NSS
Neutrosophic Sets and Systems, Vol. 53, 2023
University of New Mexico

The Basics of Neutrosophic Simulation for Converting Random Numbers Associated with a Uniform Probability Distribution into Random Variables Follow an Exponential Distribution

Maissam Jdid[1], Rafif Alhabib[2] and A. A. Salama[3]

[1] Faculty of Informatics Engineering, Al-Sham Private University, Damascus, Syria; m.j.foit@aspu.edu.sy

[2] Department of Mathematical Statistics, Faculty of Science, Albaath University, Homs, Syria; rafif.alhabib85@gmail.com

[3] Department of Mathematics and Computer Science, Faculty of Science, Port Said University, Port Said, Egypt; drsalama44@gmail.com

Abstract: When performing the simulation process, we encounter many systems that do not follow by their nature the uniform distribution adopted in the process of generating the random numbers necessary for the simulation process. Therefore, it was necessary to find a mechanism to convert the random numbers that follow the regular distribution over the period [0, 1] to random variables that follow the probability distribution that works on the system to be simulated. In classical logic, we use many techniques in the transformation process that results in random variables that follow irregular probability distributions. In this research, we used the inverse transformation technique, which is one of the most widely used techniques, especially for the probability distributions for which the inverse function of the cumulative distribution function can found. We applied this technique to generate neutrosophic random variables that follow an exponential distribution or a neutrosophic exponential distribution. This based on classical or neutrosophic random numbers that follow a regular distribution. We distinguished three cases according to the logic that each of the random numbers or the exponential distribution follows. We arrived at neutrosophic random variables that, when we use them in systems that operate according to an exponential distribution, such as queues and others, will provide us with a high degree of accuracy of results, and the reason for this is due to the indeterminacy provided by neutrosophic logic.

Keywords: Simulation - inverse transformation - uniform distribution - exponential distribution - neutrosophic exponential distribution - random numbers - random variables - neutrosophic logic.

1. Introduction

The generation of random variables that follow a certain distribution is the basis of the simulation. We can generate random events that simulate any real system by finding probability distributions that apply to the events and properties of that system, for example: "times between arrivals" in queues are random events that often follow an exponential distribution. There are several methods and algorithms for generating random variables from a given distribution [1,2,3].

To keep pace with the modern studies that emerged after the neutrosophic revolution, the logic laid down by the American mathematical philosopher Florentin Smarandache in 1995 [6,8,10,11,12,13,20] came as a

generalization of the fuzzy logic and an extension of the theory of fuzzy sets presented by Lotfi Zadeh in 1965 [7]. As an extension of that logic, A. A. Salama presented the theory of classical neutrosophic sets as a generalization of the theory of classical sets and developed, introduced and formulated new concepts in the fields of mathematics, statistics, computer science and classical information systems through neutrosophic. Logic that studies the origin, nature and field of indeterminacy so that it takes into account every idea with its opposite (its negation) and with the spectrum of indeterminacy [4]. In addition, there were several achievements of many researchers in the field of neutrosophic [5,9,14,15,16,17,18,19,21,22,23,24,25,26]. It was necessary to work on transforming the random numbers that follow a neutrosophic uniform distribution into random variables that follow a neutrosophic exponential distribution. In this research, we present a study on the process of converting random numbers that follow a regular distribution over the period [0, 1] to random variables that follow an exponential distribution, based on the definition of regular and exponential distributions according to neutrosophic logic.

2. Experimental and Theoretical Part:

In view of the great importance that the exponential distribution has in most fields of science, and in order to obtain more accurate results when using it in a field. the researchers defined this distribution according to the neutrosophic logic. The logic that enables us to deal with all the cases that we can come across during the study. In previous research [28] entitled " Fundamentals of Neutrosophical Simulation for Generating Random Numbers Associated with Uniform Probability Distribution" we reached mathematical formulas that help us in generating neutrosophic random numbers that follow the uniform distribution on the period [0, 1]. In this paper, we have developed a mechanism to obtain the neutrosophic random variables that follow an exponential distribution. This based on the random numbers that follow the uniform distribution on the period [0, 1]. This done by using the inverse transformation of the cumulative distribution function. The study included all the cases that we need during the simulation process for the systems that operate according to the exponential distribution.

