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CLARO R. CENIZA ON CONDITIONALS, PROBABILITY, AND MODALITY

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Claro R. Ceniza [1927-2001] is arguably one of the best philosophers that the Philippines has ever produced. However, it is quite unfortunate that some of his important contributions are not that well-known. This paper aims to rectify this by presenting an evaluation of his original insights on three outstanding problems in philosophy, viz., the paradoxes of material implication, the nature of probability, and the metaphysics of modality.

Keywords: Claro R. Ceniza, paradoxes of material conditionals, classical conditionals, logical theories of probability, nonreductive theories of modality

INTRODUCTION

Claro R. Ceniza [1927-2001] is arguably one of the most prolific and original philosophers that the Philippines has ever produced. His body of work covers different areas of inquiry, from abstract topics in metaphysics, epistemology, logic, philosophy of science, philosophy of language, and probability theory to more practical issues about morality, marriage, education, social transformation, nationalism, religion, and spirituality. Unfortunately, some of his significant philosophical contributions to these fields have not gained the attention they deserve. While there have been general commentaries that touched upon some aspects of his life and work, for example, Brias (1993), Castro (1993), Guevarra (1993), Gripaldo (2004), and Demeterio (2014), they barely scratched the surface and have not fully explored the significance of Ceniza's philosophical insights to the areas on which his work impacts.[1]

This paper aims to present some of Ceniza's contributions to a broader philosophically-inclined audience. I aim to provide an in-depth analysis of his significant contributions to three outstanding problems in philosophy, viz., the paradoxes of material implication, the nature of probability, and the metaphysics of modality. In particular, I evaluate the philosophical merits of his papers in these respective areas. First, I discuss what these problems imply. Then I investigate the historical development of Ceniza's papers on these topics and explore the explanatory power of the theories presented in them. Finally, I assess how these theories stand vis-à-vis other respective theories in the three specified areas of philosophy.

In the third section, I focus on Ceniza's proposed solution to the paradoxes of material implication – paradoxes that show that (i) a false proposition implies anything and (ii) a true proposition is implied by anything. According to Ceniza (1988), these paradoxes only occur in the case of *degenerate* conditionals – conditionals whose antecedent is not truth-functionally relevant to their consequent. On the other hand, *non*degenerate conditionals do not imply these paradoxes since their antecedent is (truth-functionally) relevant to their consequent. While it may be shown that such a solution succumbs to the familiar problems for any purely classical solution to these paradoxes, its key distinction between degenerate and nondegenerate conditionals is still worth exploring and extending.

In the fourth section, I look into Ceniza's contribution to the philosophical foundations of probability, in particular, his views about the nature of probability itself. Ceniza and his collaborators (1998, 70) offer an account of probability in terms of the semantics of classical propositional logic, which, as they write: "achieve(s) some kind of merger between logic (specifically, propositional calculus) and the mathematics of probability." According to this account, since the statement "The coin will land heads" has two possible truth valuations – it is either true or false – the probability that the coin will land heads is 0.5. Likewise, since the statement "The coin will land both heads and tails" is contradictory, its probability is 0. Finally, since the statement "The coin will land heads or tails" is a tautology, its probability is 1. This account is a version of the logical theory of probability that Rudolf Carnap has already endorsed in the 1950s. As such, it can be shown it inherits the same problems that Carnap's theory has faced. Despite this, however, scholars still view logical theories as respectable theories of the nature of probability.

In the fifth section, I evaluate Ceniza's views on the metaphysics of modality. One outstanding problem in this area of philosophy is whether modal notions, like possibility and necessity, could be reductively analyzed in terms of some non-modal notion. The now dominant, albeit controversial, possible worlds analysis aims to provide such a reductive analysis. According to this analysis, the proposition "Possibly, pigs fly" is true just in case there is a possible world where pigs do fly. On the other hand, the proposition "Necessarily, 2 + 2 = 4" is true just in case two and two make four in all possible worlds. Ceniza (1985) offers an analysis of modalities that does not employ the notion of possible worlds; his analysis uses the notions of compossibility and incompossibility instead. According to his analysis, the possibility of flying pigs is defined in terms of its compossibility with itself, while the necessity that two and two make four is defined in terms of its compossibility with itself and with other things (in reality). I shall show that while Ceniza's analysis is not a reductive one – since compossibility and incompossibility are modal notions – it nonetheless provides a viable (non-reductive) account of modality that dispenses with the machinery of possible worlds.

Finally, in the concluding section, I offer an overall assessment of Ceniza's contributions to these areas. I show that while they have not impacted the greater philosophical literature of these areas, the originality and ingenuity of his contributions still have to be recognized. However, before delving into his insights on these three areas of philosophy, it might be prudent to delve a bit into his philosophical background and how he views what the task of philosophy is.

CENIZA'S TWO MODELS OF DOING PHILOSOPHY

Ceniza started working on philosophical topics in 1943 when he was sixteen years old. He was influenced by Descartes's *Discourse on Method,* which he read while he was enrolled in a stenography class at Saint Paul's College in Dumaguete City. From then on, philosophy became his chief preoccupation, even during his study and practice of law in the 1950s and 1960s. He received his Bachelor of Laws in 1953 and placed 17th out of 3,000 examinees in the 1954 Philippine bar examination. During this time, he organized his philosophical ideas into the manuscript "The Rational Basis of the Problems of Philosophy," which was listed in *The Review of Metaphysics* in 1954. Sadly, there is no copy of this early work – even Ceniza has no copy of it.

In 1958, he published the pamphlet "The Relation of Man's Concept of Space to Metaphysics," which was later published as "Metaphysics" in *Silliman Review* in 1968. In 1965, after 11 years of practicing law, he quit the practice and devoted his time doing philosophy. In 1969, he obtained a Syracuse-in-Asia Fellowship that enabled him to study at Syracuse University for his M.A. and Ph.D.[2] His 1971 M.A. thesis "The Argument of Parmenides" and his 1974 Ph.D. dissertation "Some Basic Presuppositions of Classical Philosophy" form the basis of his later works on metaphysics, whose culmination is found in his final work, *Thought, Necessity, and Existence*, which was published in 2001, the year died.

Ceniza (1990, 119) sees that the task of philosophy is not only "to make sense of human experience, and thus to provide a guide to practical living and decision-making" but also to make sense of the reality as a whole and our place in it. He offers two models of how to go about doing this task. According to his jigsaw puzzle model, the task of philosophy is to make sense of reality by examining each of its bits and putting them together to get a true and objective picture of what reality ultimately is. Like a jumbled jigsaw puzzle that needs to be solved in order to see the complete picture, the reality is a puzzle that, once solved, would be seen as structured and ordered in a particular way. Moreover, just like how one solves a jigsaw puzzle by making progress from educated guesses as to which pieces go together, one solves the puzzle in reality by looking at its bits in order to see how they all hang together.

Ceniza (1990, 119) notes, however, that some philosophers have rejected the jigsaw puzzle model for being "too idealistic a notion of philosophy." These philosophers claim that reality "is perhaps unlike a jigsaw puzzle with a unique solution" because reality itself is simply meaningless – it has no objective structure that needs to be uncovered. Despite this, however, *we* are still able to give meaning to it. *We* can still make sense of reality by humanizing and personalizing it and by interpreting it using our inner, subjective reality. This meaning-giving view of philosophy is what Ceniza (1990, 120) calls the inkblot model of doing philosophy – an allusion perhaps to Rorschach's inkblot test in psychology.

