Conceptual and Metaphysical Origins and Relevance of Temporal Logic

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Abstract— **Logic has sometimes been seen as an alternative to metaphysics and to speculation. In this paper it is argued that a different story should be told when it comes to temporal logic and tense-logic in particular. A.N. Prior's first formulation of tense logic was mainly established in order to qualify the discussion of certain metaphysical and conceptual problems. Although temporal logic has now been developed in various abstract and rather technical ways, it may still serve as a great help for anyone who wants to clarify the discussion of important existential questions like the nature of time, determinism, future contingency or freedom of choice.**

Index Terms— **Temporal logic, tense logic, time, metaphysics, determinism, A.N. Prior.**

I. INTRODUCTION

A.N. Prior (1914-69) was the founding father of modern temporal logic (conceived as the idea of integrating time into logic). His original idea of a tense-logic can be dated back to 1953. According to this idea the tenses of statements (past, present, future) should be taken into serious account in logic. Actually, he argued that tense-logic is the best way of doing temporal logic. Prior continuously worked with the further development of this idea up to his death.

In the present paper we discuss what motivated Prior and why he found temporal logic extremely important. This means that the approach is mainly historical. However, we also argue that the problems that inspired Prior are still relevant.

Prior emphasized that temporal logic offers a deeper understanding of certain metaphysical and philosophical problems. In section II we consider this metaphysical foundation of temporal logic. In fact, he found two topics particularly interesting, namely firstly the tension between the doctrines of divine foreknowledge and human freedom, and secondly the problems regarding the nature of time and existence. We discuss these topics in sections III and IV respectively. In the concluding sections V and VI we consider current temporal logic and the relevance of metaphysics.

II. THE METAPHYSICAL FOUNDATIONS OF TEMPORAL LOGIC

The logic of Russell and Whitehead gave logicians in the first half of the 20th century what was seen as an effective tool for dissolving pseudo-problems in philosophy and created an exuberating atmosphere of hope that all philosophical problems would either be solved by logicians or shown by the same to be based on some nonsensical notions. Nowhere were the hopes as high as in Wittgenstein's "Tractatus" [41] from 1922, in which a model for dealing with philosophical problems was introduced:

> The correct method in philosophy would be the following: to say nothing except what can be said, i.e. propositions of natural science – i.e. something that has nothing to do with philosophy – and then, whenever someone else wanted to say something metaphysical, to demonstrate to him that he had failed to give a meaning to certain signs in his propositions. [41, p. 95]

It is indeed a unique and important role of logic, to be able to dissolve pseudo-philosophical problems, but it was, as already hinted by Russell in his introduction to "Tractatus", too ambitious of Wittgenstein to hope that metaphysics as such could be reduced to nonsense. Indeed, a story to the contrary must be told of the relationship between logic and metaphysics. This story is most evident in the invention of tense logic as it was first formulated by A.N. Prior (1914-69) in the beginning of the 1950s.

For Prior, the role of the logician was not to do away with metaphysics; it was rather to provide the metaphysician with the tools he needed in order to analyse metaphysical and other conceptual problems in a precise manner. In his own words:

> The logician must be rather like a lawyer – not in Toulmin's sense, that of reasoning less rigorously than a mathematician – but in the sense that he is there to give the metaphysician, perhaps even the physicist, the tense-logic that he wants, provided it be consistent. He must tell his client what the consequences of a given choice will be ... and what alternatives are open to him; but doubt whether he can, qua logician, do more. [26, p. 59]

It is obvious from this statement that Prior certainly did not see logic as a tool to be rid of metaphysics, but at least his tense-logic should rather be seen as service to the metaphysician in order to qualify his investigations and

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discussions. According to Prior metaphysics is not outside the scope of logic. In his opinion, even accounts of metaphysical issues should respect basic standards of logic. First of all, such accounts should be presented in a consistent manner.

Prior invented his tense-logic in the early 1950s [17]. He formulated his logical approach to tenses in terms of the tense operators, P ("it has been the case that …") and F ("it will be the case that …"). Furthermore, Prior introduced two other operators, H ("It has always been the case that …") and G ("It is always going to be the case that …"). In at least some of the systems he considered H and G could be defined in P and F, as H ≡ ~P~ and G ≡ ~F~.

Rather soon he added a metric to the tense-operators, i.e., P(n) ("it has been the case that n time units ago …") and F(n) ("it will be the case in n time units that …"). It turns out that some of the problems Prior worked with are easier stated if we have access to a formalism with metrical tense-operators.

There are at least two groups of metaphysical and existential problems that become clearer when they are stated in terms of tense-logic:

- The problems of determinism, human freedom and the contingent future. Some basic questions: How should human freedom be understood? Can a statement regarding the contingent future be true now? What is determinism and does it exclude human freedom?
- The problems regarding the nature of time and existence. Some basic questions: What is time?, What is the relation between time from within and time as seen from the outside?, Can anything exist without being present? Is there an asymmetry between past and future?