Previous studies: [1, 2, 3, 28]

If R_1, R_2 ... are a sequence of random numbers then R_i has a probability function defined as:

$$f_R(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Cumulative distribution function:

$$F_R(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

To generate x_1, x_2 ... observations of the random variable X. follow the distribution:

$$F(x) = P(X \le x) , \quad -\infty < x < \infty$$

We use the sequence of random numbers R_1, R_2 …., and the cumulative distribution function for the random variable X. Then we apply the inverse transformation method. It is the most commonly used, especially for probability distributions in which $F^{-1}(x)$ can found. It based on matching:

$$F(X) = R \qquad (*)$$

If the random variable X follows a classical exponential distribution.

Then the probability density function is:

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & x\geq 0\\ 0 & x<0\end{cases}$$

The cumulative distribution function:

$$F(x)=\begin{cases}1-e^{-\lambda x} & x\geq 0\\ 0 & x<0\end{cases}$$

We substitute in the relation (*):

$$F(X) = R$$

$$1 - e^{-\lambda x} = R \quad x \geq 0$$

By solving the previous equation, it results in:

$$x = -\frac{1}{\lambda}\ln(1-R) \qquad (**)$$

We call the equation (**): the generator equation for the random variable that follows the exponential distribution. Are of the form:

$$X = F^{-1}(R)$$

Therefore, to obtain a sequence of observations, of the random variable X that follows the exponential distribution, we use the relationship $X = F^{-1}(R)$, and the sequence of random numbers R_1, R_2.... we write:

$$X_i = F^{-1}(R_i)$$

$$X_i = -\frac{1}{\lambda}\ln(1-R_i) \quad ; \quad i = 1,2,..$$

It can be simplified to the form:

$$X_i = -\frac{1}{\lambda}\ln R_i$$

3. Results and Discussion

The current study: To generate random variables that follow an exponential distribution according to neutrosophic logic, we distinguish the following cases:

<u>First case</u>: the random numbers follow the neutrosophic uniform distribution on the period $[0+\varepsilon, 1+\varepsilon]$ and the exponential distribution in the classical form.

To generate random variables that follow the exponential distribution whose probability density function:

$$f(x) = \lambda e^{-\lambda x} \qquad x > 0$$

Cumulative Distribution Function:

$$F(x) = 1 - e^{-\lambda x} \qquad x \geq 0$$

Using the sequence of neutrosophic random numbers that follows the uniform distribution on the period $[0+\varepsilon, 1+\varepsilon]$, and which is given as $R_1 - \varepsilon$, $R_2 - \varepsilon \ldots$, we apply the relationship (*):

$$F(x) = R$$

In this case, we write:

$$F(x) = R_i - \varepsilon \Rightarrow$$
$$1 - e^{-\lambda x} = R_i - \varepsilon$$
$$e^{-\lambda x} = 1 - (R_i - \varepsilon)$$
$$-\lambda x = \ln(1 - (R_i - \varepsilon)) \Rightarrow$$
$$x = -\frac{\ln(1-(R_i-\varepsilon))}{\lambda} \qquad i = 1, 2\ldots$$

Accordingly, to obtain a sequence of observations of the random variable X using the random numbers that follow the neutrosophic uniform distribution on the period $[0+\varepsilon, 1+\varepsilon]$, which is given by the formula $R_i - \varepsilon$. We substitute, in the following relationship:

$$X_i = -\frac{\ln(1-(R_i-\varepsilon))}{\lambda} \qquad ; i = 1, 2\ldots$$

It can be simplified:

$$X_{Ni} = -\frac{\ln(R_i-\varepsilon)}{\lambda} \qquad ; i = 1, 2\ldots$$

<u>The second case:</u> classical random numbers and a neutrosophic exponential distribution.

