sqcup \beta]^+]^-$.[20]

(6) $[\alpha^{\pm} \sqcup \alpha^{\pm}]^- = \alpha^{\mp}$.[21]

To handle existential facts, I'll work with a higher-order property of *being instantiated*, and identify the fact *that something is F* with the fact *that F is instantiated*, $[\mathsf{In}\, F]^+$. Similarly, $[\mathsf{In}\, G]^-$ is the fact *that G isn't instantiated*, which will play the role of the negative existential fact *that nothing is G*. Universal facts are similar: the fact *that everything is F* is the fact *that nothing is non-F*, $[\mathsf{In}\, (\lambda x[Fx]^-)]^-$. The fact *that all Fs are Gs* is the fact that nothing has the property of *being F but not G*, $[\mathsf{In}\, (\lambda x[[Fx]^+ \sqcup [Gx]^-]^+)]^-$.

These facts might seem to be perfectly suited to play the role of sentences in our worldmaking language. But things are not quite so simple. We cannot build ersatz worlds directly from facts for the simple reason that there aren't enough *actual* facts to go around (and, as actualists, this entails that there aren't enough facts to go around simpliciter). If it's a fact that I'm sitting in the library, then it's not a fact that I'm lying in bed (it's not that kind of library). Yet I could have been lying in bed, and so we need a sentence in our worldmaking language to represent this possibility. Rather than building worlds directly from facts, I'll work with *ersatz facts*: set-theoretic representations of facts. Let the triple $(F, a, 1)$ represent the fact $[Fa]^+$ and the triple $(F, a, 0)$ represent the fact $[Fa]^-$. These ersatz facts contain the property *Fness* and the particular a themselves, rather than abstract representations of them.[22]

I'll set aside a specific set-theoretic entity, denoted '$\wedge$', for forming conjunctive ersatz facts. It has type $\mathit{fact} \longrightarrow (\mathit{fact} \longrightarrow \mathit{fact})$, so that, if x and y are ersatz facts,

18. Note that $[(\lambda x([[Fa]^- \sqcup [Gx]^-]^+)b]^-$ also exists precisely when either a is F or b is G. According to the theory of reduction set out in Jago 2010, both $\ulcorner \lambda x[[Fx]^- \sqcup [Gb]^-]^+ \urcorner$ and $\ulcorner [(\lambda x([[Fa]^- \sqcup [Gx]^-]^+)b]^- \urcorner$ reduce to $\ulcorner [[Fa]^- \sqcup [Gb]^-]^- \urcorner$. I take this to imply that all three terms denote one and the same fact.

19. (4–6) follow from (1–3) plus the theory of reduction-as-identity in Jago 2010.

20. Note that it isn't in general the case that $[[\alpha \sqcup \beta]^- \sqcup \gamma]^- = [\alpha \sqcup [\gamma \sqcup \beta]^-]^-$.

21. Here, '$\alpha^{\pm}$' and '$\alpha^{\mp}$' are schematic for pairs of terms which differ only in that one ends in '+', the other in '–'.

22. In effect, we've adopted Lewis's *lagadonian* approach (Lewis 1986), in letting each particular, property and relation name itself in our language.

$(\wedge x)$ is an ersatz property and $((\wedge x), y, 1)$ is an ersatz fact: the conjunction of x and y. The disjunction of x and y is $((\wedge \bar{x}), \bar{y}, 0)$, where $\overline{(x, y, 1)} = (x, y, 0)$ and $\overline{(x, y, 0)} = (x, y, 1)$. I'll adopt the convention of writing '$x \wedge y$' instead of '$((\wedge x), y, 1)$' and '$x \vee y$' instead of '$((\wedge \bar{x}), \bar{y}, 0)$'. There is thus a very tight relation of representation between ersatz facts and genuine facts. Let's define a function * from ersatz facts to fact-terms as follows, where neither 'z' nor 'v' has the official form '$\wedge x$':

$$\begin{aligned}
(x \wedge y)^* &= \ulcorner [x^* \sqcup y^*]^+ \urcorner \\
(x \vee y)^* &= \ulcorner [(\bar{x})^* \sqcup (\bar{y})^*]^- \urcorner \\
(z, v, 1)^* &= \ulcorner [z^* v^*]^+ \urcorner \\
(z, v, 0)^* &= \ulcorner [z^* v^*]^- \urcorner
\end{aligned}$$

