Truthmaker Account of Propositions

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1 Introduction

The *truthmaker account of propositions* identifies a proposition with the set of its possible truthmakers. That is, a proposition $\langle A \rangle$ is a set of possible entities, each of which makes it true that A (or would do, were it to exist). The proposition *that I am sitting* is the singleton containing the state of affairs *that I am sitting*. The proposition *that someone is sitting* is the set containing all possible states of affairs of the form *that x is sitting*. (Refinements and extensions of the view are offered in §2.)

The truthmaker account is a recent development and is primarily a metaphysical analysis of what propositions are, although it has antecedents in the technical literature on modal and relevant logics (§3). It can be seen as a generalisation of the possible worlds account (Lewis 1986; Stalnaker 1976b). On that view, a proposition $\langle A \rangle$ is a set of possible worlds and is *true at* a world w just in case $w \in \langle A \rangle$. On the truthmaker approach, a proposition $\langle A \rangle$ is a set of possible entities of any kind and is *made true by* an entity x just in case $x \in \langle A \rangle$. Then proposition $\langle A \rangle$ is true at world w just in case there is some entity $x \in \langle A \rangle$ which exists at w; and $\langle A \rangle$ is (actually) true just in case any of its members (actually) exist. Throughout, I'll use 'truthmakers' to mean possible (including actual) truthmakers. (I'll discuss how we might understand (merely) possible truthmakers in §7.5.)

To be viable as a metaphysical analysis of propositions, one must accept that there is at least one (possible) truthmaker available for each (possible) truth. This is the view known as *truthmaker maximalism*. Many prominent truthmaker theorists, including Armstrong (2004), Cameron (2008), Schaffer (2010), and Rodriguez-Pereyra (2005; 2006), accept the view (as do I: Jago 2018; Barker and Jago 2012). It faces metaphysical difficulties, as Molnar (2000) and others make clear (§7.1). One may, however, adopt a more instrumental stance: 'truthmaker' talk delivers a theory that's useful in logic and linguistics, and gives us a useful notion of a proposition; but we needn't take the ontology seriously. (Something like this is Fine's stance (p.c.).)

I'll begin by setting out the view a little more carefully (§2) and briefly discussing how it developed from previous accounts (§3). I'll then assess arguments in favour of the view (§4). Perhaps the most persuasive case is that the view has a number of useful logical and linguistic applications. I'll spend some time setting these out in §5 and §6. These are by no means all the applications: Fine (2017a;b; forthcoming) discusses more. Finally, in §7, I'll turn to some of the most pressing metaphysical issues raised by the approach.

Throughout, I'll occasionally compare the truthmaker approach to the *possible worlds* and *structured propositions* accounts. King (2012) gives a good overview of these other accounts.

2 Formulating the View

We shouldn't identify a proposition with the set of its actual truthmakers. This would mistakenly equate all false propositions with the empty set, and would equate a necessary truth $\langle A \vee \neg A \rangle$ with whichever is its true disjunct, which may well be contingent. A proposition must include merely possible, as well as actual, entities (just as the possible worlds account appeals to merely possible worlds). (Just how to make sense of this thought is not at all straightforward. I'll discuss some options in §7.5.) So the first refinement of the view is: a proposition is the set of all those entities which make it true, or would make it true, were they to exist.

That approach makes sense when each of a proposition's truthmakers is a single entity. But that isn't always the case. Propositions can be made true by pluralities. ⟨There are wombats⟩ is made true by each individual wombat, but also by pairs of wombats, triples of wombats, and, quite generally, by wombat pluralities of any size. A plurality of entities as a whole can be a truthmaker for a proposition. We need to respect the difference between pluralities and their members, else we would mistakenly equate ⟨there are at least three wombats⟩ with ⟨there are wombats⟩. We also need to account for propositions like ⟨there are wombats and platypuses⟩, made true by pluralities of different kinds of thing. In general, we need some way of 'coding' pluralities as members of propositions.

One approach codes pluralities through their mereological sums. ⟨There are wombats⟩ is then the set of all possible wombats and all possible wombats sums; ⟨there are at least three wombats⟩ is the set of all possible three-or-more wombat sums; and ⟨there are wombats and platypuses⟩ is the set of all possible wombat-and-platypus sums. One nice feature of this approach is that it allows us to make sense of *partial truth*, as in, ⟨there are wombats and platypuses⟩ is partly true when there exists a wombat but no platypus. (I'll say more in §6.2.)

Not every set counts as a proposition. Suppose that x is a possible truthmaker for $\langle A \rangle$ and that y is a full ground for x. (That means, roughly, that when y exists, x exists in virtue of y's existing. See Fine 2012 for the details.) Then y should count as a possible truthmaker for $\langle A \rangle$ too. So membership of a proposition should be closed under grounds of its members: if $x \in \langle A \rangle$ and y fully grounds x, then $x \in \langle A \rangle$ too.