For Ceniza (1990, 120), both the jigsaw puzzle and inkblot models of philosophy have their respective uses. The inkblot model is useful in thinking about highly interpretative and arguably, human-centered disciplines like history, political theory, ethics, arts, literature, and the humanities. Since "subjectivity" is a powerful element in these disciplines, a philosophical method that puts a premium on personal, psychological, and social factors must be employed in them. On the other hand, the jigsaw puzzle model might be a better fit for the natural sciences, mathematics, logic, and other highly *im*personal disciplines (Ceniza 1990, 121). Since "objectivity" is the distinguishing feature of these disciplines, using a philosophical method where evidence, justification, and truth are important is rather beneficial.

One might wonder whether Ceniza's two models of philosophy imply the distinction between analytic (Anglophone) philosophy and (various types of) Continental philosophy. After all, analytic philosophers are deemed as scientifically-minded individuals seeking to make sense of objective reality, while Continental philosophers are the literary-type who aim to interpret the inner, subjective world.

The distinction between analytic philosophy and Continental philosophy is rather misleading since it seems to involve a cross-classification of philosophical method and geography. While it is true that analytic philosophy is often characterized by its commitment to certain standards of objectivity, argumentation, clarity, and rigor, it is simply false that *all* philosophers from the Continent reject this. After all, some of the founders of analytical philosophy – Gottlob Frege, Ludwig Wittgenstein, and the members of the Vienna Circle –were from the Continent. On the other hand, while it is true that some Continental philosophers are *literati*, it is false that some Anglophone philosophers are not; consider the likes of Iris Murdoch, Martha Nussbaum, and others (Williams 2003, 23; Critchley 2001, 32). Though this distinction was still in vogue, it seems to be of no consequence on how Ceniza views his two models. His models are grounded on a methodological distinction between "objective" and "subjective" ways of thinking about reality and our place in it and not on the confused distinction between analytic philosophy and Continental philosophy.

Having considered Ceniza's two models of philosophy, let us now turn to how he utilized the jigsaw puzzle model to think about some problems in technical philosophy. Let us start with his work on the paradoxes of material implication.

CENIZA ON THE PARADOXES OF MATERIAL IMPLICATION

Ceniza (1988) proposes a solution to the paradoxes of material implication. In order to understand this solution, we must first discuss what material implications are in the first place. Material implications (or conditionals) are any proposition of the form "If P then Q" or "P implies Q" (where P is the antecedent of a conditional and Q is its consequent). In logic textbooks, conditionals are formalized by the right-arrow symbol "→" or the hook or horseshoe symbol "⊃."[3] For our purposes, we will use the latter for our formalization.

According to classical (two-valued) logic, $P \supset Q$ is true just in case its antecedent is false, or its consequent is true. The standard semantics for material conditionals is implied by the familiar truth-table for ⊃ (where "1" means true and "0" means false):

P	⊃	Q
1	1	1
1	0	0
0	1	1
0	1	0

Table 1: Truth-table for material conditionals

Table 1 tells us that a conditional has value 1 in three possible cases: (i) both P and Q have value 1, (ii) P has value 0, but Q has value 1, or (iii) both P and Q have value 0. The only case where it has a value 0 is when P is 1, but Q is 0.

This semantics, however, implies the following seemingly counterintuitive consequences:

(1) A false proposition implies any proposition; and,
(2) A true proposition is implied by any proposition.

More formally,

(1*) $\sim P \supset (P \supset Q)$
(2*) $Q \supset (P \supset Q)$

These are the paradoxes of material implication.[4] To illustrate why they are paradoxical, suppose the true proposition “2 + 2 = 4.” Then, given (2*), it would make a counterintuitive material conditional like “If the moon is made of green cheese, then 2 + 2 = 4” true. On the other hand, suppose the false proposition “2 + 2 = 5.” Then, given (1*), it would make “If 2 + 2 = 5, then the moon is made of green cheese” true as well.

There have been different solutions offered to this problem throughout the history of logic. Here we focus on two solutions related to Ceniza’s proposal, viz, Clarence I. Lewis’s strict implication theory and the theory of “relevant” implication. Lewis (1918, 291) distinguishes between the “material” sense of implication, which leads to the paradoxes, and the “strict” sense of implication, “which is based upon an entirely different meaning of ‘implies’ – one more in accord with the customary uses of that relation in inference and proof.” For him, the strict implication is a *necessary* conditional (formalized as “P ⥽ Q” or “$\Box(P \supset Q)$”) where it is *impossible* for P to have value 1, while Q has value 0. Alternatively, Q is strictly implied by P just in case Q is deducible (or provable) from P. He argues that this stricter notion of implication avoids the paradoxes and captures our “ordinary” notion of implication.

Lewis’s strict conditional is not straightforwardly *classical*. To define conditionals in terms of modal terms like necessity and impossibility is to extend classical logic[5] in the realm of modal logic. Since strict conditionals are necessary conditionals, it follows that the modal notion of necessity is doing some semantic work in Lewis's theory. As such, to ascertain the truth of strict conditionals, one must also ascertain the truth of necessities. For Lewis, this is done using different axiomatic systems of modal logic. In current semantic theory, this is done via possible worlds semantics.

Lewis's solution, however, invites its own paradoxes. Suppose the necessarily true proposition "There is an infinite number of prime numbers." This implies that

(i) If Manila is in the Philippines, then there is an infinite number of prime numbers.

(ii) If there is no infinite number of prime numbers, then Manila is in Thailand.

Clearly, (i) and (ii) are false even if they fulfill Lewis's requirement for strict conditionals (Priest 2008, 72-74).

The second solution to the paradoxes we will consider was proposed by relevant logicians like Richard Routley and his collaborators in the 1970s and 1980s. According to Routley *et al.* (1982, 3), the paradoxes are generated by classically valid inferences like introduction and elimination rules for disjunction and conjunction and the rule of disjunctive syllogism.[6] For example, (1*) can be generated by the following proof:

1 P & ~P	assumption (a putatively false proposition)
2 P	1, conjunction-elimination
3 ~P	1, conjunction-elimination
4 P v Q	2, disjunction-introduction
5 Q	3, 4 disjunctive syllogism
6 ~P ⊃ (P ⊃ Q)	2-5, conditional proof

For them, to block the inference that leads to the paradoxes, one must reject disjunctive syllogism; this implies accepting a relevant non-classical logic. Moreover, they point out that what is wrong with these paradoxes is that the antecedent and consequent are on completely unrelated topics (Mares 2020). This is apparent in conditionals like "If 2 + 2 = 5, then the moon is made out of cheese." Despite being true in classical logic, this conditional is false in relevant logic since its antecedent is not relevant to its consequent.

Relevantists define "relevance" here in a very technical way; they use the machinery of possible worlds semantics in defining it.[7] Without going too much into the details, the rough idea is that a *relevant* conditional "P > Q" is true just in case the antecedent has at least one propositional variable in common with the consequent (Mares 2020). That is, the antecedent and consequent share a common informational thread (in all possible worlds).[8] For example, the conditional "If Manila is in the Philippines, then Manila is in Southeast Asia" is a true relevant conditional since its antecedent and consequent are arguably about a common topic. As such, it is evident that P > Q has different semantic properties from P ⊃ Q. Unlike the former, the latter is straightforwardly truth-functional (as evidenced by its standard truth table). Unlike the latter, the former does not imply the paradoxes of material implication.

