'NO SUCCESS LIKE FAILURE …': EINSTEIN'S QUEST FOR GENERAL RELATIVITY, 1907–1920

MICHEL JANSSEN

1. INTRODUCTION

In 1905, Einstein published what came to be known as the special theory of relativity, extending the Galilean-Newtonian principle of relativity for uniform motion from mechanics to all branches of physics.[1] Two years later he was ready to extend the principle to arbitrary motion. He felt strongly that there can only be relative motion, as is evidenced, for instance, by his opening remarks in a series of lectures in Princeton in 1921, published in heavily revised form the following year (Einstein 1922c). A typescript based on a stenographer's notes survives for the first two, non-technical lectures. On the first page of this presumably *verbatim* transcript we find Einstein belaboring the issue of the relativity of motion in a way he never would in writing:[2]

> Whenever we talk about the motion of a body, we always mean by the very concept of motion relative motion … These conditions are really quite trivial … we can only conceive of motion as relative motion; as far as the purely geometrical acceleration is concerned, it does not matter from the point of view of which body we talk about it. All this goes without saying and does not need any further discussion (CPAE 7, Appendix C, [p. 1]).

Although Einstein insists that these points are trivial, we shall see that they are not even true. What makes his comments all the more remarkable is that by 1921 Einstein had already conceded, however grudgingly, that his general theory of relativity, worked out between 1907 and 1918, does *not* make all motion relative.

In a paper entitled "Is "general relativity" necessary for Einstein's theory of gravitation?" published in one of the many volumes marking the centenary of Einstein's birth, the prominent relativist Sir Hermann Bondi (1979) wrote: "It is rather late to change the name of Einstein's theory of gravitation, but general relativity is a physically meaningless phrase that can only be viewed as a historical memento of a curious philosophical observation" (181).[3]

Einstein obviously realized from the beginning that there is a difference between uniform and non-uniform motion. Think of a passenger sitting in a train in a railway station looking at the train next to hers. Suppose that—with respect to the station—one train is moving while the other is at rest. If the motion is uniform

and if the only thing our passenger sees as she looks out the window is the other train, there is no way for her to tell which one is which. This changes the moment the motion is non-uniform. Our passenger can now use, say, the cup of coffee in her hand to tell which train is moving: If nothing happens to coffee, the other one is; if the coffee spills, hers is.

The key observation on the basis of which Einstein nonetheless sought to extend the relativity principle to non-uniform motion is that, at least locally, the effects of acceleration are indistinguishable from the effects of gravity. Invoking this general observation, our passenger can maintain that her train is at rest, even if her coffee spills. She can, if she is so inclined, blame the spill on a gravitational field that suddenly came into being to produce a gravitational acceleration equal and opposite to what she would otherwise have to accept is the acceleration of her own train.[4] This was the idea that launched Einstein on his path to general relativity (Einstein 1907j, Part V). A few years later, he introduced a special name for it: The *equivalence principle* (Einstein 1912c, 360, 366).

This principle by itself does not make non-uniform motion relative. As Einstein came to realize in the course of the work that led him toward the new theory, two further conditions need to be met.

The first condition is that it should be possible to ascribe the gravitational field substituted for an object's acceleration on the basis of the equivalence principle to a material source—anything from the object's immediate surroundings to the distant stars. Otherwise, acceleration with respect to absolute space would simply be replaced by the equally objectionable notion of fictitious gravitational fields. If this further condition is met, however, the gravitational field can be seen as an epiphenomenon of matter and all talk about motion of matter in that field can be interpreted as short-hand for motion with respect to its material sources (Maudlin 1990, 561). This condition was inspired by Einstein's reading of the work of the 19th-century Austrian philosopher-physicist Ernst Mach (Barbour and Pfister 1995, Hoefer 1994, Renn 2007a).

The second condition is that all physical laws have the same form for all observers, regardless of their state of motion. In particular, this should be true for the gravitational *field equations*, the equations that govern what field configuration is produced by a given distribution of sources. This form invariance is called *general covariance*.[5] Einstein had great difficulty finding field equations that are both generally covariant and satisfactory on all other counts (Renn 2007a, Vols. 1–2). He originally settled for field equations of severely limited covariance. He published these equations in a paper co-authored with the mathematician Marcel Grossmann (Einstein and Grossmann 1913). They are known among historians of physics as the *Entwurf* (German for 'outline') field equations after the title of this paper. The precursor to general relativity with these field equations is likewise known as the *Entwurf* theory. In the course of 1913, Einstein convinced himself that the restricted covariance of the *Entwurf* field equations was still broad enough

to make all motion relative. In a vintage Einstein maneuver, he even cooked up a fallacious but ultimately profound argument, known as the *hole argument* (see Section 3 and note 62), purporting to show that generally-covariant gravitational field equations are inadmissible (Einstein and Grossmann 1914a). By the end of 1914 he felt so sure about the *Entwurf* theory that he published a lengthy self-contained exposition of it (Einstein 1914o).[6] In the fall of 1915, however, he recognized that the *Entwurf* field equations are untenable. In November 1915, with the Göttingen mathematician David Hilbert breathing down his neck,[7] Einstein dashed off a flurry of short papers to the Berlin Academy, in which he proposed, in rapid succession, three new field equations of broad and eventually general covariance (Einstein 1915f, g, h, i).[8] The final generally-covariant equations are known today as the *Einstein field equations*. Einstein subsequently replaced the premature review article of 1914 by a new one (Einstein 1916e). This article, submitted in March and published in May 1916, is the first systematic exposition of general relativity.[9] When Einstein wrote it, he was laboring under the illusion that, simply by virtue of its general covariance, the new theory made all motion relative.

The other condition mentioned above, however, was not met: General covariance in no way guarantees that all gravitational fields can be attributed to material sources. In the fall of 1916, in the course of an exchange with the Dutch astronomer Willem de Sitter, Einstein was forced to admit this. He thereupon modified his field equations (without compromising their general covariance) by adding a term with the so-called *cosmological constant* (Einstein 1917b). Einstein's hope was that these new field equations would not allow any gravitational fields without material sources. In a brief but important paper in which he silently corrected some of his pronouncements on the foundations of general relativity in the 1916 review article, Einstein (1918e, 241) introduced a special name for this requirement: *Mach's principle*.[10] A few months later, it became clear that even the modified equations do not satisfy this principle. Within another year or so, Einstein came to accept that general relativity, the crowning achievement of his career, did not banish absolute motion from physics after all.

This, in a nutshell, is the story of Einstein's quest for general relativity from 1907 to about 1920. His frustrations were many. He had to readjust his approach and his objectives at almost every step along the way. He got himself seriously confused at times, especially over the status of general covariance (see Section 3).[11] He fooled himself with fallacious arguments and sloppy calculations (Janssen 2007). And he later allegedly called the introduction of the cosmological constant the biggest blunder of his career (Gamow 1970, 149–150).[12] There is an uplifting moral to this somber tale. Although he never reached his original destination, the bounty of Einstein's thirteen-year Odyssey was rich by any measure.[13]

First of all, what is left of absolute motion in general relativity is much more palatable than the absolute motion of special relativity or Newtonian theory.[14]

Einstein had implemented the equivalence principle by making a single field represent both gravity and the structure of space-time. In other words, he had rendered the effects of gravity and acceleration (i.e., the deviation from inertial motion) indistinguishable by making them manifestations of one and the same entity, now often called the inertio-gravitational field. If Mach's principle were satisfied, this field could be fully reduced to its material sources and all motion would be relative. But Mach's principle is not satisfied and the inertio-gravitational field exists in addition to its sources. When two objects are in relative non-uniform motion, this additional structure allows us to determine whether the first, the second, or both are actually moving non-uniformly. In this sense, motion in general relativity is as absolute as it was in special relativity. In his Princeton lectures, however, Einstein (1956, 55–56) argued that there is an important difference between the two theories: In general relativity, the additional structure is a *bona fide* physical entity that not only acts but is also acted upon. As Misner, Thorne, and Wheeler (1973, 5) put it in their textbook on general relativity: *"Space acts on matter, telling it how to move. In turn, matter reacts back on space, telling it how to curve."*

By 1920, Einstein had probably recognized that Mach's principle was predicated on an antiquated 19th-century billiard-ball ontology (Hoefer 1994, Renn 2007b). In the field ontology of the early 20th-century, in which matter was ultimately thought of as a manifestation of the electromagnetic and perhaps other fields, it amounts to the requirement that the gravitational field be reduced to these other fields. A recognition of this state of affairs may have helped Einstein make his peace with the persistence of absolute motion in general relativity. Instead of trying to reduce one field to another, he now tried to unify the two. This can clearly be seen in *Ether and relativity*, the inaugural lecture Einstein gave upon accepting a visiting professorship in Leyden in 1920. Einstein was not pandering to his revered senior Dutch colleague Hendrik Antoon Lorentz when he presented the inertio-gravitational field in this lecture as a new relativistic incarnation of the ether eliminated by special relativity (Einstein 1920j).

Special relativity combines the electric and the magnetic field into one electromagnetic field, and space and time into space-time. General relativity combines the gravitational field and the space-time structure into one inertio-gravitational field. It thus made sense to try to combine the electromagnetic field and the inertio-gravitational field into one unified field. Einstein spent the better part of the second half of his career searching in vain for a theory along the lines of general relativity that would accomplish this.[15]

Even though general relativity does not eliminate absolute motion, the case can be made that it does eliminate absolute space(-time). In the classic debate between Newton (through his spokesperson Samuel Clarke) and Leibniz (Alexander 1956), these two notions seemed to stand or fall together. Modern philosophy of space

and time has made it clear that they do not. The appearance that they do is due to a conflation of two related but separate issues (Earman 1989, 12–15).

The first issue is the one we have been considering so far: Is all motion relative or is some motion absolute? This question, as we just saw, ultimately boils down to the question whether or not the space-time structure is something over and above the contents of space-time. To the extent that it is still meaningful to distinguish space-time from its contents once the former has been identified with a physical field (Rynasiewicz 1996), one would have to answer this question affirmatively. This in turn implies that absolute motion persists in general relativity.

The second issue concerns the ontological status of space-time. Is the space-time structure supported by a substance, some sort of container, or is it a set of relational properties, like the marriage of me and my wife?[16] The two views thus loosely characterized go by the names of *substantivalism* and *relationism* (or *relationalism*), respectively. Fairly or unfairly, Newton's name has been associated with substantivalism as well as with absolutism about motion, Leibniz's name with relationism as well as with relativism about motion. It is possible, however, to be an absolutist about motion while being a relationist about the ontology of space-time. Although the jury is still out on the latter count, the ontology of space-time, represented by the inertio-gravitational field in general relativity, is probably best understood in relational rather than substantival terms. In that case, however, the causal efficacy implied by the slogan that space-time both acts and is acted upon cannot be that of a substance.

If the verdicts on these two issues stand as final, the centuries-old debate between Newtonians and Leibnizians will have ended in a draw: Newtonians were right that there is absolute motion, Leibnizians were right that there is no absolute space. Accordingly, the best arguments in support of their respective positions would both be correct. Newton's rotating-bucket experiment (see Section 4) shows that rotation is absolute; Leibniz's mirror or shift argument (see Section 3) shows that space is relational. One can argue, however, that the terms of the debate have changed so drastically since the 17th century that it does not make much sense anymore to belatedly declare winners and losers (Rynasiewicz 1996).[17]

The central argument for the claim that general relativity vindicates relationism can be seen as a modern version of Leibniz's shift argument and is based on Einstein's resolution of the hole argument through the so-called *point-coincidence argument* (see Section 3). Originally, the hole argument was nothing but a fig leaf to cover up the embarrassing lack of covariance of the *Entwurf* field equations (Janssen 2007). The point-coincidence argument likewise started out as an expedient to silence two correspondents, who took Einstein to task for publishing generally-covariant field equations without explaining what was wrong with the hole argument (see note 63). Despite their inauspicious origins, both arguments have enjoyed a rich afterlife in the literature on the philosophy of space and time.

This illustrates my general point that Einstein's quest for general relativity was anything but fruitless.

This becomes even clearer when we shift our attention from foundational issues to physics proper. Even though the equivalence principle could not be used for its original purpose of making all motion relative, Einstein did make it the cornerstone of a spectacular new theory of gravity that is still with us today. The insight that space-time and gravity should be represented by one and the same structure may well turn out to be one of the most enduring elements of Einstein's legacy (Janssen 2002b, 511–512). In addition to laying the foundation for this theory, Einstein, among other things, explained the anomalous advance of the perihelion of Mercury (Einstein 1915h),[18] successfully predicted both the bending of light in gravitational fields and its gravitational redshift (Einstein 1907j, 1911h, 1915h),[19] launched relativistic cosmology (Einstein 1917b),[20] suggested the possibility of gravitational waves (Einstein 1916g, 1918a),[21] gravitational lensing,[22] and frame-dragging (Einstein 1913c, 1261–1262),[23] came up with the first sensible definition of a space-time singularity (Einstein 1918c),[24] and caught on to the intimate connection between covariance and energy-momentum conservation (Einstein 1914o, 1916o) well before Emmy Noether (1918) formulated her celebrated theorems connecting symmetries and conservation laws inspired by this particular application in general relativity (Rowe 1999, Janssen and Renn 2007).[25] Even Einstein's "biggest blunder"—the cosmological constant—has made a spectacular comeback in recent years. It can be used to give a simple account of the accelerated expansion of the universe. These results more than compensate for Einstein's failure in his quest for general relativity.

It is this quest, however, that will be the main focus of this chapter.[26] Between 1907 and 1918, Einstein made at least four different attempts to make all motion relative. In Sections 2–5, I cover these attempts and explain how and why they failed. This raises an obvious question: How do we make sense of the success of Einstein's theory of gravity given that some of the main considerations that led him to it turned out to be misguided (Renn 2007b, 21–23)? In the concluding Section 6, I identify three factors that may help answer this question. First, throughout the pursuit of his lofty philosophical goals, Einstein never lost sight of the more mundane physics problem at hand, namely how to reconcile the basic insight of the equivalence principle, the intimate connection between inertia and gravity, with the results of special relativity.[27] Second, in developing his new theory, Einstein relied not only on his philosophical ideas but also on an elaborate analogy between the electromagnetic field, covered by the well-established theory of electrodynamics, and the gravitational field, for which he sought a similar theory (Renn and Sauer 2007, Janssen and Renn 2007). Finally, as we shall see in the course of Sections 2–5, Einstein, who could be exceptionally stubborn, displayed a remarkable flexibility at several key junctures where his philosophical predilections led to results that clashed with sound physical principles, such as the conservation

laws for energy and momentum. None of this is to say that Einstein's philosophical objectives only served as a hindrance in the end. Without them Einstein would probably have taken a more conservative approach, making gravity just another field in the Minkowski space-time of special relativity rather than part of the fabric of space-time itself. As we shall see in Section 6, however, Einstein himself showed, through his contributions to a theory first proposed by the Finnish theorist Gunnar Nordström, that even this more conservative approach eventually leads to a connection between gravity and space-time curvature as in general relativity (Einstein and Fokker 1914).[28]

2. First attempt: The equivalence principle

One day in 1907, at the patent office in Berne, while working on a review article on his original theory of relativity (Einstein 1907j), it suddenly hit Einstein: Someone falling from the roof of a house does not feel his own weight. As he wrote in a long unpublished article intended for *Nature* on the conceptual development of both his relativity theories, this triggered "the best idea of [his] life."[29] It is illustrated in Fig. 1. The upper half shows Einstein looking out the window and meeting the eyes of a man who moments earlier fell off his scaffold as he was cleaning windows a few floors up. Einstein is at rest in the gravitational field of the earth, the man is in free fall in this field, accelerating toward the pavement. For the duration of the fall, he is experiencing something close to weightlessness. Although to this day few have actually experienced this condition first-hand, we have all at least experienced it vicariously through footage of astronauts in free fall toward Earth as they orbit the planet in a space shuttle. Einstein only had his imagination to go on. If it were not for air resistance, the unfortunate window cleaner, like the astronaut in orbit around the earth, would feel as if he were hovering in outer space, far removed from any gravitating matter. Moreover, on Galileo's principle that all bodies fall alike, he would fall with the same acceleration as his bucket and his squeegee. These objects would thus appear to be hovering with him. In short, moving with the acceleration of free fall in a gravitational field seems to be physically equivalent to being at rest without a gravitational field. Likewise, being at rest at one's desk, resisting the downward pull of gravity, seems to be physically equivalent to sitting at the same desk in the absence of a gravitational field but moving upward with an acceleration equal and opposite to that of free fall on earth. An astronaut firing up the engines of her rocket ship in outer space will be pinned to her seat as if by a gravitational field (an experience similar to the one we have during take-off on a plane). Einstein used observations like this for an extension of sorts of the relativity principle for uniform motion to non-uniform motion.