Two further conditions are optional. We may require that propositions be upwards-closed with respect to mereological summation: if $x, y \in \langle A \rangle$ then $x \sqcup y \in \langle A \rangle$. (This allows propositions to include sums which don't exist at any possible world.) We may also want to ensure that propositions are *convex*: if $x, z \in \langle A \rangle$ and there's an 'in between' y, such that $x \sqsubseteq y \sqsubseteq z$, then $y \in \langle A \rangle$. (Fine (2017b) discusses convexity in relation to content; Fine and Jago (2018) discuss convexity in the context of truthmaker semantics.)

If a set satisfies the chosen conditions, it counts as a proposition. That allows many, many arbitrary sets to count as propositions. Take the closure of an arbitrary set, $\{x_1, \dots, x_n\}$, under the chosen conditions. This is the proposition *that* x_1, *or* …, *or* x_n *exists*. There may be additional ways to characterise it. In many cases, there will be multiple complimentary ways to characterise a given proposition.

We might want propositions to encode information about their possible falsemakers, in addition to information about their possible truthmakers. Such entities are pairs of sets of entities, the first set containing all possible truthmakers and the second containing all possible falsemakers of the proposition. Such entities are often called *double propositions*. (Fine (2017a;b; forthcoming) calls these 'bilateral' propositions. Mares (2004a;b) discusses double propositions in the context of relevant logic.) They are, in effect, pairs of the propositions under the previous *single* conception. A little notation here is useful. $|A|^+$ is the set of all possible entities which, were they to exist, would make it true that A. $|A|^-$ is the set of all possible entities which, were they to exist, would make it false that A. Then $|A|^+$ is the single proposition *that* A, $|A|^-$ is the single proposition *that* $\neg A$, and the pair $(|A|^+, |A|^-)$ is the double proposition *that* A. One nice feature here is that the negation of a double proposition $(|A|^+, |A|^-)$ is got by swapping its order: $(|A|^-, |A|^+)$.

3 Precursors

The idea that propositions can be thought of as sets of things relating to their truth goes back to the early days of modal logic, and particularly to the influence of Carnap and Montague. In possible worlds semantics, it is often useful to consider the set of possible worlds at which a sentence is true (often denoted '$|A|$'). Such sets were often called 'propositions' and are known to some as 'UCLA propositions' (particularly by those influenced by Carnap, Montague, and Kaplan at UCLA). The approach was adopted as a philosophical analysis of what propositions are by Lewis (1986) and Stalnaker (1976b), both of whom argued for realism (of different kinds) about possible worlds. I'll call this the *possible worlds* account of propositions.

The modal logic/UCLA influence was also felt in the development of semantics for relevant logic. According to Dunn and Hardegree (2001, 150), Anderson (one of the founders of relevant logic) was using the term 'UCLA proposition' in the mid-60s, and the concept was certainly in circulation at the advent of the standard Routley-Meyer semantics for relevant logics (Routley and Meyer 1972a;b; 1973). Their semantics uses the terminology of 'scenario' or 'set-up', rather than 'possible world', with the key difference being that scenarios are allowed to be incomplete and inconsistent (whereas possible worlds are not). Relative to an incomplete scenario, a sentence may be neither true nor false. An incomplete (but consistent) scenario may be thought of as a proper part of a possible world, and so something like a state of affairs. On this reading, a relevant logician's 'UCLA' conception of a proposition comes close to the truthmaker account. (The idea of a *double proposition* (§2) also has its origins in the relevant logic literature: see Mares 2004a;b.)

Another pertinent tradition is that of *situation semantics* (Barwise and

Etchemendy 1987; Barwise and Perry 1983; Barwise and Seligman 1997; Devlin 1991), developed as an alternative to possible worlds semantics. This approach 'construe[s] propositions as sets of world parts, rather than complete possible worlds' (Kratzer 2007). These 'world parts' are *situations*, which (under some interpretations) can be viewed as possible states of affairs. This is a very similar notion of a proposition to the one from relevant logic just discussed. (And indeed, there is plenty of interplay between situation semantics and semantics for relevant logic: see Mares 1996 and Restall 1995.) Moltmann (2015) develops a linguistic framework similar to situation semantics, but explicitly in terms of truthmakers. However, Moltmann (2003) rejects the use of propositions in the analysis of attitude reports (see §6.4 below).