Whether or not relevantists have satisfactorily solved the paradoxes of a material implication is currently up for debate. One problem often raised against them is their rejection of (some valid inferences in) classical logic; in particular, disjunctive syllogism. For some logicians, this inference seems to be a good deductive inference. For example, to conclude that Manila is in the Philippines from the premises, "Either Manila is in the Philippines or in Thailand" and "Manila is not in Thailand" seems valid. Thus, relevant logicians need to show why such an inference is objectionable.[9]

Both the Lewisian and the relevantist solutions to the paradoxes of material implication imply a *non*-classical solution. The latter implies the rejection of some inferences in classical logic, while the former implies an extension of classical logic into modal logic. It is interesting to note that Ceniza's solution seems to echo both theories. For him, "The paradoxes of material implication have been called 'paradoxes' of a sort because they seem to allow truth to material implications where the antecedents and consequents, respectively, have no relevance to each other." What he aimed to prove is "that in cases of true material implications, their antecedents and consequents, respectively, have some relevance to each other" (Ceniza 1988, 510). Unlike Lewis and the relevant logicians, Ceniza's solution is not an extension of two-valued logic, nor does it reject any of the valid inferences in classical logic.[10]

What exactly does Ceniza's solution to the paradoxes imply? According to Ceniza (1988, 517), conditionals come into two sorts: a degenerate conditional and a nondegenerate conditional. Degenerate conditionals are true "merely because their antecedents are false, or else merely because their consequents are true." That is, if the truth of P ⊃ Q only depends on the limit values of P or Q, then the conditional is degenerate.[11] On the other hand, the truth of nondegenerate conditionals is "determined...empirically from the constant conjunction of states of affairs described by the antecedent and the consequent or else is derived from more general truths." That is, if the truth of P ⊃ Q depends on the values of both P and Q, then the conditional is nondegenerate.

For Ceniza, the paradoxes of material implication only arise in the case of degenerate conditionals because their antecedents and consequents are not truth-functionally relevant to each other. For example, if P is self-contradictory, then the resulting conditional P ⊃ Q is true regardless of what the value of Q may be. *Mutatis mutandis*, if Q is a tautology, then P ⊃ Q is true regardless of what the value of P is. In the case of nondegenerate conditionals, such paradoxes do not occur since their antecedents are always truth-functionally relevant to their consequents. In order to make explicit how the antecedent of a material conditional is truth-functionally relevant to its consequent, Ceniza proposes the following definition for the material conditional:

C⊃ P ⊃ Q =df. P ≡ (P & Q)
(where ≡ is the symbol for the biconditional)

Let us call this sort of conditionals "Ceniza conditionals" or "C⊃" for short. According to Ceniza (1988, 519), C⊃ "makes the logical relation between the antecedent and consequent of a material implication clearer than before." To illustrate, consider again the conditional "If 2 + 2 = 5, then the moon is made out of cheese." If this conditional is defined in terms of C⊃, then it will be translated as "2 + 2 = 5 iff (if and only if) 2 + 2 = 5 and the moon is made out of cheese." Notice that while this biconditional is true, it is only degenerately so since its truth only depends on the falsity of 2 + 2 = 5. Likewise, given C⊃, the conditional "If the moon is made out of cheese, then 2 + 2 = 4" is defined as "The moon is made out of cheese iff the moon is made out of cheese and 2 + 2 = 4." The truth of this is only derived from the degenerately false proposition that the moon is made out of cheese. This then explains away the paradoxes of material implication.

In the case of nondegenerate conditionals, on the other hand, C⊃ makes the logical relation between antecedents and consequents more explicit. For example, given C⊃, the conditional "If it rains, the ground is wet" will be translated as "It rains iff it rains and the ground is wet." The truth of this does not only depend on one fact but on the conjunction of the fact that it rains and the fact that the ground is wet. Moreover, consider the conditional "If you are mugged in Quiapo, then you are mugged in Manila." Given C⊃, the resulting biconditional "You are mugged in Quiapo iff you are mugged in Quiapo, and you are mugged in Manila" is true given that Quiapo is in Manila and all Quiapo-muggings *are* Manila-muggings. Again C⊃ provides the right result in this case.

Ceniza's 1988 paper develops from his earlier 1980 paper "Are there Paradoxes of Material Implications?". In that paper, Ceniza (1980a, 8) discusses problems that allegedly result from the paradoxes of material implication. As he writes, "the so-called paradoxes of material implications… have created at least four problems in logic." For him, these problems include (i) a problem about the counterfactual analysis of disposition terms, (ii) a problem about what constitutes the confirmatory evidence for universal statements and the related problem about the interpretation of the square of opposition in predicate logic, (iii) the paradoxes strict implication, and (iv) the problem about the validity of arguments from inconsistent premises. Given his solution to the paradoxes, Ceniza is hopeful that these other problems will be resolved as well since they all spring from the nature of conditionals.

There are certain worries that could be raised against Ceniza's solution and, by extension, against some of the deliverances his solution promises. First, his solution implies that the truth of P ⊃ Q ultimately depends on whether P and Q are true. But this seems to imply that in order to ascertain the truth of a conditional, one must already know the truth of its constituents. *Contra* Ceniza, this seems to discount the *hypothetical* nature of conditionals.

Relatedly, his solution seems to fail in accounting for nondegenerate conditionals with relevant but false antecedents and consequents. Consider the conditional, "If Duterte has a Harvard law degree, then he has a law degree." Since we know that both its antecedent and consequent are false, Ceniza's solution implies the truth of the biconditional "Duterte has a Harvard law degree iff Duterte has a Harvard law degree, and he has a law degree;" hence the truth of the original conditional. [12]

Second, Ceniza's solution seems to fail in the case of counterfactual conditionals. Counterfactuals are conditionals in the subjunctive mood of the form "Had P happened, then Q would have happened." Let us grant that Ceniza's solution works for indicative conditionals (like the ordinary material conditional). However, how would it work for a counterfactual like "Had Duterte lost the 2016 election, the Philippines would have been in a better state"? While this counterfactual seems to be nondegenerately true, its truth does not depend on the truth of the biconditional "Duterte lost the 2016 election iff Duterte lost the 2016 election, and the Philippines is in a better state."

This worry extends to Ceniza's formulation of the counterfactual analysis of disposition terms. According to Ceniza (1980a, 8), a disposition term like "solubility" may be defined as a conditional "If x is placed in water, it could be observed to dissolve it." However, since this conditional is not a counterfactual, this formulation is wrong. The right formulation is "If x had been placed in water, x would have been dissolved." Thus, since C⊃ does not provide the right result for counterfactuals, the problem about the counterfactual analysis of disposition terms remains.

Finally, Ceniza's solution seems to imply an infinite regress. C⊃ implies that in all non-opaque cases, P ⊃ Q may be substituted by P ≡ (P & Q). The latter formula, however, is made up of two further conditionals:

C1 P ⊃ (P & Q),
C2 (P & Q) ⊃ P.

Thus, they are open to the paradoxes of material implication. C1 implies the paradoxes:

C1* ~P ⊃ (P ⊃ (P & Q)),
C1** (P & Q) ⊃ (P ⊃ (P & Q)).

On the other hand, C2 implies the paradoxes:

C2* ~(P & Q) ⊃ ((P & Q) ⊃ P),
C2** P ⊃ ((P & Q) ⊃ P).

To resist these paradoxes, one may again employ C⊃, and reformulate C1 and C2 as

C1` P ≡ (P & (P & Q))
C2` (P & Q) ≡ ((P & Q) & P)

But C1` and C2` are again biconditionals that could be broken into:

C1`` P ⊃ (P & (P & Q))
C1``` (P & (P & Q)) ⊃ P

C2`` (P & Q) ⊃ (((P & Q) & (P & Q)) & P)
C2``` ((P & Q) & P) ⊃ (((P & Q) & P) & (P & Q))

Again, all of these are material conditionals susceptible to the paradoxes. In order to again resist the paradoxes at this second level, these four conditionals will be translated as biconditionals using C⊃, which leads to another set of paradoxes. This then results in an infinite regress of counter-paradoxes and counter-biconditionals.

