LINGUISTIC MULTIDIMENSIONAL SPACES

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PREFACE

This book extends the concept of linguistic coordinate geometry using linguistic planes or semi-linguistic planes. In the case of coordinate planes, we are always guaranteed of the distance between any two points in that plane. However, in the case of linguistic and semi-linguistic planes, we can not always determine the linguistic distance between any two points. This is the first limitation of linguistic planes and semi-linguistic planes.

Hence, finding a linguistic distance or line in a ling plane or a semi-linguistic plane given two ling points In that plane is not guaranteed. We may have a distance defined or may not have a distance defined. Given this, ling coordinate planes do not function like classical coordinate planes. Further, it is impossible to find the concept of finding the ling area of a triangle or trapezium or, quadrilateral is also not possible, as the very concept of ling distance is only a ling term. This is also a considerable limitation of ling planes compared to coordinate planes. However, the vital point is finding the area of a triangle A = (–1, 2), B = (3, –4) and C = (0, –7). We see what the triangle in reality looks like when it has negative values. Is the triangle ABC an imaginary one, etc, is a relevant question for children. We do not decry coordinate geometry in any way; however, we want to record the limitations of linguistic geometry. Finally, we develop the concept of multidimensional (r, n − r) semi-linguistic coordinate geometry ($r < n$; $1 \leq r < n$ and dimension is n) and multidimensional linguistic coordinate geometry. The only notable factor about multidimensional linguistic coordinate geometry is these elements $x = (x_1, \ldots, x_n)$, where each x_i is a linguistic word that can be used for feature extraction. Sometimes, one is more familiar with working with linguistic terms than numbers. This book is organized into three chapters. Chapter One discusses linguistic variables, sets and planes to make this book self-contained. Chapter 2 introduces the new notion of two-dimensional linguistic and semi-linguistic coordinate geometry. Several properties and limitations of dimensional linguistic planes and (1, 1) semi-linguistic planes are discussed. The final section introduces the notion of (r, n − r) ($1 \leq r < n$; and dimension is of the multidimensional linguistic and linguistic spaces. Several interesting results are obtained. Its limitations or shortcomings are discussed. One can get linguistic lattice structures by defining min/max operators on the n-dimensional linguistic coordinates. However, in problems where feature extractions/study is to be made, these linguistic terms will be better suited than real or fuzzy numbers.

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Chapter One

LINGUISTIC VARIABLES, SETS AND PLANES

In this chapter we introduce the basic concepts of linguistic (ling) variables, ling sets / terms and ling. planes. The main motivation for introducing these concepts is to develop the notion of linguistic analytical geometry. Even in the case of ling analytical geometry we have lots of limitations.

The notion of ling variables happens to be not only different from classical number theory. For numbers are very specific. Number systems are natural numbers, integers, rationals, reals and complex numbers.

We have defined the notion of modulo planes or mod planes.

Mod planes are infinitely many [26], their properties and research have been carried out on mod planes in [26-7].

We wish to state they are only planes in the first quadrant. Likewise, we have only used the concept of ling planes to be only planes in the first quadrant.

The main reason for this is the zero is nothing so when one tries to say the two lines intersect at the nothing it is 0, several non mathematicians cannot accept it. More so even if one accepts the fact one starts from the empty set still one doubts why should one have the negative numbers which are non-existing and only found in the scales artificially made by us. Measuring cannot be done on negative values. However, at this juncture we wish to state that negation of a word is like negative numbers. To be more linguistically appropriate we can say opposites; truth is negation false, good is negation bad, very good is negation of very bad, tall is negation of short and so on.

It is pertinent to record for any person it is easy to understand truth its negation (or opposite) is false than understanding 3 and –3 or 9 and –9 and so on. Thus these ling terminologies are more natural than the so called negative numbers.

So when a mathematics of this form is developed linguistically it will be a boon to common man as well as a better benefit to non mathematics researchers. For it helps engineers, medical experts, socio scientists, economists and so on and above all primary students

Approach of linguistic concepts are more natural than the number theoretical concepts even for children. We do not at any stage ask anyone to lay of number theoretic concepts; our only interest is to introduce some concepts parallel to the notion of linguistic concepts so that one can use any one of them for research/learning at any stage of life.

So here in this book we develop some concepts of linguistic analytical geometry analogous to classical analytical geometry. Another reason for introducing these linguistic theory is that in Artificial Intelligence (AI) in most cases a machine is trained only using words of natural or formal languages which is defined as objects or nouns and not by mere numbers at the early stages.

Keeping all these facts in mind we proceed onto to define the notion of linguistic variable (ling variable). A linguistic variable is one which is collection of words or sentences or both [21-25].

However, we do have some restrictions in the very basic definition of ling variables. Ling variables are considered as a collection of linguistic words (ling words) referred to as linguistic set or linguistic terms of the ling variable.

This linguistic set / term can be finite or an infinite set.

We will not be using the definition of [41-43] which sees ling variables as 5 tuples which also includes the fuzzy membership. We simply view a ling variable as a collection of ling terms or ling set of words or ling words.

We will first illustrate this situation by a few examples.

Example 1.1. Suppose we consider the linguistic variable 'age' then we can say from youngest – just born to the oldest – just nearing 100 or so.

So we can say all other persons in different ages can be say like; very very very young, very very young, so on young, just young and so on to middle aged, just middle aged, so on to, just old, old, very old, so on to oldest.

So we can say the ling set / term as [youngest, oldest]. However, number theoretic representations of the age of people will be represented by [0, 100] where 0 years corresponds to the just born and 100 to the oldest if they live up to it in an ordinary way.

It may so happen that in very special cases some may live over 100 years but in general the highest is put as 100 years.

We say the ling variable in general which represents age of persons is associated with the ling set / term which is infact a ling interval, [youngest, oldest] which we define as a linguistic continuum.

We also observe that the ling variable 'age' is time dependent it varies steadily with time.

Suppose on the other hand if we want to consider the linguistic variable age say of 9 persons it is very natural we cannot ask those 9 persons what is their age but we however want to put forth their age so assigning numbers which correspond to years may not be an accurate one.

We cannot assign exact numbers as age of them for it may not be proper.

However, we can looking at them assign ling words for their age say like youth, very very young for a toddler in the group, old for a old man in that group and so on.

Thus if we denote the 9 persons by $\{p_1, p_2, \ldots, p_9\}$ then we give the association of ling term to represent their age by looks as follows.

p_1 – very young

p_2 – old

p_3 – youth

p_4 – just middle age

p_5 – very old

p_6 – old

p_7 – middle age

p_8 – very young

p_9 – youth

So for this set of 9 persons we has the associated ling term / set which is given by

{very young, old, youth, just middle age, very old, middle age}.

We represent them as a ling set of cardinality 6.

$\{p_1, p_8\}$ – very young

$\{p_2, p_6\}$ – old

$\{p_3, p_9\}$ – youth

$\{p_5\}$ – very old

$\{p_4\}$ – just middle age

$\{p_7\}$ – middle age

So these six ling terms describes well the age of these 9 persons. This will be easily understood even by a layman.

So depending on the context of the problem we assign the ling set. Clearly the ling variable age is a time dependent concept, for it directly says so many years, months and days of a person.

Example 1.2. Consider the ling variable colour of the eyes of different nationalities all over the world. Clearly one cannot represent this ling variable in any form other than by the ling term / set which is given by

S = {black, brown, dark brown, light brown, hazel, gray, green, blue, light green}.

Clearly this ling variable, colour of the eyes of different nationalities cannot be represented otherwise; further this is independent of time.

So some of the concepts which are ling variables do not have numerical representation.

We provide yet another example of a ling variable.

Example 1.3. Let us consider the colour of people all over the world, this is clearly a ling variable.

The ling set / term associated with this ling variable cannot be given any numerical values. It can only be a set of colours given by

S = {black, dark brown, light brown, brown, white, yellowish, wheat colour, ebony colour}

this ling set cannot be infinite, neither is it time dependent, it is fully time independent.

Now consider Example 1.1 the ling variable age is time dependent and is an infinite continuum but the other two ling - variables colour of eyes and colour of people all over the world are time independent and are only finite discrete sets which cannot be totally ordered.

Further these two ling variables cannot be given numerical representations and they are time independent.

Next we consider the ling variable performance aspects of students in a classroom. This ling variable is also time independent.

It can be any class, any time and any school or college in the world. Because it is a class room the teacher cannot physically assess them by a test or an exam. It is not like evaluating answer papers of students. It is the way they respond in classroom like

{attentive, cooperative, isolated, indifferent, talkative, silent, chatter box, does not listen, listens well, disciplined, well behaved and so on}.