In the following we shall briefly comment on the role of tense-logic in relation to these metaphysical and conceptual problems as they have been conceived by Prior and by the generations of logicians and scholars after him.

III. THE PROBLEMS OF DETERMINISM, HUMAN FREEDOM AND THE CONTINGENT FUTURE

Prior's main motivation in his work with the development of tense-logic was his struggle with the problem of determinism and free choice. Even though he grew up in the Methodist Church of New Zealand his attraction to the Calvinism of Karl Barth led him to join the Presbyterian Church as he began his studies at Otago University. He was an active member of the Student Christian Movement and published many articles on theology in the 1930s in the Magazine Open Windows where he for a period also served as editor. [10] He even wrote a "Logic of Calvinism" [35], although he never published the work.

As a Calvinist and an active member of the Presbyterian Church in New Zealand, Prior was very much aware of the tension between two classical doctrines in traditional Christian thinking, the doctrine of divine foreknowledge and the doctrine of human freedom. This is evident from the papers in his Nachlass such as [31], [33], and [35].

Prior assumed that the problem of time and free will can be formalised in terms of logic. Although he had a serious crisis of faith in 1942 [9] and also found it increasingly difficult to accept the traditional Calvinistic rejection of freedom of choice, and although he left the Church for good in the 1950s, he kept arguing that the analysis of the metaphysical problem concerning the logical tension between the two doctrines is important, and he argued that the problem can be significantly qualified and clarified if it is discussed in terms of logic. During the 1950s he suggested various versions of the tense-logical formalisation of the problem, and he studied it within a framework of modality. [17] In this framework we need a necessity operator, □, which should be understood as "it is now unpreventable that …". In addition, we need a possibility operator, ◊, which may be defined as ~□~.

In 1957 Saul Kripke (by then only 17 years old) suggested the notion of branching time in a letter to Prior, [19]. Prior found the idea of alternative future possibilities very interesting and useful as an illustration of the formal consequences of the doctrine of human freedom. If time is branching and the future is open, it means that we have to take models like the following into account.
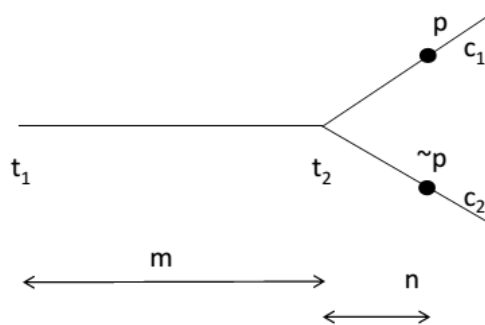

Figure 1. An illustration of the idea of alternative future possibilities in terms of Saul Kripke's notion of branching time.

In this model time branches at t_2 into two different chronicles, c_1 and c_2. This means that it depends on the decision at t_2 (and earlier) whether p or ~p will be the case n time units later. The general denial of the possibility of future alternatives would be determinism, i.e., that for any n and p:

(Det) $\Box \sim F(n)p \lor \Box F(n)p$

One of Prior's main interests was the various versions of the classical argument from the doctrine of divine foreknowledge to determinism. For several years Prior again and again discussed the tense-logical representation of this argument. One of his most important papers in tense-logic, "The formalities of omniscience", was published in 1962 and dealt with this problem (see [28, pp. 39-59]). In the following we shall consider a slightly modernised version of the argument with which Prior worked.

The doctrine of divine foreknowledge can be formalised in the following manner:

(Div) $F(n)p \equiv P(m)F(n+m)p$

The idea is that $F(n)p$ is true at t_2 if and only $F(n+m)p$ was true at any arbitrary earlier time t_1 m time units earlier. If n=0 (Div) becomes

(Div') $p \equiv P(m)F(m)p$

This means that p is the case now, if and only if it was the case at any earlier time, m time units ago, that p would be the case in m time units. (The theological counterpart of this is of course that God knows any true proposition at any time.)

In order to establish the argument, a few other principles are needed in addition to standard propositional calculus (PC), general substitution (Subst), and the rule of necessitation (Nec):

(□C) $\Box(p \supset q) \supset (\Box p \supset \Box q)$
(□P) $P(m)p \supset \Box P(m)p$
(DF) $F(n)p \lor F(n)\sim p$

(□C) is a standard axiom in classical modal logic. (□P) is the assumption that what was the case is now unpreventable, in the sense that nothing we can do now can influence what was true in the past. (DF) is the disjunctive principle regarding the future, which could also be expressed as

(DF') $\sim F(n)p \supset F(n)\sim p$

i.e., if p is not going to be the case in n time units then ~p is going to be the case in n time units.