Perhaps one way out of the infinite regress is to show that the revenge paradoxes arise only if the values of P and Q are *limit* values of "just true" and "just false." If there are other (real) values between these limit values, then the revenge paradoxes need not arise. Since these other values imply a gap between the limit values, it follows that some statements may have a value in this gap. Given this extension, statements are not only true or false, their value may also be in the gap. Consequently, conditionals may be degenerately true, nondegenerately true, false, or in the gap. Accordingly, using C⊃, biconditionals may also be degenerately true, nondegenerately true, false, or in the gap. Conditionals and biconditionals in the gap will not generate the paradoxes since they are neither just true nor just false. As such, they escape the problem of infinite regress.

To venture on a counterfactual, had he encountered the infinite regress problem, I am sure that Ceniza would not opt for the gappy solution provided above. That some statements are in the gap implies a non-classical logic that Ceniza does not really countenance. After all, he was working with a two-valued logic throughout his career. One thing is for sure, though, his distinction between degenerate and nondegenerate conditionals is an inspired contribution to the paradoxes of material implication. Unlike the Lewisian and the relevant logics solutions, Ceniza's distinction does not require abandoning classical logic. But whether the resources of his solution could address the infinite regress that results from C⊃ is still up for debate and needs to be further explored.

CENIZA ON THE NATURE OF PROBABILITY

Having discussed Ceniza's work on the paradoxes of material implication, let us now turn to his work on probability theory. However, before discussing Ceniza's contribution to this field, it might be helpful to provide a background of the problem on which he and his collaborators were focused.

We all know that the probability that a tossed fair coin will land heads is 0.5. We also know that if nine out of ten logic students are smart, then the likelihood that a particular logic student is smart is 0.9. But what do these probability statements mean? What is probability in the first place?[13] According to the founders of modern probability theory, Fermat and Pascal, probability is the ratio between a positive (favorable) outcome and the total number of equally possible outcomes. More formally,

Definition of Probability $Pr(A) = f/n$

(where Pr is the probability function, A is any proposition, f is the favorable outcome, and n is the total number of possible outcomes.)

Thus, the probability that a tossed fair coin will land heads is 0.5 since there is one positive outcome (the coin landing heads) out of two possible outcomes (the coin landing heads or tails). This simple definition extends to other sorts of probability statements. For example, "The probability that an ace of spades will be picked in a standard deck of cards is 1/52" is actually about a conjunction of two things: that the card that will be picked is an ace, and it is a spade. The statement is also true since there is just one ace of spades in the standard 52-deck. Furthermore, consider the statement, "The probability that a rolled die will turn up 1 or 6 is 2/6 (or 1/3)." This is obviously a disjunction, and it is actually true since there are two favorable outcomes (viz., the die will turn up 1 or 6) and a total of six equally possible outcomes (since a regular die has six sides). We could also have a probability of a negative statement. Consider the sentence, "The probability that this die will not turn up 5 is 5/6". Again, this statement is true since the probability that a die will turn up 5 is 1/6; its negation must be 5/6. Finally, consider a conditional sentence "If I already have an ace of spades at hand, the probability that I will get another ace is 3/51." This statement is again true since if I already have an ace at hand, then there are only 3 remaining aces from the now 51-deck.

Computing the probability of conjunctions, disjunctions, negations, and conditionals is captured by the standard probability calculus, which was first axiomatized by the Russian mathematician Andrey Kolomogorov in the 1930s (Hájek 2019). Kolmogorov's calculus starts with three basic ideas (axioms): (i) there is a universal set of statements or propositions, (ii) probabilities (of statements or propositions) are real numbers between 0 and 1, (ii) probabilities are additive, i.e., Pr(A v B) = Pr(A) + Pr(B).

From these ideas, Kolmogorov develops the foundations of probability theory. Here are some of the key points of the standard probability calculus. [14]

1 If A is a tautology, then Pr(A) = 1.
2 If A is self-contradictory, then Pr(A) = 0

If A is a logical truth like "Either it will rain or not," then its probability is 1. If A is a self-contradictory of the form "P & ~P," then its probability is 0.

3 If A and B are logically equivalent, then Pr(A) = Pr(B)

If A is the statement "The coin will land heads" and B is the statement "The coin will not land tails, then both A and B will have the same probability (supposing, of course, that "landing heads" is logically equivalent to "not landing tails").

4 Pr(~A) = 1 - Pr(A)

That is, the probability of a negative statement is the difference between the maximum probability value (1) and the probability of the statement being negated. For example, the probability that the picked card is not an ace is 1 - Pr(the card is an ace), which is 51/52.

5 If A and B are independent statements, then Pr(A & B) = Pr(A) x Pr(B)
6 If A and B are not independent, then Pr(A & B) = Pr(A) x Pr(B given A)

Two statements are independent if the probability of one does not raise or lessen the probability of the other. For example, the probability that I will randomly pick out an ace and the probability that I will pick out a spade are independent of each other. Thus, the probability that I will randomly pick out an ace of spades from a standard deck of cards is just the product of the two probabilities. On the other hand, two statements are not independent if the probability of one raises or lessens the probability of the other. For example, the probability that I will randomly pick out a pair of aces from a standard deck of cards is (4/52) x (3/51).

7 If A and B are mutually exclusive statements, then Pr(A v B) = Pr(A) + Pr(B)
8 If A and B are not mutually exclusive, then Pr(A v B) = Pr(A) + Pr(B) - Pr(A & B)

Two statements are mutually exclusive if they are inconsistent with each other. For example, that a rolled die will turn up 5 *and* will turn up 6 is inconsistent since both cannot happen at the same time. Thus, the probability that a rolled die will turn up 5 and turn up 6 is 0. However, the probability that die will either turn up 5 or 6 is just the sum of the probabilities of each disjunct, which is 2/6 or 1/3. On the other hand, if two statements are not mutually exclusive (i.e., they can happen at the same time), then the probability that either will happen is just the difference of the sum of their respective probabilities and the probability that they will both happen. For example, the probability that at least one head will come in two tossed coins is ((1/2) + (1/2)) - (1/2) x (1/2), which is 3/4.

9 Pr(B | A) = Pr(A&B)/Pr(A)

That is, the probability that B will happen on the condition that A has already happened is the ratio between the probability that both A and B will happen and the probability that A will happen. For example, the probability that the die lands even, given that it landed on 2, is 1 since if the die landed on 2, it already landed on an even number. On the other hand, the probability that the die lands 2, given that it landed on even, is 1/3 since if the die landed on even, then there are one out of three chances that the die landed on 2 (since there are just three even numbers in a regular die (viz., 2, 4, and 6)).

While the standard probability calculus outlined so far provides a mathematical understanding of how probabilities work, it has not provided an answer to the question about the nature of probability. To this end, we now turn to the philosophical theories about the nature of probability and Ceniza's contribution to this discussion.

There are three main concepts of probability: (i) an epistemic concept that is meant to measure objective evidential support relations (for example, when one judges that it will probably rain based on the evidence of an overcast sky) (ii) a concept of an agent's credence or rational degree of confidence or belief (for example, when one claims that it will probably rain based solely on one's subjective assessment), and (iii) a physical, mind-independent concept that applies to various physical systems (for example, that a radium atom will probably decay in 10,000 years regardless of whether someone holds this belief or not) (Hájek 2019; compare with Skyrms 2000, 137). These three concepts are distinct, but they are closely related. One's credence that a radium atom will decay in 10,000 years must be based on some objective fact found in the world. Similarly, the objective fact that an overcast sky will probably lead to rain must inform one's credence that it will probably rain. The *desiderata* of any philosophical theory of probability is that it must be able to explain (i) how these three concepts relate and fit in well with the probability calculus and (ii) which of these is fundamental and which are derivative.

There have been five dominant philosophical theories of probability in the literature (Hájek 2019). They are the classical theory championed by Pascal, Fermat, Laplace, and others; the subjectivist theory proposed by De Morgan, F. P. Ramsey, and others; the frequency theory developed by Venn, von Mises, J. J. C. Smart, and others; the propensity theory defended by Karl Popper; and the logical or evidential theory defended by Keynes, Carnap, and others.[15] As we shall see later, Ceniza's theory is a version of the last theory in this list.