Fig. 1 depicts the four physical states described above. Both in situation (I), near the surface of the earth, and in situation (II), somewhere in outer space, the man on the left (a) and the man on the right (b) can both claim to be at rest *as long*
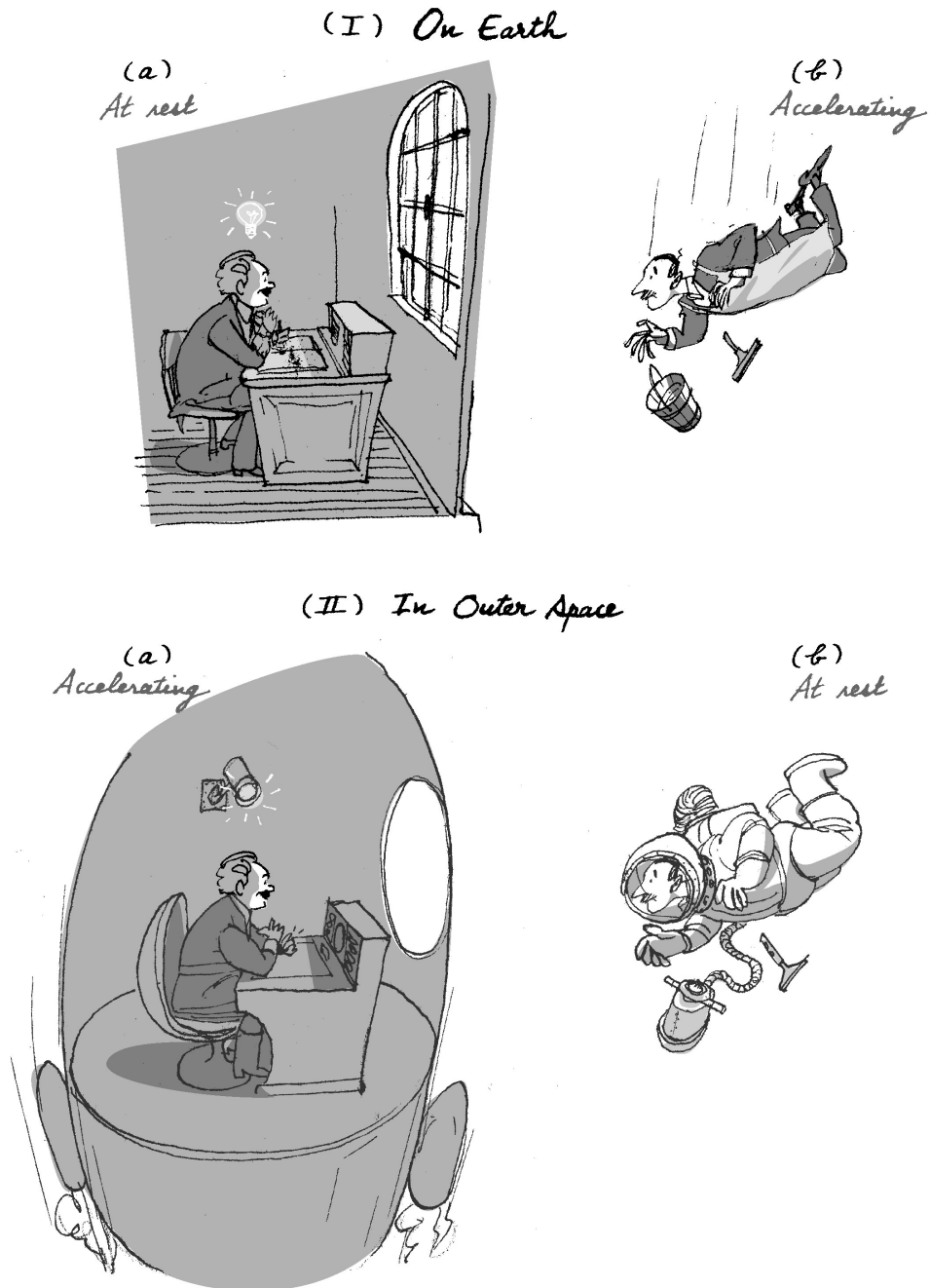
(I) On Earth

(a) At rest

(b) Accelerating

(II) In Outer Space

(a) Accelerating

(b) At rest

FIGURE 1. The equivalence principle.
This and all other diagrams in this chapter by Laurent Taudin.

as they agree to disagree on whether or not there is a gravitational field present.
For Einstein sitting at his desk in situation (I), there is a gravitational field, he is
at rest, and the other man is accelerating downward. For the falling man, there is
no gravitational field, he is at rest, and Einstein is accelerating upward. Situation
(II) fits the exact same description.

This extended relativity principle, however, is very different from the relativity
principle for uniform motion. The situations of two observers in uniform motion
with respect to one another are physically fully equivalent. This is not true for
non-uniform motion. Resisting and giving in to the pull of gravity (Ia and Ib,
respectively) feel differently; so do accelerating and hovering in outer space (IIa
and IIb, respectively). In fact, the equivalence captured in Fig. 1 is not between
different observers in the same situation—i.e., between observers (a) and (b) in
situation (I) or (II)—but between different situations for the same observers—i.e.,
between situations (I) and (II) for observer (a) or (b).

We call the uniform motion of one observer with respect to another relative
because the situation is completely symmetric. It is therefore arbitrary in the final
analysis (even if hardly ever in practice) which one we label 'at rest' and which
one we label 'in motion'. There is no such symmetry in the case of non-uniform
motion. Non-uniform motion is thus not relative in the sense that uniform motion
is. What is relative in this sense in the situations illustrated in Fig. 1 is *the presence
or absence of a gravitational field.* Situations (I) and (II) can both be accounted
for with or without a gravitational field. From the perspective of observer (a),
both situations involve a gravitational field; from the perspective of observer (b),
there is none in either.

If we try to extend the descriptions of situations (I) and (II) to include all
of space, the equivalence of the description with and the description without a
gravitational field breaks down. Contrary to what Einstein thought in 1907, we
cannot fully reduce inertial effects, the effects of acceleration, to gravitational
effects. As mentioned in the introduction, however, general relativity in its final
form does trace inertial and gravitational effects to the same structure, the inertio-
gravitational field.

In Newtonian physics particles get their marching orders, figuratively speaking,
from the space-time structure and from forces acting on them. According to New-
ton's first law (the law of inertia), a particle moves in a straight line at constant
speed as long as there are no forces acting on it. This is true regardless of its size,
shape, or other properties. Forces cause a particle to deviate from its inertial path.
By how much depends on its susceptibility to the particular force (e.g., an elec-
tric force will only affect charged particles) and on its resistance to acceleration.
The marching orders issued by forces are thus specific to the particles receiving
them. There is one force in Newtonian physics, however, giving marching orders
that are as indiscriminate and universal as those issued by the space-time struc-
ture: Gravity. Newton accounted for this universality by setting *inertial mass*, a

measure for a particle's resistance to acceleration, equal to *gravitational mass*, a measure for its susceptibility to gravity.[30] Newton did some pendulum experiments to test this equality, now known as the *weak equivalence principle.* It was tested with much greater accuracy in a celebrated experiment of the Hungarian physicist Baron Loránd von Eötvös (1890). Einstein was still unaware of this experiment in July 1912. He first cited it in his 1913 paper with Grossmann (CPAE 4, 340, note 3).

The equality of inertial and gravitational mass, without which Galileo's principle that all bodies fall alike would not hold, is an unexplained coincidence in Newtonian physics. To Einstein it suggested that there is an intimate connection between inertia and gravity. The universality of gravity's marching orders makes it possible to move gravity from the column of assorted forces to the column of the space-time structure. General relativity combines the space-time structure (more accurately: The inertial structure of space-time) and the gravitational field into one inertio-gravitational field. This field specifies the trajectories of particles on which no additional forces are acting. Einstein thus removed the mystery of the equality of inertial and gravitational mass in Newton's theory by making inertia and gravity two sides of the same coin.

In the passage from the unpublished *Nature* article of 1920 referred to at the beginning of this section, Einstein drew an analogy with electromagnetism to explain the situation:

> Like the electric field generated by electromagnetic induction, the gravitational field only has a relative existence. *Because, for an observer freely falling from the roof of a house, no gravitational field exists while he is falling, at least not in his immediate surroundings* (CPAE 7, Doc. 31, [p. 21], Einstein's italics).[31]

He had explained the example from electromagnetism in the preceding paragraph.[32] It is the thought experiment, illustrated in Fig. 2, with which Einstein (1905r) opened his first paper on special relativity.

Consider a bar magnet and a conductor—say, a wire loop with an ammeter—in uniform motion with respect to one another. In pre-relativistic electrodynamics, it made a difference whether the conductor or the magnet is at rest with respect to the ether, the medium thought to carry electric and magnetic fields. In case (a)—with the conductor at rest in the ether—the magnetic field at the location of the wire loop is growing stronger as the magnet approaches. Faraday's induction law tells us that this induces an electric field, producing a current in the wire, which is registered by the ammeter. In case (b)—with the magnet at rest in the ether—the magnetic field is not changing and there is no induced electric field. The ammeter, however, still registers a current. This is because the electrons in the wire are moving in the magnetic field and experience a Lorentz force that drives them around in the wire. It turns out that the currents in cases (a) and
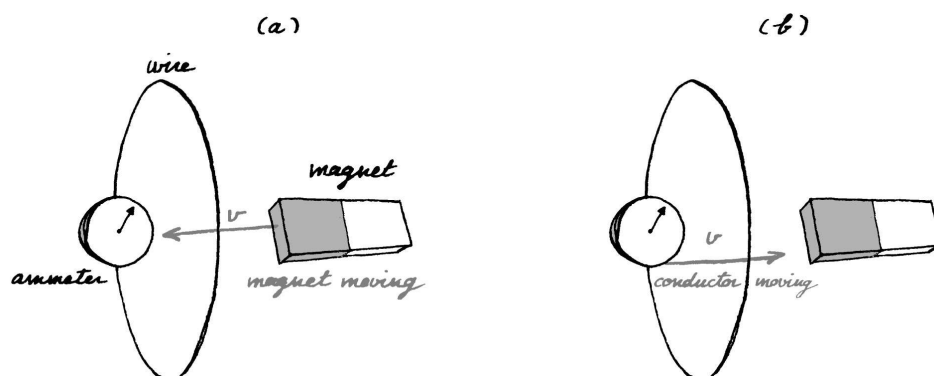
FIGURE 2. The magnet-conductor thought experiment

(b) are exactly the same, even though their explanations are very different in pre-relativistic theory.

Einstein found this unacceptable. He insisted that situations (a) and (b) are one and the same situation looked at from two different perspectives. It follows that the electric field and the magnetic field cannot be two separate fields. After all, there is both a magnetic and an electric field in situation (a), while there is no electric field in situation (b). Einstein concluded that there is only an electromagnetic field that breaks down differently into electric and magnetic components depending on whether the person making the call is at rest with respect to the magnet or with respect to the conductor. The equivalence principle in its mature form can be formulated in the exact same way. There is only an inertio-gravitational field that breaks down differently into inertial and gravitational components depending on the state of motion of the person making the call.[33] This is what Einstein meant when he wrote in 1920 that "the gravitational field only has a relative existence." This statement must sound decidedly odd to the ears of many modern relativists. The modern criterion for the presence or absence of a gravitational field—does the so-called *curvature tensor* have non-vanishing components or not?—leaves no room for disagreement between different observers (see Section 3).

It took Einstein more than a decade to articulate the mature version of the equivalence principle (Einstein 1918e).[34] In the meantime, the general insight that acceleration and gravity are intimately linked had guided Einstein on his path to the new theory. The equivalence principle, seen now as a heuristic principle, allowed him to infer effects of gravity from effects of acceleration in Minkowski space-time, the space-time of special relativity (Norton 1985).[35] The mature equivalence principle retroactively sanctioned such inferences, at least qualitatively. From the point of view of general relativity, the space-time structure of special relativity is nothing but a specific inertio-gravitational field.

Rotation in Minkowski space-time formed the starting point of the most fruitful application of this type of reasoning. The inertial effects due to centripetal acceleration (which one experiences, for instance, when trying not to be thrown off a merry-go-round) can, in the spirit of the equivalence principle, be re-interpreted as due to a centrifugal gravitational field. The situation is illustrated in Fig. 3.
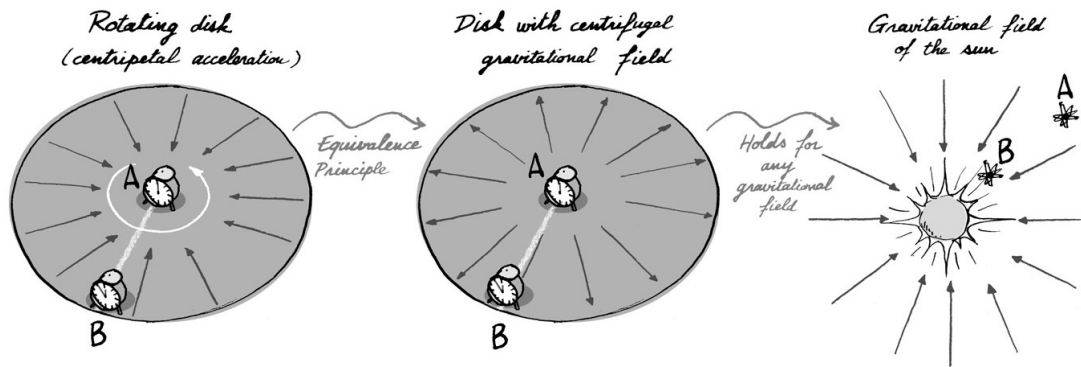

Figure 3. The rotating disk

The first drawing shows a circular disk rotating in Minkowski space-time.[36] The inward pointing arrows represent the centripetal acceleration. They give the direction in which the velocity of a person on the rotating disk is *changing*. The second drawing shows the same disk from the point of view of this person, who, appealing to the equivalence principle, considers herself at rest in a centrifugal gravitational field. This field is represented by outward pointing arrows. Special relativity tells us what happens in the situation in the first drawing. The equivalence principle tells us that the same things will happen in the peculiar gravitational field in the second. By determining in this manner what special relativity has to say about this particular gravitational field, we can expect to gain insights about gravitational fields in general, such at the gravitational field of the sun shown in the third drawing in Fig. 3. Such insights gave Einstein valuable clues about features of a new theory of gravity that goes beyond Newton's.

First, we examine the consequences of the special-relativistic effect of time dilation for gravitational theory. Compare two clocks on the rotating disk, clock A at the center and clock B on the circumference. B is moving, while A is practically at rest (it is spinning on its own axis with a velocity much smaller than that of B). According to special relativity, moving clocks tick at a lower rate than clocks at rest. One revolution of the disk thus takes less time on B than it takes on A. This is just a variant of the famous twin paradox in special relativity.[37] The equivalence principle tells us that the gravitational field pointing from A to B in the second drawing likewise causes clock B to tick at a lower rate than clock A. The same will be true for the gravitational field of the sun pictured in the third drawing.

The ticking of a clock will slow down as it is lowered in the sun's gravitational field. The frequency of light emitted by atoms will be subject to this same effect. Hence, the frequency of light emitted by an atom close to the sun (at B) will be lower than the frequency of light emitted in the same process by an identical atom farther away from the sun (at A). If an atom is lowered in a gravitational field, the frequency of the light it emits will shift to the red end of the spectrum. This phenomenon is known as *gravitational redshift*. The conclusion of this simple argument based on the equivalence principle is confirmed by general relativity in its final form.[38]

An equally simple argument establishes that gravity will bend the path of light.[39] Suppose a light signal is sent from the center A of the rotating disk in the direction of the line connecting A and B, which is painted on the disk. The light will travel in a straight line, but, since the disk is rotating under it, it will *not* follow the line AB. The light will cross the circumference slightly behind B. The equivalence principle tells us that the light will follow this exact same path across the disk at rest with the centrifugal gravitational field shown in the second drawing in Fig. 3. It will start out in the direction AB but veer off to the right (i.e., in the direction opposite to that of the disk's rotation in the first drawing). The light will travel along a path that is bent. What is true for this particular gravitational field will be true for gravitational fields in general. This conclusion is confirmed, at least qualitatively, by general relativity in its final form. The phenomenon is known as *light bending*. When British astronomers announced in 1919 that the effect had been detected during a solar eclipse, it made headlines on both sides of the Atlantic. Einstein became an overnight sensation, the world's first scientific superstar.[40]

I turn to the consequences of the special-relativistic effect of length contraction for gravitational theory. Suppose we put measuring rods on the radius and on the circumference of the rotating disk. According to special relativity, moving objects contract in the direction of motion. This does not affect the length of the rods on the radius since the radius is perpendicular to the motion of the disk. The length of the rods on the circumference, however, will be affected. The number of rods that a person on the rotating disk can fit on the disk's circumference is thus greater than the number of rods that a person standing next to the disk can fit on a circle under the rotating disk with the same diameter. Have both observers measure the ratio of the circumference and the diameter of the disk. The person next to the disk will find the Euclidean value π. The person on the disk will find a ratio greater than π. The equivalence principle tells us that someone in the centrifugal gravitational field in the second drawing in Fig. 3 will likewise find a value greater than π. This means that the spatial geometry in this particular gravitational field is non-Euclidean. We should expect this to be true for gravitational fields in general. The rotating disk is, in all likelihood, what first suggested to Einstein to represent gravity by curved space-time (Stachel 1989).[41] This in turn suggested a new way

of trying to make all motion relative. Before turning to this new attempt, I briefly discuss how Einstein came to abandon his original idea of reducing all non-uniform motion to gravity.

In 1912, partly in response to a special-relativistic theory of gravity published by Max Abraham (1912),[42] Einstein proposed his first formal new theory of gravity based on the equivalence principle. Up to that point, he had only explored isolated applications of the principle. The centerpiece of Einstein's theory was its gravitational field equation. One requirement the equation had to fulfill was that the static homogeneous gravitational field corresponding to uniform so-called Born acceleration in Minkowski space-time be a vacuum solution (i.e., a solution for the case without any gravitating matter). The equation that Einstein (1912c) initially published met this requirement. As Einstein quickly discovered, however, the equation violated energy conservation. The equivalence of energy and mass, expressed in special relativity's most famous equation, $E = mc^2$, demands that all energy, including the energy of the gravitational field itself, acts as a source of gravity. In the original field equations of Einstein's 1912 theory only the mass-energy of matter entered as a source. Einstein (1912d) had to add a term representing the mass-energy of the gravitational field itself. Unfortunately, the gravitational field corresponding to Born acceleration is only locally a vacuum solution of these amended equations. This made Einstein reluctant to add the extra term (ibid., 455–456). It meant that the equivalence principle, even for constant acceleration and static homogeneous gravitational fields, only held in infinitely small regions of space (Norton 1984, 106).[43] Einstein faced a choice between the philosophical promise of the equivalence principle to make all motion relative and the physical requirement of energy conservation. He opted for the latter: Physics trumped philosophy.