Then an ersatz fact x says: $\ulcorner x^* \urcorner \frown \ulcorner \text{exists} \urcorner$.[23]

This gives us a very flexible worldmaking language, with one serious limitation. Since each entity is represented by itself, we have just one name for each actual particular. As a consequence, we will not be able to capture fine-grained epistemic or doxastic states (or content corresponding to Fregean senses). If Greg is Meg, then we will have just one name for Greg in our language, and hence we will not be able to distinguish between a belief that Greg likes eggs and a belief that Meg likes eggs. To rectify the issue, we will replace each name a in our language with denumerably many ordered pairs (a, n) for $n \in \mathbb{N}$, each of which is a name for (i.e., refers to) a. Ersatz facts still represent genuine facts in a very tight way: we need only add a base clause to our translation, for any name (a, n) in the language:

$$(a, n)^* = a.$$

Worlds are then sets of ersatz facts: every non-empty set of ersatz facts is a world. Since we are thinking of ersatz facts as sentences, we can classify worlds as we would usually classify sets of sentences in a first-order language. A world is *prime* iff it contains a disjunct of each disjunction it contains. A world is *maximal* iff it is prime and contains each instance of excluded middle. A world is *logically possible* (with respect to some logic) iff it is both maximal and consistent (with respect to that logic's derivability relation).

In this section, I presented a theory of genuine facts, and a language of ersatz facts which represent the genuine facts. Worlds were constructed as sets of ersatz facts. Even though worlds are not constructed directly from genuine facts, the theory of genuine facts I've presented benefits the construction in two ways. In section §6, I show how it gives rise to natural subspaces of worlds, which are useful for modelling notions of what is said. Before that, in §5, I use the theory of facts to address the problem of representing non-actual particulars and properties.

23. We can't define * from ersatz facts to genuine facts, since there aren't enough actual genuine facts. But, speaking loosely, the idea is that x represents the genuine fact x^* as existing.

5 The Problem of Aliens

Lewis (1986) discusses a deep problem for constructions of ersatz worlds of the kind I've just given: the problem of representing 'alien' particulars and properties. In this section, I'll show how the theory of facts from above allows us to avoid the problem. In the case of properties, the problem is this. Suppose that nothing is actually an F, but it could have been that something is an F. Then, the argument goes, the property of *Fness* does not actually exist, but it could have. An actualist then has no property of *Fness* within her worldmaking language, and so is unable to describe the possible situation in which something is F.[24]

The theory of facts presented above provides a neat solution to this worry. First, we need consider only the case in which *Fness*, in the envisioned possibility, is a fundamental property, i.e., an abstraction from a single fundamental fact. A non-fundamental property G is an abstraction from a conjunction of fundamental facts. If facts involving those fundamental properties exist, then so does their mereological sum, and hence so do all properties abstracted from it, including G. Hence the objection must be that there is a missing *fundamental* property F. Hence, if *Fness* does not actually exist, there is no fundamental fact $[Fx]^+$ (for any x).

I hold that that the world is complete with respect to the fundamental facts: that is, if it is a fundamental matter whether Fa, then either $[Fa]^+$ or $[Fa]^-$ exists.[25] Hence, for each actual particular x such that it is a fundamental matter whether x is F, the fact $[Fx]^-$ actually exists (for x isn't F). Since there are such xs, there actually exist such facts. It follows that $\lambda x[Fx]^-$, the property abstracted from this fact, actually exists too. This property, *lacking Fness*, names itself in our worldmaking language (as all properties do), and so we can form ersatz facts such as $(\lambda x[Fx]^-, a, 1)$ and $(\lambda x[Fx]^-, a, 0)$. The latter represents the genuine fact $[(\lambda x[Fx]^-)a]^-$ which, according to the theory of reduction and identity of Jago 2010, is identical to $[Fa]^+$, the fact that a is F.[26] This was just the possible state of affairs we wanted to represent.[27]