According to the classical theory of probability, probability is nothing more than what the definition "Pr(A) = f/n" tells us. It is the ratio between the favorable outcomes and the total number of equally possible outcomes. As Laplace writes:

> The theory of chances consists in reducing all events of the same kind to a certain number of equally possible cases, that is to say, to cases whose existence we are equally uncertain of, and in determining the number of cases favorable to the event whose probability is sought. The ratio of this number to that of all possible cases is the measure of this probability, which is thus only a fraction whose numerator is the number of favorable cases, and whose denominator is the number of all possible cases (as quoted in Hájek 2019).

There are problems with the classical theory, though, chief among which is that "The notion of 'equally possible' cases face the charge of either being a category mistake (for 'possibility' does not come in degrees) or circular (for what is meant is really 'equally probable'" (Hájek 2019).

For subjectivists about probability, on the other hand, probability does not measure the occurrence or non-occurrence of events; rather, it measures an agent's rational credence or degree of confidence in a proposition, and this credence leads the agent to perform an action (Hájek 2019). For example, when I say, "There is a 0.7 chance that it will rain this afternoon", I reflect my confidence in the proposition "It will rain this afternoon". And to show this confidence, to show that I am willing to bet that this proposition is right, I may bring an umbrella.

That an agent can subjectively assign probabilities does not guarantee that the assigned probabilities are rational. For example, I may assign a 0.7 chance that it will rain this afternoon but also assign a 0.6 chance that it will not rain. But these assignments are incoherent since they violate a fundamental principle of the standard probability calculus, viz., that the union of Pr(A) and Pr(~A) must be 1. Moreover, since my probability assignments are incoherent, I may succumb to a *Dutch book* – "a series of bets bought and sold at prices that collectively guarantee loss; however, the world turns out" ((Hájek 2019); see also (Joaquin 2017, 40)). To be a rational probability assignment, and in order not to succumb to a Dutch book, an agent's credence must abide by the rules of the probability calculus.

One major flaw in the subjectivist theory of probability is that it "presuppose(s) the existence of necessary connections between desire-like states and belief-like states, rendered explicit in the connections between preferences and probabilities" (Hájek 2019). For example, it presupposes that there is a connection between my desire not to be duped by a Dutch book and my degree of belief that it will rain tomorrow. But there are cases where this connection fails. Imagine a case of an agent who misrepresents his degree of confidence or a case of a compulsive gambler who bets on any proposition. In either case, the agent's beliefs and desires come apart.

According to the defenders of the frequency theory of probability, probability is not an *a priori* measure of the ratio formula (like what the classical theory suggests), nor is it a measure of an agent's degree of confidence (like what the subjectivist theory claims). Rather, it is the *actual* relative frequency of the favorable outcomes over the total number of outcomes (Hájek 2019). For example, the probability that a fair coin lands heads is not to be determined *a priori* but by conducting actual coin-tossings and observing that they actually land heads about fifty percent of the time.

The frequency theory, however, faces many problems. One problem is about biased experiments. Suppose that we conducted a coin-tossing experiment, where we tossed a fair coin 100,000 times. Suppose that 90,000 of these tosses landed on heads. According to the frequentist, we should say that the probability of this coin landing heads is 0.9. However, surely, this yields the wrong result (Hájek 2019). Moreover, given this data, we must say that the coin might not be as fair as we thought it would be.

Another problem for frequentists is the notorious problem of the single case – a problem that was raised by Popper (Hájek 2019). Suppose we tossed a fair coin but never saw how and where it landed. Suppose further that the coin was never found. According to the frequentist, the probability that it landed on heads cannot be determined. Now suppose that we found the coin and saw that it landed on heads. But we kept the coin and never tossed it again. According to the frequentist, the probability that this coin landed on heads, in this case, is 1. Surely, the frequentist made the wrong verdict in both cases. If the coin is a fair one, then the probability that it will land heads is 0.5 regardless of the actual outcomes of the coin tossings.

The problem of the single case led Popper and other theorists to propose the propensity theory of probability. The propensity theory tells us that "probability is…a physical propensity, or disposition, or tendency of a given type of physical situation to yield an outcome of a certain kind, or to yield a long run relative frequency of such an outcome" (Hájek 2019). For example, that there is a 0.5 probability that a fair coin will land heads when tossed is brought about by the coin's dispositional properties and the dispositional properties of the physical systems, e.g., the air pressure, the tossing machine (like a human being's hand), etc., surrounding the coin-tossing event.

Moreover, these physical propensities explain the 0.5 frequency of actual coin-tossings.

For propensity theorists, propensities are supposed to be causal features of physical systems that explain probability. For example, fair coins land heads fifty percent of the time because of their physical properties. But "we do not know enough about what propensities are," let alone how they relate to probabilities (Hájek 2019). Of course, propensities may be thought of as theoretical or explanatory posits. If so, then they are nothing more than fictions. Furthermore, when propensity theorists claim that physical propensities explain the frequencies in experimental setups (like coin-tossing experiments), they assume a kind of stability or uniformity in those setups. However, that assumption, as David Hume has argued, implies the problem of justifying induction (Hájek 2019).

Let us take stock. So far, we have discussed the four main theories of probability: the classical theory, the subjectivist theory, the frequency theory, and the propensity theory. However, we have yet to discuss Ceniza's contribution to the discussion. The theory that Ceniza and his collaborators offered in their 1998 paper reflects a portion of the logical theory of probability that Carnap already developed in the 1950s.[16] We could thus understand the latter's proposal in terms of Carnap's theory.

Carnap's main project is to explicate two concepts of probability: (i) probability as "the degree of confirmation of a hypothesis *h* with respect to an evidence statement *e*," and (ii) probability as "the relative frequency (in the long run) of one property of events or things with respect to another" (Carnap 1962, 19). But he thinks that while both concepts are objective and integral in the sciences, they nonetheless talk about two different *explicanda* of probability (i.e., two different ideas of probability) (Carnap 1962, 55).

What makes Carnaps' theory a *logical* theory of probability is how he treats probability as degrees of confirmation.[17] For Carnap, the degree of confirmation *c* is a function of a hypothesis *h* given the evidence *e* such that for any measure *m,* the ratio of *m*(*h* & *e*) and *m*(*e*) holds (Hájek 2019). More formally,

Degree of Confirmation $c(h \mid e) = m(h \,\&\, e)/m(e)$.

As one may notice, this formula is just an instance of the conditional probability discussed above. That is, if *m* is a probability measure, then the degree of confirmation of *h* given *e* is just the probability of (*h* & *e*) over the probability of *e*. For example, the degree of confirmation that it will rain later given an overcast sky is measured by the probability that it does rain and the sky is overcast over the probability that the sky is overcast. However, what is a probability measure, what does it range over, and how does it function in the degree of confirmation formula?

A probability measure for Carnap ranges over the disjunction of state-descriptions -- i.e., of all *admissible* descriptions of the state of (some logically independent portions of) the world (Hájek 2019). Consider the state-descriptions of all the possible states (or permutations) of three fair coins.

	Coin 1	Coin 2	Coin 3
A	Heads	Heads	Heads
B	Heads	Heads	Tails
C	Heads	Tails	Heads
D	Heads	Tails	Tails
E	Tails	Heads	Heads
F	Tails	Heads	Tails
G	Tails	Tails	Heads
H	Tails	Tails	Tails

Table 2: State-descriptions of three fair coins

(A) tells us that coins 1, 2, and 3 landed on heads; (B) tells us that coins 1 and 2 landed on heads while coin 3 landed on tails; (C) tells us that coins 1 and 3 landed on heads while coin 2 landed on tails; and so on. From these state descriptions, we may observe the following *structure descriptions* (i.e., a generalized statement about the state-descriptions) (Hájek 2019):

(i) There is one state (A) where all three coins land heads;
(ii) There is one state (H) where all three coins land tails;
(iii) There are three states (B, C, E) where two coins land heads and one coin lands tails;
(iv) There are three states (D, F, G) where two coins land tails and one coin lands heads.