The metaphysical literature on truthmaking began in earnest with Fox (1987) and Mulligan et al. (1984), and hit full stride with Armstrong (2004) and Rodriguez-Pereyra (2005; 2006). The introductions to Beebee and Dodd 2005 and Lowe and Rami 2009 are excellent overviews of the area. This literature says nothing about propositions as constructs from their truthmakers, as far as I know. Indeed, Armstrong (2004, 5) talks of truthmaking as a 'cross-categorial' relation holding between worldly entities and non-worldly propositions, which are 'abstractions … from beliefs, statements and so on' (Armstrong 2004, 13). A connection between truthmaking and relevant logic briefly appears in Restall 1996 and van Fraassen 1969, although there's no analysis of propositions there.

As far as I know, the first explicit analysis of propositions as sets of truthmakers appears in Fine 2015, and is continued in Fine 2017a;b; forthcoming. (Fine (2015) speaks of 'verifiers' and 'falsifiers', but is clear that his key semantic notions are 'making true' and 'making false'.) Fine's focus is on logical and linguistic aspects of the idea. Jago (2017; 2018) discusses metaphysical aspects of the approach and formulates the first (as far as I know) positive arguments in favour of the view. The remainder of this article draws heavily on ideas in Fine 2017a;b; forthcoming and Jago 2017; 2018.

4 Arguments for the View

Arguments for a particular theory of propositions are often pragmatic in nature. They set out a number of jobs a good theory should be able to do, then argue that only the preferred theory can do those jobs (or perhaps, that it does them better than the others). The truthmaker account is no exception. Perhaps the strongest argument for it is based on what it can do. I review those features in §5 and §6. The argument is then that all these useful applications fall very naturally out of the truthmaker conception of a proposition, but not out of other theories.

I'll now discuss two arguments which purport to show that the truthmaker approach is the correct account of propositions, independently of its useful applications. The first focuses on the notion of *truth conditions*. In brief, it goes as follows.

(1) Propositions are truth conditions.

(2) Truth conditions are sets of possible truthmakers.

(3) So, propositions are sets of possible truthmakers.

Premise (1) is usually offered as an analytic truth about what propositions are. Many theorists would accept it. (2), by contrast, is highly controversial. We might approach it by first asking what a *condition* is. One approach takes a condition to be the entity expressed by a that-clause in a construction such as 'on condition that you do your homework first'. But this is unhelpful, since we take that-clauses to denote propositions. Then (1) says that propositions are propositions and (2) says that propositions are sets of possible truthmakers, the very thesis under consideration. On this reading, the argument makes no progress in establishing the conclusion.

On a different understanding, a condition is something which differentiates between the situations in which the condition is met and those in which it is not. More precisely, it is a function taking possible situations as input and outputting 'yes' or 'no' (or 'met' or 'not met', or '1' or '0', or something similar). The condition *that you do your homework*, for example, outputs 'yes' to all and only those situations in which you do your homework. Assuming classical logic, this function is equivalent to the set of situations for which the function answers 'yes' (since the 'no' situations are then given by the set-theoretic complement of that set). So we arrive at the view that conditions, and hence propositions, are sets of possible situations.

The remaining question is: which notion of situation is appropriate when we speak of *truth* conditions? One option is that they are possible worlds. This delivers the traditional possible worlds theory of propositions. But we might instead opt for a more fine-grained notion of situations, aligning them with states of affairs. That move supports (2). In favour of it, one might claim that the condition *that you do your homework* is not the same condition as *that you do your homework and* $1 + 1 = 2$, and hence that conditions are not sets of possible worlds. The claim is certainly intuitive but, whilst the notion of a condition is up for debate, it is hard to see why a defender of the possible worlds approach would accept it. So ultimately, this line of argument may be inconclusive.

We could instead change (1) to: propositions are truth*maker* conditions, as Jago (2017) does. Then (3) follows quickly, given the argument identifying conditions with sets of possible entities. But this revised premise doesn't enjoy the intuitive appeal that the original (1) does, and is hard to justify as being analytic of what we mean by 'proposition'. Alternatively, we could weaken (2) and (3) to say that truth conditions (and hence propositions) are sets, either of possible worlds or of possible states of affairs. We could then use an additional argument to rule out the possible worlds view of propositions, allowing us to infer that propositions are sets of possible states of affairs. The additional argument might be that there are distinct but necessarily equivalent propositions (see §5.1) and hence that propositions are not sets of possible worlds.

I promised a second argument in favour of the truthmaker approach to propositions. Here it is (adapted from Jago 2018):

(4) Propositions are entities which, by their very nature, are true or false.

(5) To be true just is to be made true; and to be false just is to be made false.

(6) So, propositions are entities which, by their very nature, are made true or made false.

(7) Propositions are sets.

(8) The nature of a set is given by its membership.

(9) So, the members of a proposition are its truthmakers and falsemakers.