Let's have a sequence of random numbers R_1, R_2 … that follows a uniform distribution on the period $[0, 1]$, and we want to generate random variables that follow a neutrosophic exponential distribution.

Probability density function of the neutrosophic exponential distribution [4]:

$$f_N(x) = \lambda_N e^{-x\lambda_N} \qquad 0 < x < \infty$$

The cumulative distribution function given by:

$$NF(x) = 1 - e^{-x\lambda_N}$$

Using the relation (*):

$$NF(x) = R \Rightarrow$$
$$1 - e^{-x\lambda_N} = R \Rightarrow$$
$$e^{-x\lambda_N} = 1 - R$$

$$x = -\frac{\ln(1-R)}{\lambda_N} \qquad \text{Or:} \qquad x = -\frac{\ln R}{\lambda_N}$$

Accordingly, to obtain a sequence of observations of the random variable X, "which follow the neutrosophic exponential distribution". Using the random numbers that follow the uniform distribution on the period $[0, 1]$, we substitute in the relationship:

$$X_{Ni} = -\frac{\ln R_i}{\lambda_N} \qquad \text{i} = 1, 2\ldots$$

<u>The third case:</u> the random numbers follow the neutrosophic uniform distribution and the neutrosophic exponential distribution.

To find the relationship through which we get: random variables that follow the neutrosophic exponential distribution starting from the sequence of neutrosophic random numbers that follow the regular distribution on the period $[0+\varepsilon\ ,1+\varepsilon]$, which are given as follows:

$R_1 - \varepsilon, R_2 - \varepsilon, \ldots.$

We apply the relationship (*):

$$F(x) = R$$

$$1 - e^{x\lambda_N} = R - \varepsilon \Rightarrow$$

$$X = \frac{-1}{\lambda_N}\ln[1-(R-\varepsilon)]$$

or in the form:

$$X = \frac{-\ln(R-\varepsilon)}{\lambda_N}$$

Therefore, to obtain a sequence of observations of the random variable X that follows the neutrosophic exponential distribution using the random numbers that follow the neutrosophic uniform distribution on the period $[0+\varepsilon\ ,1+\varepsilon]$, we substitute in the relation:

$$X_{Ni} = \frac{-\ln(R_i-\varepsilon)}{\lambda_N} \qquad \text{i} = 1, 2\ldots$$

4. Application Example:

Suppose we have a system that operates according to an exponential distribution whose probability density function is $f(x) = 2e^{-2x}$; $x \geq 0$. We want to conduct a neutrosophic simulation of this system. Where the indeterminate $\varepsilon = [0\ ,\ 0{,}03\ ]$. Here we need to generate neutrosophic random numbers. Therefore, we use one of the cases:

<u>First case:</u> The exponential distribution is classical, its probability density function is $f(x) = 2e^{-2x}$; $x \geq 0$, and neutrosophic random numbers (We get it by one of the methods studied in the research [28]).

In this example, we generate random numbers that follow the neutrosophic uniform distribution on the period $[0+\varepsilon\,,1+\varepsilon]$, That is, we generate random numbers according to one of the known methods. Here we will use the "mean-squared" method, by taking the seed $R_{\circ}=1276$. We get the random numbers:

$$R_1=0{,}6281\quad,\quad R_2=0{,}4509\quad,\quad R_3=0{,}3310\quad,\quad R_4=0{,}0951$$

By using the rule that we reached in previous research [28] to convert classical random numbers into random numbers that follow the neutrosophic uniform distribution on the period $[0+\varepsilon\,,1+\varepsilon]$. In addition, take the given indeterminacy $\varepsilon=[0\,,\ 0{,}03\,]$. We get the neutrosophic random numbers:

$$R_{N\circ}=[0{,}0976\,,0{,}1276]\quad,\quad R_{N1}=[0{,}5981\,,0{,}6281]\quad,\quad R_{N2}=[0{,}4209\,,0{,}4509]$$
$$R_{N3}=[0{,}3010\,,0{,}3310]\quad,\quad R_{N4}=[0{,}0656\,,0{,}0956]$$