What about alien particulars, such as my merely possible older brother? A standard approach is to represent such possibilities using bundles of properties. I have a few worries about this approach. We want to represent merely possible entities as having at least some of their properties contingently: I could have had an older brother who could have had no brothers. So a merely possible particular will not be named by the same bundle at each world which represents that particular. It

24. She might try to *stipulate* that *Fness* actually exists after all, but at the risk of making property-talk completely obscure. Since *Fness* isn't actually instantiated, it cannot be an Aristotelian or Armstrongian universal, a trope or a set-of-possibilia.

25. If this were not the case—if there were fundamental 'fact gaps'—then some fundamental truths would lack a specific truthmaker. Since one of the main reasons for introducing positive and negative facts is to provide specific truthmakers for all all truths, this is an important tenet of the theory rather than an optional extra.

26. Since *non-Fness* is possessed by all and only those things which lack *Fness*, lacking *non-Fness* amounts to possessing *Fness*. The theory thus embodies a kind of double-negation elimination.

27. Note that this argument does not slip the alien property F through the back door: I'm not claiming that *Fness* actually exists after all. What the argument shows is that we can represent F-facts without having a word for *Fness*. Our word for *non-Fness* (which actually exists) will suffice.

will be named by many bundles, standing in counterpart (resemblance) relations to one another. Can such property bundles have concrete particulars as counterparts? If so, then the theory allows that I could have had an identical twin who could have been me, even though I couldn't have been him.[28] If not, then the theory allows that I could have had an identical twin distinct from me, who couldn't have been me, yet who could have had my haecceity.[29] Both consequences seem troubling.[30]

Because of worries such as this, I'd prefer to avoid counterpart theory entirely. We can do this by dealing with alien particulars in the same way we dealt with alien properties. As before, we need be concerned only with missing fundamental particulars. Suppose a is one of these. Then, by a similar argument to the one above, some fact $[Fa]^-$ actually exists. We abstract $\lambda X[Xa]^-$, a higher-order property possessed by all and only those properties not possessed by a. We let this property name itself; and we represent the possible situation in which a is F by the ersatz fact $(F, \lambda X[Xa]^-, 0)$.

The worldmaking language I've presented suffices for all our 'ordinary' hyperintensional needs. With regard to any a and F, it can represent both a's having and a's lacking *Fness*. For most contexts in which we are interested, this is sufficient. Yet we face a problem concerning 'advanced' contexts. Suppose a philosopher, after reading about how facts work, comes to have beliefs both about *Fness* and about *non-Fness*. But she is a confused individual: she believes that a lacks *non-Fness* without thereby believing that a is F.[31] To capture such doxastic states, we will need worlds according to which a lacks *non-Fness*, yet according to which a is not F.

One might try to build such worlds by enriching the worldmaking language with a sentential negation, '¬' (so that, if x is an ersatz fact, then so is $\neg x$). If x says that $[Fa]^+$ exists, then $\neg x$ says that $[Fa]^+$ does *not* exist (which is not the same as saying that $[Fa]^-$ exists). Unfortunately, this approach interferes with how conjunctions represent. If $\neg x \wedge \neg y$ says 'neither x^* nor y^* exists', then $x \wedge y$ will say 'both x^* and y^* exist'. But $x \wedge y$ is supposed to say 'the conjunctive fact $[x^* \sqcup y^*]^+$ exists'. Although these two statements go together at all the logically possible worlds, they may come apart at logically impossible worlds.[32] To make this approach work, we would need to restrict our worldmaking language so that '¬' appears only at the head of a sentence.

A neater (although equivalent) approach is to forget about the negation symbol

28. To see why, let bundle B name my merely possible identical twin. Since nothing in the actual world resembles B more than I do, I will be B's counterpart at the actual world. But B will not be a counterpart of mine, since I could not have been something which lacks my haecceity.