This argument is more complex and controversial that the previous one. The best I can do in limited space here is indicate why someone might accept the premises. (4) is meant to be justified by our characterisation of propositions as those entities which are the primary bearers of the truth-values. This is a *real definition* of *proposition*, and real definitions are supposed to tell us about the nature of the relevant entity (Fine 1994; Lowe 2012). This is relatively uncontroversial, for those who accept talk in those terms.

Premise (5) is far more controversial. It might be understood as the claim that the property *being true* is identical to the property of *being made true by something* (or, equivalently, of *having a truthmaker*). Jago (2018) argues in favour of this property identification (given prior acceptance of truthmaker maximalism: §7.1). The inference from (4) and (5) to (6) has the form: x is by its nature F; $F = G$; therefore, x is by its nature G. That seems to be valid, given that 'nature' claims relate to properties, rather than modes of presentation of those properties.

Premise (7) may be justified in a number of ways. One is that propositions are either collections or abstract structures; collections are sets; and abstract structures are best understood as sets. (There isn't room here to evaluate these claims.) Premise (8) is much less controversial. It is often taken to capture the intuitive thought that sets are in some sense *defined* by their members, and is used to justify the commonly held view that sets have their members of necessity. Finally, the inference to (9) takes a little more work. (7) and (8) imply that the nature of a proposition is given by its membership; (6) implies that the nature of a proposition is to be made true or made false. The suggestion is that the only way to reconcile these claims is to identify membership of a proposition with the entities that make it true or make it false (Jago 2018, §8.5).

5 Logical Features

In this section and the next, I will briefly describe some of the advantages and applications of the truthmaker approach. I've grouped these topics under 'logical' and 'linguistic' headings, but don't read much into this.

5.1 Hyperintensionality

A concept is *hyperintensional* when it can differentiate between logically equivalent contents. *Belief* is hyperintensional, for example, for one may believe that A whilst not believing that $A \wedge T$, where 'T' is some horrendously complex logical theorem.

Being the proposition that is also a hyperintensional concept, for

(10) ⟨Puss is stretching $\vee$ $\neg$Puss is stretching⟩

(11) ⟨Bertie is barking $\vee$ $\neg$Bertie is barking⟩

are distinct but logically equivalent propositions. (10) is made true by a state of affairs involving Puss, not Bertie, whereas (11) is made true by a state of affairs involving Bertie, not Puss. They have different truthmakers and so, quite independently of our account of propositions, they are distinct. The challenge for any theory of propositions is to deliver a hyperintensional notion.

The truthmaker approach delivers a hyperintensional account, for the reason just given: (10) and (11) differ in their possible truthmakers and so are correctly treated as distinct propositions. (For the record: Russellian and Fregean structural accounts also do well on this score, whereas the possible worlds account fares badly.)

5.2 Exact and inexact truthmaking

An account of propositions should tell us when the proposition $\langle A\rangle$ is identical to the proposition $\langle B\rangle$. In particular, we would like to know what logical operations on a proposition (or on the sentence that expresses it) will preserve the identity of the proposition, and which will result in a new proposition. The possible worlds account, for example, tells us that any operation which preserves logical equivalence – such as changing the order of a conjunction's conjuncts – will not affect the identity of the proposition expressed. What does the truthmaker approach say on this score?

The truthmaker approach says: $\langle A\rangle$ is identical to $\langle B\rangle$ when they have the same possible truthmakers. Equivalently, we can say: they are identical when A is equivalent to B in the *logic of truthmaking*. There are several logics of truthmaking, however, based around two contrasting notions of truthmaking: the *exact* and the *inexact*. (The terminology is Fine's (2014). Jago (2018) discusses the relationship between different logics of truthmaking.) To be an exact truthmaker for a proposition is to suffice for its truth without any irrelevant overshoot. The exact truthmaker for ⟨Anna is knitting⟩ is the state of affairs *that Anna is knitting*. The more inclusive state of affairs, *that Anna is knitting and Bertie is snuffling*, contains more of the world than is needed to make ⟨Anna is knitting⟩ true. (That's the 'irrelevant overshoot'.) But it does suffice for that truth, since it contains the exact truthmaker *that Anna is knitting*. It is an inexact truthmaker for ⟨Anna is knitting⟩.

Inexact truthmaking obeys the *heredity* principle:

(12) If x is an inexact truthmaker for $\langle A \rangle$ and x is a part of y, then y is an inexact truthmaker for $\langle A \rangle$, too.

As a consequence, if $\langle A \rangle$ is true at world w, then w counts as an inexact truthmaker for $\langle A \rangle$.