Then we apply the following rule $\quad X_{Ni}=-\dfrac{\ln(R_i-\varepsilon)}{\lambda}=\dfrac{\ln R_{Ni}}{\lambda}\quad;\quad i=0,1,2,3,4$

We get:

$$X_{N\circ}=-\frac{\ln R_{N\circ}}{\lambda}=-\frac{\ln[0{,}0976\,,0{,}1276]}{2}=[\,1{,}0294\,,1{,}16343]$$
$$X_{N1}=-\frac{\ln R_{N1}}{\lambda}=-\frac{\ln[0{,}5981\,,0{,}6281]}{2}=[\,0{,}2325\,,0{,}2570]$$
$$X_{N2}=-\frac{\ln R_{N2}}{\lambda}=-\frac{\ln[0{,}4209\,,0{,}4509]}{2}=[\,0{,}3983\,,0{,}4327]$$
$$X_{N3}=-\frac{\ln R_{N3}}{\lambda}=-\frac{\ln[0{,}3010\,,0{,}3310]}{2}=[\,0{,}5528\,,0{,}6003]$$
$$X_{N4}=-\frac{\ln R_{N4}}{\lambda}=-\frac{\ln[0{,}0656\,,0{,}0956]}{2}=[\,1{,}1738\,,1{,}3621]$$

It is sequence of neutrosophic random observations, which follow an exponential distribution.

The second case: The neutrosophic exponential distribution, its probability density function is $f(x)=[2\,,\,2{,}03]e^{-[2\,,\,2{,}03]x}\quad;\quad x\geq 0$. Random numbers are classical.

To find the required neutrosophic random observations, we will take the random numbers that follow the uniform distribution on the period $[0\,,1]$:

$$R_{\circ}=1276\quad,\quad R_1=0{,}6281\quad,\quad R_2=0{,}4509$$
$$R_3=0{,}3310\quad,\quad R_4=0{,}0951$$

Then we apply the rule: $\quad X_{Ni}=-\dfrac{\ln R_i}{\lambda_N}\quad;\quad i=0,1,2,3,4$

We get:

$$X_{N\circ} = -\frac{\ln R_i}{\lambda_N} = -\frac{\ln 1276}{[2\ ,\ 2{,}03]} = [1{,}01421,\ 1{,}0294]$$

$$X_{N1} = -\frac{\ln R_i}{\lambda_N} = -\frac{\ln 0{,}6281}{[2\ ,\ 2{,}03]} = [0{,}2290\ ,\ 0{,}2325]$$

$$X_{N2} = -\frac{\ln R_i}{\lambda_N} = -\frac{\ln 0{,}4509}{[2\ ,\ 2{,}03]} = [1{,}5045,\ 0{,}3983]$$

$$X_{N3} = -\frac{\ln R_i}{\lambda_N} = -\frac{\ln 0{,}3310}{[2\ ,\ 2{,}03]} = [0{,}5446\ ,\ 0{,}5528]$$

$$X_{N4} = -\frac{\ln R_i}{\lambda_N} = -\frac{\ln 0{,}0951}{[2\ ,\ 2{,}03]} = [1{,}1590\ ,\ 1{,}1764]$$

It is sequence of neutrosophic random observations, which follow the neutrosophic exponential distribution.