These are a crude way of linguistically analyzing, the students in the classroom scenario. In fact if we wish to assess their ability in studies like

{good, bad, very bad, average, very good, etc} ;

then we do have a better assessment, the first one can be thought of as their behavior in class room the second one can be thought of as their capacity of learning so good or bad or so on.

The latter one can also be easily done by a teacher in classroom by asking questions after teaching them and so on to assess their goodness in studies. The former one corresponds to their behavior in classroom.

As our study pertains to performance aspects of students we have to prefer the later ling set only.

Example 1.4. Consider the ling variable yield of paddy plants. The corresponding ling set associated with this ling variable, yields is as follows:

{high, low, very high, very low, medium, just medium, just high and so on}.

We see the set can be finite or infinite depending on the experts need.

Having seen examples of ling variables and its associated ling set and their related structures we now proceed on to try to develop them with more properties.

A ling variable can be time dependent or time independent. Time dependent ling variables highly depend on the time whereas time independent variable have no relevance on time, like colour of eyes, performance of students varies depending on a set of students, class they study, school they study, the assessment beginning of the year, middle of the year or end of the year and so on so forth, it can be viewed as a very special type of ling variable.

It is important to note when we say time dependent for it is important that the age changes as time changes continuously.

Likewise the weight changes from time to time increasing, decreasing or remaining constant and then decrease or increase but however this is also a continuous curve which is time dependent. But ages is increasing where as weight of a person is continuous increasing or decreasing but it can be constant in some period of time.

Now take weather report the ling variables measuring sunshine or rain or wind in a day or week or month or year. It is important to note that sunshine (or rain or wind) is also time dependent continuous in a way very different from the ling variables ages or weight of a person. Very much so will continue till the universe exists in tact.

Now having seen ling variables that are time independent, time dependent, time dependent but having no bounds or non stopping or many other special types we will discuss some more properties of ling variables and its associated ling set / term.

A ling variable can always be associated with some ling term / set S and the set S may be totally orderable or partially orderable or unordered.

To this effect we will recall the definition of these three concepts, so that the reader can easily adopt to it in case of ling sets/terms.

For more about partially ordered set or totally ordered set or unordered set refer [8-10].

Definition 1.1. *Consider the non empty set S. If every distinct pair of elements in S is such that they are comparable that is for every pair a, b* $\in$ *S (a* $\neq$ *b) we have a* $\leq$ *b (or a* $\geq$ *b) that is a is* *less than or equal to be (or a is greater than or equal to be) then we say (S, ≤) (or S ≥) is totally a ordered set (clearly $a \leq a$ or $a \geq a$).*

For more refer [8-10].

We will provide one or two examples of them which are totally ordered set and which are not totally ordered set. The main aim of the book is to make itself self contained and understandable even by a non mathematician.

Example 1.5. Let Z be set of integers Z is a totally ordered set for every pair of distinct elements a and b in Z we have ($a \leq b$) (or $b \leq a$). Thus

$$-\infty \leq \ldots n \leq \ldots \leq -3 \leq -2 \leq -1 \leq 0 \leq 1 \leq 2 \leq 3 \leq \ldots \leq n \leq \ldots \leq \infty.$$

Clearly $\{Z, \leq\}$ is a totally ordered set.

This is an example of a totally ordered set which is of infinite order.

Now we provide an example of a totally ordered set which is of finite order.

Example 1.6. Let $S = \{\sqrt{2}, -1, 5, 3.77, -5, 9, 8, 11, 99\}$

be a set S is totally ordered as follows:

$$-5 \leq -1 \leq \sqrt{2} \leq 3.77 \leq 4.5 \leq 8 \leq 9 \leq 11 \leq 99.$$

S is also a totally ordered set but S is of finite order.

Now we see an example of a set which is not totally ordered.

Example 1.7. Let S = {a, b, c} be a set of order three. P(S) be the power set of S given by

P(S) = {ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c} = S}.

We see {a} and {b} the subsets of P(S) are not comparable. {a, b} and {a, c} in P(S) are not comparable.

However {{a}, {a, b}, {a, b, c}} ∈ P(S) are comparable infact

{a} ⊆ {a, b} ⊆ {a, b, c} is a totally ordered but P(S) is not a totally ordered set as every distinct pair of elements in P(S) are not comparable.

Thus we have examples of nonempty sets which are not totally ordered. If in this example take the set S be of infinite order and P(S) the power set of S is of infinite ordered set which is not totally ordered.

Next we proceed onto define the notion of partially ordered set.

Definition 1.2. *Let S be a nonempty set. We say S is a partially ordered set if for atleast one pair of elements a, b in S is comparable; that is a ≤ b (or b ≤ a). That is there are some elements in S which are not comparable. Thus { S, ≤ } is a partially ordered set which in general it is not a totally ordered set.*

We provide examples of them.

Example 1.8. Let S = {a, b, c, d} be a set of four elements. Let P(S) be the power set P(S) of S.

P(S) = {{ϕ}, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {d, c}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c, d} = S}.

Clearly o(P(S)) = 2^4.

We see {P(S), ⊆} is only a partially ordered set where ⊆ implies the containment of subsets.

Consider {a, b}, and {b, c, d} in P(S). Clearly they are not comparable. {b, c, d}, {a, b, c} ∈ P(S) are not comparable.

Consider {a, b} and {a, b, d} ∈ P(S); we see

{a, b} ⊆ {a, b, d}.

So we have atleast one pair of distinct elements which satisfies the containment relation; also we have distinct pair of elements which do not satisfy containment relation; hence P(S) is only a partially ordered set and not a totally ordered set. Further |P(S)| = 2^4 = 16.

So we have provided an example of set of finite order which is partially ordered.

Now we provide an example of an infinite set which is partially ordered.

Example 1.9. Let S be a linguistic set of infinite order . P(S) be the partially ordered set under '⊆' relation (it is easily verified). {P(S), ⊆} is an infinite partially ordered set.

Having seen examples of partially and totally ordered set we now proceed onto describe a relation and then define the notion of an unordered set.

Theorem 1.1*. Let { S, ≤} be a totally ordered set them { S, ≤} is also a partially ordered set. However the converse is not true.*

Proof. Since every distinct pair in {S, ≤} is ordered if it is totally ordered we have atleast a pair of distinct pair which is orderable. So {S, ≤} is partially orderable.

On the other hand if {S, ≤} is only partially orderable then by definition their can be distinct pair which are not comparable so {S, ≤} not totally orderable.

We have several such set built using power set of a set and taking the ordering relation as the containment of subsets relation.

Thus all power sets {P(S), ⊆} is only partially ordered set which is not a totally ordered set.

Now provide examples of unordered set.

Example 1.10. Let

S = {red, blue, green, black, brown, white, violet, orange, pink, yellow} be set of colours.

We see S cannot be ordered in any way. Thus this S is an example of an unordered set.

Thus there exist examples of unordered set.

Now we proceed onto give the abstract definitions of unordered set.

Definition 1.3. *Let S be a non empty set S is said to be an unordered set if their exist no distinct pair of element which are comparable.*

We have given an example of an unordered set which is of finite order.

Now we provide an example of an unordered set which is of infinite order.

Example 1.11. Let

S = {collection of all m × n matrices $1 \leq m, n < \infty$} there is one and only one m × n matrix (for example there is one and only one matrix of order 2 × 3, 1 × 3, 3 × 1, 5 × 1, 6 × 1, …, so on).

We see the collection is infinite S is an infinite set which is un ordered.

$$\text{For if} \quad x = \begin{bmatrix} 3 \\ 0 \\ 1 \\ 2 \\ 5 \\ 7 \end{bmatrix} \text{ and } y = \begin{bmatrix} 9 & 12 \\ -1 & -5 \end{bmatrix}$$

x and y cannot be compared in any way.

We have unordered sets of both finite and infinite order.

Now we provide examples of these 3 types of ling sets.

Example 1.12. Let S be the ling set associated with the ling variable age of persons.

S = [youngest, oldest] this forms the ling continuum as this interval is analogous to the interval

[0, 100] on the real line and we know [0, 100] is totally ordered interval further all closed intervals are totally orderable.

We can have a 1 – 1 correspondence between [0, 100] and [youngest, oldest].

So the ling continuum S = [youngest, oldest] is also a totally ordered ling set of infinite order.

Take very old, young ∈ S; clearly young ≤ very old.

Consider the subset

P = {old, young, just old, very old, very young, just young, middle age, youth}

⊆ [youngest, oldest] = S.

P is a proper linguistic subset of S.

We see

very young ≤ just young ≤ young ≤ youth ≤ middle age ≤ just old < old ≤ very old.

Clearly P is also a totally ordered finite linguistic subset of S.