3. Second attempt: General covariance

To implement the insight that gravity is intimately connected with the geometry of space(-time) in a formal theory, Einstein turned to the mathematics of curved surfaces developed by Gauss. As a student at the *Eidgenössische Technische Hochschule* (ETH) in Zurich, he had studied this subject relying on notes of his classmate Grossmann. As luck would have it, when Einstein realized that this was the kind of mathematics he needed, the two of them were about to be reunited at their *alma mater*. In early 1912, Einstein was appointed professor of theoretical physics at the ETH, where Grossmann was professor of mathematics. Grossmann familiarized Einstein with the extension of Gauss's theory to higher dimensions by Riemann, Christoffel, and others.[44] Einstein supposedly told his friend: "You must help me or else I'll go crazy" (Pais 1982, 212; Stachel 2002b, 107).

The central quantity in the geometry of Gauss and Riemann is the *metric tensor* or *metric* for short. In general relativity it does double duty. It gives the geometry of space-time—or, to be more precise, its *chrono*-geometry—and the potential for

the gravitational field. The description of a 3+1D locally Minkowskian curved space-time (three spatial and one temporal dimension) with the help of a metric is completely analogous to that of a 2D locally Euclidean curved surface, such as the surface of the earth.
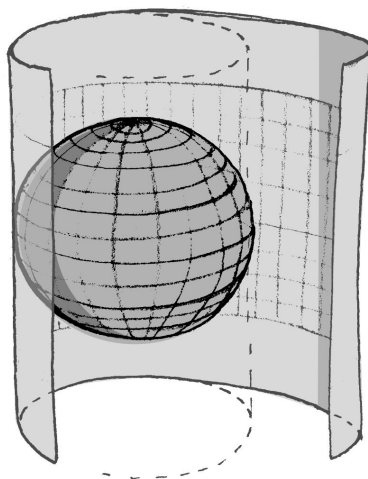

FIGURE 4. Mapping the earth

Fig. 4 shows a simple way of making a map of a miniature copy of this surface. A sheet of paper is rolled around the equator of a globe, forming a snug-fitting cylinder (as indicated by the dashed lines in the figure). The surface of the globe is projected horizontally on this cylinder mantle. The sheet is rolled out and a grid of regularly spaced horizontal and vertical lines is drawn on the part containing the image of the globe. With the help of this grid a unique pair of coordinates can be assigned to every point of the globe except for the two poles. To turn this grid into a useful map, instructions must be provided for converting distances in terms of (fractions of) steps on the grid to actual distances on the globe. In standard terminology, *coordinate distances* must be converted to *proper distances*. The conversion factors are given by the metric. They vary with direction and they vary from point to point. Right at the equator, where the map touches the globe, the conversion factors are equal to 1 in all directions. Everywhere else, the distance between lines of equal longitude is *larger* on the map than on the globe, while the distance between lines of equal latitude is *smaller* on the map than on the globe. In both cases, the discrepancy between distance on the map and distance on the globe gets larger as one moves away from the equator. Hence, the 'east-west conversion factor' gets *smaller* and the 'north-south conversion factor' gets *larger* as one moves away from the equator.

The 'east-west' component of the metric will vanish at the poles. Since all points on the horizontal line at the top of the grid correspond to the north pole,

the conversion factor multiplying the finite distances between them must be zero. The metric has a so-called *coordinate singularity* at the poles. In Section 5, we shall encounter an example of such a singularity in space-time.

For an arbitrary 2D curved surface, three conversion factors are needed at every point. For an arbitrary n-dimensional curved space(-time) this number is $\frac{1}{2}n(n+1)$. This then is the number of components of the metric that need to be specified. The standard notation for the components of the metric in general relativity is $g_{\mu\nu}$.[45] The Greek indices take on integer values from 1 to n (or, equivalently, from 0 to $n-1$). So $g_{\mu\nu}$ has a total of n^2 components, i.e., 16 in the case of 3+1D space-time. However, since the metric tensor is *symmetric* (i.e., for all values of μ and ν, $g_{\mu\nu} = g_{\nu\mu}$), only $\frac{1}{2}n(n+1)$ of those components are independent, i.e., 10 for 3+1D space-time. This means that the gravitational potential in Einstein's theory likewise has 10 components.

The metric field $g_{\mu\nu}(x^\rho)$ assigns values to the components $g_{\mu\nu}$ of the metric to points labeled with coordinates $x^\rho \equiv (x^1, \ldots, x^n)$. In 3D Euclidean space these could be the familiar Cartesian coordinates, $(x^1, x^2, x^3) = (x, y, z)$. In the case of the 2D surface in Fig. 4, the coordinates (x^1, x^2) refer to the grid drawn on the sheet. There are infinitely many other grids that can be used to assign a unique pair of coordinates to points of this or any other surface. It is not necessary (and often impossible) to cover the entire surface with one map. An atlas of partly overlapping maps will do. Any one-to-one mapping from a region of the surface to a region of the plane $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ (where $\mathbb{R}$ is the set of real numbers) will do as a map. With any map a metric field $g_{\mu\nu}(x^1, x^2)$ needs to be specified that gives the corresponding conversion factors from coordinate distances to proper distances.

Gauss made the remarkable discovery that at every point of an arbitrary 2D surface one can define curvature without reference to the surface's 3D Euclidean embedding space. He also found that this intrinsic so-called Gaussian curvature is the same function of the components of the metric field and its first- and second-order derivatives with respect to the coordinates in all coordinate systems.[46]

The transformation rules for translating the metric field and other quantities encoding the geometry of the surface from one coordinate system to another are also the same for all coordinate systems. The geometry of any curved surface can thus be described in the exact same way regardless of the choice of coordinates. In other words, the Gaussian theory of curved surfaces is *generally covariant*. The same holds for the Riemannian extension of the theory to higher-dimensional spaces, such as 3+1D space-time.[47]

Once a metric has been introduced, the length of lines in space(-time) can be computed. The lines of extremal length—the shortest ones in ordinary space, the longest ones in space-time—are called (metric) *geodesics*. In Riemannian geometry these are also the *straightest* lines, called *affine geodesics*. Which lines are the geodesics in a given Riemannian space is determined by the *geodesic equation*. This equation involves the *Christoffel symbols*, a sum of three terms, each of which

is a gradient of the metric. In electricity theory, the field is the gradient of the potential. Since the components of the metric double as the gravitational potentials in Einstein's theory, the Christoffel symbols are the natural candidates for representing the components of the gravitational field.

It was only in 1915 that Einstein adopted this definition of the gravitational field (see Section 6). In the following years the mathematicians Gerhard Hessenberg, Tullio Levi-Civita, and Hermann Weyl worked out the general concept of an (*affine*) *connection* (Stachel 2007, 1044–1046).[48] This quantity allows one to pick out the straightest lines directly, without the detour via the metric and lines of extremal length. In Riemannian geometry, the connection is given by the Christoffel symbols but it can be defined more generally and independently of the metric. Since what matters for the equivalence principle are the straightest rather than the longest lines in space-time, one can argue that general relativity is most naturally formulated in terms of the connection (Stachel 2007, 1041). Since Einstein formulated his theory in terms of the metric (and to this day textbooks tend to follow his lead), it looks as if the mathematical tools he needed were right at hand. In hindsight, it may be more accurate to say that he made do with the tools he had (Stachel 2002b, 86).

With the help of the notion of a geodesic, metric or affine, the situations illustrating the equivalence principle in Fig. 1 can readily be characterized in geometrical language. The *worldlines*, the trajectories through space-time, of an observer hovering freely in outer space far away from gravitating matter (IIb) or in free fall on earth (Ib) are (timelike[49]) geodesics, the worldlines of an observer accelerating in outer space (IIa) or resisting the pull of gravity on earth (Ia) are non-geodesics. As the examples illustrate, moving on a geodesic is physically different from moving on a non-geodesic.

In both situations, flat Minkowski space-time (II) and curved space-time (I), both observers, non-geodesic (a) and geodesic (b), can use their own worldline as the time axis of a coordinate system providing a map of the space-time region in their immediate vicinity. The metric field will be given by different functions of the coordinates for the two observers, but, because of the general covariance of Riemannian geometry, they will use the same equations involving the same functions of the metric field to describe the situation. This suggested to Einstein that the property of general covariance itself could be used to extend the principle of relativity from uniform to accelerated motion. In special relativity in its standard form, two inertial observers in uniform motion with respect to one another can use the same equations if they use special coordinate systems related to one another through special coordinate transformations called Lorentz transformations. By allowing arbitrary coordinates and arbitrary coordinate transformations, Einstein thought, one automatically extends the principle of relativity from uniform to arbitrary motion. Unlike Lorentz transformations in Minkowski space-time, however, the transformations between the coordinate systems of observers like (a) and (b)

in situations (I) and (II) in Fig. 1 are not between *physically equivalent* states of motion. We already saw this in Section 2. The point can be made succinctly in terms of the geometrical language introduced in this section: No coordinate transformation turns a geodesic into a non-geodesic or *vice versa.*

Erich Kretschmann (1917), a former student of Max Planck who had become a high school teacher, took Einstein to task for his conflation of general covariance and general relativity.[50] Given enough mathematical ingenuity, Kretschmann pointed out, just about any space-time theory, with or without absolute motion, can be written in generally-covariant form. Einstein (1918e) granted this criticism but predicted that the generally-covariant version of, say, Newtonian theory would look highly artificial compared to a theory such as general relativity that is naturally expressed in generally-covariant form. This expectation was proven wrong when generally-covariant formulations of Newtonian theory were produced in the 1920s (Norton 1993b, Sec. 5.3). Kretschmann also put his finger on the crucial difference between the invariance under Lorentz transformations of the standard description of Minkowski space-time in special relativity and the invariance under arbitrary coordinate transformations of the standard description of curved space-times in general relativity. Only the former transformations capture a symmetry of the space-time. They map the set of all inertial states—in geometrical terms: The set of all geodesics representing all possible inertial paths—back onto itself. The state of rest in one coordinate system will be mapped onto a state of uniform motion in another, but, since all such states are physically equivalent, that does not make any difference. This then is how Lorentz invariance expresses the relativity of uniform motion. General relativity allows many different space-times depending on the matter distribution. The set of all geodesics of all these space-times has no non-trivial symmetries. The theory's general covariance therefore is not associated with a relativity-of-motion principle in this way.[51]

General covariance, however, is important for the relativity of the gravitational field expressed in the mature version of Einstein's equivalence principle.[52] Once again consider Fig. 1. Both in situation (I) and in situation (II), observer (a)—Einstein, sitting at his desk, moving on a non-geodesic—will say that there is a gravitational field while observer (b)—the falling-window-cleaner/hovering-astronaut moving on a geodesic—will say that there is none. If we want to insist that there are no grounds for preferring the judgment of one over the other, it had better be the case that the laws of physics are the same for both of them. General covariance guarantees that this is true for all observers.[53]

Both in the Minkowski space-time of situation (II) and in the curved space-time of situation (I), observer (b) can, at least in his immediate vicinity, use special relativity in standard coordinates, using his own worldline as the time axis. This is because locally curved space-time is indistinguishable from flat Minkowski space-time, just as the surface of the earth or any other curved surface is locally

indistinguishable from a flat Euclidean plane. In Minkowski space-time in standard coordinates the components of the metric are constants, so all gradients and hence the Christoffel symbols are zero. Representing the gravitational field by the Christoffel symbols, observer (b) concludes, in situation (I) as well as in situation (II), that there is no gravitational field and that the inertio-gravitational effects experienced by observer (a) are due to inertial forces. For observer (a), the Christoffel symbols do not vanish, neither in situation (I) nor in situation (II), and he will ascribe the inertio-gravitational effects he experiences to gravitational forces. General covariance and the identification of the Christoffel symbols as the gravitational field thus implement the relativity of the gravitational field of the mature equivalence principle.

Many modern relativists see things differently. They would say that there is only a gravitational field in situation (I) and not in the flat Minkowski space-time of situation (II). They would also object to having the presence or absence of a gravitational field depend on which observer is making the call. In the spirit of general covariance, they would prohibit such coordinate-dependent notions and insist that only quantities transforming as tensors be used to represent physically meaningful quantities. One consequence of the transformation rules for tensors is that, if all components of a tensor vanish in one coordinate system, they vanish in all of them. The Christoffel symbols then are clearly not tensors. For many modern relativists, this disqualifies them as candidates for the mathematical representation of the gravitational field. Instead, as mentioned in Section 2, the non-vanishing of the curvature tensor is used as a coordinate-independent criterion for the presence of a gravitational field. To the end of his life, however, Einstein preferred to use the Christoffel symbols instead.[54]

By late 1912, for reasons good and bad, general covariance, or at least a covariance broad enough to cover arbitrary states of motion, had become central to Einstein's quest for general relativity. That winter he set out to find field equations for his new theory. He hoped to extract field equations of broad covariance from generally-covariant ones. The fruits of his labor, in which he was assisted by Grossmann, have been preserved in what is known as the Zurich notebook (CPAE 4, Doc. 10).[55] Despite considerable effort, he could not find physically sensible field equations of broad covariance and ruefully settled for equations of severely limited covariance. They were first published in May 1913 (Einstein and Grossmann 1913). It was only in November 1915 that Einstein replaced these *Entwurf* field equations by the generally-covariant field equations named after him. The Zurich notebook shows that almost three years earlier he had come within a hair's breadth of these generally-covariant equations. As he told some of his colleagues in 1915,[56] he had rejected them at the time because they did not seem to be compatible with energy-momentum conservation or reduce to the equations of Newtonian

gravitational theory for weak static fields. In 1913, Einstein thus saw another attempt to generalize the principle of relativity foiled because he could not get the physics to work out.

The restricted covariance of the *Entwurf* field equations, however, continued to bother him until, in late August 1913, he convinced himself through the ingenious "hole argument" that such restrictions are unavoidable.[57] Generally-covariant field equations, the argument purported to show, cannot do the basic job of uniquely determining the space-time geometry once the matter distribution has been specified. After his return to general covariance in November 1915, Einstein produced an equally ingenious escape from the hole argument, known as the "point-coincidence argument."[58]
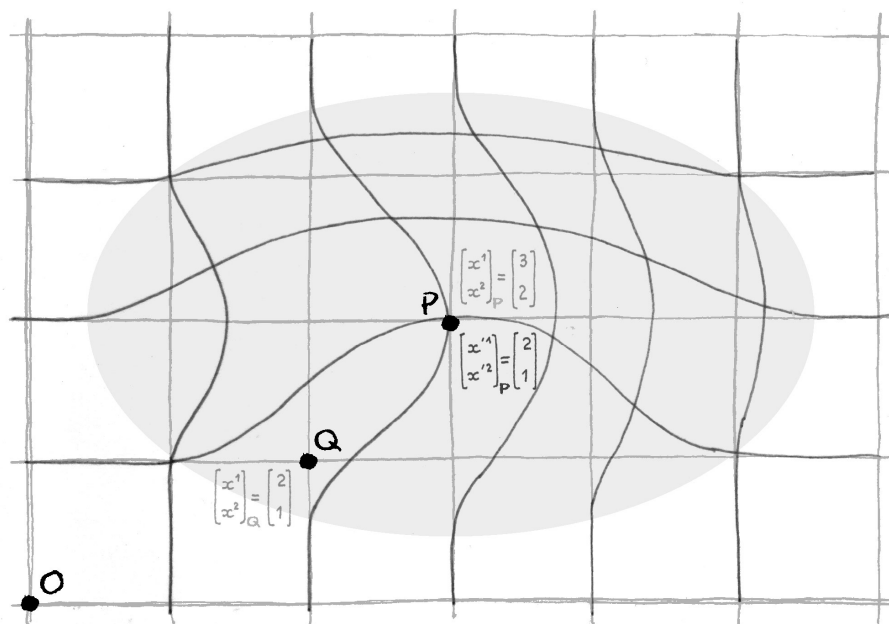

Figure 5. The hole argument

Fig. 5 illustrates how Einstein's hole argument works. It shows a 1+1D space-time (one spatial and one temporal dimension) with two coordinate systems, one with unprimed coordinates, (x^1, x^2), referring to the (lighter) grid with straight lines and one with primed coordinates, (x'^1, x'^2), referring to the (darker) grid with swiggly lines. The two grids coincide except in the shaded oval-shaped region. This region, devoid of matter, is the hole from which the hole argument derives its name. All candidate field equations are *local* in the sense that they set functions of the metric field and its derivatives, all evaluated at the same point, equal to functions describing the field's material sources evaluated at that same point. If

such equations are generally covariant, the hole argument seems to show, the matter distribution does not uniquely determine the geometry inside the hole.

The functions describing the matter distribution in this case are the same in both coordinate systems. This is because, outside the hole, the two coordinate systems coincide, and, inside the hole, these functions are identically zero. Let $g_{\mu\nu}(x^1, x^2)$, abbreviated $g(x)$, be a solution of the field equations for this particular matter distribution in terms of the unprimed coordinates. Let $g'_{\mu\nu}(x'^1, x'^2)$, abbreviated $g'(x')$, describe the same geometry in terms of the primed coordinates. If the field equations are generally covariant, this will be a solution for the same matter distribution. So far, we do not have different geometries, only different *descriptions* of the same geometry.[59] It takes one more step to get a different geometry: If $g'(x')$ is a solution, then $g'(x)$ is a solution as well. More explicitly, $g'(x')$ remains a solution for the same matter distribution *if we read the primed coordinates as referring to the straight grid rather than to the swiggly grid for which they were originally introduced.*[60]

Consider the three labeled points in Fig. 5. The point O is chosen as the origin of both coordinate grids. The coordinates of P with respect to the straight grid are $(x^1, x^2) = (3, 2)$. Its coordinates with respect to the swiggly grid are $(x'^1, x'^2) = (2, 1)$. The solution $g(x)$ assigns the metric $g_{\mu\nu}(3, 2)$ to P. The solution $g'(x')$ assigns the metric $g'_{\mu\nu}(2, 1)$ to that same point P. The curvature at P computed from those two metrics is the same. This will be true for all points in the hole. This is just a different way of saying that $g(x)$ and $g'(x')$ describe the same geometry. The solution $g'(x)$, it seems, does not. This solution assigns the metric $g'_{\mu\nu}(2, 1)$ to the point Q with coordinates $(x^1, x^2) = (2, 1)$ *with respect to the straight grid.* So the curvature assigned to one point (P) by both $g(x)$ and $g'(x')$ is assigned to another point (Q) by $g'(x)$. The solutions $g(x)$ and $g'(x)$ thus do seem to describe different geometries inside the hole. To block this violation of determinism, Einstein argued, the covariance of the field equations needs to be restricted. Field equations that preserve their form under coordinate transformations affecting only matter-free regions must be ruled out.[61]

Einstein used this argument in print on several occasions to defend the restricted covariance of the *Entwurf* field equations.[62] In November 1915, however, he published generally-covariant field equations without losing a word about the hole argument. Einstein (1915f, g, i) focused on demonstrating that his new field equations respect energy-momentum conservation and are compatible with Newton's theory in the appropriate limit. Problems on these two counts had made him forego general covariance in the first place. When his friends Michele Besso and Paul Ehrenfest reminded him of the hole argument, Einstein rolled out a new argument, the point-coincidence argument.[63] The hole argument was never mentioned in print again, but a version of this new argument was included in the first systematic presentation of the new theory a few months later (Einstein 1916e, 776–777).