Propositions can be defined relative to either the exact or the inexact notion of truthmaking. Both approaches guarantee the following identities:

$$
\begin{array}{cc}
\langle A \wedge B \rangle = \langle B \wedge A \rangle & \langle A \vee B \rangle = \langle B \vee A \rangle \\
\langle A \wedge (B \wedge C) \rangle = \langle (A \wedge B) \wedge C \rangle & \langle A \vee (B \vee C) \rangle = \langle (A \vee B) \vee C \rangle \\
\langle A \wedge (B \vee C) \rangle = \langle (A \wedge B) \vee (A \wedge C) \rangle & \langle A \wedge A \rangle = \langle A \vee A \rangle = \langle \neg\neg A \rangle = \langle A \rangle \\
\langle \neg (A \wedge B) \rangle = \langle \neg A \vee \neg B \rangle & \langle \neg (A \vee B) \rangle = \langle \neg A \wedge \neg B \rangle
\end{array}
$$

In addition, formulating propositions in terms of inexact truthmaking adds the following identities:

$$\langle A \vee (A \wedge B) \rangle = \langle A \rangle = \langle A \wedge (A \vee B) \rangle \qquad \langle A \vee (B \wedge C) \rangle = \langle (A \vee B) \wedge (A \vee C) \rangle$$

On the inexact notion, these identity principles correspond to logical equivalence in the system of *first degree entailment* (Anderson and Belnap 1963). On the exact notion, they correspond to logical equivalence in *exact truthmaker logic* (Fine and Jago 2018).

One may prefer just one of these notions, on logical or metaphysical grounds. Rodriguez-Pereyra (2006) argues that the truthmaking relation, by definition, relates a proposition to the entity *in virtue of which* it is true; and that *true in virtue of* supports the exact but not the inexact notion of truthmaking. Fine (2014; 2016) and Jago (2018) take the more concessive line and admit both notions. They understand the inexact notion in terms of the exact one: to truthmake $\langle A \rangle$ inexactly is to have as a part an exact truthmaker for $\langle A \rangle$.

6 Linguistic Features

6.1 Same-saying

Two speakers can say the same thing as one another in different ways. During the film, you say, 'this film is great'. Later, I say, 'that film was great'; someone else says, 'I agree'; someone else, 'what they said'. We all say the same thing, *that the film in question is great*, but we say it in different ways. A good theory of propositions should make good predictions about when speakers use different utterances say the same thing, by understanding *same-saying* in terms of *expressing the same proposition*. The truthmaker approach does well on this count (and certainly better than the possible worlds, Russellian, and Fregean accounts of propositions).

Same-saying is a hyperintensional concept (§5.1). Logically or mathematically equivalent utterances may be used to say different things, as in this example:

(13a) I can colour in any map with just three colours, so that no two adjacent areas have the same colour;

(13b) I can take one lemon and one orange, and thereby end up with three more fruits than I started.

Each speaker claims to be able to do *different* (and, unbeknownst to them, mathematically impossible) things. They're not saying the same thing. The same holds of logical examples:

(14a) The Liar is both true and false;

(14b) Claims about large cardinal numbers are neither true nor false.

The truthmaker approach gets the right results here: it distinguishes (a) from (b) in each pair. (The possible worlds approach, by contrast, will treat all four utterances as expressing the very same proposition.)

In the following pairs, by contrast, the truthmaker approach predicts that (a) and (b) say the same thing:

(15a) It's cold and wet;

(15b) It's wet and cold.

(16a) Cath or Dave will turn up, and Ed will turn up;

(16b) Either Cath and Ed will turn up, or else Dave and Ed will.

(17a) Either Cath doesn't like Dave or she doesn't like Ed;

(17b) Cath doesn't like both Dave and Ed.

Again, this seems to be the right result, as these pairs are intuitively clear cases of same-saying. Russellian and Fregean structural approaches get the opposite result. Since (a) and (b) in each pair differ syntactically, structural approaches treat (a) and (b) as expressing different propositions and hence as saying something different.

6.2 Partial truth

Speakers can be partly right (and partly wrong) by saying something that's partly true (but partly false). How should we analyse this concept of partial truth? The truthmaker approach offers a simple and elegant answer. Since a proposition's (full) truth consists in the existence of a truthmaker, its partial truth consists in the existence of a (proper) part of a truthmaker without a whole truthmaker. (Fine (2016) gives an alternative account in terms of *analytic containment*, based on Angell 1989.)