Infact we can say every proper ling subset of a ling set is such that if the ling set is totally ordered so is every proper ling subset of S.

Now we provide yet another example of a totally ordered finite ling set.

Example 1.13. Let

S = {tall, short, very short, medium height, very tall}

a ling set associated with the height of some set of plants.

S is a totally ordered ling set for we have the unique total ordering on S

very short ≤ short ≤ medium height ≤ tall ≤ very tall.

Thus S is a totally ordered finite ling set.

Having seen examples of finite and infinite ordered totally ordered ling sets, we proceed onto describe ling partially ordered sets.

We will first provide some examples of them.

Example 1.14. Consider the ling set S = {green, blue, black, brown, yellow, red, pink, dark brown, light brown}

be the ling set associated with the ling variable colour of the eyes of internationals all over the world.

Clearly S is not totally ordered ling set as red and yellow ∈ S but they cannot be compared as shades.

The subset P = {brown, dark brown, light brown} ⊆ S.

How every we see light brown ≤ brown ≤ dark brown so we have atleast a pair of distinct elements which can be compared; so S is a partially ordered ling set.

So we have given example of a ling set which is partially ordered. However this S is of finite order.

Suppose if we consider the set

{white, black, brown, yellow} = S the ling set associated with the ling variable complexion of a set of people.

Clearly S is not a totally ordered set or a partially ordered set, but is only a unordered ling set.

Now having seen examples of ling sets which are totally ordered, partially ordered or unordered we now proceed onto develop other properties of ling sets.

Now we wish to develop ling planes. We cannot as in case of reals or integers or rationals or complex numbers from the ling plane in a very classical way.

For us to develop the notion of ling plane need several special conditions.

1. Every ling variable cannot give way to ling set which is a ling continuum. So for as to have a ling continuum we basically need to consider only those ling variables which has its associated ling set to be a totally set which is an interval.

So all ling variables cannot give ling sets which are ling continuums.

To construct ling planes we only consider those ling variable which has its associated ling sets to be ling continuum.

We will consider the ling variable age of people, the linguistic set S associated with this ling variable is a ling continuum.

We define first semi ling planes.

For these semi ling planes one of the vertical or horizontal axis is reals and the other one is linguistic.

We will give example of them.

Example 1.15. Let S = [youngest, oldest] be the ling set associated with the ling variable age of people. S is taken as the ling vertical axis whereas the reals [0, 100] is taken as the horizontal axis. The plane so formed is defined as the semi ling plane.

This is presented in the following figure.

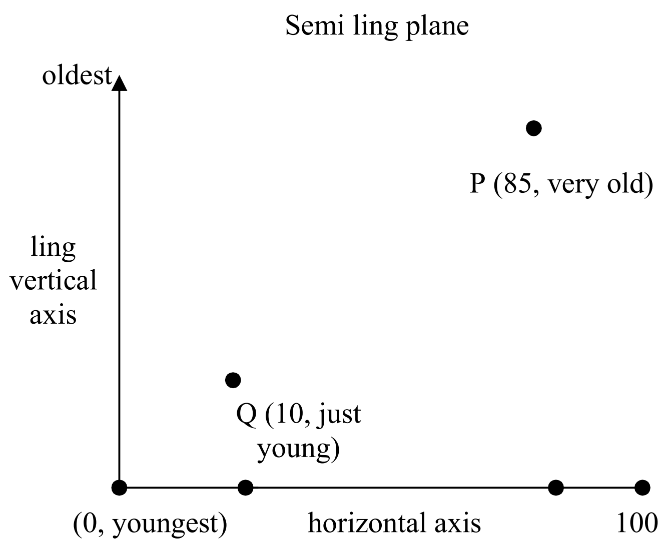


Figure 1.1

This diagram is defined as the semi ling plane.

Suppose 10 years then the ling term is as follows just young represented by Q (10, just young). Suppose we have a 85 year old; person the ling term associated with him can only be very old P(85, very old) is marked in the semi ling plane given in figure 1.1 It is not mandatory the ling set should always take the vertical axis. It can also take the horizontal axis.

The same semi ling plane can be formed by taking the vertical axis as reals [0, 100] and the ling horizontal axis as the ling continuum [youngest, oldest]. The semi ling plane is described by the following figure.

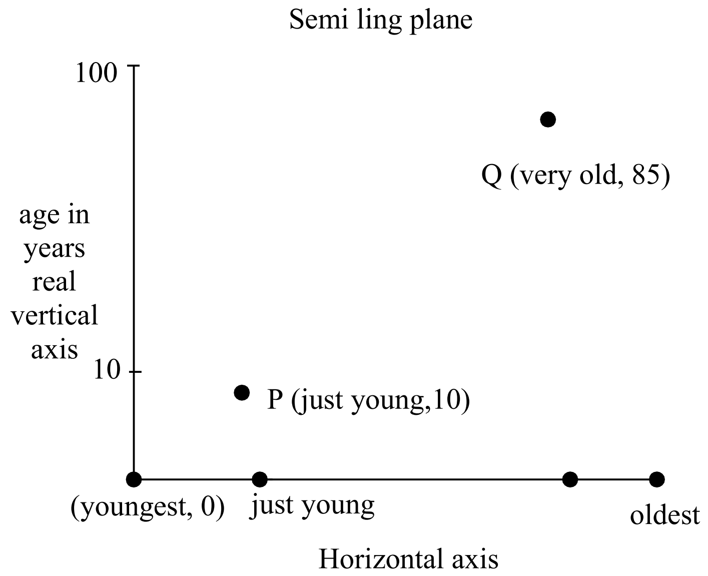


Figure 1.2

The ling term just young and 10 years is mapped as P (just young, 10) on the plane.

The ling variable very old and 85 years is plotted in the semi ling plane as Q (very old, 85) shown in figure 1.2.

We make the following observations about semi ling planes and give the difference between the semi ling planes and real planes.

i) The origin is not 0 in case of semi ling planes it can be (0, ℓ) or (ℓ, 0) where ℓ is the lowest or the least element of the continuum and 0 is the real zero.

ii) Semi ling planes can be of two types either real values on horizontal axis and ling values on the vertical axis or ling values in horizontal axis and real values on the vertical axis.

iii) The reals need not be $-\infty$ to ∞ it can be [0, n], n a finite value or [a, b]; a the lowest value and b the largest value both a and b finite real values $a \neq 0$.

iv) The most important observation is this has only one quadrant namely the 1st quadrant.

The distance between points on the semi ling plane is also a pair; one depicting the real distance and other the ling distance.

For instance the distance PQ is (very far, 75) or (75, very far) as the case maybe.

Now having seen a semi ling plane we proceed onto describe yet some more semi ling planes by examples.

Example 1.16. Let us consider the set S = [lowest, highest], the ling set / term associated with the ling variable weight of persons. The real scale in which the average weight of a person can vary in kilograms in [1 kg, 90 kg] this real continuum is taken as the other axis.

Now we give the semi ling plane connecting weight of persons along the ling vertical axis and the real scale [1, 90] in kilograms along the horizontal axis of the semi ling plane.

It is important to not the general weight of a new born can be 1 kg at the least. We do not go for the special case of when the weight of the just born is 0.5 kg or 0.3 kg, 0.7 kg and so on.

Similarly we take the highest weight to be 90 kgs, in some extra ordinary cases the weight can be 110 kg or 108.5 kg or 105 kg or so on. We do not and need not consider the exceptional cases we study only the general case.

Now the semi ling plane is given by the following figure.

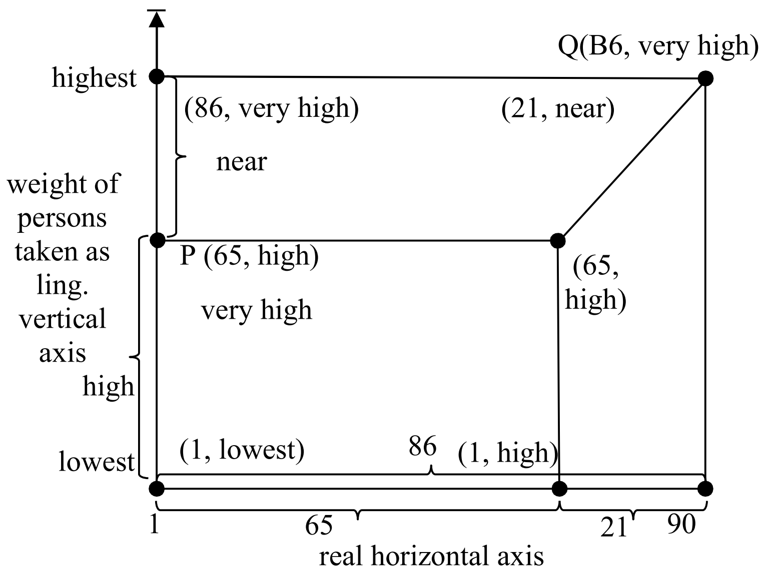


Figure 1.3

The origin of this semi ling plane is (1, youngest).