The printed version of the point-coincidence argument is disappointing. Its premise is that all we ever observe are spatio-temporal coincidences, such as the intersections of worldlines.[64] Since there is no reason to privilege one coordinatization of a set of point coincidences over any other, the argument continues, all physical laws, including the field equations, should be generally covariant.[65] This does provide an escape from the hole argument. The different geometries found for the same matter distribution agree on all point coincidences. If that exhausts all we can ever observe, we have no empirical means of telling these geometries apart. We still have indeterminism but of a benign kind. If we deny reality to anything but point coincidences, there is no indeterminism at all. This way of avoiding indeterminism, however, comes at the price of "a crude verificationism and an impoverished conception of physical reality" (Earman 1989, 186).[66]

The letters to Besso and Ehrenfest suggest a more charitable interpretation of Einstein's resolution of the hole argument. In these letters, it seems, Einstein used point coincidences to put his finger on an unwarranted implicit assumption without which no indeterminism can be inferred in the first place. Consider, once again, Fig. 5. Suppose that, in the solution $g(x)$, two worldlines cross at P. In the solution $g'(x)$, the corresponding worldlines cross at Q. This is a different state of affairs only if there is some way of identifying Q other than by referring to it as the point where these two worldlines meet. It is at this juncture that the hole argument starts to unravel.

The identity of a point, one can argue, though the issue remains controversial, lies in the sum total of the properties assigned to that point by the metric field and all matter fields. It cannot be identified or individuated independently of those properties. It only has *suchness* and no *primitive thisness* or *haecceity*. Since candidate field equations are local in the sense specified above, *all* properties assigned to P by $g(x)$ are assigned to Q by $g'(x)$. But then P and Q are only different labels for one and the same space-time point, and $g(x)$ and $g'(x)$ are only different descriptions of the same geometry. Generally-covariant field equations can be perfectly deterministic after all.

In modern terms, all fields are defined on a so-called *differentiable manifold*, which, for our purposes, one can think of as an amorphous set of points with little more than a topology defined on it. The manifold still needs to be "dressed up" by a metric field if it is to represent space-time. Metric fields such as $g(x)$ and $g'(x)$ generated in the hole argument dress up different points of the bare manifold to become a particular space-time point. If points of the bare manifold could be individuated independently of the fields defined on it, these differently dressed-up manifolds would represent distinct though empirically indistinguishable space-times and we would have (a benign form of) indeterminism. We can avoid this consequence by denying that bare manifold points can be individuated in this way. That, in turn, means that we cannot think of the bare manifold as some kind of container. The combination of the hole argument and (the sophisticated

version of) the point-coincidence argument thus amounts to an argument against a substantival and in favor of a relational account of the ontology of space-time.[67]

This argument for relationism can be seen as a modern version of a classic argument against absolute space given by Leibniz in the course of his correspondence with Clark (Alexander 1956, 26).[68] One way to make the argument is the following. Newtonian space is the same everywhere, so the location of the world's center of mass makes no observable difference. This seems to violate Leibniz's *principle of sufficient reason.* For no reason whatsoever, God had to make one point rather than another the center of mass of the universe. To avoid such consequences, Leibniz insisted on his *principle of the identity of indiscernibles.* Since it is impossible to tell two worlds apart that differ only in the position of their center of mass, they must be one and the same world. But then Newtonian space cannot be some kind of container. In the hole argument, a violation of determinism replaces the deity's violation of the principle of sufficient reason that so exercised Leibniz. In the point-coincidence argument, determinism is restored through an account of the identity and individuation of space-time points in the spirit of the principle of the identity of indiscernibles with which Leibniz restored the principle of sufficient reason. So, even though general covariance does not eliminate absolute motion, Einstein's struggles with general covariance did produce what would appear to be a strong argument against absolute space(-time).

4. Third attempt: A Machian account of Newton's bucket

When it looked as if general covariance was not to be had, Einstein explored another strategy for eliminating absolute motion. This one was directly inspired by his reading of Mach's attempt to get around a classic argument for the absolute character of acceleration, an argument based on Newton's thought experiment of the rotating bucket in the *Scholium* on space and time in the *Principia* (Cohen and Whitman 1999, 412–413). Looking back on this period, Einstein wrote:

> Psychologically, this conception [that a body's inertia is due to its interaction with all other matter in the universe] played an important role for me, since it gave me the courage to continue to work on the problem when I absolutely could not find covariant field equations (Einstein to De Sitter, 4 November 1916 [CPAE 8, Doc. 273]).[69]

Consider a bucket of water set spinning. As the water catches up with the rotation of the bucket, it will climb up the side of the bucket. Since the effect increases as the relative rotation between water and bucket decreases and is maximal when both are rotating with the same angular velocity, Newton argued, the effect cannot be due to this relative rotation.[70]

Fig. 6 illustrates a different way of making the same point. The bucket experiment is broken down into four stages, the fourth being a flourish added by later authors (Laymon 1978, 405). In stage (I) the bucket and the water are at rest. In

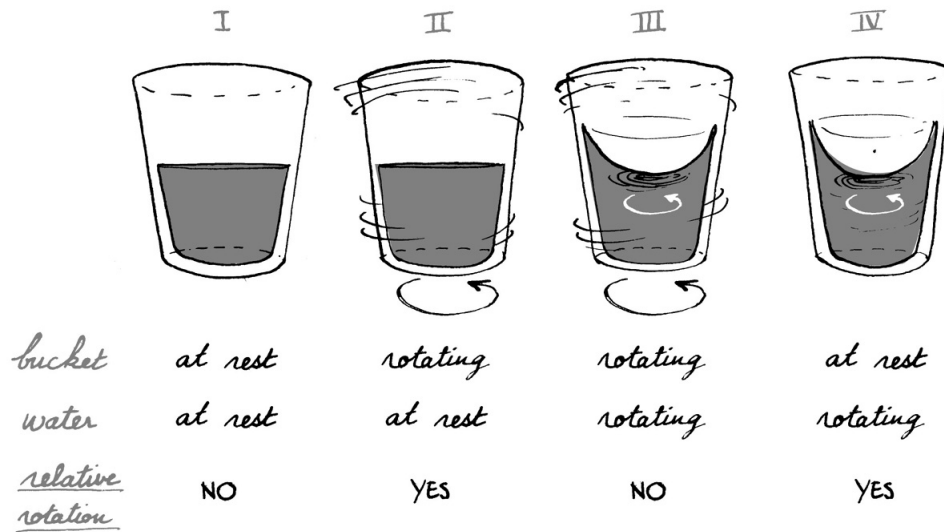|	I	II	III	IV
bucket	at rest	rotating	rotating	at rest
water	at rest	at rest	rotating	rotating
relative rotation	NO	YES	NO	YES

Figure 6. The rotating-bucket experiment

stage (II) the bucket has started to rotate but the water has yet to catch up with it. In stage (III) it has. In stage (IV) the bucket is abruptly stopped while the water continues to rotate. Comparison of these four stages shows that the shape of the water surface cannot be due to the relative rotation of the water with respect to the bucket. In stages (I) and (III) there is no relative rotation, yet the surface is flat in one case and concave in the other. In stages (II) and (IV) there is relative rotation, yet, once again, the surface is flat in one case and concave in the other.

The concave shape of the spinning water, Newton argued, is due to its rotation with respect to absolute space. Three centuries later, Mach resurrected another option briefly considered but rejected by Newton: Rotation with respect to other matter in the universe. "Try to fix Newton's bucket and rotate the heaven of fixed stars," Mach (1960, 279) asked his readers to imagine, "and then prove the absence of centrifugal forces." The implication is that it should make no difference whether the bucket or the "heaven of fixed stars" is rotating: In both cases the water surface should become concave. Mach's idea is illustrated in Fig. 7, depicting the earth, the bucket, and the water at the center of a spherical shell, much larger than shown in the figure, representing all other matter in the universe. On the left (situation I), the bucket and the water are rotating and the earth and the shell are at rest. On the right (situation II), it is the other way around.

The problem with Mach's proposal is that, according to Newtonian theory, the rotation of the shell will have no effect whatsoever on the water in the bucket, so the water surface on the right in Fig. 7 (situation II) should have been drawn flat. For most of the reign of the *Entwurf* theory and beyond, Einstein was convinced that this was a problem not for Mach's analysis but for Newton's theory and that his own theory vindicated a Machian account of the bucket experiment.

bucket and water rotating
shell and earth at rest

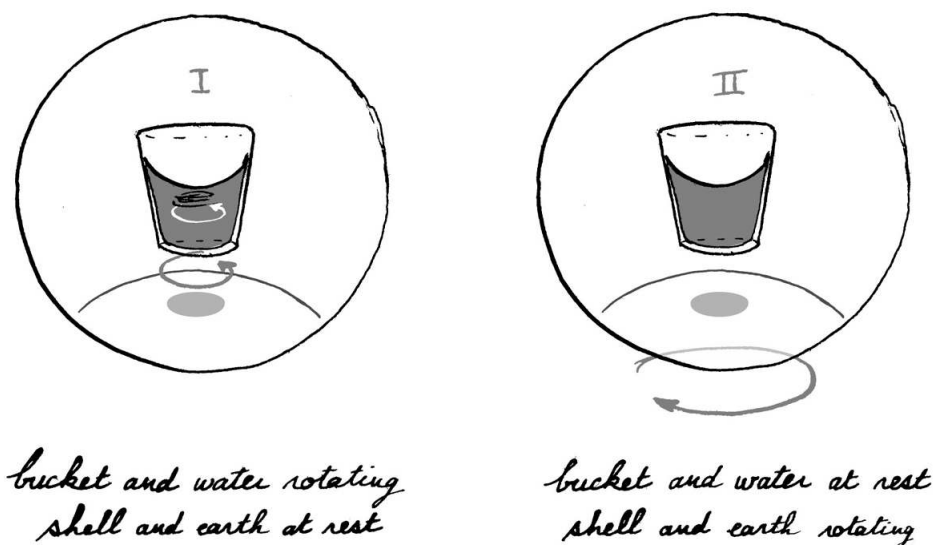bucket and water at rest
shell and earth rotating

FIGURE 7. Machian account of Newton's rotating-bucket experiment

Einstein thought, mistakenly, that his theory reduced the two situations pictured in Fig. 7 to one and the same situation viewed from the point of view of two different observers, one at rest with respect to the shell, the other at rest with respect to the bucket. He thought this followed from two more specific claims. First, the metric field of Minkowski space-time in rotating coordinates is a vacuum solution of the field equations, i.e., a solution in which there is no gravitating matter at all. Second, this is the metric field that a spherical shell rotating in the opposite direction with the same angular speed would produce near its center. We need to take a closer look at both claims as well as at the conclusion Einstein drew from them.

We can take the space-time in which we perform the bucket experiment to be Minkowskian even though the tell-tale shape of the water surface obviously depends on the gravitational field of the earth (cf. note 36). The metric field of Minkowski space-time in the standard coordinates for an observer at rest with respect to the shell is a vacuum solution of the field equations. This is true both for the *Entwurf* field equations of 1913 and the Einstein field equations of 1915. For the two situations in Fig. 7 to be equivalent, it is necessary—though not sufficient—that this metric field also be a vacuum solution, at least near the center of the shell, in the coordinates used by the observer at rest with respect to the bucket. The Einstein field equations automatically satisfy this requirement. Their general covariance guarantees that an arbitrary solution in some coordinate system remains a solution under arbitrary transformations to other coordinate systems. This is not true for the *Entwurf* field equations. Einstein had to check whether this specific solution, the Minkowski metric in standard coordinates, remains a solution

under the specific transformation to a rotating coordinate system. In this context, Einstein and Grossmann (1914b, 221) talked about "justified transformations" between "adapted coordinate systems" (i.e., adapted to the metric field). Earlier, Einstein had distinguished such transformations for specific solutions from the usual transformations for arbitrary solutions by labeling them "non-autonomous" and "autonomous," respectively.[71] This terminology reflects that the former depend on the metric field that is being transformed while the latter do not. Already in the Zurich notebook, Einstein had retreated to field equations invariant under non-autonomous transformations whenever he could not find equations invariant under ordinary autonomous transformations (Renn 2007a, Vol. 2, 495–496, 533–535).

Einstein went back and forth for more than two years on whether or not the transformation to rotating coordinates in the special case of Minkowski space-time is a justified transformation in the *Entwurf* theory; in other words, whether or not the *rotation metric*, the metric field of Minkowski space-time in rotating coordinates, is a vacuum solution of the *Entwurf* field equations (Janssen 2007). A sloppy calculation preserved in the so-called Einstein-Besso manuscript (cf. note 18) and probably dating from early 1913 reassured him that it is (CPAE 4, Doc. 14, [pp. 41–42]). In a letter to Lorentz of August 1913, Ehrenfest reported that Einstein had meanwhile done this calculation "five or six times," finding "a different result almost every time" (Janssen 2007, 833). Einstein appears to have accepted for a few months late in 1913 that the rotation metric is not a solution, but by early 1914 he had convinced himself on general grounds that it had to be.[72] In the authoritative exposition of the *Entwurf* theory of late 1914, this result, erroneous as it turns out, is hailed as a vindication of a Machian account of the bucket experiment (Einstein 1914o, 1031). In September 1915, Einstein redid the calculation of 1913 once more, this time without making any errors, and discovered to his dismay that the rotation metric is not a solution (Janssen 1999). He thereupon carefully reexamined the *Entwurf* theory, discovered a flaw in a uniqueness argument for the *Entwurf* field equations that he had published the year before, and used the leeway this gave him to introduce new field equations of broad covariance preserving their form under ordinary autonomous transformations to rotating coordinates (Janssen and Renn 2007).

The rotation metric was now a vacuum solution of the field equations. Is it also the metric field that a rotating shell produces near its center? It is not, neither according to the *Entwurf* field equations nor according to the Einstein field equations. To calculate the metric field for a given matter distribution one typically needs boundary conditions, the values of the metric field at spatial infinity. When Einstein calculated the metric field of a rotating shell in 1913, he uncritically took those values to be Minkowskian (CPAE 4, Doc. 14, [pp. 36–37]; Einstein 1913c, 1260–1261). He thus started with Minkowski space-time and calculated only how the rotating shell would curve this Minkowski space-time in its interior.

This curvature, it turns out, is much too small to make the water surface concave. More importantly, treating the effect of the rotating shell as a small perturbation of the metric field of Minkowski space-time defeats the purpose of producing a Machian account of the bucket experiment. Only a small part of the metric field is due to the rotating shell this way; most of it is due to absolute space-time, albeit of the Minkowskian rather than the Newtonian variety. To put it differently, only a small part of the inertia of particles near the center of the shell is due to their interaction with the rest of the matter in the universe, represented by the shell; most of it is determined by the absolute Minkowskian space-time. The theory thus fails to satisfy what Einstein (1913c, 1261) called the "hypothesis of the relativity of inertia" (see also Einstein 1912e and Einstein 1917b, 147).[73] This problem will arise for any physically plausible boundary conditions. At this point, Einstein clearly had a blind spot for the role of boundary conditions in his theory, something that would come back to haunt him (see Section 5).

As long as the rotation metric is a solution of the field equations, however, the relativity of the gravitational field expressed by the mature equivalence principle does hold for a bucket rotating in Minkowski space-time. The analysis is similar to that of the rotating disk in Section 2. Consider situation (I) on the left in Fig. 7 from the perspective of two observers, one at rest with respect to the shell and one at rest with respect to the bucket. As we just saw, the latter perspective on situation (I) is *not* the same as situation (II) depicted on the right in Fig. 7. For one thing, the water surface should have been drawn almost flat in situation (II). Furthermore, the metric field, which is not represented in Fig. 7, is very different in the two situations. Focus on situation (I). For an observer at rest with respect to the shell, the components of the metric field are constants, there is no gravitational field, the concave shape of the water surface is due to inertial forces, and the particles forming the shell are hovering freely in outer space. For an observer at rest with respect to the bucket, the components of the metric vary and there is a gravitational field. As we go to infinity, the values of the metric field become infinite, a clear indication that we are not dealing with situation (II) in which these values are assumed to remain perfectly finite. These degenerate non-physical values of the metric field need not bother us here, since we are only interested in the large but finite region occupied by the shell. For the observer at rest with respect to the bucket, the shape of the water surface is due to gravitational forces while the particles forming the shell are in free fall in this gravitational field.[74] Note that there is no need for cohesive forces keeping the particles of the rotating shell together, another clear indication that we are not dealing with situation (II), which does require such cohesive forces. To drive home this point one last time, note that the gravitational field for the observer rotating with the bucket in situation (I) does *not* have the shell as its source.

Einstein conflated the situation on the left in Fig. 7, redescribed in a coordinate system in which the bucket is at rest, with the very different situation on the right.

He believed accordingly that the metric field of a rotating shell would automatically be the rotation metric as long as the field equations used to compute this field preserve their form under the transformation to a frame rotating with the bucket. As he told Besso in July 1916, it is "obvious given the general covariance of the [field] equations," that the metric field near the center of a rotating ring, a case analogous to that of a rotating shell, is just the rotation metric. It is therefore, he added,

> of no further interest whatsoever to actually do the calculation. This is of interest only if one does not know whether rotation-transformations are among the "allowed" ones, i.e., if one is not clear about the transformation properties of the equations, a stage which, thank God, has definitively been overcome (Einstein to Besso, 31 July 1916 [CPAE 8, Doc. 245]).