Recall from §2 how pluralities of truthmakers are coded as mereological sums. ⟨There are wombats and platypuses⟩ is the set of all possible wombat-and-platypus sums. Similarly,

(18) ⟨There are wombats and talking donkeys⟩

is the set of all possible wombat-and-talking donkey sums. In our world, there are wombats but no talking donkeys and so no wombat-and-talking donkey sums. So (18) is partly but not fully true. For although it lacks a truthmaker, the wombat-part of a truthmaker exists. In the same way, if we understand

(19) There are talking donkeys

conjunctively, as saying that there are things that talk and are donkeys, then our theory says it is partly true. But we are not thereby forced to say that it is partly true that Trump is a great president, on the basis that he is president. For in general, 'great *F*' does not mean 'is great and is *F*'. The latter is partly true of any *F*, but 'great *F*' may not be (as in the example).

To get the best results from this approach, we should understand states of affairs as mereological atoms. Although Anna and *sitting* are constituents of the state of affairs *that Anna is sitting*, they are not mereological parts of it, for both constituents may exist, even when Anna is not sitting (Armstrong 1997, 115). So Anna's existence, on its own, does not make it partly true that Anna is sitting. If she's standing, it's wholly false that Anna is sitting.

6.3 Aboutness and subject matter

The truthmaker approach allows for a neat characterisation of a proposition's *subject matter*, or what it is about Osorio-Kupferblum (2016); Yablo (2014). We might characterise ⟨Bertie is snuffling⟩ as being about Bertie and *snuffling*, or we might characterise it as being about whether Bertie is snuffling. (I take these to be distinct but complementary ways of talking about *aboutness*.) I will take the latter notion to be primary and the former to be understood in terms of it. As a first approximation, we might understand what a proposition is about in terms of the states of affairs that are its members. We then define the objects and properties it is about – Bertie and *snuffling*, in our example – as those that appear as constituents of any of those states of affairs. However, this will give us the strange result that $\langle A\rangle$ and $\langle \neg A\rangle$ have different and incompatible subject matters (since the possible truthmakers for $\langle A\rangle$ and $\langle \neg A\rangle$ do not overlap). This is the wrong result: $\langle A\rangle$ and $\langle \neg A\rangle$ are incompatible precisely because they say opposite things about the *same* subject matter.

We improve matters by taking the subject matter of $\langle A\rangle$ to be the set of all its possible truthmakers and falsemakers: $|A|^+ \cup |A|^-$. (If we are working with double propositions, we obtain $\langle A\rangle$'s subject matter by 'flattening' $\langle A\rangle$ into a single set, $|A|^+ \cup |A|^-$.) This approach gives the correct results for negation: $\langle A\rangle$ and $\langle \neg A\rangle$ coincide on their subject-matter. But it allows that $\langle A \wedge B\rangle$ and $\langle A \vee B\rangle$ can have different subject matters. They differ in their truthmakers (and falsemakers) because conjunction pairwise sums together elements from $|A|^+$ and $|B|^+$, whereas disjunction takes their union, $|A|^+ \cup |B|^+$. This gives incorrect results for subject matter: both $\langle A \wedge B\rangle$ and $\langle A \vee B\rangle$ are about whatever $\langle A\rangle$ is about, plus whatever $\langle B\rangle$ is about. They differ in what they say about that subject matter, but not in the subject matter itself. To avoid this consequence, we can take a subject matter to be the sum of a proposition's truthmakers and falsemakers, $\bigsqcup(|A|^+ \cup |A|^-)$. (Here, $\bigsqcup X$ is the mereological sum of all entities in X.)

We can talk of the subject matter of things other that propositions: speeches, stories, historical accounts, and philosophical papers all have a subject matter. We can understand their subject matter as the sum of the subject matters of all the propositions they express. Or we could think in terms of the conjunction of all the propositions they express, and take their subject matter to be the subject matter of that giant conjunction. We get the same result either way: the subject matter of a conjunction is identical to the summed subject matters of the conjuncts.

6.4 Attitude reports

One application for which the truthmaker approach is (probably) *not* suitable is the analysis of attitude reports. A truthmaker, as commonly understood, is a worldly entity such as a state of affairs. Truthmakers do not, in general, involve modes of presentation. What makes it true that George Eliot wrote *Middlemarch* also makes it true that Mary Anne Evans wrote *Middlemarch*, for they were one and the same person. So (assuming the necessity of identity), the proposition that George Eliot wrote *Middlemarch* will be identical to the proposition that Mary Anne Evans wrote *Middlemarch*, according to the truthmaker approach. Yet one can believe that George Eliot wrote *Middlemarch* without believing that Mary Anne Evans wrote *Middlemarch*. It seems that attitude reports are beyond the scope of the truthmaker approach. (The Russellian and possible worlds approaches do no better, whereas the Fregean approach was designed specifically to handle cases like this.)

One response is that attitude ascriptions are best analysed, not in terms of relations to propositions, but as part of a worlds-based theory of epistemic states which uses impossible as well as possible world (Jago 2014a;b). Moltmann (2003) offers another approach.