Suppose middle age man weight 65 kg then

P(65, middle aged)

is marked on the semi ling plane is as follows:

Let Q(86, very heavy) is shown in the figure 1.3.

Now we draw perpendicular from P and Q to real horizontal axis and ling vertical axis.

The distance between P and Q is (21, near).

We observe the origin of the semi ling plane is (1, youngest). It is only in the first quadrant.

We see depending on the ling variable the origin is fixed on the semi ling plane.

We give some more example of semi ling planes.

Example 1.17. Consider the ling set S associated with the ling variable height of persons. The height can be in the ling continuum [shortest, tallest] where shortest is the least element and the greatest ling term is the tallest.

However in general the height of a person numerically varies from [1, 7] (1 foot to 7 feet) where 1 is the least height or the shortest and 7 feet is the tallest possible one.

The semi ling plane can have either the real values as the vertical axis or horizontal axis.

Similarly the ling terms [shortest, tallest] can be taken either as the vertical axis or the horizontal axis.

The semi ling planes that can be associated with these is given by the following figures.

The origin is taken as (1, shortest) or (shortest, 1) depending on the semi ling plane with real horizontal axis or with real as vertical axis respectively.

The semi ling planes are given in the following.

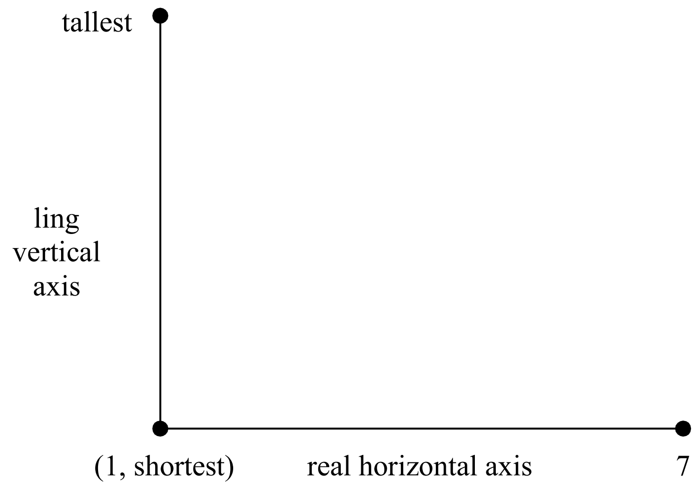


Figure 1.4

This is the semi ling plane with ling vertical axis and real horizontal axis.

We give the 3 semi ling pairs A(3, short), B(5, tall) and C(6, very tall) marked on the semi ling plane.

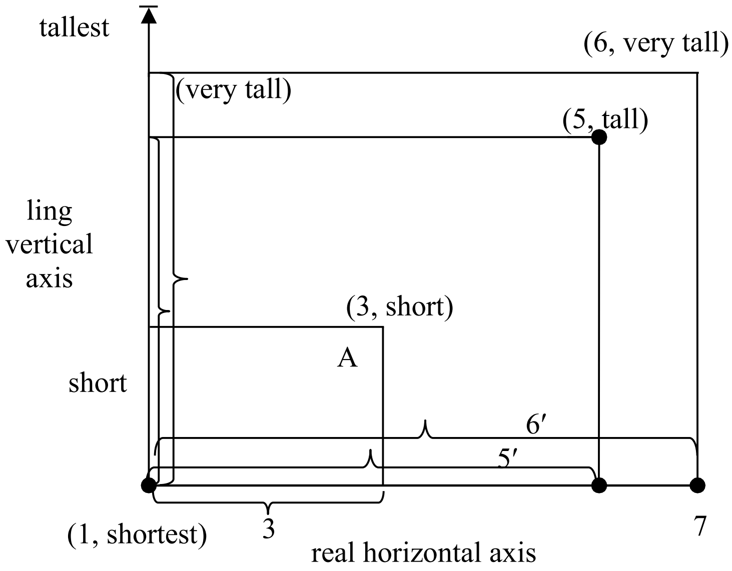


Figure 1.5

Now can we have a semi ling plane connecting them. We cannot have a semi ling plane but we can mark them as a graph in a very special way.

Since our main aim of this book is to introduce the new notion of linguistic analytical geometry while defining ling planes or semi ling planes it is mandatory that both the variables on the horizontal and vertical axis must form an interval which is a continuum.

Now the origin is dependent on the real and ling variables further they are defined only on the first quadrant. Be it ling planes or semi ling planes we do not have all the four quadrants.

Next we proceed onto describe by examples the notion of ling planes. For semi ling planes we have one of the axis to be real but in case of ling planes both the axis must be linguistic.

First we provide a few examples of them.

Example 1.18. Let us consider the ling variable age and height of people all over the world.

The ling set / continuum associated with the ling variable age is given by S = [youngest, oldest]. The ling set / continuum associated with the ling variable height of a person is, R = [shortest, tallest].

Taking on the ling vertical axis the age of a person and by taking ling horizontal axis the height of a person we obtain the ling plane which is described by the following figure.

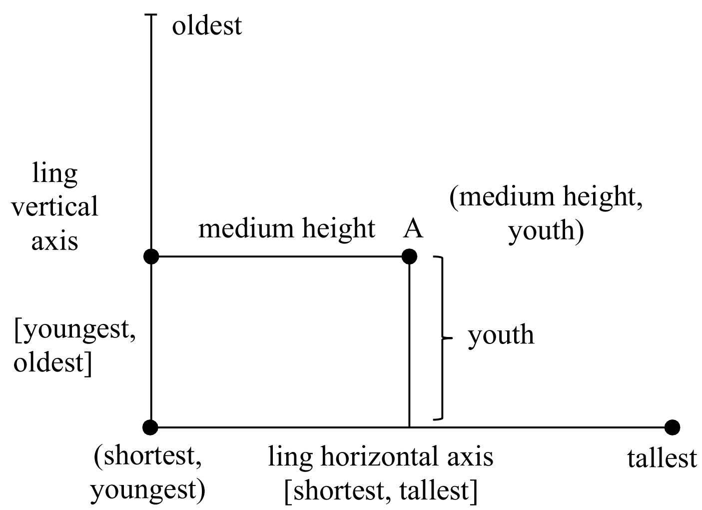


Figure 1.6

It does not matter if we exchange the ling vertical and horizontal axis.

The other ling plane figure is given in the following.

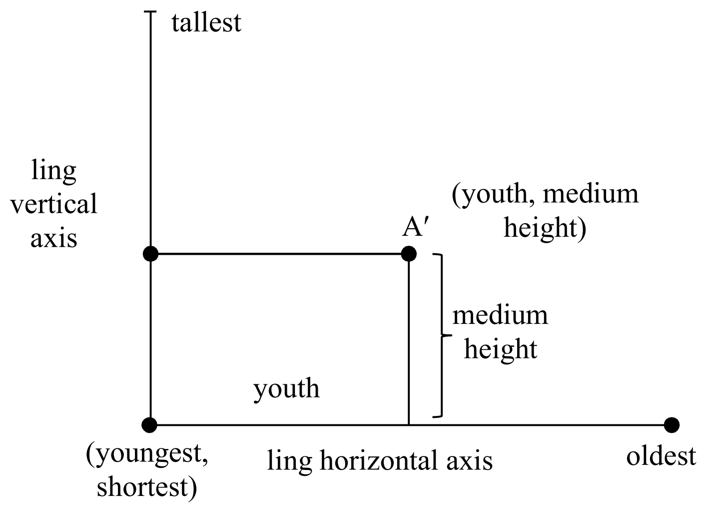


Figure 1.7

It is important to note when we exchange ling axes the origin changes from

(shortest, youngest) to (youngest, shortest).

Suppose we have the ling pair A (medium height, youth) then A is represented in figure 1.6.

Clearly A cannot be accommodated in figure 1.7.

However the ling pair A′ (youth, medium height) can find its place only on the ling plane given in figure 1.7.

We will provide yet another example of ling plane.

Example 1.19. Consider the ling variables yield of paddy plants and height of paddy plants.

Taking along the ling horizontal axis the ling continuum associated with the ling variable yield and that of the ling continuum associated with ling variable height of plants as the ling vertical axis we get the ling plane.

The ling continuum S associated with the ling variable height is given by [shortest (most stunted), tallest] = S. The ling continuum associated with ling variable yield is given by P = [very poor, very good].

The ling graph is as follows.