Given these principles, Prior's argument can be formulated in the following manner:

1. $F(n)p \supset P(m)F(n+m)p$	(from Div)
2. $P(m)F(n+m)p \supset \Box P(m)F(n+m)p$	(from □P & Subst)
3. $P(m)F(n+m)p \supset F(n)p$	(from Div)
4. $\Box(P(m)F(n+m)p \supset F(n)p)$	(from 3 & Nec)
5. $\Box P(m)F(n+m)p \supset \Box F(n)p$	(from 4 & □C)
6. $F(n)p \supset \Box F(n)p$	(from 1, 2, 5 & PC)
7. $F(n)\sim p \supset \Box F(n)\sim p$	(from 7 & Subst)
8. $\Box F(n)p \lor \Box F(n)\sim p$	(from 6, 7, DF & PC)

Versions of this argument have been known since antiquity, [17]. However, Prior was the first to formulate the argument in terms of a formal tense-logical and modal language.

The formalisation of the argument shows that if we want to avoid the acceptance of determinism, (Det), we have to reject at least one of the principles (Div), (□P) and (DF) given a classical logical framework.

In his early writings Prior found the rejection of (DF) rather attractive and conceived it as related to the debate concerning a third truth-value of statements about the contingent future. However, later he had to conclude that the idea of a third truth-value is very problematic, [17]. However, Prior found that if we reject (Div), it will also be rather natural to deny (DF) since in that case we are denying that any question regarding the

contingent future in principle can be answered in a precise manner. In consequence, Prior maintained that there are two obvious ways to go:

1) The rejection of the general validity of (□P).
2) The rejection the doctrine of divine foreknowledge, (Div) and the disjunctive principle regarding the future (DF).

Much of Prior's own work in tense-logic as well as much of the later work in tense-logic deal with the further analysis of these two possibilities. In the following we shall discuss 1) and 2) briefly. Prior termed them the Ockhamist and the Peircean solution, respectively. He himself preferred the latter.

Although Prior often formulated the problem regarding the above argument in a theological framework, he was also aware of the fact that the problem can be stated in a broader philosophical way: Is it possible for an indeterminist to hold consistently that a future contingent can be true? Formulated in this manner the question appears secular and not theological, although it still has a clear metaphysical flavour to it.

A. The Ockhamist solution: Rejection of (□P)
Prior's Ockhamist solution is based on a branching time model of time. We assume a temporal structure (TIME,<), with TIME as the set of temporal moments, and < a partial ordering of the members of TIME. We can then define chronicles as linear and maximal subsets of TIME.

Truth is in this context conceived as a function, π, defined on TIME $\times$ Φ, where Φ is the set of propositional variables over which propositional variables of the logical system can range. This means that for any pair (t,q) where t is a temporal moment and q is a propositional constant of the logical language is given a truth value $\pi(t,q)$ as either 0 (false) or 1 (true). On this basis Prior defined the truth values of complex expressions recursively for a pair (t,c), where t is a moment in the chronicle c, i.e., $t \in c$:

$t, c \models q$	if q is a propositional constant with $\pi(t,q) = 1$
$t, c \models \sim\varphi$	if it is not the case that $t, c \models \varphi$
$t, c \models F\varphi$	if there is a $t' \in c$ with $t < t'$, such that $t', c \models \varphi$
$t, c \models P\varphi$	if there is a $t' \in c$ with $t' < t$, such that $t', c \models \varphi$
$t, c \models \Box\varphi$	if $t, c' \models \varphi$ for any c' with $t \in c'$

(We assume that the truth value of any statement in the formal language is either 0 or 1, and that it is 0 if it does not follow from the recursive definition that it is 1.)

A metric may be added to this semantical model in an obvious manner.

It should be noted that in Prior's Ockhamistic solution, truth is not only defined relative to temporal moments, but rather to pairs, (t,c), of a temporal moment t and a chronicle c to which t belongs.

Given this model it is obvious that (□P) has to be rejected. In order to verify this we may consider Fig. 1. In this case it is

obvious that $P(m)F(n+m)p$ is true, and that $\Box P(m)F(n+m)p$ is false for (t_2,c_1).

B. The Peirce solution: Rejection of (Div) and (DF)

Prior was not satisfied with the Ockhamist solution, since it turns out that the price of such a solution is that propositions about the past, just like ones about the future, get a 'wait and see' character. For Prior this was a reason to reject the Ockhamistic solution. He developed instead a solution which he, in honour of the great American logician, Charles Sanders Peirce (1839-1914), named the Peircean solution, since early versions of the view can in fact be found in Peirce's works, [21].