The third case: The neutrosophic exponential distribution, its probability density function given by the following $f(x) = [2\ ,\ 2{,}03]e^{-[2\ ,\ 2{,}03]x}$; $x \geq 0$. The neutrosophic random numbers from the figure $R_i - \varepsilon$. To find neutrosophic random observations. We take the neutrosophic random numbers used in the first case and apply the following rule:

$$X_{Ni} = \frac{-\ln R_{Ni}}{\lambda_N} \quad ; \quad R_{Ni} = R_i - \varepsilon \quad ; \quad i = 0{,}1{,}2{,}3{,}4$$

We get:

$$X_{N\circ} = \frac{-\ln R_{N\circ}}{\lambda_N} = \frac{-\ln[0{,}0976\ ,\ 0{,}1276]}{[2\ ,\ 2{,}03]} = [1{,}01421,\ 1{,}16343]$$

$$X_{N1} = \frac{-\ln R_{N1}}{\lambda_N} = \frac{-\ln[0{,}5981,\ 0{,}6281]}{[2\ ,\ 2{,}03]} = [0{,}2291,\ 0{,}2570]$$

$$X_{N2} = \frac{-\ln R_{N2}}{\lambda_N} = \frac{-\ln[0{,}4209\ ,\ 0{,}4509]}{[2\ ,\ 2{,}03]} = [0{,}3924,\ 0{,}4327]$$

$$X_{N3} = \frac{-\ln R_{N3}}{\lambda_N} = \frac{-\ln[0{,}3010\ ,\ 0{,}3310]}{[2\ ,\ 2{,}03]} = [0{,}54465,\ 0{,}6003]$$

$$X_{N4} = \frac{-\ln R_{N4}}{\lambda_N} = \frac{-\ln[0{,}0656\ ,\ 0{,}0956]}{[2\ ,\ 2{,}03]} = [1{,}1564\ ,\ 13621]$$

It is sequence of neutrosophic random observations, which follow the neutrosophic exponential distribution.

5. **Conclusions**:

Through the previous study, we found that, to generate a sequence of neutrosophic random observations that follow an exponential distribution, using a sequence of random numbers that follow a uniform distribution. We use one of the following cases, according to the case under study: The first

case: neutrosophic random numbers, i.e. They follow the neutrosophic uniform distribution on the period $[0+\varepsilon, 1+\varepsilon]$, and the exponential distribution in the classical form.

The second case: random numbers that follow the uniform distribution on the period $[0, 1]$, and the neutrosophic exponential distribution.

The third case: the neutrosophic random numbers, i.e., they follow the neutrosophic uniform distribution on the period $[0+\varepsilon, 1+\varepsilon]$, and the neutrosophic exponential distribution.

By using techniques used in classical logic. In this paper, we used the inverse transformation technique. In addition, we found that for every random number (neutrosophic or classical) there is a random variable that follows the neutrosophic exponential distribution, which enables accurately simulate the systems that follow the exponential distribution. That is through the accuracy that neutrosophic logic provides us when studying any system according to its hypotheses.

In the near future, we are looking forward to preparing studies that will enable us to generate neutrosophic random variables that follow other probability distributions such as the Weibull distribution, the geometric distribution, and others.

Funding: "This research received no external funding".

References

Al-Ali .I. M, Operations Research, Tishreen University Publications. 2004. .(Arabic version).

Bugaha J.S , Mualla.W , Nayfeh.M , Murad.H , Al-Awar.M.N - Operations Research Translator into Arabic ,The Arab Center for Arabization, Translation, Authoring and Publishing,Damascus,1998.(Arabic version).

Berry.A.M.A - Modeling and Simulation - King Saud University, 2002. . (Arabic version).

Alhabib.R, Ranna.M, Farah.H and Salama, A. A, PhD thesis. University of Aleppo, 2019. (Arabic version).

Smarandache, F - Translated by Alhabib.R - Introduction to Neutrosophical Statistics - Translated into Arabic Education Publishing – 2020.

A.A. Salama- F. Smarandache- Neutrosophic crisp set Theory Educational- Education Publishing 1313 Chesapeake, Avenue, Columbus, Ohio 43212- 2015.

ZADEH .L. A. Fuzzy Sets. Inform. Control 8 (1965).