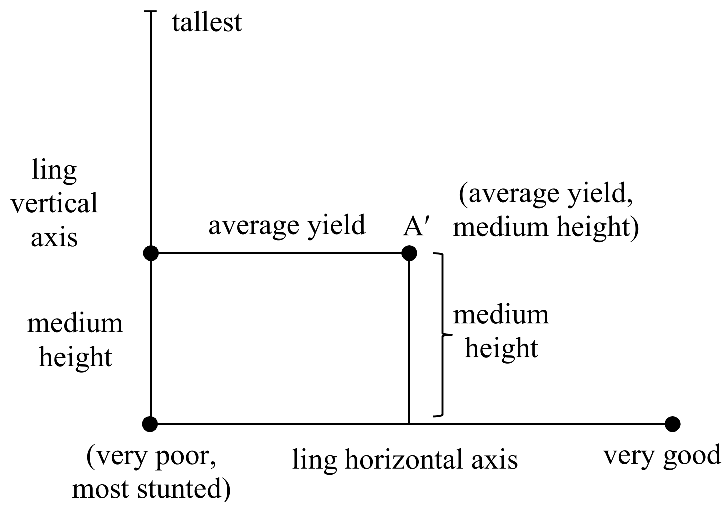


Figure 1.8

Consider the ling pair of points given by

A′ (medium height, average yield).

We map A on the ling plane as follows.

However if we take along the ling horizontal axis the ling variable yield and that of the ling variable height of plants along the ling vertical axis, we obtain the following plane.

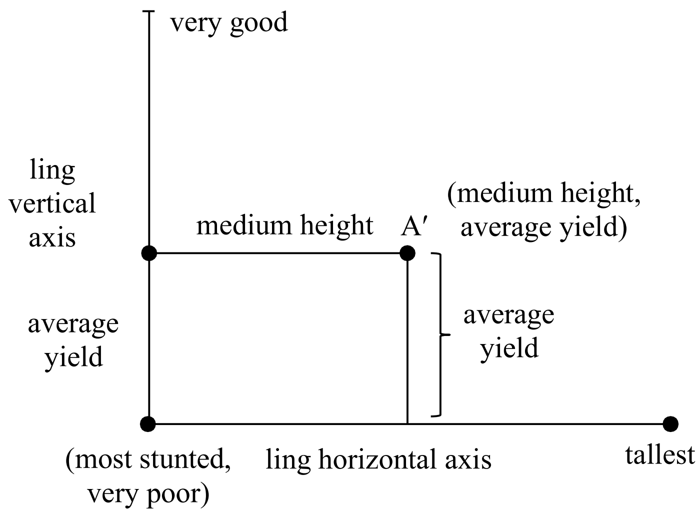


Figure 1.9

We give yet another example of a ling plane.

Example 1.20. Let us consider the ling variable performance aspects of students in the class room and the capabilities of the teacher in making students perform well/ill is taken as the ling variable associated with the teacher.

The ling set or continuum S associated with ling variable performance of students is given by S = [worst, very good].

The ling set or continuum R associated with the ling variable teachers capabilities is given by R = [very bad, good].

Now the ling plane with the ling set S as its ling horizontal axis and with the ling set R as its ling vertical axis we have the following figure.

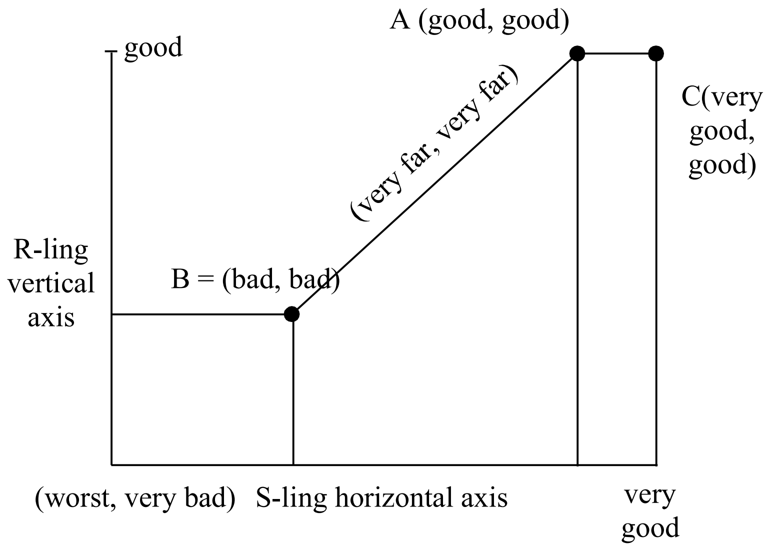


Figure 1.10

(good, good) = A be the ling pair which states if the teacher is good the student is also good.

Let B = (bad, bad) be the ling pair which states a bad student implies the teacher is bad.

We represent the two ling point pairs A and B on the ling plane.

The ling distance between the ling pairs A and B is given by (very far, very far).

Further we see when the teacher is good she / he can produce good to very good students.

Now we can also have the ling plane with the ling horizontal axis as performance of teacher and ling vertical axis as performance of students. We have the following figure.

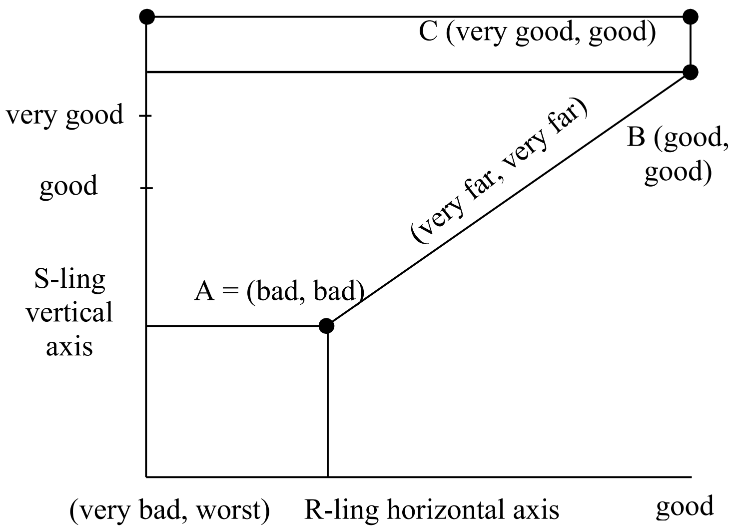


Figure 1.11

So to every pair of related ling variables we have two ling planes. The origin in both the cases are different or interchanged.

Now we have defined real planes and ling planes are different from semi ling planes. For the ling planes or semi ling planes to exist it is mandatory the ling set associated with the ling variables mush yield a ling interval / continuum which is totally ordered set. Otherwise the ling plane or semi ling plane cannot exist. Another fact is that the origin vary with the ling variables.

Now we describe ling points on ling planes and semi ling planes.

Example 1.21. Let us study the amount rain of a particular country on rainy season for 15 days. The ling set / continuum S associated with the ling variable measuring rain is given by the interval [very scattered, very heavy]. The real variable is number of days [1, 15]. Taking [1, 15] as the real vertical axis and S as the ling horizontal axis we have the following semi ling plane.

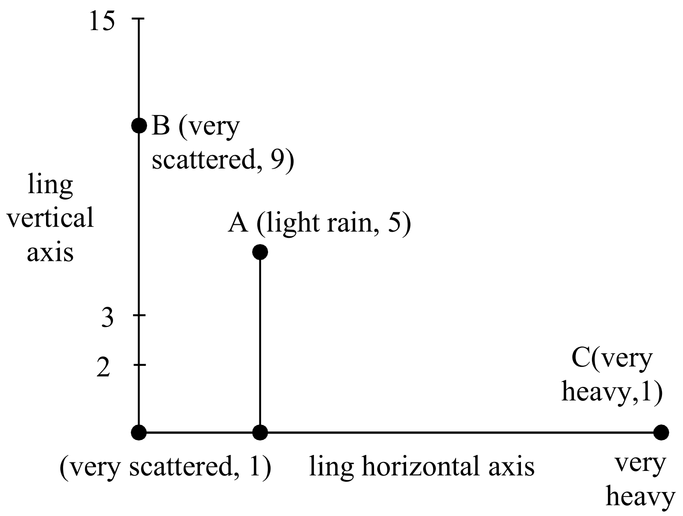


Figure 1.12

On 5th day the rain was very light, so A (light rain, 5).

On 9th day the rain was very scattered given by the semi ling pair B(very scattered, 9) represented in the semi ling plane given in figure 1.12.

On first day the rain way very heavy so C(very heavy, 1) is mapped or represented in the semi ling plane given in figure.

This is the way semi ling pairs are represented in the semi ling plane given in figure 1.12.