We may assign the following initial probability weights to each of the structure descriptions: each of (i) and (ii) has a 1/8 probability, while each of (iii) and (iv) has a 3/8 probability.

Suppose we claim that at least one coin will land on heads, call this hypothesis *h*. Suppose further that coins 1 and 2 already landed on heads; call this evidence *e*. According to Carnap's formula, the degree of confirmation that at least one coin will land on heads is 1 since there are already two coins that landed on heads. Now suppose that we change our hypothesis to "There are at least two coins that will land on heads." Suppose that *e* now is the fact that at least one coin landed on heads. Then according to Carnap's formula, the degree of confirmation of *h* given *e* is 4/7.

From the simple illustration of the three fair coins, we could define all the other relevant (unconditional) probabilities. For example, the probability that there is at least one coin that will land on heads is 7/8 since there is just one state (H) out of eight possible states where none of the coins landed on heads. On the other hand, the probability that at most two coins will land on heads is 1/2 since there are only four possible states (A, B, C, E) where this will hold true.

Let us now turn to the contribution of Ceniza *et al.* in this discussion. It is important to note that while Ceniza *et al.* (1998) did not delve into Carnap's or any other philosophers' idea of confirmation, they touched on Carnap's account of how probabilities are grounded on admissible state-descriptions. Ceniza *et al.* begin their system with the idea that probability statements could be translated as statements in propositional logic. For example, suppose we have two probability statements "There is a 1/3 chance that John will win the elections" and "There is a 2/3 chance that Paul will win the elections." Let us label the first one as "P" and the second one "Q." According to their proposed analysis, P is translated as "There is one out of three chances that the statement 'John will win the elections' is true," while Q is translated as "There are two out of three chances that the statement 'Paul will win the elections' is true" (Ceniza *et al.* 1998, 70-71). We could now cash out these two translations in terms of a probability table (where 1 is true and 0 is false):

P	Q
1	1
0	1
0	0

Table 3: Probability Table

Given this, we could show that their analysis obeys the rules of standard probability calculus. For example, Pr(~P) = Pr(1 - 1/3) = 2/3. Supposing that P and Q are independent, then Pr(P & Q) = Pr(1/3) x Pr(2/3) = 2/3. Supposing that P and Q are mutually exclusive, then Pr(P v Q) = Pr(1/3) + Pr(2/3) = 1. We could also show that Pr(P v ~P) = 1 since the sum of Pr(P) and Pr(~P) is 1. Moreover, we could also show that Pr(Q & ~Q) = 0 since the product of Pr(Q) and Pr(~Q) is 0. Finally, the analysis also obeys the rule for conditional probability. Suppose we want to know the probability that John will win elections given that Paul wins. The analysis will give the right verdict in this case: Pr(P | Q) = Pr(1/3)/Pr(2/3) or about 1/2.

Notice that the probability table (and the probability formulas that could be derived from it) could be translated in terms of Carnap's state-descriptions if we let state-descriptions (the structure description derived from them) refer to truths (in the world) rather than to states of affairs. Thus, the Cenizan statement "P is 1/3 true" is translated in Carnapian terms as "There is exactly one state (of three) where John won the elections." Conversely, Carnapian descriptions could be translated as Cenizan statements. For example, "There is exactly one state where all three coins will land on heads" could be translated as "The statement 'All three coins will land on heads' is 1/8 true." This means that both theories are logically equivalent; as such, the theory proposed by Ceniza *et al.* inherits the same problems raised against Carnap's theory.

One problem against Carnap's theory is how to determine the *admissible* state-descriptions in the first place (Hájek 2019). For Ceniza *et al.*, this translates to the problem of determining the admissible truth values in the probability table. Both theories imply that probability ranges over a set of states of affairs (or truth conditions). In the case of the three coins example, the set includes eight possible states; in the case of John and Paul winning the elections, the set includes three possible truth conditions. However, there are surely other states or truth conditions that were not included in these sets. For example, the state where one coin exploded once tossed or the other conditions where neither John nor Paul won the elections. If such states or truth conditions were added to the sets, then the probability assignments would change. Such change, however, may violate the standard rules of probability theory. For example, instead of having the probability that a coin will land on heads as 1/2, we could have a scenario where it is 1/3.

Carnap and Ceniza may reply that we could bar these states and conditions from being included in the set of *admissible* states or truth conditions since we have an *in-principle* argument to bar them. For example, they might say that a fair coin landing on heads is just 1/2 simply because that is just the nature of fair coins or that the probability determination is done *a priori*. The latter reason collapses the logical theory of probability with the classical theory, which has its own problems. The former reason collapses the logical theory with the propensity theory, which again has its own problems. So, neither choice might be tenable for Carnap or Ceniza. Be that as it may, logical theories (of the Carnapian or the Cenizan brand) are still live options in the literature of the philosophy of probability and are worth exploring further (Franklin 2001).

CENIZA ON THE METAPHYSICS OF MODALITY

In this penultimate section, let us focus on one philosophical topic to which Ceniza has devoted most of his philosophical career, viz., the topic of metaphysics of modality. The main issue in this area is whether modal notions, like possibility and necessity, could be analyzed in terms of non-modal notions. As Sider (2003) puts it, "whether modal notions can be reductively defined." Ceniza (1985) is critical of the standard possible worlds analysis of modality and offers a different – albeit nonreductive – analysis of modality in terms of the concepts of compossibility and incompossibility. Before getting into his proffered analysis, however, let us further explore what the main issue implies.

The issue about the metaphysical underpinnings of modal judgments seeks the *thing* that makes modal statements (i.e., statements that have modal notions) like "Possibly, pigs fly" and "Necessarily, 2 + 2 = 4" true or false ((Garrett 2017, 37; compare with (Sider 2003)). Let us formalize the modal concepts as follows: "□P" for "Necessarily, P" and "◇P" for "Possibly, P." Let us define "Impossibly, P" as "□~P" or "~◇P" and "P is contingent" as "◇P & ~□P" or "◇P & ◇~P." Let us further distinguish between two kinds of modal statements: *de dicto* modalities and *de re* modalities. *De dicto* statements are statements where the modal operator governs the whole statement. For example, when we say that "Necessarily, the number of planets in the solar system is odd" (or more formally, "□[(the x: Nx) Ox]"), we are making a *de dicto* statement. On the other hand, *de re* statements are statements where the modal operator governs the property of the things talked about in the statement. For example, when we say that "The number of planets in the solar system is such that it is necessarily odd" (or more formally, "(the x: Nx) [□Ox]," we are making a *de re* statement. Intuitively, the *de dicto* statement is false. The number of planets might have been even, so it is false that it is necessarily odd. On the other hand, the *de re* statement seems true. It claims *that* the number that actually numbers the planets is necessarily odd; since we actually have 9 planets, then it is true that the number of planets in the solar system is odd (Sider 2003). But what makes these modal intuitions true or false?

The now dominant analysis of modality, due to Saul Kripke (1959) and others, employs the notion of possible worlds in order to account for the truth of modal statements. According to this analysis, □P is true iff P is true in all possible worlds; ◇P is true iff P is true in some possible world (Garrett 2017, 42). Thus, "Possibly, pigs fly" is true just in case there is at least one possible world where pigs do fly. On the other hand, "The number of planets in the solar system is such that it is necessarily odd" is true just in case in all possible worlds where the number of planets in the solar system is 9, the number of planets is odd.