29. To see why, again let bundle B name my merely possible identical twin and let B' be the bundle just like B but also containing my haecceity. Given their close similarity, B' is a counterpart of B whereas I am not (given the assumption that bundles never have non-bundles as counterparts).

30. Perhaps a counterpart theorist will accept one or both of these consequences. In that case, she is free to adopt the picture I've described to far, with alien particulars represented by property bundles.

31. Perhaps one's beliefs simply *can't* be like that. But given our interest in hyperintensional contexts, I don't want to assume that the contents *possessing Fness* and *lacking non-Fness* will always align, in any context whatsoever.

32. It is impossible for a conjunctive fact to exist without its conjuncts existing; so that is exactly what some impossible worlds represent as being the case.

'¬' and, instead, construct *double worlds*. A double world w is a pair of sets of ersatz facts, (W^+, W^-). If $x \in W^+$, then w says 'fact x^* exists'. If $x \in W^-$, by contrast, then w says 'fact x^* does not exist'. For logically possible worlds, W^+ and W^- are mutually exclusive and jointly exhaustive of the worldmaking language, and $x \in W^+$ iff $\bar{x} \in W^-$. We relax this requirement for the logically impossible worlds.

In this section, I've used the theory of genuine facts from §4 to address Lewis's problem of alien properties and particulars. In the next section, I'll discuss the second advantage which the theory of facts brings to our construction of worlds, concerning notions of *what is said*.

6 Content and Same-Saying

The worldmaking language presented above allows for very fine-grained representations. Accordingly, we have a space of very fine-grained worlds at our disposal. For any two sentences A and B, we have a world which represents that A but not that B. However, as discussed in §2, some notions of content require certain logical constraints to be met. If one says that $A \wedge B$, for example, one thereby says that $B \wedge A$: the content asserted is identical; what alters is the way in which it is asserted. Given that our worlds are so fine-grained, can we use them to say anything interesting about such notions of content? In this final section, I show that we can.

Suppose, just for the moment, that we have at our disposal a space of genuine facts which includes merely possible as well as actual facts; and suppose we construct worlds from these facts. How would this space of worlds differ from the one we developed above? It would differ in two main respects. First, if $a = b$, then facts about a are identical to facts about b, i.e. $[Fa]^{\pm} = [Fb]^{\pm}$. So in the language build from genuine facts, we would have but one name for each particular (i.e., the particular itself). Therefore, we could not use the corresponding space of worlds to deal with Frege-problems. Second, genuine facts obey certain logical principles. The genuine fact *that* $A \wedge B$ is identical to the fact *that* $B \wedge A$, for example. Ersatz facts, by contrast, allow us to break these rules.

With actualist hat firmly back on, let's now focus on the space of worlds constructed from ersatz facts which behave *as if* they were constructed from genuine facts.[33] We do so by restricting our attention to those worlds which obey

33. For the actualist, constructing a space of worlds from genuine facts is a rather pointless exercise, since there's so many merely possible states of affairs she can't represent in that way.

the rules just mentioned, as follows:

$$x \wedge y \in w \text{ iff } x \in w \text{ and } y \in w \quad \text{(a)}$$
$$\neg(x \wedge y) \in w \text{ iff } \neg x \in w \text{ or } \neg y \in w \quad \text{(b)}$$
$$x \vee y \in w \text{ iff } x \in w \text{ or } y \in w \quad \text{(c)}$$
$$\neg(x \vee y) \in w \text{ iff } \neg x \in w \text{ and } \neg y \in w \quad \text{(d)}$$
$$\neg\neg x \in w \text{ iff } x \in w \quad \text{(e)}$$
$$x \in w \text{ iff } x\left[{}^{(a,n)}/_{(a,m)}\right] \in w.^{34} \quad \text{(f)}$$

At such worlds, conjunction and disjunction are associative, commutative and idempotent operators which distribute over one another and obey the De Morgan laws. Such worlds correspond to distributive lattices, but need not be full Boolean algebras. We do not have that $x \in w$ iff $\neg x \notin w$, so we allow that for some fact x, neither $x \in w$ nor $\neg x \in w$; and we allow that both $x \in w$ and $\neg x \in w$.