Validity of statements according to the Peircean solution may in fact be defined in terms of the Ockhamistic solution. The Peircean solution may in fact be introduced as fragment of the Ockhamist solution. There is no Peircean equivalent to the Ockhamistic future operator, F. The Peircean future operator simply corresponds to the Ockhamist $\Box F$, i.e.

$$F_{Peirce} =_{def} \Box F_{Ochamistic}$$

This means that if we want to evaluate (Div) according to the Peircean solution it corresponds to an Ockhamistic evaluation of the formula $\Box F(n)p \equiv P(m) \Box F(n+m)p$. It is easy to see that this is not a valid thesis. Figure 2 clearly illustrates this:
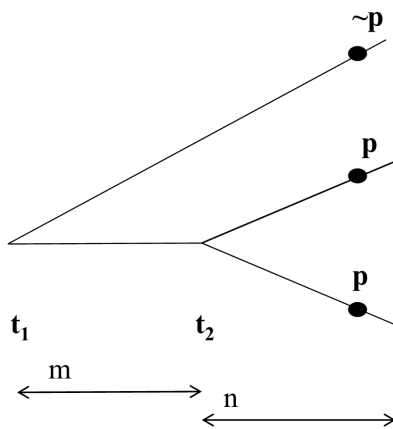

Figure 2. This example illustrates the invalidity of (Div) according to the Peircean solution.

Prior found the Peircean solution much more attractive than the Ockhamistic solution, mainly because it would allow him to keep ($\Box P$) as an axiom. He defended his view in his 1962 paper, "Formalities of Omniscience" [28, pp. 39-59]. This paper is arguably his most important contribution to analytic theology and, as mentioned above, and – as mentioned above – also one of his most important papers on temporal logic as such. This work is still very relevant in the context of the theological and metaphysical problem. According to W. Hasker:

The modern controversy over the compatibility of divine foreknowledge and human freedom, begun in the 1960s by Nelson Pike and A.N. Prior, has so far failed to reach a satisfactory conclusion. [7, p. 64]

Prior may thus be seen as the founder of what Hasker has called Open Theism, the view that God does not have knowledge of future contingent propositions, although according to this view, we may still claim that God knows everything which is true now.

C. The contingent future

According to Prior's Peircean solution a statement regarding the future, like "I am going to be in London tomorrow at noon", can only be true now, if it is necessary now. If it is still possible that my plans for tomorrow may or may not turn out as expected, the statement is simply false according to this solution. Actually the same holds for the statement "I am going to be absent from London tomorrow at noon". Given that the statements are contingents, both $F(n)p$ and $F(n)\sim p$ will be false now according to Prior's Peircean solution. In consequence, $\sim F(n)p$ and $\sim F(n)\sim p$ will both be true according to this solution. Clearly, this also means that anybody who accepts Prior's Peirce solution will have to make a clear distinction between $\sim F(n)p$ and $F(n)\sim p$, i.e., between "It is not the case that I am going to be in London tomorrow at noon" and "It is the case tomorrow at noon that I am not in London". Such a sophisticated distinction is of course conceivable in principle, but it has to be admitted that many language users will find it hard to incorporate it in a common sense approach to natural language understanding.

Another weakness related to Prior's Peircean solution has to do with the fact that the solution identifies "I am going to be in London tomorrow at noon" and "I am necessarily going to be in London tomorrow at noon", i.e., $F(n)p$ and $\Box F(n)p$. Again this appears odd from a common sense point of view, since we'll normally want to make a clear semantical difference between the following three statements:

$\Diamond F(1)p$: "I am possibly going to be in London tomorrow at noon"
$\Box F(1)p$: "I am necessarily going to be in London tomorrow at noon"
$F(1)p$: "I am going to be in London tomorrow at noon"

In Prior's Ockhamist solution there is in fact a clear semantical difference between these three statements. However, according to this solution it would also wrong to claim that a statement about the contingent future is true now. The reason is that according to Prior Ockhamistic truth-values depend not only on temporal moments, but also on the chronicles through the moment. Using the example in Fig. 1 as an illustration, the proposition $F(1)p$ is true at t_2 for c_1, but false at t_2 for c_2. According to Prior's Ockhamistic solution it will not be meaningful to claim that the statement that I am going to be in London tomorrow at noon is true now, given that there are other possibilities now.

It should be pointed out that William of Ockham (1285-1347) himself would probability not have accepted Prior's

Ockhamistic solution fully. Ockham clearly accepted the doctrine of divine foreknowledge even for statements regarding the contingent future, [40]. Apparently, Prior was aware of this problem and tried to find as version of his Ockhamistic solution closer to Ockham's original position. Malpass [12] has found a note in Prior's Nachlass which Prior most likely wrote when he prepared the book, "Past, Present and Future" [26], but which he for some reason chose not to include in the book. The note spells out a model for time where the actual future is "picked out in red":

> In these models the course of time (in a rather broad sense of this phrase) is represented by a line which, as it moves from left to right (past to future), continually divides into branches, so that from any given point in the diagram there is a unique route backwards (to the left; to the past) but a variety of routes forwards (to the right; to the future). In each model there is a single designated point, representing the actual present moment; and in an Ockhamist model there is a single designated line (taking one only of the possible forward routes at each fork), which might be picked out in red representing the actual course of events. [32]

It is interesting that in this early paper Prior introduced what Nuel Belnap and Michael Green [2] much later termed "the thin red line".