Now having seen how semi ling points are represented in the semi ling plane we proceed onto describe representation of points on ling planes.

We will illustrate this situation by an example.

Example 1.22. Let us study the temperature as well as rainfall on certain days.

Thus the two ling variables under consideration are temperature and rainfall.

Let then corresponding ling sets be [lowest, highest] and [scattered, very heavy] respectively.

We can have the ling plane by taking rainfall as the ling horizontal axis and temperature on the ling vertical axis, which is given by the following figure 1.13.

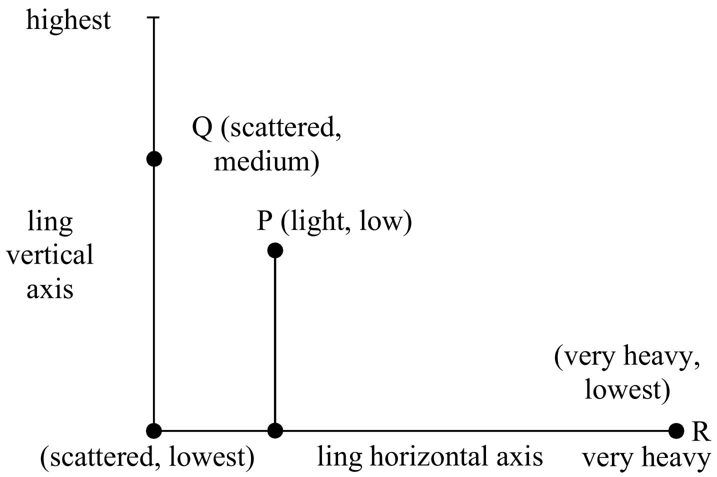


Figure 1.13

We plot the three ling points on the ling plane.

P(light, low), Q(very heavy rain, lowest) and

R (scattered, medium) are represented in the plane.

Thus having seen the representation of the ling pair of points on the ling plane and semi ling points on semi ling planes we proceed onto describe in the next chapter ling distance or semi ling distance between a given pair of ling points or semi ling points respectively.

When we try to work with linguistic variables we see we need to have another linguistic variable which should pave way for same type of indeterminancy in its structure like a decision about the validity of a life prisoner acquisition of being a murderer when he has not done it, thus a legal problem.

However this study needs one more dimension. So it cannot be mapped on the plane. That is why a stared problem is proposed in chapter II of this book under problem session.

We also discuss how a triangle, square or quadrilateral are formed.

Next we proceed onto describe the notion of multi dimensional ling spaces.

In the following we suggest a few problems to the reader. Working with these will make the reader well versed in these ling concepts.

SUGGESTED PROBLEMS

1. Give some four examples of ling variables whose associated ling set/term is of finite order and is not totally ordered.

2. Give an example of ling variables whose associated ling terms / sets are only partially ordered set.

3. Give some examples of ling variables whose ling terms / sets are only totally ordered sets.

4. What are the basic conditions on the ling variables for semi ling planes to exist?

5. Do we have semi ling planes for all ling variables? Justify your claim?

6. What are the special features enjoyed by semi ling planes and how is it different from the classical real plane?

7. Can we say all semi ling planes have the same origin? Substantiate your claim.

8. What is the main difference between the ling plane and usual real planes?

9. Give 3 examples of ling planes.

10. Prove the origin of the ling plane depends on the linguistic variables used by the expert / researcher.

11. Can we prove all ling variables cannot give way to ling planes?

12. What is the speciality enjoyed by the ling variable for it to have ling planes associated with it?

13. Show the method of defining distance between pair of linguistic points is very different from the method of finding distance between pairs points in the real plane.

14. How is the concept used in (13) in case of semi ling planes will be different from real plane?

15. Show by example ling planes are very different from real planes in structure.

16. Is it true if I and J are ling intervals continuums with $I \times J$ the ling plane associated with it then if $I_1 \subseteq I$ and $J_1 \subseteq J$ are ling subintervals, then $I_1 \times J_1$ is a ling subplane of $I \times J$? Figuratively they can be represented by the following figure 1.14.

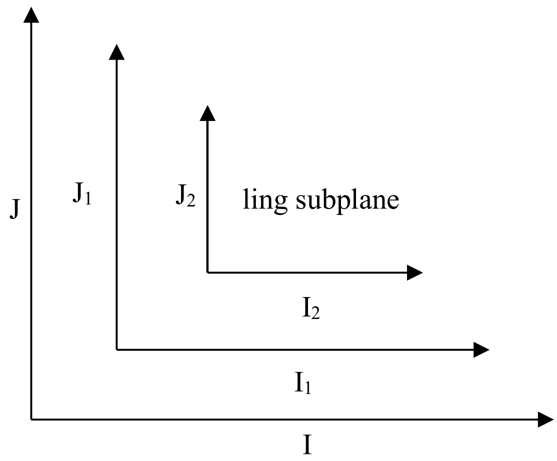


Figure 1.14

So prove or illustrate by an example if I × J is a ling plane then I × J has infinite number of ling subplanes and so on.

17. Consider the two ling sets associated with two variables performance aspects of workers and the industries status as profit or loss or high or low production.

    i) Draw the two ling planes associated with it.

    ii) Represent 4 different pairs of ling points on it.

    iii) Find 3 ling subplanes of this ling plane.

18. Suppose I × J is a ling plane with J and I associated with the vertical ling axis and horizontal ling axis respectively.

    i) Prove $I_1 \times J$ where $I_1 \subseteq I$ is also a ling subplane of I × J.

ii) Prove $I \times J_1$ where $J_1 \subseteq J$ is a ling subplane of $I \times J$.

iii) Prove $I_1 \times J_1$ is also a ling subplane of $I \times J$.

iv) If P_1 and Q_1 are two ling pairs of points in $I \times J$.

a) Find distance between P_1 and Q_1.

b) Prove ling distances are always ling pairs from the set
[nearest, farthest] × [nearest, farthest].

Chapter Two

TWO DIMENSIONAL LINGUISTIC PLANES

Now we in the first chapter introduced the notion of ling points, ling planes and ling lines. Ling lines can be formed only if the ling variable yield a ling set which is a ling continuum. Only if the ling lines are formed then only one can construct ling planes otherwise the question of forming ling planes in impossible.

We will illustrate this situation by some examples.

Example 2.1. Let us consider the ling variable L age of people. The ling set associated with the ling variables is given by S_1 = [youngest, oldest].

Clearly S is a continuum defined as the ling continuum.

If we consider also the ling variable height of people.

The ling set associated with them is

$$S_2 = [\text{shortest, tallest}].$$

If we accept the real continuum for age and height then we have the ling real subinterval of $[-\infty, \infty]$ as $P_1 = [0, 100]$ in years and $P_2 = [1, 7]$ in feet respectively.

We will describe the semi ling plane and ling plane. Both of them are represented only in the first plane.

It is important to keep on record we do not use the whole plane but only the first quadrant.

The semi ling plane using the real subinterval [0, 100] and the ling continuum, [youngest, oldest] is given in figure 2.1.

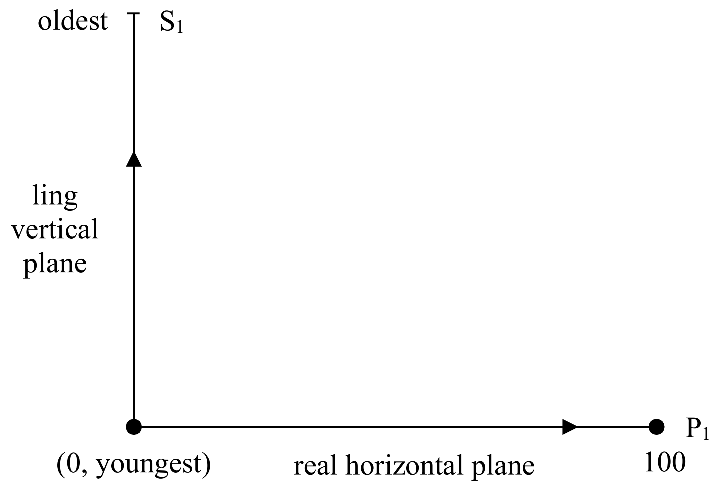


Figure 2.1

or we can have using P_1 and S_1 the semi-ling plane is given in figure 2.2.