Call the space of all such worlds $\mathcal{S}_1$. Sets of $\mathcal{S}_1$-worlds provide a notion of content appropriate to many uses of 'proposition' and 'what is said' in the philosophical literature.[35] This is an *object-based* notion of content, in the sense that, if Greg is Meg, the $\mathcal{S}_1$-content of 'Greg is F' is identified with the $\mathcal{S}_1$-content of 'Meg is F'. Suppose Anna and Bob are arguing, Anna insisting that the person over there is Greg, whereas Bob insists that it's Meg. There's clearly a sense in which they're not really disagreeing at all, for they are both correctly identifying the person they see.[36] This is the notion of same-saying captured by sets of $\mathcal{S}_1$-worlds.

We have other notions of content available to us. Let $\mathcal{S}_2$ be the class of worlds which obey rules (a–e) above, but ignore rule (f). Such worlds may contain x without containing $x\left[{}^{(a,n)}/_{(a,m)}\right]$. They thus behave as if they were constructed from genuine facts, albeit individuated at the level of *sense*, rather than *reference*. Sets of $\mathcal{S}_2$-worlds give us a *competence-based* notion of content. Anna and Bob are competent uses of English; their disagreement over whether the person they see is Greg or Meg reflects their lack of empirical knowledge (that Greg is Meg), rather than a lack of competence in their uses of 'Greg' and 'Meg'. Their assertions are thus assigned distinct competence-based contents. In this sense, Anna and Bob say different things.

Both notions of content (and so both notions of same-saying) are hyperintensional: (classically) logically equivalent utterances may be used to say different things. We thus have notions of content which can make sense of an argument between, say, a classical and an intuitionistic mathematician, and which can distinguish between the content of dialethist approaches to the liar and arbitrary contradictory utterances. Nevertheless, certain logical equivalences are guaranteed to preserve both notions of content. In each of the following pairs, for example, each utterance is assigned the same (object-based or competence-based) content:

35. On this use, what is said is narrower than what is implicated (or conveyed) by an utterance. It is what Salmon (1991) calls *what is said* in the strict and philosophical sense, as opposed to the loose and popular sense.

36. They would have been disagreeing had their assertions been metalinguistic ones, 'that person is called 'Greg'' and 'that person is called 'Meg''. But that wasn't what they said.

(7) A: it's sunny and hot

B: it's hot and sunny

(8) A: Cath or Dave will go, and Ed will go

B: either Cath and Ed will go, or else Dave and Ed will

(9) A: either Cath doesn't like Dave or she doesn't like Ed

B: Cath doesn't like both Dave and Ed

This is just what we want, since each example is intuitively a case of same-saying.

In this section, I've argued that, even though we have a very fine-grained space of worlds, useful subspaces of worlds naturally fall out of it. Two such subspaces arise when we treat worlds as if they were constructed from genuine facts. These can be used to theorise about notions of what is said, or expressed, by an utterance. We thus have a theoretical handle on at least one of the main uses of 'proposition' in the literature. Depending on what phenomenon we are trying to capture, 'proposition' may pick out either a set of $\mathcal{S}_1$-worlds or a set of $\mathcal{S}_2$-worlds. Other uses of 'proposition' (and of 'content') require finer-grained contents.[37] I discuss such notions of content elsewhere (Jago 2008). These can be defined on our space of worlds; in fact, the theory given in Jago 2008 *requires* a space of worlds as fine-grained as the one constructed here. This isn't the place to discuss that theory; I mention it only to highlight how the space of worlds I've constructed can be used to capture diverse notions of content.

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37. In §3, I argued that belief ascriptions require very fine-grained contents—finer than the contents provided by $\mathcal{S}_1$ or $\mathcal{S}_2$. Compositional values must be at least this fine-grained (§2).

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