7 Metaphysical Issues

In this section, I survey five metaphysical issues faced by the truthmaker approach.

7.1 Truthmaker maximalism

If propositions are sets of their possible truthmakers, then all true propositions must have truthmakers. This is *truthmaker maximalism*. Without it, we must surrender the analysis of a proposition's truth in terms of its membership; and worse, we would mistakenly conflate distinct propositions. (I'll explain why shortly.) Maximalism is a hard thesis to defend, in light of 'negative' truths, such as, ⟨there are no penguins in my attic⟩. Molnar (2000) argues that no metaphysically respectable entity can act as a truthmaker for such truths. If he's right, then all such 'negative' propositions *necessarily* lack truthmakers, and so the truthmaker account mistakenly treats them all as the same proposition. That's why the truthmaker account requires maximalism. Fortunately for the truthmaker approach, Armstrong (2004), Barker and Jago (2012), Cameron (2008), Lewis and Rosen (2003), and Schaffer (2010) all offer plausible ontologies which support maximalism. Jago (2013) evaluates the relative merits of those approaches to the problem.

7.2 Distinct necessary falsehoods

When $\langle A\rangle$ and $\langle B\rangle$ are distinct propositions, we want to distinguish between the necessarily false propositions $\langle A \wedge \neg A\rangle$ and $\langle B \wedge \neg B\rangle$. We can't do this by identifying propositions with sets of possible truthmakers, since these propositions don't have any. This is the problem of distinguishing between necessary falsehoods.

One solution is to adopt double propositions (§2) for, in general, $\langle A \wedge \neg A\rangle$ and $\langle B \wedge \neg B\rangle$ will differ in their possible falsemakers. A falsemaker for $\langle A \wedge \neg A\rangle$ is whatever truthmakes either $\langle A\rangle$ or $\langle \neg A\rangle$ (or both), and similarly for $\langle B \wedge \neg B\rangle$. But double propositions have problems of their own (§7.3) and so it is worth investigating what can be done with single propositions. One approach is to allow impossible states into the ontology. Suppose our ontology includes all possible states and that those states are closed under mereological summation. Then, for some contingent A, there will be states *that* A and *that* $\neg A$ and hence their sum, which I take to the the state *that* $A \wedge \neg A$. So this approach naturally delivers impossible states. (We needn't infer that contradictions can be true. For if *that* A and *that* $\neg A$ never appear at the same possible world, then neither will *that* $A \wedge \neg A$.)

Other impossible cases are not explained so easily, however. A truthmaker for the necessarily false proposition $\langle 1 = 2\rangle$ is not naturally understood as the sum of two possible states. On the double proposition approach, we might take the numbers 1 and 2, together, as the falsemaker for $\langle 1 = 2\rangle$. Then $\langle 1 = 2\rangle$ can be distinguished from, say, $\langle 3 = 4\rangle$, since they will have different falsemakers. But on the single proposition approach, propositions must be constructed from truthmakers. What would an impossible truthmaker for $\langle 1 = 2\rangle$ look like? Must we accept primitive yet impossible states in our ontology? That seems unappealing.

It might seem that the solution is to adopt double propositions. But they have problems of their own, as we'll now see.

7.3 The polarity problem for double propositions

A double proposition is an ordered pair of a set of truthmakers and a set of falsemakers. But in which order do these sets come? Is the proposition $\langle A\rangle$ the ordered pair $(|A|^+, |A|^-)$, or is it $(|A|^-, |A|^+)$? For the purposes of doing semantics, either approach is fine and we may simply stipulate one of these to be the correct approach. But if we ask the metaphysical question, 'what *are* double propositions?', we should have some reason for preferring one approach over the other. The problem is that there are no such reasons to be had.

Consider the pair, $(\{$*that Bertie is snuffling*$\}, \{$*that Bertie isn't snuffling*$\})$. Is this the double proposition *that Bertie is snuffling*? Or is it *that Bertie isn't* *snuffling*? There cannot be any intrinsic differences in the composition of those sets to mark the difference, for the negation of a proposition $\langle A \rangle$ consists in those very same sets, $|A|^+$ and $|A|^-$, but with the order switched: $|\neg A|^+ = |A|^-$ and $|\neg A|^- = |A|^+$. So it seems we need to stipulate which set in the pair comes first, the truthmakers or the falsemakers. Yet there's nothing in the nature of propositions, or in the nature of *truth*, which dictates any priority between truth and falsity. The problem seems to be insoluble. Those who want to accept double propositions must bite the bullet.