The possible worlds semantics offers a reductive analysis of modality. It is reductive in that the notion of *possible* worlds is not a modal one. To translate modal statements into the possible worlds semantics is to take such statements as quantified statements over worlds. For example, if "Necessarily, P" is true, then it means that P is true in *all* possible worlds. If "Possibly, P" is true, then it means that P is true in *some* worlds. If "Impossibly, P" is true, on the other hand, then it means that there is *no* world where P is true. Clearly, quantification over worlds is not a modal notion. As such, the possible worlds semantics reductively define modalities.

Moreover, possible worlds semantics offers a metaphysical view where modalities are not fundamental features of reality. If one is a fundamental feature of reality – a feature that carves reality at its natural joints, then the concept that signifies it is primitive and unanalysable (Sider 2011, 5). Since modalities are definable in terms of truths in (quantified) possible worlds, it follows that they are analyzable concepts. As such, they are not fundamental features of reality. That is, there are no necessities or possibilities in reality; there are only truths in (quantified) worlds (Sider 2003). This latter thought, however, invites a question about the ontological status of these possible worlds.

When we say that "Possibly, P" is true just in case *there is* a world where P is true; we are ontologically committed to the existence of possible worlds. The idea of statements implying ontological commitments is nothing new. It implies that quantified (existential) statements of the form "There is an x..." are committed to the existence of the thing that the quantifier ranges over (Joaquin 2013). Or, as W. V. Quine puts it, "A theory is committed to those and only those entities to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory be true" (Quine 1948, 33). Since possible worlds semantics implies quantification over possible worlds, it also implies a commitment to their existence.

This is one reason that philosophers like David Lewis (1986) advocate modal realism – the view that our world is just but one of an infinite number of causally isolated concrete possible worlds.

The commitment to the existence of possible worlds is not without problems. For one, to argue that possible worlds exist because possible worlds semantics implies quantification over worlds might be a nonstarter. Of course, there are statements that quantify over *Tikbalangs* or *Aswangs* – that there are *Tikbalangs* living in that tree or that all *Aswangs* are blood-thirsty. However, such statements do not imply their existence. This point led other philosophers to think of possible worlds as mere abstract or fictional entities, like sets of statements or *ersatz* worlds (Sider 2003). Being abstract or fictional entities, possible worlds are nonexistent. But despite this, they are still useful as theoretical devices in analyzing modal statements – much like how nonexistent quarks are useful theoretical posits in current quantum mechanics.

It is interesting to note that despite not citing any relevant literature on possible worlds semantics for modal logic, Ceniza arrived at an objection against it. He writes, "Possible worlds in possible worlds model of modal logic... are only weakly existent entities" (Ceniza 1980b, 11). For him, since possible worlds are equivalent to a bare logical possibility, they just weakly exist. What he means by this is that all this talk about possible worlds is nothing more than specifying the vastness of logical space. For example, to say that "Possibly, pigs fly" is to specify the possible states of affairs where pigs do fly. This, however, is nothing more than specifying under what conditions is the statement "Pigs fly" is true and under what conditions it is false. For him, bare logical possibilities do not give us the strong sense of existence that a metaphysical theory of modality needs to account for. The strong sense of existence that Ceniza has in mind here requires causal efficacy – the difference-making property that a strongly existent thing has. Since possible worlds do not have such a property, they are weakly existent. *Sans* the objection against Ceniza's criterion for strong existence that was raised above, we might venture to ask what Ceniza's account of modality is.

Ceniza (1985) offers an analysis of modal terms in terms of compossibility and incompossibility. He does this in two steps. First, he defines "compossibility" and "incompossibility." Second, he defines the modal terms in terms of them. Let us see how these steps proceed. For Ceniza (1985, 1), two things are compossible iff "they are capable of coexisting or of being-so together." That is, p and q are compossible iff it is not necessary that p's existence implies the nonexistence of q (Ceniza 1985, 4). More formally,

Compossibility $\quad Cpq$ iff $\sim N(p \supset \sim q)$.

On the other hand, p and q are incompossible if they are not compossible. That is,

Incompossibility $\quad \sim Cpq$ iff $N(p \supset \sim q)$.

Let us highlight some key concepts in these definitions. First, one may mistakenly assume that (in)compossibility is mere (in)consistency. Consistency is a property of statements such that a set of statements are consistent so long as all the statements in the set can be true at the same time. Compossibility, on the other hand, is a property that things have – that two or more things are compossible iff they can coexist at the same time. Thus, while compossibility implies consistency, consistency does not imply compossibility. Second, the "N" in Ceniza's definition must not be taken as a purely modal notion since it is not a *mode of truth*. It is not the same as "necessarily" in "Necessarily, 2 + 2= 4" because if it is, then the definition of modal terms that Ceniza is proposing will be viciously circular. What "N" refers to is more of a metaphysical concept of necessity. It is a *mode of being-so* of things.

So, what do Ceniza's definitions imply? One may think of them in terms of Ceniza's strong sense of existence; p and q are compossible iff the existence of p does not causally imply the nonexistence of q. On the other hand, p and q are incompossible iff the existence of p causally implies the nonexistence of q. For example, that this chair is brown implies that being brown and being a chair are compossible things since both could coexist at the same time. Moreover, the brownness of this chair implies that it cannot be not brown because being brown and being not-brown are incompossible things; the existence of one implies the nonexistence of the other.

Given these two definitions, Ceniza defines possibility as follows:

Possibility $\Diamond P$ =df. Cpp =df. $\sim N(p \supset \sim p)$ =df. $\sim N \sim p$.

That is, the statement P is possible iff the entity p (of which P is about) is compossible with itself. This implies that p's nonexistence is not (metaphysically) necessary (Ceniza 1985, 4-7). For example, "Possibly, pigs fly" just means that a flying pig is compossible with itself: its nonexistence is not metaphysically impossible.

Ceniza defines impossibility as a direct contradiction of possibility:

Impossibility $\sim\Diamond P$ =df. $\sim$Cpp =df. $N(p \supset \sim p)$ =df. $N\sim p$.

That is, the statement P is impossible iff the entity p (of which P is about) is incompossible with itself. This implies that p is metaphysically nonexistent (Ceniza 1985, 4-7). For example, "It is impossible that there are male vixens" just means that being a male vixen is incompossible with itself. Since being a vixen implies being a female fox, it is simply metaphysically impossible that there are female foxes that are males. That is, the statement "There are male vixens" is putatively false.

Notice that the definitions of possibility and impossibility rely on the entity being compossible or incompossible with itself. In defining contingency and necessity, on the other hand, Ceniza (1985, 7-11) defines them in terms of how entities are or are not compossible with other entities. He defines contingency as follows:

Contingency $\Diamond P \ \& \sim\Box P$ =df. Cpp & $\exists q$ [Cqq & $N(p \supset \sim q)$]

That is, the statement P is contingent iff the entity p (that P is about) is compossible with itself but is also incompossible with at least one other entity q. For example, “This chair is brown” is contingent since while it is compossible with itself, it is incompossible with it not being brown. Of course, the chair being not brown is a possible – hence, a compossible – thing itself. But that is also a contingent thing, according to Ceniza.

Finally, Ceniza defines necessity as a contradiction of contingency:

Necessity $\Box P$ =df. $Cpp \,\&\, \forall q\, [Cqq \supset \sim N(p \supset \sim q)]$

That is, P is necessary iff the entity p (that P is about) is compossible with itself and also compossible with all other entities. For example, “Necessarily, 2 + 2 = 4” implies that the mathematical entity “2 + 2 = 4” is not only compossible with itself but is also compossible with any other compossible entity, say of the brown chair, such that the conjunction of “2 + 2 = 4 and this chair is brown” is true.