Smarandache, F. Introduction to Neutrosophic statistics, Sitech & Education Publishing, 2014.

Atanassov .k, Intuitionistic fuzzy sets. In V. Sgurev, ed., ITKRS Session, Sofia, June 1983, Central Sci. and Techn. Library, Bulg. Academy of Sciences, 1984.

Smarandache, F, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy , Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA,2002.

Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM, 1999.

Smarandache, F, Neutrosophic set a generalization of the intuitionistic fuzzy sets. Inter. J. Pure Appl. Math., 24, 287 – 297, 2005.

Salama, A. A, Smarandache, F, and Kroumov, V, Neutrosophic crisp Sets & Neutrosophic crisp Topological Spaces. Sets and Systems, 2(1), 25-30, 2014.

Smarandache, F. & Pramanik, S. (Eds). (2016). New trends in neutrosophic theory and applications. Brussels: Pons Editions.

Alhabib.R, The Neutrosophic Time Series, the Study of Its Linear Model, and test Significance of Its Coefficients. Albaath University Journal, Vol.42, 2020. (Arabic version).

Alhabib.R, Ranna.M, Farah.H and Salama, A. A, Neutrosophic Exponential Distribution. Albaath University Journal, Vol.40, 2018. (Arabic version).

Alhabib.R, Ranna.M, Farah.H and Salama, A. A, studying the random variables according to Neutrosophic logic. Albaath-University Journal, Vol (39), 2017. (Arabic version).

Alhabib.R, Ranna.M, Farah.H and Salama, A. A, Neutrosophic decision-making & neutrosophic decision tree. Albaath-University Journal, Vol (40), 2018. (Arabic version).

Alhabib.R, Ranna.M, Farah.H and Salama, A. A, Studying the Hypergeometric probability distribution according to neutrosophic logic. Albaath- University Journal, Vol (40), 2018.(Arabic version).

Salama, A. A and F. Smarandache. "Neutrosophic crisp probability theory & decision making process." Critical Review: A Publication of Society for Mathematics of Uncertainty, vol. 12, p. 34-48, 2016.

Alhabib .R, M. Ranna, H. Farah and A. A Salama, "Foundation of Neutrosophic Crisp Probability Theory", Neutrosophic Operational Research, Volume III , Edited by Florentin Smarandache, Mohamed Abdel-Basset and Dr. Victor Chang (Editors), pp.49-60, 2017.

Alhabib .R, M. Ranna, H. Farah and A. A Salama.(2018). Some neutrosophic probability distributions. Neutrosophic Sets and Systems, 22, 30-38, 2018.

Aslam, M., Khan, N. and Khan, M.A. (2018). Monitoring the Variability in the Process Using the Neutrosophic Statistical Interval Method, Symmetry, 10 (11), 562.

Aslam, M., Khan, N. and AL-Marshadi, A. H. (2019). Design of Variable Sampling Plan for Pareto Distribution Using Neutrosophic Statistical Interval Method, Symmetry, 11 (1), 80.

Victor Christianto , Robert N. Boyd , Florentin Smarandache, Three possible applications of Neutrosophic Logic in Fundamental and Applied Sciences, International Journal of Neutrosophic Science, Volume 1 , Issue 2, PP: 90-95 , 2020.

Alhabib .R, A. A Salama, "Using Moving Averages To Pave The Neutrosophic Time Series", International Journal of Neutrosophic Science (IJNS), Volume III, Issue 1, PP: 14-20, 2020.

Smarandache .F, Introduction to Neutrosophic Measure Neutrosophic Integral and Neutrosophic Probability, 2015. http://fs.gallup. unm.edu/ eBooks-otherformats.htm.

Jdid .M, Alhabib.R and Salama.A.A, Fundamentals of Neutrosophical Simulation for Generating Random Numbers Associated with Uniform Probability Distribution, Neutrosophic Sets and Systems, 49, 2022.

Received: Sep 7, 2022. Accepted: Dec 24, 2022