D. The thin red line and Molinism
After Prior's death in 1969 several tense-logicians and philosophers interested in temporal reasoning continued investigating the problems related to the idea of the contingent future. One of the scholars who continued the study of the metaphysical aspects of the discussion initiated by Prior was Alvin Carl Plantinga (born 1932) who, like Prior, wanted to defend the notion of free choice. In the book, "Nature of Necessity" [22], which he published in 1974, he defended the compatibility of the existence of evil with the existence of an omniscient, omnibenevolent and omnipotent deity. However, in order to carry out his argument Plantinga found that the Ockhamistic solution was not fully satisfactory. [23] In addition, he had to assume not only a true future at every moment in the actual history, but also at any counterfactual moment. Anthony Kenny, could, at a conference, enlighten Plantinga, that his argument in favour of free will relies on the ideas that Luis Molina (1535-1600) had defended centuries ago:

> At a council for Philosophical Studies Summer Institute (held at Calvin in 1973), I gave a version of the Free Will Defense in which I took it for granted both that there are counterfactuals of freedom and that God's omniscience includes his knowing their truth values. Anthony Kenny was present at the institute and declared that I was a 'Molinist'. I wasn't sure whether that was commendation or condemnation; but as it turned out the whole subject of counterfactuals of freedom and God's knowledge of them

had been debated and explored at length in the 16th century. [38, p. 50].

Molina's view on divine foreknowledge and human freedom has since received much attention in philosophy, logic and theology. The idea of a true contingent future even in the counterfactual case, has been linked to what Molina himself terms 'Scientia Media'[11]. In fact a formal Molinistic solution has been developed which should be seen as one of the most interesting alternatives to Prior's 'classical' Ockhamistic and Peircean solutions.[1]

It should be mentioned that Prior not only knew about the importance of Molina's idea of 'Scientia Media' in relation to the debate concerning human freedom and divine foreknowledge, but he actually worked on a Molinistic model for future contingency. In an unpublished article on The Westminster confession, which Prior most likely wrote in 1942, in a comment on section II of the confession which deals with predestination, Prior wrote: "Section II refers to a controversy then current about the nature of God's foreknowledge. Calvinists held that God's knowledge was of two kinds — a knowledge of what was possible, and a knowledge of what was and would be actually the case. And His knowledge of what did and would actually happen boiled down to a knowledge of what He himself had purposed to do, since everything that happened did so as a result of His free decision and decree. Jesuits and Arminians, however, taught that there was a third kind of divine knowledge in between these two, a *scientia media* which was neither a knowledge of what was merely possible nor a knowledge of what He himself had decreed, but a knowledge of what was bound to happen because certain other things had happened, or because He had decreed certain other things. Some of the *consequences* of His decree were thus conceived as outside His control, though not beyond His foresight." (The Prior collection, Bodleian Library, Oxford.)

It seems clear, that Prior would have found the introduction and formalisation of Molina's ideas very interesting. However, Prior dismissed this model on theological grounds, and this would most likely have been the best explanation for why he did not consider Scientia Media as a model for future contingency.

It should also mentioned that Thomason and Gupta in 1980 published a paper, "A Theory of Conditionals in the Context of Branching Time" [38], in which they came rather close to Molina's ideas. However, it seems this work has been rather unnoticed among philosophers interested in the metaphysical problems related to the idea of branching time.

In a very important paper from 1994 Nuel Belnap and Michael Green have discussed the idea of a true future, i.e. the idea of a thin red line. [2] They argued that this idea can only make sense if there is a true future in any counterfactual as well as in any actual case. They pointed out that this seems to be what we often assume in the common sense reasoning on which natural language understanding rely. Belnap and Green

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[1] Other alternatives have been suggested such as R. Thomason's idea of super-valuations and the works of Alberto Zanardo, [17], [18], [37], [42], [43].

refer to the following natural language example on the tossing of a coin:

The coin will come up heads. It is possible, though, that it will come up tails, and then later it will come up tails again (though at this moment it could come up heads), and then, inevitably, still later it will come up tails yet again. [2, p.379]

The branching time diagram in Figure 3 displays the situation described by Belnap and Green:
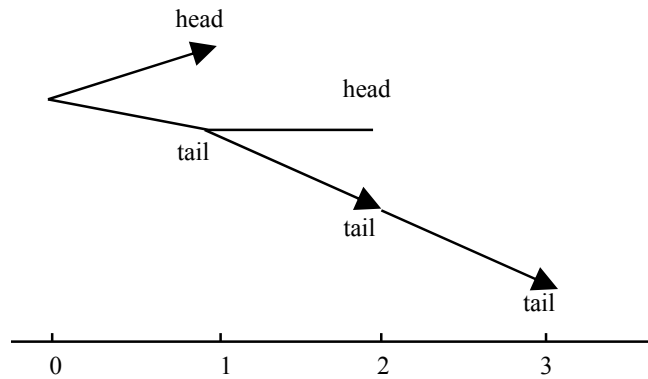

Figure 3. A model for branching time with arrows indicating the true futures on actual, as well as counterfactual futures.