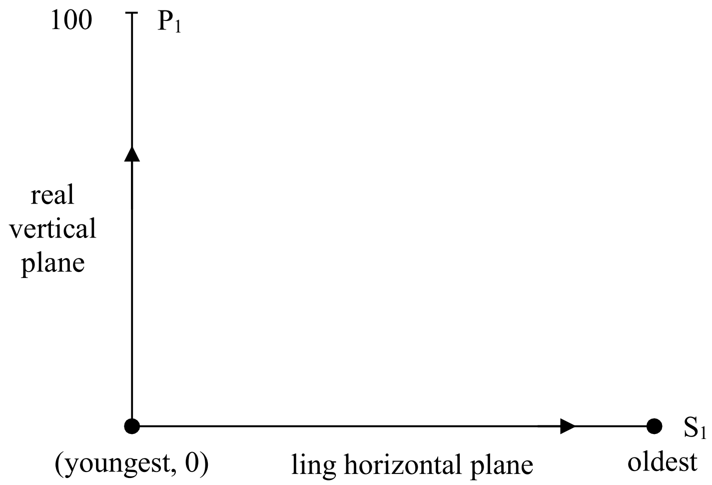


Figure 2.2

Similarly the semi ling plane using P_2 and S_2 is given by the figure 2.3.

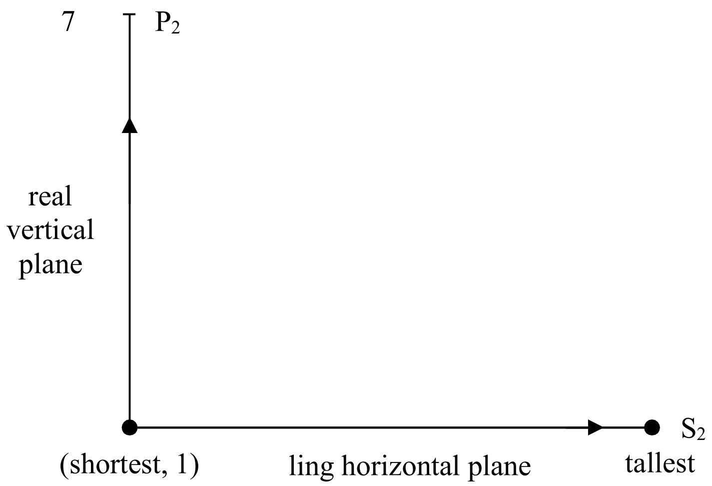


Figure 2.3

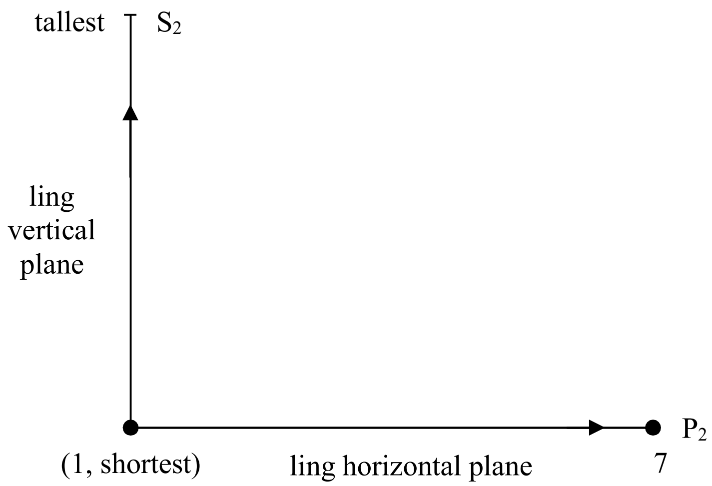


Figure 2.4

All the four semi ling planes. Now we can represent the ling planes using the ling continuum.

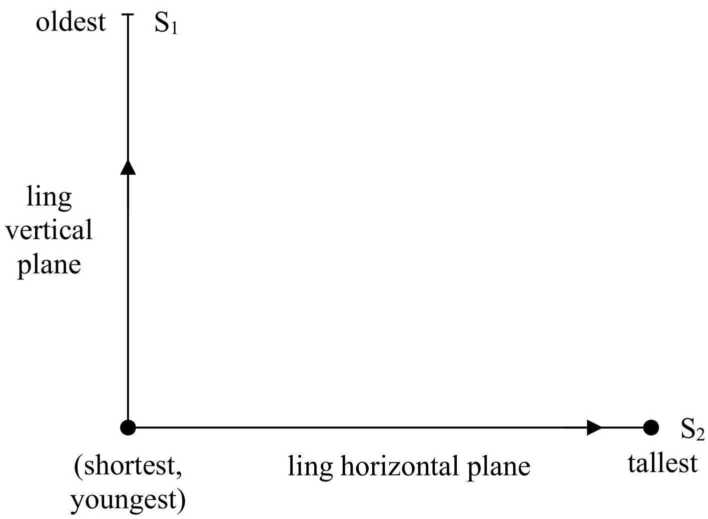


Figure 2.5

This is a ling plane with ling lines S_L and S_2.

Now we proceed onto give yet another representation of the ling plane using S_1 and S_2.

This is given in the following figure 2.6.

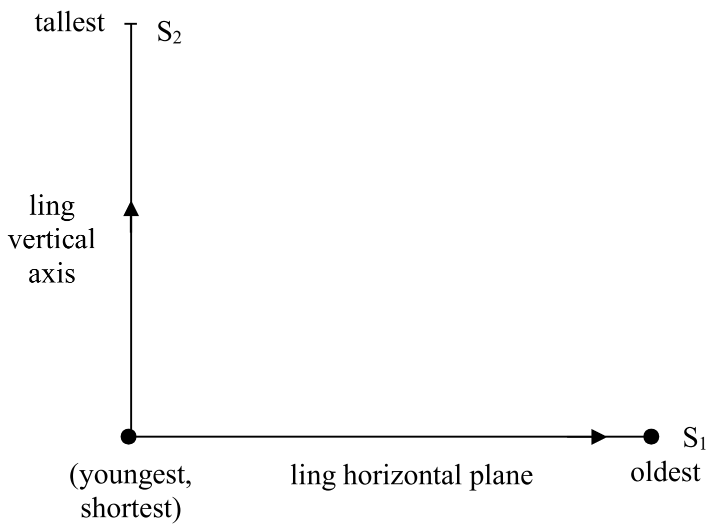


Figure 2.6

The properties of these ling planes and their limitations and their uses will be discussed deeply in the following chapter. However we have discussed their properties.

Further we have also defined and developed higher dimensional ling and semi ling planes in chapter III.

Since this is only two dimensional plane we wanted to make the reader feel at home so we develop here ling planes and semi ling planes just like the real planes.

Further only for the sake a comparison and understanding and to help them find the differences we have introduced by examples ling and semi ling planes.

Now the reader would have observed that these semi ling spaces and ling planes do not have all the four quadrants they all find their place only in the first quadrant.

Now we find a method of getting ling planes similar to the real plane.

For the reader to know the similarities and limitations we for the first time develop semi ling planes and ling planes which has four quadrants.

Example 2.2. Let us suppose the ling variable is the height of people then the ling continuum associated with this ling variable is

$$S_1 = [\text{shortest, tallest}]$$

and that of the real sub interval or real sub continuum associated with the variable height is

$$P_1 = [1, 7] \text{ measured in feet.}$$

The semi ling plane is formed by taking the semi ling origin as

(medium height, 3.5) or (3.5, medium height)

given by the following figures.

The semi ling plane is given by the figure 2.7.

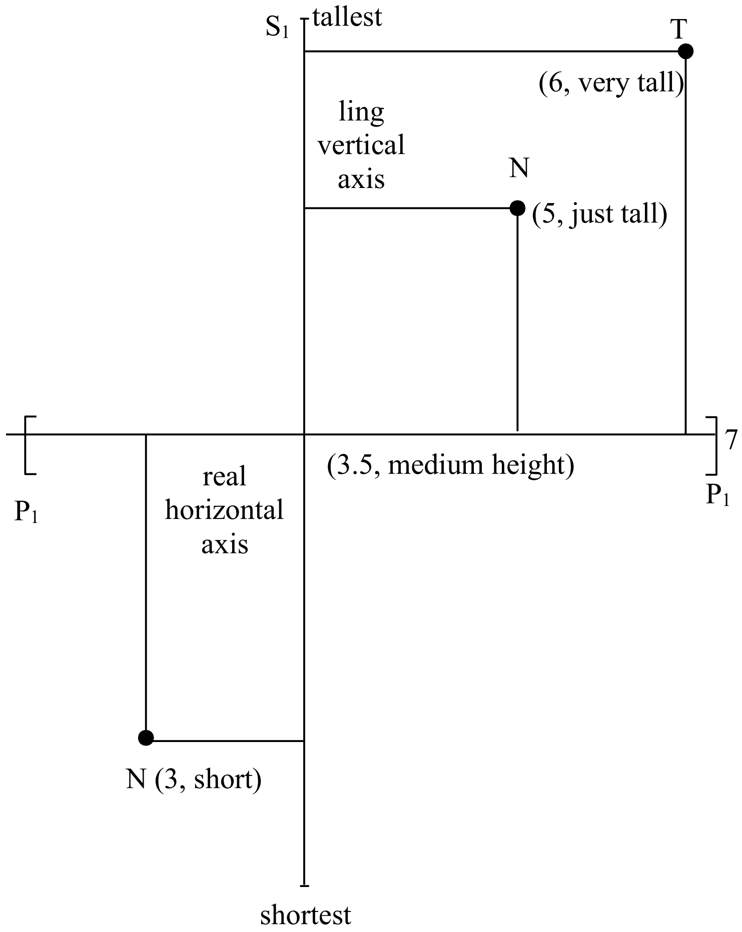


Figure 2.7

We make the following observations.

i) We divide the ling continuum into two with medium height in the center. As far as we are concerned the ling vertical axis below the real horizontal axis gives the usual negative part of the real axis. But here none of the ling values are negative but dividing by centre of the ling continuum we do so. However we see the negation of tallest is shortest and the negation of the shortest is tallest. The negation of very short is very tall and that the negation of very tall is very short and so on.