One upshot of Ceniza’s analysis is that it validates some of the standard modal truths without recourse to possible worlds semantics. For example, it validates the idea that whatever is true is possible (formally, $P \supset \Diamond P$). Here is a proof of this statement using Ceniza’s analysis.

1	$\sim(P \supset \Diamond P)$	assumption for *reductio*
2	P	1, negated conditional
3	$\sim\Diamond P$	1, negated conditional
4	$N{\sim}p$	3, definition of impossibility
5	$\sim P$	4, definition of impossibility
6	$P \,\&\, \sim P$	2 contradicts 5
7	$P \supset \Diamond P$	2-6, *reductio ad absurdum*

Despite the deliverances of Ceniza’s analysis, however, there is one important worry that may be raised against it. Ceniza’s analysis implies that modalities are defined in terms of compossibility and incompossibility. As Ceniza defines them, two things are compossible iff they *can* coexist with each other. They are incompossible iff they *cannot*. But being *able* to coexist *is* a modal notion. They are modal not just in the metaphysical sense that Ceniza cashes out “N” but also in the sense that they modify truths. For example, that a chair’s brownness is compossible with itself could easily be translated as the modal statement “The chair *could be* brown.” Thus, Ceniza’s analysis fails to be reductive since his *analysans*, compossibility, and incompossibility, still have modal import. Ceniza might reply that his proposal implies a non-reductive account of modalities. If so, then well and good since defining modalities in terms of compossibility and incompossibility would yield the same results as the possible worlds account does, even without countenancing the existence of such worlds.

CONCLUSION

In this paper, I have discussed Ceniza’s contributions to the paradoxes of material implication, the nature of probability, and the metaphysics of modality.

Despite their shortcomings, they are still noteworthy contributions to these fields. His solution to the paradoxes of material implication may yield a vicious regress, but his distinction between degenerate and nondegenerate conditionals still offers a way of looking at the nature of implication without abandoning classical logic. His idea of using truth conditions as an *analysans* is an inspired one and can be seen as logically equivalent to Carnap's state-description theory; although both theories have problems, they are still very much live options. Finally, his proposal of defining modalities in terms of compossibility and incompossibility, albeit still non-reductive in nature, offers an alternative account of modalities to the standard possible worlds semantics used in the literature.

However, it is sad to note that Ceniza's contributions in these fields have not impacted the greater philosophical literature. A quick citation check of Ceniza's articles indicates an absence of works that refer to or cite them. Perhaps this is not a conclusive evidence of the lack of philosophical impact of Ceniza's ideas, but it is a good indication of it. I see two reasons for this. First, Ceniza was a product of his time. He was writing at a time when philosophers only knew of each other's work through international conferences, publications in top journals, and correspondences. In short, he lived during a time when knowing philosophers personally mattered in the profession. That is, one's work will likely be cited by philosophers whom he or she knows personally. This is not to say that having a professional network does not matter now. Of course, it does! What I am only saying is that networking in professional philosophy then was more of a face-to-face rather than an online endeavor.

Evidence tells us that Ceniza did not have a strong philosophical network. He was not keen on presenting at international conferences; he would rather prepare lectures for his students. He did publish one paper in a top philosophy journal. However, one paper is insufficient to make a splash in the field. There are exceptions, of course (think of Edmund Gettier's case), but though there are such exceptional cases, the rule still generally applies. Finally, though Ceniza has a long-standing correspondence with Irving Copi (evidenced by the latter mentioning the former in the acknowledgments in his logic book), this again is not sufficient to ensure a philosophical impact. At the time that Ceniza was publishing his works, Copi was arguably a middle-weight philosopher compared to the likes of Kripke and Quine.

The second reason I think that Ceniza's work has not caught on in the wider philosophical community is that the topics he was working on are technical in nature. The logic of conditionals, probability theory, and modal logic are among the more abstract fields of analytic philosophy. And these fields were dominated by a closely-knit community of Anglo-American philosophers. To be part of this community, one must not only be a good philosopher but he or she must also be exceptional. As original and ingenious as Ceniza's ideas may be, he was still an outsider that needed to "fit in" in that he must prove his philosophical worth to the forerunners of the field. Unfortunately, he does not have enough philosophical pedigree for this (again, this is evidenced by his published works not citing the "right" works in the field).

The technical nature of works might also be the reason why Ceniza has not impacted the local philosophical community in the Philippines. As he was working on topics deeply entrenched in logical and mathematical formulas, local philosophers of his time might not have had the philosophical training to appreciate his ideas. After all, since philosophy in the country throughout the 1970s to the 1990s was not steeped in symbolic logic, only a handful of local philosophers may have been interested in the topics that Ceniza worked on in the first place. Actually, this hesitancy in dealing with technical philosophy may still be true of 21st-century Filipino philosophers, but hopefully, things are changing given the advancements in our philosophy curriculum and the access we have of works in the wider philosophical community.

I do hope that this paper has given you, dear reader, an idea of Ceniza's main contributions to the three areas of philosophy discussed. I do hope that you have recognized them as innovative and noteworthy contributions to philosophy and as marks of a great Filipino philosopher that must be emulated. Ceniza's contributions may not be that well-known, but they still warrant not only a place in the pantheon of the pioneers of Filipino Philosophy but also a place in the wider philosophical community.[18]

NOTES

1. The exceptions, perhaps, are Castro (1993) and Gripaldo (2004) who have truly appreciated the extent of Ceniza's views on the paradoxes of material implication and the metaphysics of modality, respectively.

2. Biana (2022) explored how Ceniza's travels may have influenced his overall philosophy.

3. For example, Sider (2010) uses the arrow symbol, while Priest (2008) uses the hook.

4. Disputes about the paradoxes of material implication have been recorded in the writings of the Stoics and some logicians in the medieval period ((Sanford 1989); (Priest 2008, 12)). Its modern formulation however can be traced to Hugh MacColl (1908) and Whitehead and Russell (1910/1956).

5. By classical logic, we mean a two-valued logic where the law of excluded middle and the law of non-contradiction hold.

6. Conjunction-elimination implies that one can validly infer P from the conjunction P & Q. On the other hand, disjunction-introduction implies that one can validly infer P v Q from P. Finally, disjunctive syllogism implies that one can validly infer Q from the disjunction P v Q and ~P (i.e., the negation of P).

7. For a survey of different semantic models for relevant logics, see (Mares 2020).

8. Priest (2008) offers an alternative possible worlds semantics for relevant logic.

9. Several attempts have been made to meet this problem, see (Garrett 2014, 142-149).

10. It is curious to note, however, that Ceniza (1988) makes no reference to Lewis or to any relevant logician. In fact, he only cites one work in the whole paper, viz., the fifth edition of Irving Copi's *Symbolic Logic*.

11. *Principia Mathematica* has a similar description for degenerate conditionals, see (Whitehead and Russell 1910/1956, xvi).

12. This is a worry that a referee has pointed out.

13. As Alan Hájek (2019) phrases these questions: "What kinds of things are probabilities, or more generally… What makes probability statements true or false?"

14. The discussion of the standard probability calculus follows Skyrms (2000, ch. VI).

15. Hájek (2019) reports that David Lewis's best-system approach to probability is becoming another viable option. However, we will not touch on Lewis's theory in this paper.

16. However, Ceniza *et al.* (1998) made no reference to Carnap's work. Nonetheless, the fundamental ideas of Carnap's theory can be seen in their proposed theory.

17. Popper pointed out, and which Carnap acknowledges, that Carnap's theory of confirmation involves an equivocation. On the one hand, confirmation may refer to firmness or how probable *h* is given *e*; on the other hand, confirmation may also refer to an increase in firmness or how much the probability of h is increased when *e* is acquired. This is a subject of the Popper-Carnap controversy that we will not touch on in this paper. For a discussion of this issue, see (Fitelson 2006).

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