Formally, we need a function for picking out the true future at any temporal moment, actual as well as counterfactual. Nuel Belnap and others [3] have argued that such a solution is metaphysically problematic, and we should stick to Prior's Ockhamistic solution although it makes it difficult to model the semantics of examples such as the one mentioned in Fig. 3. Others have argued that the Molinistic solution is metaphysically acceptable and may be even attractive; [4], [13], [15], [16].

IV. THE PROBLEMS REGARDING THE NATURE OF TIME AND EXISTENCE

In addition to the metaphysical problems regarding human freedom, contingency, determinism etc., Prior was highly interested in the questions concerning the nature of time and existence. Some of the important questions of this kind have to do with the relations between the so-called A-series (past, present, future) and B-series (before, after, simultaneous with). Should time mainly be conceived in terms of the A- series or in terms of the B-series? A related question has to do with the so-called passage of time: Is it real, or is it just something apparent? [16], [17] In the paper, 'Some Free Thinking about Time', he wrote:

So far, then, as I have anything that you could call a philosophical creed, its first article is this: I believe in the reality of the distinction between past, present, and future. I believe that what we see as a progress of events is a

progress of events, a coming to pass of one thing after another, and not just a timeless tapestry with everything stuck there for good and all... [34].

According to Prior's philosophical creed, the tenses (past, present and future) represent a very important aspect of reality.

A. Presentism and the concept of time

One of the philosophical problems he worked with was an attempt to understand the notion of the present. In a talk given at the inauguration of the International Society for the Study of Time in Oberwolfach in Germany, Prior presented what has later become one of the go-to articles for a definition of so-called presentism. In the article, published after his death by his good friend Anthony Kenny, Prior defines the present as:

"they [the real and the present] are one and the same concept, and the present simply *is* the real considered in relation to two particular species of unreality, namely the past and the future." [27]

Whereas many have followed Prior in defending the view that somehow the present simply is the real, they often also distance themselves from Prior's own definition. Indeed, one of them, Quintin Smith goes as far as calling Prior's definition self-contradictory. [35] One does well in remembering, though, that Prior's article is a publication of a paper prepared for a talk, and not the final article. We have been able to locate the notes to the talk in Prior's Nachlass and from these it becomes clear that Prior did not intend to affirm a relation between the real and the unreal. [8] Prior's definition of the present openly invites metaphysical questions. What, if anything, are we ontologically committed to when using tense operators if the domain of discourse over which they range is unreal? Here a novel solution developed by Prior suggests illumination, even though Prior did not get the change to develop it thoroughly himself.

What Prior is after in defining the present as the real is, among other things, the same which Augustin notes when he says "What, then, is time? If no one asks me, I know: if I wish to explain it to one that asketh, I know not" [1]. He is thus not reducing the present to another concept, and thus doing away with time. He is simply affirming that time is a part of the ultimate reality and cannot be reduced to anything less. What Prior affirms is that reality itself is dynamic, and that which is present is real. This immediately raises the question how we are to understand and talk about the future and the past. What is that over which the operators P and F range? On the one hand, we cannot get a deeper answer than that which will be the case, and that which has been the case. On the other hand, more can be said than this. In the notes to "The Notion of The Present", it seems that Prior considered to investigate it further:

There is, indeed, a great deal more law and regularity about the logical behavior of the prefixes "it has been that" and "it

will be that" than there is about the logical behavior of "it is imagined that". [8]

Eventually he skipped the idea, and chose to discuss how his idea of the present squared with the theory of relativity instead. The law like behaviour of tensed prefixes was never directly addressed by Prior, but this should probably be seen in the context of his idea of instant propositions.

B. The concept of time and hybrid logic

The metaphysical idea behind hybrid logic is Prior's idea of an instant proposition. [16], [20]. An instant proposition is, in hybrid logic, treated as a nominal, a name for a certain instant in which all other instants in time is present. If we thus consider the now to be everything which now is the case, and take the present to be all that exists, then the now entails more than the present does. The Now conceived as an instant proposition implies everything which is true including all true statements on what will be, what has been and what could have been. It is a very interesting idea that the Now (like any other instant proposition) in its inner structure reflects everything in the whole branching time system.

Using the idea of instant propositions Prior was able to give a very interesting account of the relation between the so-called A- and B- series. Thereby he was also able to offer a tense-logical and rather attractive answer to the classical questions regarding the nature of time. [16], [17], [18], [20].