Correspondence between Einstein and the Austrian physicist Hans Thirring in 1917 reveals that this misconception persisted for at least another year and a half. When Thirring first calculated the metric field inside a rotating shell, he was puzzled, as he told Einstein,[75] that he did not recover the rotation metric, as he expected on the basis of remarks in the introduction of Einstein's (1914o) exposition of the *Entwurf* theory. In his reply Einstein failed to straighten out Thirring and in a follow-up letter he explicitly confirmed Thirring's expectation.[76] By the time he published his final results, Thirring (1918, 33, 38) had realized that the metric field inside a rotating shell and the rotation metric correspond to completely different boundary conditions. He cited Einstein (1917b) and De Sitter (1916b) in this context. As we shall see shortly, the role of boundary conditions was at the heart of the debate between Einstein and De Sitter. Yet, Einstein did not breathe a word about them in his letters to Thirring.[77]

Thirring's work serves as a reminder that, as with Einstein's first two attempts, something good came of Einstein's third failed attempt to eliminate absolute motion. Following up on his study of the effect of a rotating hollow shell on the metric field *inside* of it, Thirring studied the effect of a rotating solid sphere on the metric field *outside* of it (Lense and Thirring 1918). Einstein (1913c, 1261) had also pioneered calculations of this effect, now known as "frame dragging."[78] In April 2004, NASA launched a satellite carrying the special gyroscopes of an experiment called *Gravity Probe B* aimed at detecting it. The data analysis has not been completed as this volume goes to press, but the scientists involved are confident that the experiment will confirm the predictions of general relativity.

5. Fourth attempt: Mach's principle and cosmological constant

The period from late 1915 to the fall of 1916 can be seen as an idyllic interlude in Einstein's quest for general relativity. The first systematic exposition of the theory dates from this period (Einstein 1916e). This widely-read article is probably

one of the reasons that the impression has lingered that with general relativity Einstein succeeded in banishing absolute motion from physics.[79] With the new field equations of November 1915, the entire theory was generally covariant at last. Einstein believed that this automatically extended the relativity principle for uniform motion, associated with Lorentz invariance, to arbitrary motion (see Section 3). He also believed that it sufficed for a Machian account of Newton's bucket experiment (see Section 4). Kretschmann disabused him of the first illusion in 1917; De Sitter of the second in the fall of 1916.[80]

General relativity retains vestiges of absolute motion through the boundary conditions at infinity needed to determine the metric field for a given matter distribution. During a visit to Leyden in the fall of 1916, Einstein was confronted with this problem by De Sitter. The solution he initially proposed was so far-fetched that he never put it in print. We only know of it through the ensuing correspondence[81] and through two papers of De Sitter (1916b,c).[82]

To ensure that the metric field has the same boundary conditions for every observer, Einstein argued, the value of all its components at spatial infinity must be either 0 or ∞. He imagined there to be masses outside the visible part of the universe that would contribute to the metric field in such a way that these degenerate values turn into Minkowskian values at the edge of the observable universe. De Sitter derided this proposal. This was a cure worse than the disease. It just replaced Newton's absolute space by invisible masses. What if better telescopes made more of the universe visible? Would these special masses then have to be pushed even farther out?

Einstein came to accept these criticisms. As he told De Sitter in February 1917: "I have completely abandoned my views, rightfully contested by you, on the degeneration of the $g_{\mu\nu}$. I am curious to hear what you will have to say about the somewhat crazy idea I am considering now."[83] This "crazy idea" was actually quite ingenious: If boundary conditions at spatial infinity are the problem, why not eliminate spatial infinity? Einstein thus explored the possibility that the universe is spatially closed.[84] He considered the simplest example that he could think of. In the *Einstein universe*, as this first relativistic cosmological model came to be known, the spatial geometry is that of the 3D hyper-surface of a hyper-sphere in 4D Euclidean space. This hyper-surface is analogous to the ordinary 2D surface of an ordinary sphere in 3D Euclidean space. It is also analogous to a circle, the 1D boundary of a round disk in 2D Euclidean space.

Suppressing two spatial dimensions, we can visualize the spatially closed 1+1D Einstein universe as a circle of some large radius R persisting through all eternity, forming an unbounded cylinder mantle, as illustrated in Fig. 8. The Einstein universe is therefore also known as the *cylinder universe*. It is a *static* world. The diameter R of the cylinder does not change over time. De Sitter emphasized a few months later that our universe is almost certainly *not* static.[85] Einstein ignored these warnings.
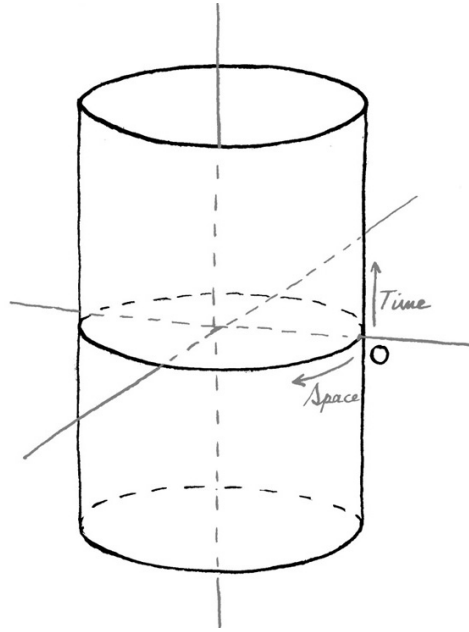
Figure 8. Einstein's cylinder universe

Before Einstein could use the cylinder universe as a new solution to the problem of boundary conditions, he had to check whether it was allowed by his theory, and, if so, for what matter distribution. He computed the components of the metric field of the cylinder universe in a convenient coordinate system and inserted them into the field equations. In this coordinate system the matter distribution is at rest and fully characterized by its mass density ρ. In general, the matter distribution is described by the ten independent components of the so-called (stress-)energy-momentum tensor $T_{\mu\nu}$. The energy density—or, equivalently, the mass density—is just one of those.

The result of Einstein's calculation was that the metric field of the cylinder universe is *not* a solution of the field equations as they stood. It *is* a solution, however, of slightly altered equations. A term proportional to $g_{\mu\nu}$, the so-called *cosmological term*, needs to be added. The proportionality constant lambda—nowadays used both in lower (λ) and in upper (Λ) case—is the infamous *cosmological constant*. It has to be exceedingly small so as not to disturb general relativity's agreement with Newton's theory of gravity in the limit of slow motion and weak fields. The cosmological constant determines both the radius R and the mass density ρ of the cylinder universe: $\lambda = 1/R^2 = \kappa\rho/2$ (where kappa is Einstein's gravitational constant). The radius of the cylinder universe is thus constant and large, its mass density constant and small.

When Einstein first considered tinkering with his field equations, he must have anticipated renewed criticism from De Sitter. While abandoning his nebulous

distant masses, he was now helping himself to an arbitrary new constant of nature. Mathematically, it turns out, the cosmological term is a natural addition to the Einstein field equations, but that was not immediately clear. In the paper in which he introduced the cosmological constant, however, Einstein (1917b) masterfully preempted the predictable charge of arbitrariness.

The title of the paper, "Cosmological considerations on the general theory of relativity," suggests that Einstein's aim was simply to apply his new theory to cosmology. Today the paper is indeed remembered and celebrated for launching modern relativistic cosmology. It did have a hidden agenda, however, which Einstein revealed in a letter to De Sitter about a month after its publication:

> From the standpoint of astronomy, I have, of course, built nothing but a spacious castle in the sky. It was a burning question for me, however, whether the relativity thought can be carried all the way through or whether it leads to contradictions. I am satisfied now that I can pursue the thought to its conclusion, without running into contradictions. Now the problem does not bother me anymore, whereas before it did so incessantly. Whether the model I worked out corresponds to reality is a different question (Einstein to De Sitter, 12 March 1917 [CPAE 8, Doc. 311]).

This hidden agenda explains why the order of presentation in the paper is the opposite of the order in which the results presented had been found. In the *context of discovery*, to borrow Hans Reichenbach's (1938, 6–7) terminology, Einstein had conceived of the cylinder universe first, had added the cosmological term to make sure the model is allowed by the field equations, and had only then started to worry about making the extra term plausible. In the *context of justification*, preempting the kind of criticism he could expect from De Sitter, Einstein argued for the extra term first and then showed that the field equations with the cosmological term do indeed allow the cylinder universe.

Einstein's justification for adding the cosmological term turned on an analogy with Newtonian cosmology.[86] To prevent a static universe from collapsing, he argued, a gravitational repulsion needs to be added, both in Newtonian theory and in general relativity. The cosmological term provides this repulsion. Arthur S. Eddington (1930) was the first to point out in print that the equilibrium thus produced in Einstein's cylinder universe is unstable. Much to the surprise of modern commentators (Weinberg 2005, 31), Einstein failed to recognize this.

What did De Sitter make of Einstein's new proposal? In response to the letter from which I quoted above, he wrote:

> As long as you do not want to force your conception on reality, we are in agreement. As a consistent train of thought, I have nothing against it and I admire it. I cannot give you my final approval before I have had a chance to calculate with it (De Sitter to Einstein, 15 March 1917 [CPAE 8, Doc. 312]).

Five days later, De Sitter had done his calculations. They had led him to an alternative solution of Einstein's amended field equations. He communicated this result in a letter to Einstein,[87] which served as the blueprint for a paper submitted to the Amsterdam academy shortly thereafter (De Sitter 1917a).
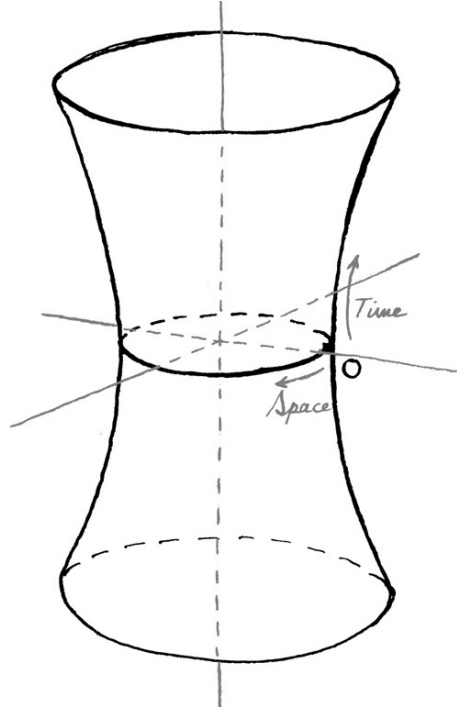

Figure 9. De Sitter's hyperboloid universe

Following a suggestion by Ehrenfest, De Sitter considered a natural analogue of the cylinder universe in which time is treated in a similar way as the three spatial dimensions. This *De Sitter universe* has the space-time geometry of the 3+1D hypersurface of a hyper-hyperboloid in 4+1D Minkowski space-time. It is therefore also known as the *hyperboloid universe*. Fig. 9 shows a lower-dimensional version of this space-time, the 1+1D surface of a hyperboloid embedded in 2+1D Minkowski space-time. All points on the hyperboloid have the same spatio-temporal distance to its center in the embedding space (the origin of the coordinate axes shown in the figure). A hyperboloid in 2+1D Minkowski space-time is thus the analogue of a sphere in 3D Euclidean space.[88]

As Einstein had done for the cylinder universe, De Sitter checked whether the hyperboloid universe was allowed by the field equations with the cosmological term. He found that it was, provided that the radius R of the 'waist' of the hyperboloid satisfies the relation $\lambda = 3/R^2$ and the mass density ρ equals zero everywhere. De Sitter had thus found a vacuum solution of the new field equations.

This defeated the purpose of Einstein's introduction of the cosmological term. The inertia of test particles in De Sitter's hyperboloid universe is due to space-time rather than to their interaction with distant matter. It was crucial for Einstein's new attempt to implement the relativity of arbitrary motion that this be impossible. As he wrote to De Sitter:

> It would be unsatisfactory, in my opinion, if a world without matter were possible. Rather, it should be the case that the $g_{\mu\nu}$-field is *fully determined by matter and cannot exist without the latter.* This is the core of what I mean by the requirement of the relativity of inertia (Einstein to De Sitter, 24 March 1917 [CPAE 8, Doc. 317]).

De Sitter got Einstein's permission to quote this passage in a postscript to his paper (De Sitter 1917a). The second sentence is the first explicit statement of what Einstein (1918e) dubbed "Mach's principle" the following year. If this principle were satisfied, absolute motion would finally be eradicated. A body's motion is defined with respect to the metric field. If Mach's principle is true, this field is nothing but an epiphenomenon of matter and all talk about motion with respect to it is nothing but a *façon de parler* about motion with respect to the matter generating it (Maudlin 1990, 561). Vacuum solutions were therefore anathema and Einstein immediately set out to find grounds to dismiss the one De Sitter had purportedly found.

Einstein eventually fastened on to the so-called *static form* of the solution, an alternative way of mapping the hyperboloid universe in which it can more readily be compared to Einstein's cylinder universe (De Sitter 1917b, c).[89] The hyperboloid universe looks anything but static in Fig. 9. Consider horizontal cross-sections of the hyperboloid. These circles represent space at different times. Going from the distant past to the distant future, we see that these circles get smaller until we reach the waist of the hyperboloid and then get larger again. It thus looks as if the hyperboloid universe contracts and then re-expands. One has to keep in mind, however, that this conclusion is based on an arbitrary choice of space and time coordinates.

Fig. 10 shows an alternative coordinatization of the hyperboloid universe. In these *static coordinates*, De Sitter's universe, shown on the right, is represented by a cylinder, just as Einstein's, shown on the left. In both worlds, space is represented by a circle of radius R at all times. In these coordinates, the spatial part of the metric field of the De Sitter universe is exactly the same as that of metric field of the Einstein universe in its standard coordinatization. The temporal part, however, is different.

Compare the components g_{44} of the two metric fields, the conversion factors from coordinate time to proper time, at $t = 0$. The situation will be same for any other value of t. Space at $t = 0$ is represented by the circles through O and P, the positions at that time of an observer and of the 'horizon' or 'equator' for that observer, respectively. In the Einstein universe, the time conversion factor is the
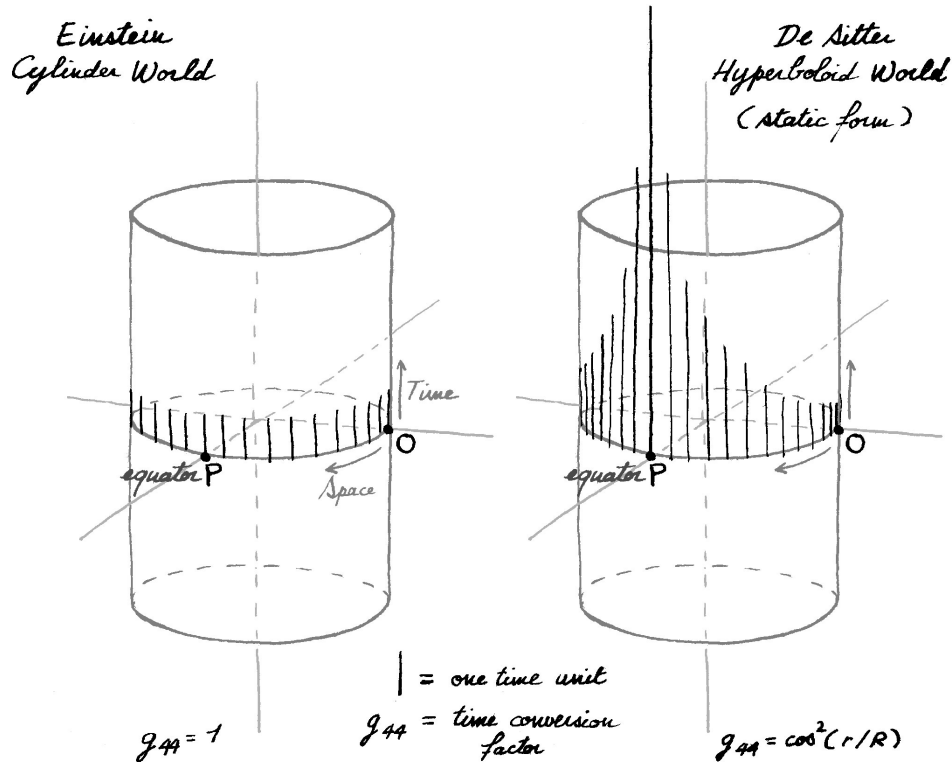
Figure 10. Comparing Einstein's cylinder universe and De Sitter's hyperboloid universe

same everywhere: $g_{44} = 1$. For all points on the circle, one unit of proper time, represented by the vertical line segments in Fig. 10, corresponds to one unit of coordinate time. In the De Sitter universe, the time conversion factor varies from point to point: $g_{44} = \cos^2(r/R)$ (where the distance r from point O runs from 0 to πR). It is equal to 1 for $r = 0$, then steadily decreases until it vanishes at the horizon P at distance $r = (\pi/2)R$. As indicated on the right in Fig. 10, when we go from O to P, segments of coordinate time of increasing length correspond to one unit of proper time. At the horizon P we need a segment of infinite length.

Einstein used this odd behavior of the temporal component of the metric to argue that the De Sitter world is not empty after all. That the vertical line segments in the drawing on the right in Fig. 10 get longer and longer as we go from O to P means that it takes an increasing amount of coordinate time for a clock to advance one unit of proper time. It thus looks as if clocks are slowing to a crawl as they approach the horizon. This is reminiscent of the gravitational redshift experienced by clocks brought ever closer to some massive object (see Section 2). Einstein concluded that a large amount of matter must be tucked away at the horizon in

the De Sitter universe. The main difference between the Einstein universe and the De Sitter universe, he thought, was that in the former matter was spread out evenly, while in the latter it was concentrated at the horizon.