7.4 Expressing all the truths

Both single and double propositions face a difficulty with propositions such as:

(20) ⟨Propositions exist⟩

(21) ⟨Sets exist⟩

One might expect the truthmakers for (20) to be all propositions, and truthmakers for (21) to be all sets. Indeed, that result falls out of a general principle: existential truths are made true by the truthmakers for their instances. But this is incompatible with (20) and (21) themselves being sets (or pairs of sets), for no set can contain itself. Moreover, (21) would have to contain all sets, which no set can.

The first worry can be overcome by adopting non-well founded set theory (Aczel 1988), which allows sets to contain themselves as members. But this doesn't help with the second problem: the collection of truthmakers for (21) is 'too big' to be a set. Taking propositions to be classes (which may contain all sets) won't help, since the problem is then faced by the proposition ⟨classes exist⟩.

One may reply that these problems occur already in our theories of quantification. Quantified sentences like

(22) All sets have $\varnothing$ as a subset

require a domain of quantification – a set – which contains all the entities quantified over by those truths. On the face of it, (22) quantifies over all sets. For that to be so, all sets (including the domain of quantification itself) must be included in the domain of quantification, which is ruled out both by the ban on self-membership and by the non-existence of a set of all sets. These issues are deeply puzzling. One line of thought is that whatever semantic mechanism is at work in (22) can be used to understand propositions like (20) and (21).

7.5 What are merely possible truthmakers?

I've claimed that propositions are sets, or pairs of sets, of possible (and perhaps impossible) entities. Typically, these entities are states of affairs. On pain on contradiction, not all of those states of affairs can obtain at once. But what on earth is a state of affairs that does not obtain? Here are three potential options.

Option 1: There exist merely possible concrete states of affairs, making up other possible worlds. 'Obtaining' (relative to world w) means existing at (as part of) world w. Non-obtaining states of affairs (relative to our world) are otherworldly states of affairs.

Option 2: Some states of affairs do not exist (but remain legitimate objects of quantification). The obtaining states of affairs are those that exist.

Option 3: There exist 'ersatz' states of affairs, in addition to the concrete ones. An ersatz state of affairs obtains when it corresponds to some concrete state of affairs.

These approaches are modelled on the main options in the metaphysics of possible worlds. The first takes its cue from the genuine modal realism of Lewis (1986), McDaniel (2004), and Yagisawa (2010). On this approach, all possible worlds are ontologically on a par with our own. The second is a broadly Meinongian approach, defended (in the case of worlds) by Priest (2005). The third approach is based on ersatz modal realism (Adams 1974; Stalnaker 1976a), on which possible worlds other than our own are actually existing ersatz representations.

The genuine realist approach (option 1) seems to say that reality is inconsistent. It says that the actual states of affairs *that Anna is sitting* and there merely possible one *that Anna is not sitting* both exist. If the existence of a state of affairs makes the corresponding proposition true, the contradictory propositions ⟨Anna is sitting⟩ and ⟨Anna is not sitting⟩ are both true, and chaos ensues. To be viable, the genuine realist approach must index states of affairs to possible worlds. Reality can consistently contain the states of affairs *that Anna is sitting at* w_1 and *that Anna is not sitting at* w_2. But then it becomes hard to state the thesis, that all possible states of affairs exist. For that thesis is not indexed to any possible world. It is supposed to be a truth about all possible worlds at once. The issues here are similar to the *problem of advanced modalizing* for genuine realism about possible worlds (Divers 1999; Jago 2016).

Option 2 allows that some entities do not exist. On this broadly Meinongian view, it makes sense to talk about and quantify over entities which lack existence. The suggestion is that merely possible states of affairs be placed in this category. To avoid the inconsistency problem above, the Meinongian will say that only existing states of affairs make a proposition true. Although both *that Anna is sitting* and *that Anna is not sitting* are real, only the former exists, and so ⟨Anna is sitting⟩ is true and ⟨Anna is not sitting⟩ is false. An unfortunate consequence of this move is that it makes the view false. If only existing entities act as truthmakers, then ⟨some states of affairs do not exist⟩ has no truthmaker and so is false. That's deeply problematic for this approach.

Ersatz states of affairs (option 3) merely *represent* real states of affairs. They themselves do not constitute something's being the case and they do not make propositions true (other than propositions about the existence of ersatz states of affairs). So they avoid the worries for options 1 and 2. However, since ersatz states of affairs may exist without corresponding to any real state of affairs, they seem to be very similar to propositions. Perhaps we can understand the ersatz state of affairs *that Fa* as the ordered pair containing F and a themselves, in that order (and similarly for more complex states of affairs). But such entities are (or are very similar to) *Russellian structured propositions* (King 1995; 1996; Salmon 1986; 2005). It seems problematic to give a theory of propositions in terms of entities which, according to a rival theory, are themselves propositions. There are further issues with this approach, discussed in Jago 2017.

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