C. Jonathan Edwards' principle of existence and nature

In his view on existence and time, Prior was inspired by the ideas of Jonathan Edwards who was one of the first philosophers Prior heard of. [29, p. 73] Even as a Calvinist Prior disagreed with Edwards on the issue of predestination. Prior presents Edwards' argument against free choice in this way:

> The general question to which Edwards here addresses himself is 'whether any event whatsoever, and *volition* in particular, can come to pass *without a cause* of its existence', and among other arguments for a negative answer he has a *reductio ad absurdum*, arguing that if an act of will can occur without a cause, then anything at all, no matter how fantastic, can occur without a cause. [29, p. 73]

Edwards had argued against free will and the ideas of human volition. In Edward's words, as they are quoted by Prior:

> The peculiar nature of that thing called volition, can do nothing, can have no influence, while it is not. And afterwards it is too late for its influence: for then the thing has made sure of existence already, without its help. [29, p. 75]

Over the years the belief in human freedom became more and more important to Prior. In fact, he held that this view if related to his basic belief in the tensed view of time. Referring to the latter he wrote:

> This belief, or prejudice of mine, is bound up with a belief in real freedom. One of the big differences between the past and the future is that once something has become past, is, as it were, out of our reach – once a thing has happened, nothing we can do can make it not to have happened. But the future is to some extent, even though it is only to a very small extent, something we can make for ourselves." [34]

This clearly meant that Prior had to reject Edwards' arguments against indeterminism and human freedom. On the other hand, Prior continued to find in Edwards' philosophy "a certain metaphysical logic with which we may still grapple profitable." [29, p. 73] On the basis of Edwards' ideas, Prior even suggested an argument in favor of what he termed limited indeterminism. He based his argument on a principle of existence and nature that he had found in Edwards' philosophy:

> nothing has any nature until it is there, so that whatever a things nature may explain or permit, it cannot explain or permit the things starting to be. [29, p. 75]

With a reference not only to Edwards but also to Aquinas, Prior argued that we should make a distinction between "coming into existence" and "starting to exist". Prior rejected the idea of the former, and accepted the idea of the latter. Clearly, if we accept that a volition can come into being (or come to pass), we will have accepted that it has a nature and maybe even a sort of existence before it is there. According to Prior this odd idea should obviously be rejected. On the other hand, it might make sense to claim that a volition is starting to exist i.e. there for the very first time.

Prior's accepted Edwards' principle concerning existence and nature and in order to evade determinism he took a nominalist stance on volitions, as an event, a person makes in a given circumstance. What explains an action is, contrary to Edwards' criticism, not the nature of a volition in any abstract sense, but rather the disposition of an actual substance or an actual person. In Prior's own words:

> We cannot and do not need to say that it is the nature of 'volitions', or of certain volitions, which makes their coming-to-be possible but not necessary, and the nature of other non-existent or not-yet-existent occurrences which makes their coming-to-be either necessary or impossible. It is rather that there are certain already-existing objects which have certain capacities, and some which lack them, and none which have certain other capacities. Persons, say, have the power, without the necessity, of doing *X* in certain circumstances; for oysters, on the other hand, doing *X* may be necessary or impossible; and *Y*, say turning into a dragon, may be something which no existing object has the power to do. [29, p. 77]

This means that a person's free action should not be explained as any other physical event, but rather in terms of the powers of the person. In general Prior wanted to see indeterminism as related to physical objects and not just to abstract events:

Given this metaphysical apparatus, it seems perfectly possible to say that some things, but not all things, have alternative possibilities of reaction to one and the same stimulation. It is 'open', we might say, to a disturbed electron to jump to orbit A, and equally open to it to jump to orbit B, but perhaps not open to it to jump to orbit C. In other words, its dispositions may be such that with certain provocations it will 'jump to orbit A or to orbit B', without having any determinate disposition to jump to orbit A, or any to jump to orbit B. Its jumping to orbit A rather than to orbit B, which we can call if we like the coming-to-be of a jump to orbit A rather than of one to orbit B, will then be a circumstance without a cause. And it may be that the only circumstances without causes are the ways that electrons jump from orbit to orbit. But the explanation of this fact will lie, not in the nature of those non-existent or not-yet-existent jumps, but in the nature of the existing electrons (and of other existing things). [29, p. 77]

What helps Prior escape Edward's argument against indeterminism is a certain nominalistic stance on "jumping", "volition", or for that matter, "headaches". Such terms should be understood in a nominalistic manner. They do not exist independently such as persons and physical objects. Prior writes:

I have admitted that we can, if we like, describe a head's starting to ache as a headache's starting to exist; but what must be insisted upon, if we are to answer Edwards, is that this change of key is not metaphysically illuminating but metaphysically obfuscating. This 'existence' and this 'starting to exist' of things like headaches is a purely Pickwickian and eliminable existence and starting-to-exist, and we explain what is meant by a headache's starting to exist by saying that it just means a head's starting to ache, not vice versa. [29, p. 76-77]