On the postcard on which Einstein first spelled out this line of reasoning, De Sitter scribbled in the margin: "That would be distant masses yet again!"[90] And on the back he elaborated: "How large does the "mass" of this matter have to be? I suspect ∞! I do *not* adopt such matter as ordinary matter. It is *materia ex machina* to save Mach's dogma" (ibid., my italics; the pun—ex *Mach*ina—was probably unintended). For all the exasperation one senses in these comments, De Sitter could not put his finger on the error in Einstein's argument.

The analysis of the static form of the De Sitter solution strengthened Einstein in his belief that the field equations with cosmological term do not allow vacuum solutions. In March 1918, he submitted two short papers in response to De Sitter's challenge to his latest attempt to eliminate absolute motion. In the first, Einstein (1918e) reworked the foundations of his theory[91] and officially introduced Mach's principle. In the second, he conjectured that the De Sitter solution, an apparent counter-example to Mach's principle,

> may not correspond to the case of a matter-free world at all, but rather to that of a world, in which all matter is concentrated on the surface $r = (\pi/2)R$:[92] This could well be proven by considering the limit of a spatial matter distribution turning into a surface distribution (Einstein 1918c, 272).

It was Weyl who took up the challenge of producing such a proof.[93]

Less than two months later, on the very same day that Einstein sent Weyl a letter in which he expressed his satisfaction over the latest version of this proof, another mathematician, Felix Klein, sent Einstein a letter in which he showed that the singular behavior of the metric field of the De Sitter world in static coordinates is just an artifact of those coordinates.[94] It may come as a surprise that this had not been clear to all parties involved right away. As we saw above, De Sitter had found his solution by considering a completely regular hypersurface embedded in a 4+1D Minkowski space-time. It follows that any singularity in any coordinate representation of the solution has to be a coordinate singularity and cannot be an intrinsic singularity (cf. the poles in the example in Fig. 4). Einstein and De Sitter had, in fact, recognized the degeneration of the metric field of the hyperboloid universe in *other* coordinates as pathologies of those coordinates.[95] And in his paper on the De Sitter solution, Einstein (1918c) had taken a significant first step toward formulating a sensible criterion to distinguish intrinsic singularities from coordinate singularities.[96] Yet, despite all of this, Einstein did not immediately appreciate Klein's point. In his response he wrote that Weyl had just furnished the proof for his conjecture that there must be a large amount of mass at the horizon of the hyperboloid universe.[97]
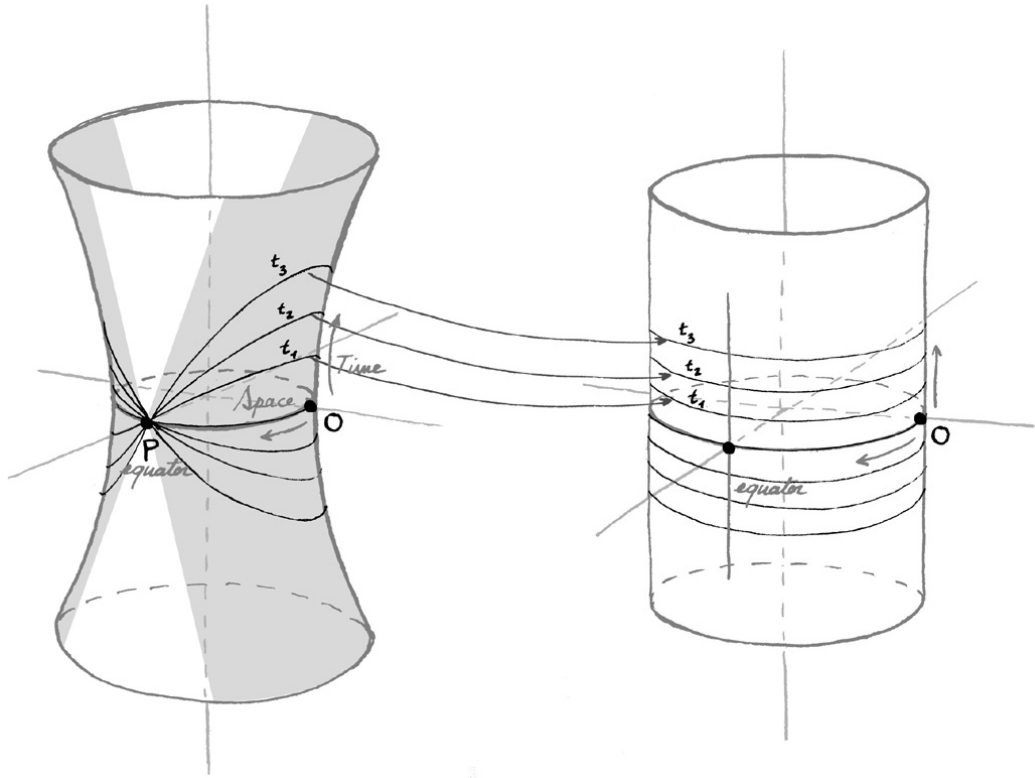
FIGURE 11. Mapping a double-wedge region of De Sitter's hyperboloid universe onto a cylinder

In his next letter, Klein reiterated the point of the previous one in simpler terms.[98] Klein's reasoning is illustrated in Fig. 11.[99] The figure shows geometrically how to get from the original hyperboloid (Fig. 9) to the static form of the De Sitter solution (Fig. 10). This is done through a clever choice of time slices of the hyperboloid. Imagine that the plane cutting the hyperboloid horizontally at the waist, i.e., the plane through the circle with O and P on the left in Fig. 11, can pivot around the coordinate axis of the embedding space-time going through P. Rotate this plane from -45° to $+45^o$ around this axis and let its successive cross-sections with the hyperboloid represent time slices from past to future infinity. In the figure, these cross-sections look like ellipses that get ever more elongated as their angle with the horizontal plane increases until they degenerate into a pair of parallel lines for angles of $\pm 45^o$. In terms of the metric of Minkowski space-time, however, for all angles between -45° and $+45^o$, they have the exact same shape as the circle that forms the hyperboloid's waist (recall that all points on the hyperboloid have the same spatio-temporal distance to its center in the embedding space-time). Stacking up these circles we arrive at the cylinder mantle on the right in Fig. 11, which is just the static form of the De Sitter solution.

As Fig. 11 shows, these static coordinates only cover the shaded double-wedge-shaped region of the hyperboloid. More importantly for our purposes, we can now see why the time conversion factor in these coordinates vanishes at the two points on the edge of these wedges. The times slices all intersect at P (like the lines of equal longitude on earth at the poles). This one point on the hyperboloid thus gets mapped onto a vertical line on the cylinder mantle (like the horizontal line representing the poles on the map in Fig. 4). The distance between different points on this line needs to be multiplied by zero to reflect that they all represent the same point P on the hyperboloid. This is why $g_{44} = 0$ at P. There is nothing special about P. We could go through the exact same argument using a different set of axes in the embedding space-time and g_{44} would be zero at some other pair of points. Contrary to what Einstein and Weyl believed at the time, there is no mass anywhere in the De Sitter universe.

To Einstein's credit, he immediately accepted this dire consequence of Klein's analysis once Klein had explained it to him in terms he understood. On the half-empty *verso* of Klein's letter, Einstein drafted his response. Testifying to his supreme surefootedness as a writer, the draft does not contain a single deletion and is virtually identical to the actual letter sent a few days later. The letter begins:

> You are completely right. The De Sitter world in and of itself is free of singularities and all its points are equivalent. A singularity only arises from the substitution which gives the transition to the static form of the line element … My critical comment on the De Sitter solution stands in need of a correction; there actually is a singularity free solution of the gravitational equations without matter (Einstein to Klein, 20 June 1918 [CPAE 8, Doc. 567]).

Einstein then retreated to the position that the De Sitter solution could still be ruled out as a model of our universe precisely because it cannot be turned into a static model without the introduction of a singularity.

Einstein never published a correction to his critical note on the De Sitter solution. But he lost his enthusiasm for Mach's principle—and for the cosmological constant that had been the price he paid for it—once he had been forced to admit that the De Sitter solution is a counter-example. Looking back at this period the year before he died, Einstein wrote:

> In my view one should no longer speak of Mach's principle at all. It dates back to the time in which one thought that the "ponderable bodies" are the only physically real entities and that all elements of the theory which are not completely determined by them should be avoided. (I am well aware of the fact that I myself was long influenced by this *idée fixe*) (Einstein to Felix Pirani, 2 February 1954).[100]

Although this statement dates from a much later period, the disenchantment with Mach's principle can already be discerned in *Ether and relativity*, in which Einstein (1920j) presented the metric field as a new kind of ether, thus abandoning the requirement that the metric field be reducible to matter. This development was greeted enthusiastically by De Sitter (Hoefer 1994, 329).

This marks the end of Einstein's crusade against absolute motion. After four failed attempts he finally threw in the towel. Around 1920, he embarked on a new project, the unification of the inertio-gravitational field and the electromagnetic field through the extension of general relativity in various different directions.[101] This project he pursued until his dying days (Pais 1982, 479).

6. Post mortem: How Einstein's physics kept his philosophy in check

It should be clear by now that general relativity does not generalize the relativity principle of special relativity from uniform to non-uniform motion. The combination of the equivalence principle and general covariance leads to what can be called the relativity of the gravitational field—the recognition that an effect due to gravity for one observer can be due to inertia for another—not to the relativity of arbitrary motion. Einstein's theory also does not vindicate Mach's suggestion that Newton's bucket experiment could be accounted for in terms of relative motion with respect to distant matter. Nor is the theory such that the metric field can be reduced to its material sources, as demanded by what Einstein called Mach's principle. General relativity thus failed to fulfill many of the high hopes Einstein had nourished during the long years he had spent in search of it. The consoling thought in all of this is that Einstein had found a tremendously successful new theory of gravity.

The analysis so far may have left the impression that it was sheer luck that Einstein arrived at this theory at the end of his journey. Many of the guideposts he had relied on along the way had, after all, listed a destination that was nowhere to be found. The aim of this concluding section is to dispel this impression. I want to highlight three factors that help explain the success of Einstein's search for a new theory of gravity despite the failure of many of his philosophical objectives. First, Einstein did not just want to eliminate absolute motion, he also wanted to reconcile some fundamental insights about gravity with the results of special relativity and integrate them in a new broader framework. Second, when these efforts led him to the introduction of the metric field, he carefully modeled its theory on the successful theory of the electromagnetic field of Maxwell and Lorentz. Third, whenever his philosophical agenda clashed with sound physical principles, Einstein jettisoned parts of the former instead of compromising the latter. In short, throughout his quest for general relativity, Einstein checked whether the philosophical goals he had set himself could be realized in a physically sensible theory.

Einstein's later recollections, especially those in the lecture in Glasgow mentioned at the beginning of this chapter (Einstein 1933, cf. note 2), leave little doubt that his interest in gravity predated, if not by much, his hope that the subject might hold the key to the relativity of arbitrary motion. Special relativity made Newton's theory of gravity unacceptable. Like other physicists around 1905, Einstein sought to replace this theory, based on instantaneous action-at-a-distance, by a new theory in which, as in the electrodynamics of Maxwell and Lorentz, action is mediated by fields propagating with the speed of light. Working out the law for the force this field exerts on a test particle, Einstein arrived at

> a result which raised my strong suspicions. According to classical mechanics, the vertical acceleration of a body in the vertical gravitational field is independent of the horizontal component of its velocity ... But in the theory I advanced, the acceleration of a falling body was not independent of its horizontal velocity (Einstein 1933, 286–287).[102]

The acceleration of a falling body will likewise depend on the horizontal velocities of its constituent parts and thus on "the internal energy of a system" (ibid.). This is at odds with Galileo's principle that the acceleration of free fall is the same for all bodies. Recognizing this conflict, Einstein seems to have had an epiphany:

> This law, which may also be formulated as the law of the equality of inertial and gravitational mass, was now brought home to me in all its significance. I was *in the highest degree amazed* at its existence and guessed that *in it must lie the key to a deeper understanding of inertia and gravitation* (Einstein 1933, 287, my emphasis).

Einstein's interest thus shifted from the conflict between special relativity and Newtonian action-at-a-distance, on which his contemporaries continued to focus, to the conflict between special relativity and Galileo's principle (Renn 2007b, 61). Einstein quickly gave up on the attempt to develop a theory of the gravitational field within the framework of special relativity. Such a theory, he felt "clearly failed to do justice to the most fundamental property of gravitation" (Einstein 1933, 287). What he would come to call the equivalence principle would have to be the cornerstone of a truly satisfactory new theory of gravity. In his Glasgow lecture, as in the article intended for *Nature* of 1920 (see note 31), Einstein still presented the equivalence principle as intimately connected with the relativity of arbitrary motion, but that persistent and unfortunate association does not diminish its value as a constraint on Einstein's theorizing about gravity.

In Einstein's 1912 theory for static gravitational fields, a variable speed of light plays the role of the gravitational potential. Einstein thus gave up one of the two postulates of special relativity, the light postulate, in his effort to extend the other, the relativity postulate (Einstein 1912h, 1062). From the point of view of the *Entwurf* theory, the precursor of general relativity proposed the following

year, the variable speed of light of the 1912 theory is one of the components of the metric field. From this point of view, gravity had thus already become part of the fabric of space-time in the 1912 theory. Space-time is no longer the Minkowski space-time of special relativity.

Later that same year, Nordström (1912) published a paper in which he proposed a theory of gravity that stays within the confines of special relativity. In this theory, gravitational interaction, like electromagnetic interaction, is conceived of in terms of a field in Minkowski space-time. In a note added in proof, Nordström (1912, 1129) informs the reader that Einstein had already told him (in a letter that is no longer extant) that this theory runs afoul of the general problem with horizontal velocities mentioned above: The acceleration of free fall of a body rotating in a horizontal plane would be less than that of the same body without such rotation. Nordström initially shrugged off the objection, insisting the effect was too small to be measured.

Einstein took an active part in the further development of Nordström's theory.[103] A sizable fraction of the first two papers by Nordström (1912, 1913a) on his new theory went into deciding on the quantity that should represent the material source of the gravitational field. Nordström settled on the energy density. In his part of the *Entwurf* paper, Einstein approvingly passed on the suggestion of Max Laue, then at the University of Zurich, that it should be the so-called *trace* of the energy-momentum tensor instead (Einstein and Grossmann 1913, 21).[104] Acknowledging both Einstein and Laue, Nordström (1913b, 533) adopted this suggestion. This established a first parallel between the theories of Einstein and Nordström. In Einstein's theory, the ten independent components of the energy-momentum tensor act as the material source for the ten independent components of the metric field. In Nordström's amended theory, a scalar constructed out of the energy-momentum tensor acts as the material source for the one-component gravitational potential.

Even in the modified version of the Nordström theory, the acceleration of free fall of a body depends on its horizontal velocity, as it must in any special-relativistic theory of gravity (see note 102). The Einstein-Laue amendment, however, did remove the dependence of the acceleration on a body's rotation and on the kinetic energy of its constituent particles. The general treatment of stressed bodies by Laue (1911a,b), of which Nordström (1913a) had already made extensive use, shows that such dependence disappears once the internal forces that keep a body from flying apart are taken into account.[105] This illustrates a more general point. A new theory of gravity had to incorporate the insights of special relativity and these insights went well beyond the prohibition against instantaneous action-at-a-distance or the inclusion of gravity among its own sources required by the equivalence of mass and energy. The work by Laue and others on the relativistic mechanics of continua, in which the (stress-)energy-momentum tensor takes center stage, was especially important in this regard. In an unpublished review article on special relativity written in 1912, Einstein appropriately called this development

"the most important new advance in the theory of relativity" (CPAE 4, Doc. 1, [p. 63]).[106]

Based on one gravitational potential and flat space-time, Nordström's theory was much simpler than the *Entwurf* theory with its ten gravitational potentials and curved space-time. In defense of his own more complicated theory, Einstein concocted a clever thought experiment showing that Nordström's theory violated energy conservation, albeit only under highly artificial circumstances (Einstein and Grossmann 1913, 21–22).[107] Einstein conceded, however, that his main reason for preferring the *Entwurf* theory was that it generalized the relativity principle to arbitrary motion (ibid.). This was a remarkable admission. To generalize the relativity principle, Einstein thought, a theory of broad covariance was needed (see Section 3). Yet so far he had been unable to establish whether the limited covariance of the *Entwurf* theory was broad enough for his purposes. That he nonetheless preferred his own theory over Nordström's shows that he had the courage of his convictions; that he carefully examined and even contributed to the strengthening of Nordström's theory shows that he was not dogmatic about them.

This same open-mindedness is on display in a lecture that Einstein (1913c) gave in Vienna in September 1913. Einstein compared and contrasted the Nordström theory and the *Entwurf* theory, giving roughly equal time to both. He had meanwhile found a way to restore energy conservation in his competitor's theory. This made it a perfectly viable alternative to the *Entwurf* theory. True to his belief that Galileo's principle held the key to a new theory of gravity, Einstein had no interest in theories in which this principle does not hold. For this reason, he made no mention of the gravitational theory proposed by Gustav Mie (1913), who predictably took exception in question time (Einstein *et al.* 1913, 1262–1263).[108]

To decide between the Nordström theory and the *Entwurf* theory empirically, Einstein (1913c, 1262) told his audience in Vienna, one had to wait for a solar eclipse. In Nordström's special-relativistic theory, light propagates in straight lines at constant speed. It is not bent by gravity. Nordström's theory thus respects the equality of inertial and gravitational mass but does not implement the equivalence principle.[109] After all, it follows directly from the latter that gravity does bend light (see Section 2, Fig. 3). The *Entwurf* theory predicts an effect half the size of that predicted by general relativity (Einstein 1915h, 834). Einstein expressed the hope that the solar eclipse of August 1914 would bring the decision between the two theories.[110] In the meantime, other arguments would have to do. Whereas in the *Entwurf* paper, Einstein had presented his theory's broader covariance as its main advantage, he now pointed to the relativity of inertia, which, he argued, was realized in the *Entwurf* theory but not in the Nordström theory (Einstein 1913c, 1260–1261). This reflects the shift in Einstein's strategy for eliminating absolute motion between May and September 1913 (cf. the quotation at the beginning of Section 4). In the present context, the important point is that, both in the *Entwurf* paper in May and in the Vienna lecture in September, Einstein argued

that philosophical considerations gave his theory the edge over Nordström's while acknowledging that in terms of more mundane physical considerations it was a toss-up.