According to Prior the idea that persons and things in general have powers and alternative possibilities is closely related to the tensed view of time. In his own words:

If we adopt a 'substance' metaphysic, for this or any other reason, we must of course do it properly, and be prepared to wear its further trimmings and trappings. Substance-talk, for example, is *tensed* talk. The use of the 'earlier' and 'later' relations in making temporal references belongs basically to the event-and-process language, and if it is made fundamental we get a world-picture of events arranged in an unchanging string. [29, p. 78]

For Prior, the nominalistic stance towards abstract events like the above mentioned, is tied together with a tensed view on reality. The before-after logic suggests instead a "world picture of events arranged in an unchanging string." [29, p. 78] The rejection of this interpretation means that the above branching time diagrams (Fig.1, Fig. 2, and Fig. 3) should be understood in a somewhat nominalistic manner. The chronicles etc. are just abstract constructions. Strictly speaking, according to Prior only the present exists.

If we reject this before-after picture of the world though, then what we find, at the moment subsequent to the present moment of history in which an indetermined action takes place, is an acting individual with an openness to "one and the same stimulation." [29, p. 77] This openness is part of the substance with dispositions in accordance with its nature.

Prior's treatment of Edward's thoughts on existence and time in [29], as well as in [26], shows not just the inspiration Prior found in Edwards, but also how nominalism, for Prior, plays an important role in spelling out a tensed view of metaphysics that secures a dynamic view of time and the free will.

V.   CURRENT TEMPORAL LOGIC

Over the years since Prior's first formulations of tense-logic, it has turned out that tense-logic (as well as temporal logic in general) is very useful in computer science. Actually, Prior himself was aware of such perspectives. In relation to his discussion of a tense-logic with operators corresponding to "yesterday" and "tomorrow", he wrote:

The usefulness of systems of this sort does not depend on any serious metaphysical assumption that time *is* discrete; they are applicable in limited fields of discourse in which we are concerned only with what happens next in a sequence of discrete states, e.g. in the workings of a digital computer. [26, p. 66]

Such applications of temporal logic are practical, and as Prior states here not directly related to metaphysical questions or problems. If we want to make a system using the operators corresponding to "yesterday" and "tomorrow", we can clearly do so without addressing the conceptual and metaphysical question about the nature of time.

After Prior's death, his temporal logic was imported and adjusted in computer science. This was first done in the 1970s by Amir Pnueli (1941-2009) who found great inspiration is Prior's work. Temporal logic has since become a very important field in computer science and in 1996 Amir Pnueli received the Turing Award for "seminal work introducing temporal logic into computing science and for outstanding contributions to program and systems verification" (see http://amturing.acm.org). Since then there has been a lot of research activity in the field. A number of conferences have been organized, and a long list of papers and books have been published in order to study the application of temporal logic in computer science.

It is indeed fascinating that a discipline that was originally designed in order to analyse problems in philosophy, metaphysics, and even theology, has become useful in computer science.

VI. CONCLUSION: THE RELEVANCE OF METAPHYSICS IN CURRENT TEMPORAL LOGIC

It is evident that there are many practical applications of temporal logic in computer science. It will, however, be a serious mistake to assume that this is all there is to be said about the actual usefulness of temporal logic. As we have seen, temporal logic – and in particular tense-logic – was created in order to analyse some important conceptual and metaphysical questions regarding the relations between time and existence. Many such problems are still very important. If we want to obtain a deeper understanding of reality we have to study these problems carefully. Prior suggested doing so using formal methods. He once formulated his strategy in the following manner:

And I think it important that people who care for rigorism and formalism should not leave the basic flux and flow of things in the hands of existentialists and Bergsonians and others who love darkness rather than light, but we should enter this realm of life and time, not to destroy it, but to master it with our techniques [undated note; kept in the Prior Collection, Bodleian Library, Oxford].

There is no reason to believe that there is nothing more to gain from using Prior's strategy in order to analyse these problems. On the contrary, we have every reason to believe that this approach is still extremely attractive when it comes to the study of a number of deep problems regarding life and time. Philosophers interested in such problems and logicians interested in the formal structures should work together in order to develop techniques that can throw light on these important problems. This means that the use of logic can make discussions on problems like determinism and human freedom much clearer.

As Prior and many others have pointed out logic alone cannot decide whether we should prefer the Peircean, the Ockhamistic, the Molinistic or some other solution. Logic can tell us whether a certain model is consistent or not, and by logical analysis we may learn what follows from the various models. But the question of which particular solution we should prefer will still depend on metaphysical and philosophical reflection. In Prior's own words: "In doing metaphysics there is still no substitute for 'the choice of the soul'; or, if you like, prejudice." [28, p.284] However, this does make less important to use temporal logic in order to make it clear what alternative positions are in fact open to us.

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