The following year, Einstein produced a much stronger argument in favor of moving beyond special relativity. He showed that Nordström's theory could readily be reformulated as a theory in which, as in his own theory, gravity is incorporated into the space-time structure. This possibility was first brought out by the escape Einstein found from his own argument against the Nordström theory in the *Entwurf* paper. Einstein communicated this escape to Nordström, who presented it in his next paper on his theory, dutifully acknowledging his source (Nordström 1913b, 543–545).[111] Einstein argued that the only way to guarantee energy conservation in the Nordström theory was to assume a universal dependence of the dimensions of physical systems and the duration of physical processes on the gravitational potential.[112] Because of this universality, clocks and rods would no longer measure times and distances in the flat Minkowski space-time posited by the theory but times and distances in some curved space-time.[113]

Early in 1914, in a joint paper with Lorentz's former student Adriaan D. Fokker, Einstein reformulated Nordström's theory using Riemannian geometry (Einstein and Fokker 1914). In this reformulation of Nordström's theory, as in Einstein's own theory, the metric field describes both the gravitational potential and the space-time geometry. The metric field in the Nordström theory is determined by a generally-covariant equation and an additional condition.[114] The generally-covariant equation sets the so-called curvature scalar, a quantity constructed out of the Riemann curvature tensor involving first- and second-order derivatives of the metric field, equal to the trace of the energy-momentum tensor. The structure of this equation is thus similar both to the *Entwurf* field equations and to the Einstein field equations. Unlike these equations, however, the equation in the Nordström theory only has one component. To determine the ten independent components of the metric, one needs the additional condition that any metric field allowed by the theory can be written in the form of the product of a function of the space-time coordinates and the constant components of the metric for Minkowski space-time in the standard form used by inertial observers in special relativity. This extra condition guarantees that the velocity of light is a constant, as it should be in a special-relativistic theory. It is also what rules out the light bending required by the equivalence principle. The *conformal factor*, as the function multiplying the standard Minkowski metric is called, is just the gravitational potential in the original formulation of the Nordström theory. The field equation of the original formulation is recovered if this special form of the metric field is inserted into the generally-covariant equation.

The reformulation of the Nordström theory by Einstein and Fokker shows that even this most satisfactory of special-relativistic theories of gravity eventually leads

beyond special relativity. As Norton (1992b, 1993a) as well as Giulini and Straumann (2006, Sec. 5) emphasize, the new formulation turns gravity from a field in flat Minkowski space-time to part of the fabric of curved space-time. As Einstein and Fokker (1914, 321) put it themselves, their reformulation shows that the Nordström theory is covariant under a group of transformations broader than the class of Lorentz transformations characterizing special relativity. In Einstein's thinking at the time, this was tantamount to a generalization of the relativity principle. The main difference between Nordström's theory and his own *Entwurf* theory then was that the latter not only extended the relativity principle but also implemented the equivalence principle, reducing the equality of inertial and gravitational mass to an essential unity—a *Wesensgleichheit* (see note 34)—of gravity and inertia.

Regardless of how the point is made, the recasting of Nordström's theory in terms of Riemannian geometry bolstered Einstein's confidence that he was on the right track with a theory like the *Entwurf* theory based on the metric tensor. Considerations of how to reconcile the physical insights represented by Galileo's principle and special relativity, which had led to Einstein's interest in Nordström's theory in the first place, ended up pointing in the same direction as the considerations about extending the relativity principle that had guided Einstein in his formulation of the *Entwurf* theory.

As noted in Section 3, Einstein gave up the search for field equations of broad covariance in 1913 because he could not find any that were compatible both with energy-momentum conservation and with the results of Newtonian theory in the case of weak static fields. When he finally did publish field equations of broad and eventually general covariance in 1915, Einstein accordingly made sure that they passed muster on both counts. What I did not mention so far is that Einstein used these requirements not just to check whether they were met by candidate field equations he was considering but also to generate candidates specifically designed to meet them. This is how Einstein arrived at the *Entwurf* field equations in the Zurich notebook (Renn 2007a, Vol. 2, 706–711). Like the relativity principle and the equivalence principle, these physical principles thus guided Einstein in his theory building.

In a similar vein, Einstein relied strongly on the analogy with electrodynamics, both for the further elaboration of the *Entwurf* theory and for the transition to the new theory in November 1915.[115] Much of Einstein's work on the *Entwurf* theory in 1913–1914 went into recasting it in a form in which it could readily be compared with electrodynamics. This is nicely illustrated by the Vienna lecture. Einstein (1913c, 1249–1250) began by explaining that one should expect the transition from Newton's theory to a new theory of gravity to be similar to the transition from Coulomb's electrostatics to Maxwell's electrodynamics. In the body of the lecture, Einstein presented the *Entwurf* field equations in a form that closely matches the field equations for the electromagnetic field. He consistently used the equations in this form in subsequent publications (Janssen and Renn 2007, 847). Like Maxwell's

equations, the *Entwurf* field equations in this new form set the divergence of the field equal to their source.[116] The equations governing the transfer of energy-momentum between the gravitational field and matter can likewise be written in a form that is similar to the corresponding equation in the case of the electromagnetic field. It was, in fact, on the basis of these parallels that Einstein originally identified the gravitational field as the gradient of the metric field.

The following year, Einstein developed a more general formalism to analyze various properties of the *Entwurf* field equations (Einstein and Grossmann 1914b, Einstein 1914o).[117] He derived a set of conditions in this formalism that, on the one hand, determine under which (non-autonomous) transformations the field equations are invariant and, on the other, ensure that the field equations imply energy-momentum conservation.[118] The central quantity in this formalism is the so-called Lagrangian. Specification of the Lagrangian for the gravitational field is tantamount to the specification of the vacuum field equations. Einstein modeled the Lagrangian for the gravitational field in the *Entwurf* theory on the Lagrangian for the electromagnetic field in Maxwell's theory. It is essentially the same quadratic expression in the components of the field in both cases.

When, sometime in October 1915, Einstein finally came to accept that the rotation metric is not a vacuum solution of the *Entwurf* field equations (see Section 4), he held on to his general formalism, including the expression for the Lagrangian in terms of the gravitational field. He only changed the definition of the field from the gradient of the metric to the Christoffel symbols (see Section 3). The resulting new field equations were of broad covariance. Purely mathematical considerations had already led Einstein to consider these equations three years earlier. They can be found in the Zurich notebook. At that time, physical considerations had steered Einstein away from these equations and toward the *Entwurf* field equations. Now the formalism that Einstein had developed for the *Entwurf* theory, relying heavily on the analogy with electrodynamics, not only led him back to the equations rejected earlier, but also provided him with all the guidance he needed to demonstrate that they are compatible with energy-momentum conservation after all. Moreover, the connection between energy-momentum conservation and the covariance of the field equations, one of the key insights enshrined in his general formalism, gave Einstein the decisive clue for solving the other problem that had defeated him before, namely to show that these field equations reproduce the results of Newtonian theory in the case of weak static fields. With both these problems taken care of, Einstein rushed his rediscovered field equations into print (Einstein 1915f). Within days he realized that they were still not quite right. Guided once again by his general formalism, Einstein fixed the remaining problems in two further communications to the Berlin academy in November 1915 (Einstein 1915g, i).

This whole chain of events was triggered by Einstein's redefinition of the gravitational field. One can thus understand his assessment at the time that the old

definition had been a "fateful prejudice" (Einstein 1915f, 782) and that the new one had been the "key to the solution."[119] Einstein later downplayed the importance of the physical considerations encoded in his general formalism for the transition from the *Entwurf* field equations to the Einstein field equations. The way he came to remember it was that he had chosen the new equations purely on grounds of mathematical elegance (Janssen and Renn 2007, Sec. 10).

Ultimately, it was probably the convergence of physical and mathematical lines of reasoning that reassured Einstein that the field equations of his fourth communication of November 1915 were the right ones. Confident that no further corrections would be needed, he could afford to poke fun at the way victory had at long last been achieved. As he told Ehrenfest in late December: "It's convenient with that fellow Einstein, every year he retracts what he wrote the year before."[120] This self-deprecating comment nicely captures the flexibility we have seen Einstein exhibit at several junctures on his road to the new theory. Three days later, Einstein likewise told his Polish colleague Władysław Natanson: "I once again toppled my house of cards and built a new one."[121] In terms of Einstein's philosophical objectives, the new structure indeed turned out to be yet another house of cards. As a physical theory, however, it has proved to be remarkably sturdy and durable.

Acknowledgments

Notes

[1]See Norton's contribution to this volume as well as Appendix A.

[2]Otherwise, Einstein's introduction of general relativity here is similar to the one he gave over a decade later in a lecture in Glasgow. In the published text of the latter we read: "[O]nly a relative meaning can be assigned to the concept of velocity" and "[f]rom the purely kinematical point of view there was no doubt about the relativity of all motions whatsoever" (Einstein 1933, 286). Page references to this lecture are to the reprint in Einstein (1954).

[3]See also Fock (1959, xviii). Bondi's remarks and similar ones by Synge, another leading relativist of the same era (see note 35), are quoted and discussed by Schücking and Surowitz (2007, 19). In his article Bondi tried to preempt criticism of sacrilegiousness: "one may surely admire and embrace Einstein's theory of gravitation while rejecting his route to it, however heuristically useful he himself found it" (Bondi 1979, 180).

[4]Einstein (1918k) produced an account of the twin paradox along these lines (Janssen 2005, 64, note 23). Gustav Mie used the example of the passenger in the accelerating train to criticize this way of extending the relativity of uniform motion to accelerated motion in the discussion following a lecture by Einstein in Vienna in 1913 (Einstein *et al.* 1913, 1264; cf. note 52). See also Weyl (1924, 199; cf. note 33).

[5]In Section 3 below, we shall see how Einstein came to equate general covariance with general relativity. For an insightful review of the decades-long debate over the status of general covariance, see Norton (1993b).

[6]For expressions of his strong confidence in the theory at this point, see Einstein to Heinrich Zangger, [after 27 December 1914] and 11 January 1915 (CPAE 10, [Vol. 8, Doc. 41a] and [Vol. 8, Doc. 45a]).

[7]See Sauer (1999, 2005b), Renn and Stachel (2007) and Brading and Ryckman (2008) for comparisons of the relevant contributions of Einstein and Hilbert.

[8]For a reconstruction of these developments, see Janssen and Renn (2007).

[9]Along with the Princeton lectures (Einstein 1922c) and his popular book on relativity (Einstein 1917a), this is Einstein's best known exposition of general relativity. It is included in *The Principle of Relativity*, an anthology still in print today (Einstein *et al.* 1952). For detailed commentary, see Janssen (2005) and Sauer (2005a).

[10]The characterization of Einstein's project given above in terms of the equivalence principle, Mach's principle, and general covariance follows this paper. This was the first time that Einstein explicitly separated the three notions involved. In a footnote he conceded that he had not clearly distinguished the relativity principle, identified with general covariance, from Mach's principle before (Einstein 1918e, 241). See Lehner (2005) for a somewhat different take on the changes in the status of and the relation between these three principles in Einstein's thinking in this period.

[11]This is nicely captured in the title of a paper by Earman and Glymour (1978), "Lost in the tensors," even though the paper itself was quickly superseded by papers of Stachel (1980) and Norton (1984). For the first publication of the former and a reprint of the latter, see Howard and Stachel (1989).

[12]For more on the checkered history of the cosmological constant, see Smeenk's contribution to this volume.

[13]The story of Einstein's quest for general relativity thus simultaneously confirms the first and refutes the second part of the observation in Bob Dylan's 1965 song *Love minus zero/no limit* that "there's no success like failure and that failure is no success at all."

[14]One can argue that absolute motion is already less objectionable in special relativity than it was in Newtonian theory (Dorling 1978). Once again, consider the two passengers whose trains are in non-uniform motion with respect to one another. According to Newtonian theory, these two observers, using ideal rods and clocks (i.e., ideal in the sense of measuring intervals in the space and time posited by Newtonian theory), will arrive at equivalent *descriptions* of the motion of the other observer, the only difference being the direction of the motion. Yet, the *effects* of the motion (e.g., whether or not the coffee in their cups spills) is different for the two observers. This, Einstein (1916e, 771–773) pointed out, amounts to a violation of the principle of sufficient reason: Two motions that look the same have different effects. This, Einstein suggested, is what makes absolute motion so objectionable. If this were all there is to the problem of absolute motion, Einstein had already solved it in 1905 (Dorling 1978). According to special relativity,

given the behavior of ideal clocks and rods posited by the theory, the two observers will describe the motion of the other observer differently. Contrary to what Einstein claims in the passages from the lectures in Princeton and Glasgow quoted at the beginning of this chapter, in both special and general relativity it *does* matter, even "from the purely kinematic point of view" or in terms of the "purely geometrical acceleration" (i.e., acceleration as determined by ideal rods and clocks in the space-time posited by the relevant theory) "from the point of view of which body we talk about [non-uniform motion]." Since the two motions under consideration here *look* different according to special relativity, it is not surprising that their effects are different as well. There is no violation of the principle of sufficient reason. For further discussion, see Janssen (2005, 62–63).

[15]For discussion of Einstein's work on unified field theory, see Sauer's contribution to this volume.

[16]This analogy nicely illustrates that being a set of relational properties does not make a structure any less real. One need only think of adultery.

[17]See, e.g., DiSalle (2006) for an attempt to parse the philosophical debate over these issues in a new way.

[18]For accounts of Einstein's work on the perihelion problem, see the editorial note, "The Einstein-Besso Manuscript on the Motion of the Perihelion of Mercury" (CPAE 4, 344–359), Earman and Janssen (1993), and Janssen (2003).

[19]See Earman and Glymour (1980a,b) and CPAE 9 (introduction, secs. III–V) for discussions of these two classical tests of general relativity (the third being the prediction of an additional advance of the perihelion of Mercury of some 43 seconds of arc per century [see the preceding note]).

[20]See Smeenk's contribution to this volume.

[21]See Kennefick's contribution to this volume.

[22]See Renn *et al.* (1997); Renn and Sauer (2003).

[23]Pfister (2007) convincingly argues that Einstein actually deserves most of the credit for what is usually referred to as the Lense-Thirring effect (Lense and Thirring 1918).

[24]See Earman and Eisenstaedt (1999) and Earman (1995).

[25]See, e.g., Brading (2002) for analysis of Noether's theorems.

[26]In his contribution to this volume, Friedman places the development of general relativity in the context of the history of the philosophy of geometry.

[27]For a concise overview of the development of general relativity that largely focuses on this strand of the story rather than on the failed quest for general relativity, see Giulini and Straumann (2006).

[28]See Norton (1992b, 1993a) and, drawing on this work, Giulini and Straumann (2006, Sec. 5, 145–151). The first of Norton's two papers is reprinted in Renn (2007a, Vol. 3, 413–542) along with translations of the original papers by Nordström (1912, 1913a,b).

[29]CPAE 7, Doc. 31, [p. 21]. A literal translation of the German original ("der glücklichste Gedanke meines Lebens") would be: "the happiest thought of my life," where 'happy' is to be taken in the sense of 'fortunate'. Einstein also told this story in a lecture in Kyoto in 1922 (Abiko 2000, 15).

[30]To be more precise, this is a particle's *passive* gravitational mass, a measure of how strongly it is attracted by other particles. Its *active* gravitational mass measures how strongly it attracts other particles. These two quantities also have the same numerical value.

[31]In the next sentence, Einstein admittedly still suggested that this consideration leads to an extension of the relativity principle to non-uniform motion. Old habits die hard. For further discussion, see Janssen (2002b, 507–508).

[32]The relevant passage is quoted and discussed in sec. 4.1 in Norton's contribution to this volume, where a variant of the thought experiment shown in Fig. 2 is analyzed.

[33]In a remarkable semi-popular article, Hermann Weyl (1924, 198–199) put the notion of what he called a "guiding field" that cannot be split uniquely into inertial and gravitational components at the center of his discussion of the foundations of general relativity.

[34]As Einstein (1918e, 241) put it, "Inertia and gravity are of the exact same nature." Six years earlier, Einstein (1912h, 1063) had already written about the equivalence of inertial and gravitational mass and the equivalence of a static gravitational field and the acceleration of a frame of reference using the same term, *wesensgleich*, which I translated as "of the exact same nature" (Norton 1992b, 447, note 42). Page references to this paper are to the reprint in Renn (2007a, Vol. 3).

[35]In the preface of his textbook on general relativity, J. L. Synge admitted that he had never been able to define the equivalence principle in a way that would not make it either trivial or false, but he still recognized its heuristic value: "The Principle of Equivalence performed the essential office of midwife at the birth of general relativity ... I suggest that the midwife be now buried with appropriate honours and the facts of absolute space-time faced" (Synge 1960, ix–x). He spoke for many when he observed that "the word 'relativity' now means primarily Einstein's theory and only secondarily the obscure philosophy which may have suggested it originally" (ibid., ix). Cf. the comment by Bondi quoted in the introduction.

[36]We can still continue to think of the situation on an ordinary merry-go-around even though that involves a gravitational field perpendicular to the plane of the disk, while in Minkowski space-time there is no gravitational field at all. Since we only consider what happens in the plane of the disk, the difference is of no consequence for our arguments.

[37]See sec. 2.7 in appendix A on special relativity for an analysis of the twin paradox. This appendix also provides elementary explanations of time dilation and length contraction.

[38]The experimental verification of the effect was much more contentious. See Hentschel (1993); Pound (2000, 2001); and CPAE 9, xxxvii–xl.

[39]Einstein himself established this by considering linear acceleration rather than rotation.

[40]See Rowe's contribution to this volume.

[41]See also Einstein's correspondence with the mathematician Vladimir Varićak about the rotating disk and related issues in 1910–1912 (Sauer 2007).

[42]For discussion of Abraham's theory and Einstein's criticism of it, see the editorial note, "Einstein on Gravitation and Relativity: The Static Field" (CPAE 4, 122–128), and Renn (2007d).

[43]Page references to this paper are to the reprint in Howard and Stachel (1989). Three years earlier, in response to criticism by Max Planck of the definition of constant acceleration in his 1907 review article, Einstein (1908b) had already been forced to accept a restriction of the principle to bodies at rest in the accelerated frame (Schücking and Surowitz 2007, 7).

[44]The Göttingen mathematician Felix Klein later noted that this had been a rather one-sided introduction to the field (Renn 2007a, Vol. 2, 611, note 212). For an account of how his collaboration with Grossmann began, see Einstein's Kyoto lecture (Abiko 2000, 16, cf. note 29).

[45]Richard Feynman once boasted that he had found his way to the 1957 conference on general relativity at Chapel Hill by asking a cab driver to take him to the same place the man had taken others going 'gee-mu-nu, gee-mu-nu' (Feynman and Leighton 1985, 258–259).

[46]Gaussian curvature would be meaningful to critters constrained to the surface, such as ants crawling along on it. Without leaving the surface, they could ascertain that they live on a curved surface by measuring the angles of triangles drawn on it and noting that these angles do not add up to π.

[47]Since space-time is locally Minkowskian or pseudo-Euclidean, it is, strictly speaking, only *pseudo-Riemannian*.

[48]For an English translation of the key parts of Levi-Civita's (1917) paper on the subject, see (Renn 2007a, 1081–1088). With the help of the connection the old Gaussian interpretation of curvature in terms of *angular excess* of geodetic triangles (cf. note 46) was replaced by the modern interpretation in terms of *parallel displacement* (Janssen 1992). In curved spaces a vector transported parallel to itself around a closed loop will no longer point in the same direction as the original vector.

[49]Cf. Appendix A, Sec. 2.5

[50]Norton (1999a), building on some of his earlier work (Norton 1992a), argues that the conflation was the result of the collision in Einstein's work of two different traditions in geometry, one going back to Klein's famous Erlangen program, the other going back to Riemann (Janssen 2005, 61–62).

[51]For further discussion of Kretschmann's paper, see Norton (1992a, 1993b) and Rynasiewicz (1999). See Anderson (1967, Secs. 4.2–4.4) for a classic discussion of covariance groups and symmetry groups.

[52]Mie (1917) defended a similar view of the role of general covariance in Einstein's theory (see CPAE 8, Doc. 346, note 3). Mie completely agreed with Kretschmann (1917) and recommended the latter's paper to Einstein (Mie to Einstein, 17 February 1918 [CPAE 8, Doc. 465]).

[53]Dieks (2006) defends Einstein against the charge of conflating general covariance and general relativity by arguing that his goal was to eliminate preferred frames of references in the sense of laws taking on a special form in them rather than in the sense of their special states of motion.

[54]See, e.g., Einstein to Max von Laue, 12 September 1950 (AE 16 148), quoted by Stachel (2002a, 256) in a supplementary note to a reprint of his paper on the rotating disk (Stachel 1989). Einstein likewise saw no problem representing the energy and momentum of the gravitational field by a quantity that is not a tensor. Since the mature equivalence principle makes the presence of a gravitational field coordinate-dependent, it is only natural that its energy and momentum are too. Einstein (1918f) defended his pseudo-tensor of gravitational energy-momentum against criticism of various colleagues, including Levi-Civita, Lorentz, and Klein. Part of this debate over the pseudo-tensor is covered by Cattani and De Maria (1993). Trautman (1962) provides a concise overview of subsequent work on energy and momentum conservation in general relativity.

[55]This notebook is the centerpiece of Renn (2007a, Vols. 1 and 2), where it is presented in facsimile with a transcription and a detailed commentary. High-quality scans of the notebook are available at the *Einstein Archives Online*.

[56]See Janssen and Renn (2007, 913–914) for the relevant passages from letters to Michele Besso, Hilbert, and Arnold Sommerfeld.

[57]An embryonic version of the hole argument can be found on a page in Besso's hand dated 28 August 1913 (Janssen 2007).

[58]For historical discussion of these arguments, see, e.g., Stachel (1980); Norton (1984, 1987); Howard (1999); Janssen (2007). For philosophical debate, see, e.g., Earman and Norton (1987); Stachel (1986, 2002c); Earman (1989); Maudlin (1990); Rynasiewicz (1992); Howard (1999); Saunders (2003); Rickles and French (2006); Pooley (2006). For a good introduction to the debate and further references, see the entry on the hole argument in the on-line *Stanford Encyclopedia of Philosophy* (Norton 2008).

[59]Before Stachel (1980) and Norton (1984), commentators, including Pais (1982, 222), thought that *this* was the indeterminism lurking in the hole argument and they therefore dismissed Einstein's argument as a beginner's blunder of someone who has just learned Riemannian geometry.

[60]This extra step is already present in the embryonic version of the hole argument mentioned in note 57 (Janssen 2007, 821–823).

[61]That it is far from trivial to spot the flaw in this argument was forcefully demonstrated by the discovery of page proofs of Hilbert's (1916) first paper on general relativity, which show that even the great mathematician had originally fallen for it (Corry *et al.* 1997).

[62]Einstein and Grossmann (1914a, 260; 1914b, 217–218); Einstein (1914e, 178; 1914o, 1067).

[63]See Einstein to Ehrenfest, 26 December 1915 and 5 January 1916 (CPAE 8, Docs. 173 and 180) and Einstein to Besso, 3 January 1916 (CPAE 8, Doc. 178).

[64]Einstein in all likelihood got the notion of point coincidences from a paper by Kretschmann (1915) that was published just days before Einstein wrote the letter in which the new argument makes it first appearance (Howard and Norton 1993, 54). As can be inferred from the manuscript mentioned in note 57, Einstein had rejected a similar escape from the hole argument two years earlier (Janssen 2007, Sec. 4).

[65]Two years later, Einstein elevated this observation to the statement of the relativity principle itself: "The laws of nature are nothing but statements about spatio-temporal coincidences; they therefore find their only natural expression in generally-covariant equations" (Einstein 1918e, 241).

[66]Howard (1999) draws attention to the harmful influence of this reading of the point-coincidence argument in philosophy of science. It was not just readers of Einstein's 1916 review article, however, who interpreted the argument this way. It was also how Lorentz interpreted the original argument in Einstein's letters to Ehrenfest, Lorentz's successor in Leyden. It was this version of the argument that convinced Lorentz of the need of general covariance (Kox 1988, Janssen 1992).

[67]The argument, however, does leave the determined substantivalist plenty of wiggle room. First, the account of identity and individuation that it is based on remains controversial: Can identity truly be a matter of *suchness* alone or does it always involve some *thisness* as well (Maudlin 1990)? Second, the argument specifically targets substantivalists committed to the reality of bare manifold points as the ultimate carriers of all physical properties. One can argue that this is not the right way to assign physical meaning to bare manifold points (Wilson 1993). Or one could opt for a more sophisticated form of substantivalism that avoids commitment to the reality of bare manifold points. Both moves, however, would seem to end up blurring the distinction between substantivalism and relationism (Rickles and French 2006, 3–4).

[68]See Earman (1989, Ch. 6) for discussion

[69]For other discussions of Einstein's efforts to implement Machian ideas in his new theory of gravity, see, e.g., Barbour (1992, 2007), Hoefer (1994, 1995), Barbour and Pfister (1995), and Renn (2007c).

[70]For Newton the bucket experiment was first and foremost an argument against the Cartesian concept of motion rather than an argument for absolute acceleration (Laymon 1978; Huggett 2000, Ch. 7). For discussion of the responses of Huygens, Leibniz, Berkeley, Kant, Maxwell, Mach, and Poincaré to Newton's bucket experiment, see Earman (1989, Ch. 4).

[71]Einstein to Lorentz, 14 August 1913 (CPAE 5, Doc. 467).

[72]Einstein to Lorentz, 23 January 1915 (CPAE 8, Doc. 47).

[73]In the early 1960s, the small increase of inertia that Einstein did find as a result of interaction with distant matter was shown to be an artifact of the coordinates he used (Torretti 1978, 20).

[74]The gravitational field will exert both centrifugal and Coriolis forces. The latter are twice the size of the former and point in the opposite direction, thus keeping the particles in orbit (Janssen 2005, notes 24 and 44).

[75]Thirring to Einstein, 11–17 July 1917 (CPAE 8, Doc. 361).

[76]Einstein to Thirring, 2 August 1917 and 7 December 1917 (CPAE 8, Docs. 369 and 405). See also Einstein to Eduard Hartmann, [27 April 1917] (CPAE 8, Doc. 330).

[77]What is also puzzling is that Einstein mentioned that he was working on the problem of boundary conditions in a letter to Besso of 14 May 1916 (CPAE 8, Doc. 219), i.e., several months *before* the exchange with De Sitter.

[78]See also the Einstein-Besso manuscript (CPAE 4, Doc. 14, [pp. 18–24, 32–35, 41–42, 45–49]). For further discussion, see Pfister (2007).

[79]Another factor, I suspect, is that early critics may have been reluctant to take Einstein to task on this score for fear of being lumped in with Anti-Relativists, whose attacks on Einstein had more nefarious motives (see Rowe's contribution to this volume and Wazeck [2009]).

[80]See the correspondence between Einstein and De Sitter in 1916–1918 published in CPAE 8 and the editorial note, "The Einstein-De Sitter-Weyl-Klein Debate" (ibid., 351–357). For another concise account of the developments discussed in this section, see Giulini and Straumann (2006, sec. 6.6).

[81]De Sitter to Einstein, 1 November 1916; Einstein to De Sitter, 4 November 1916 (CPAE 8, Docs. 272 and 273).

[82]The second of these was the second installment of a trilogy in the *Monthly Notices* of the *Royal Astronomical Society* that first introduced British scientists to Einstein's new theory (De Sitter 1916a, c, 1917c).

[83]Einstein to De Sitter, 2 February 1917 (CPAE 8, Doc. 293).

[84]He had mentioned this possibility the year before in the letter to Besso cited in note 77.

[85]De Sitter to Einstein, 1 April 1917 (CPAE 8, Doc. 321).

[86]For more detailed discussion of these considerations, see Norton (1999b) and Smeenk's contribution to this volume. Drawing on his historical analysis of the difficulties with Newtonian cosmology, Norton (1995, 2003) shows that there is a class of cosmological models in which the arbitrariness of the split between inertial and gravitational effects expressed in the mature equivalence principle amounts to a true relativity of acceleration. The particles in relative acceleration toward one another in these models all move on geodesics.

[87]De Sitter to Einstein, 20 March 1917 (CPAE 8, Doc. 313).

[88]Cf. Appendix A, Sections 2.5 and 2.6 and Fig. 27.

[89]See also De Sitter to Einstein, 20 June 1917 (CPAE 8, Doc. 355).

[90]Einstein to De Sitter, 8 August 1917 (CPAE 8, Doc. 370).

[91]This may be the reason that this paper was published in *Annalen der Physik*, in which Einstein (1916e) had published his big review article, while most of his papers on general relativity during this period appeared in the proceedings of the Berlin academy (Janssen 2005, 60).

[92]In the 1+1D version of the model the horizon consists of two points rather than a 2D surface.

[93]Einstein corresponded with Weyl about this proof and it is referred to in the section on cosmology in the first edition of *Space-time-matter* (Weyl 1918, Sec. 33). Although Weyl thus helped Einstein defend Mach's principle, he later explicitly distanced himself from it (see, in particular, the popular article mentioned in note 33).

[94]Einstein to Weyl, 31 May 1918; Klein to Einstein, 31 May 1918 (CPAE 8, Docs. 551 and 552).

[95]Einstein to De Sitter, 24 March 1917, and De Sitter to Einstein, 1 April 1917 (CPAE 8, Docs. 317 and 321).

[96]For discussion, see, e.g., Earman (1995, sec. 1.2) or Earman and Eisenstaedt (1999, sec. 3).

[97]Einstein to Klein, [before 3 June 1918] (CPAE 8, Doc. 556).

[98]Klein to Einstein, 16 June 1918 (CPAE 8, Doc. 566).

[99]The following analysis follows Schrödinger (1956).

[100]This passage and a similar passage from the autobiographical notes (Einstein 1949a, 29) are quoted and discussed by Hoefer (1994, 330) and Renn (2007c, 61).

[101]See Sauer's contribution to this volume.

[102]See Norton (1993a, 6–11) for a reconstruction of how Einstein presumably derived the result that the acceleration of a falling body decreases when it is moving sideways. The paper also provides diagrams illustrating the effect (ibid., Figs. 1 and 2). Einstein first alluded to this problem with special-relativistic theories of gravity in print in the course of his polemic with Abraham (Einstein 1912h, 1062–1063). Renn (2007b, 55) presents a more elementary argument that shows that the acceleration of free fall must depend on a body's horizontal velocity in any special-relativistic theory of gravity (see also Giulini [2006, 16]). Consider two trains traveling in opposite directions on parallel tracks with constant speeds. The moment a passenger in one train comes face to face with a passenger in the other train, they each drop some object from the same height. The relativity of simultaneity implies that, if the objects were to hit the floor of the respective trains simultaneously according to the passenger in one train, they would not do so according to the passenger in the other train. Since the situation of the two passengers is completely symmetric, it follows that the objects must hit the floor one after the other for both passengers. One easily verifies that both observers will claim that the object they themselves dropped hit the floor first (consider Fig. 26 in Appendix A and let the events P and Q represent the two objects hitting the floor of their respective trains).

[103]My discussion of Einstein's engagement with the Nordström theory follows Norton (1992b, 1993a). For insightful further discussion, see Giulini (2006).

[104]The trace of the energy-momentum tensor is the sum of its diagonal components, T_{11}, T_{22}, … It turns out that this quantity is invariant under Lorentz transformations. In other words, it transforms as a scalar under such transformations.

[105]For detailed discussion, see Norton (1992b, Secs. 9–10, 437–450) and Giulini (2006, Sec. 6).

[106]For further discussion of these developments, see, e.g., Janssen and Mecklenburg (2006, 107–111).

[107]See Norton (1993a, Sec. 5) for discussion of Einstein's objection and a helpful diagram (ibid., Fig. 5).

[108]The following year, Mie (1914) published a sharp critique of the *Entwurf* theory. By 1917, however, he had abandoned his own theory and accepted general relativity, though not Einstein's interpretation of general covariance in terms of relativity of motion (see note 52). See Smeenk and Martin (2007) for an introduction to some of Mie's papers on gravity presented in English translation in Renn (2007a, Vol. 4, 633–743).

[109]In modern terms, Nordström's theory satisfies the weak but not the strong equivalence principle.

[110]The following year, Erwin Freundlich, a Berlin astronomer and Einstein's protégé, set out for the Crimea to observe this eclipse, but then World War I broke out and Freundlich was interned by the Russians. Another expedition was rained out (Earman and Glymour 1980a, 60–62). In a sense, this was a fortunate turn of events for Einstein since the effect predicted by the *Entwurf* theory was too small (Earman and Janssen 1993, 129).

[111]The paper was submitted from Zurich, so Nordström would have had the opportunity to discuss his theory in person with both Einstein and Laue (Norton 1992b, 455).

[112]See Norton (1993a, Sec. 6) for discussion and another helpful diagram (ibid., Fig. 6). Giulini (2006, 26) takes issue with Einstein's claim that this is the only way to save energy conservation in the Nordström theory.

[113]This is similar to the situation in Lorentz's ether theory of electrodynamics out of which special relativity grew. By 1899, Lorentz assumed that any moving system contracts by a factor depending only on the ratio of the system's velocity with respect to the ether and the velocity of light and that any process in a moving system takes longer than the same process in the system at rest by that same factor (Janssen 2002a, 425). As a result, clocks and rods in Lorentz's theory

measure times and distances in the Minkowski space-time posited by special relativity rather than times and distances in the Newtonian space-time posited by Lorentz's own theory.

[114]See Norton (2007, 775–776) for discussion. Einstein thought that the *Entwurf* field equations could likewise be recovered from the combination of generally-covariant equations and additional conditions (Einstein and Fokker 1914, 328). In the case of the *Entwurf* theory, Einstein knew the additional conditions (these were the conditions for "adapted coordinates" mentioned in Section 4) but not the generally-covariant equations that together with these conditions would give the *Entwurf* field equations. See Janssen and Renn (2007, 842–843; 866–867) for further discussion of Einstein's understanding during this period of how field equations could be extracted from generally-covariant equations, which by themselves were inadmissible as field equations on account of the hole argument.

[115]Einstein had already used the electrodynamical analogy when he was looking for a generalization of his 1912 theory for static gravitational fields to stationary fields. See, in particular, Einstein (1912e), where the electrodynamical analogy is mentioned in the title, and Einstein to Besso, 26 March 1912 (CPAE 5, Doc. 377).

[116]There is still an important difference between the two cases: The source of the electromagnetic field is the electric charge-current density, while the source of the gravitational field is the energy-momentum density, both of matter *and of the gravitational field itself.* In his contribution to this volume, Kennefick emphasizes the importance of both analogies and disanalogies between gravity and electromagnetism in the later debate over gravitational waves.

[117]The discussion of the developments of 1914–1915 below is based on Janssen and Renn (2007). Our account of how Einstein found the generally-covariant field equations now named after him deviates at key points from the one given in a classic paper by Norton (1984). For a short version of our new still controversial account, see Janssen (2005, 75–82).

[118]Two years later, Einstein (1916o) used this same formalism to show that the general covariance of the Einstein field equations is directly related to energy-momentum conservation (Janssen and Renn 2007, Sec. 9).

[119]Einstein to Sommerfeld, 28 November 1915 (CPAE 8, Doc. 153).

[120]Einstein to Ehrenfest, 26 December 1915 (CPAE 8, Doc. 173).

[121]Einstein to Natanson, 29 December 1915 (CPAE 8, Doc. 175).

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