Now we analyse the real subcontinuum [1, 7] for us the negation of 1 is 7 and that of 7 is 1. But we see the medium height in real can be only 5 and not 3.5.

So there is a drawback in the semi ling plane with four quadrants for we cannot otherwise get the negation of every element. The limitations of this semi ling plane is so.

For algebraist who seek perfection may accept this ling plane.

However medium height is by no means the middle weight in real subcontinuum [1, 7].

Now we can yet have another semi ling plane for the ling variable height which will take for the ling variable the horizontal axis and for the vertical axis we take the real subcontinuum [0, 7].

It is pertinent to observe in case of real plane we have only one but in case of semi ling plane we have two and the vertical and horizontal axis are not the same viz $[-\infty, \infty]$ figure 2.8 give the semi ling plane with real vertical axis and horizontal ling axis.

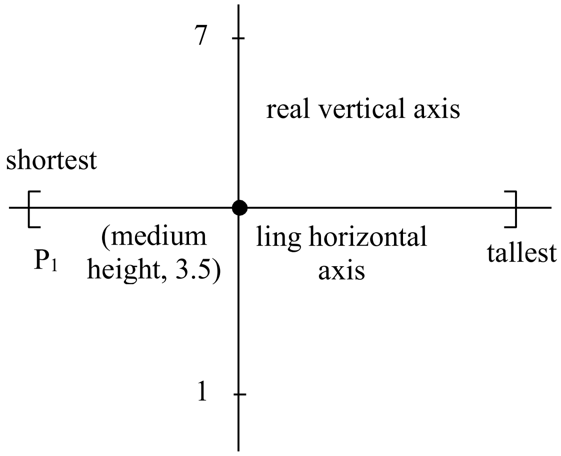


Figure 2.8

Having seen the semi ling planes we will mark some of the semi ling points on them.

Consider the 3 semi ling points

M(3, short), N(5, just tall) and T(6, very tall)

in the semi ling plane given in figure 2.7.

Now we can plot the semi ling points. In fact we can also find the semi ling distances from these points.

The d_l(NT) = d_l ((6, very tall), (5, just tall))

= (1, near).

The semi ling distance

d_l(MT) = d_l((3, short), (6, very tall))

= (3, very far)

We find the semi ling distance

$d_l(MN) = d_l$ ((3, short), (5, just all))

= (2, far).

We make the following observations.

The difference between two points in the real plane and the semi ling plane is that the distance between two points in the real plane is a positive real value whereas in the case of semi ling plane the distance between any two semi ling points is an ordered pair were in figure 2.7 the first component is a positive real number ling between [1, 7] and the second component is the ling value measuring the distance in the ling interval [nearest, farthest].

So we cannot in semi ling planes give a numerical value.

This is the major differences between the real plane and the semi ling plane.

Now we give the ling plane associated with the ling variables height and age of people.

We have already given the ling continuum associated with the ling variable height is [shortest, tallest] = L_1 and the mid value is the medium height.

Now the ling continuum associated with the ling variable age is given by L_2 = [youngest, oldest].

The ling mid value of L_2 is middle age.

The ling plane with four quadrants is given by the following figure 2.9.

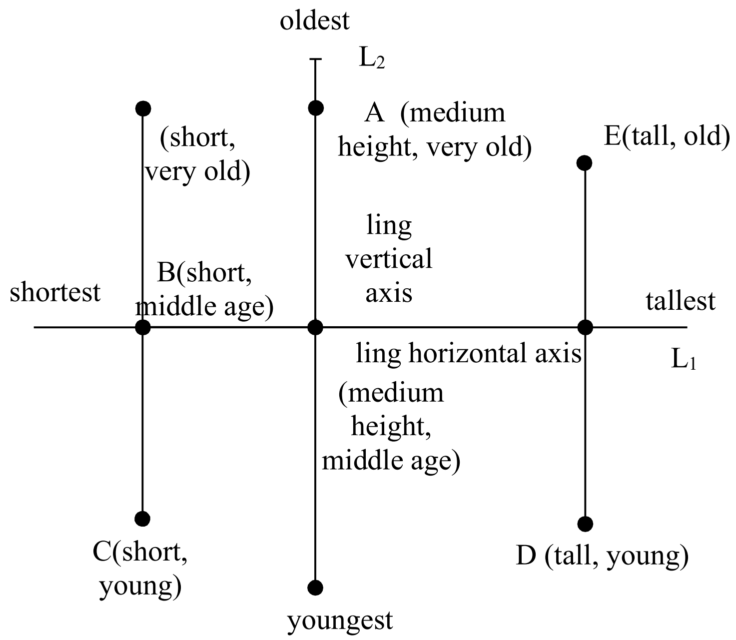


Figure 2.9

Now we have yet another ling plane where ling variable height is taken along the vertical axis and the ling variable age is taken along the horizontal axis.

Only in case of ling planes as the ling origin varies as we interchange the axis we explicitly represent the figure associated with two ling planes.

The ling plane has the ling origin

(middle age, medium height)

is given in the following figure 2.10.

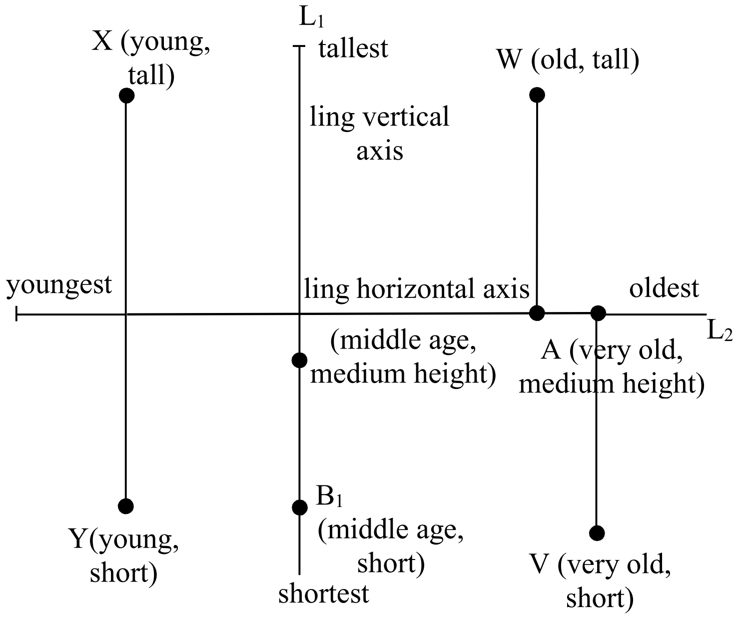


Figure 2.10

Consider the following ling point in the ling plane given by figure 2.9.

A (medium height, very old). This ling point finds its plane on the vertical ling axis.

Now consider the ling point B(short, middle age).

This ling point finds its place in the second quadrant of the ling plane.

Consider the ling point C(short, young). This is marked in the ling plane in the 3rd quadrant.

Now if we take D a ling point D = (tall, young) on the ling plane.

We see it takes its position in the fourth quadrant of the ling plane given in figure 2.9.

Consider the ling point E(tall, old). This ling point E find its plane in the first quadrant.

We have represented these ling points in the ling plane related to the ling origin (medium height, middle age).

Now will see how the changes occur if we take the ling plane with the ling origin (middle age, medium height) and plot the following points in the ling plane given in figure 2.10.

A_1 = (very old, medium height).

The ling pair find its place on the ling horizontal axis adjoining the 1st and fourth quadrant.

Consider the ling pair B_1 = (middle age, short).

This point finds its place on the ling vertical axis adjoining the 3rd and 4th quadrants.

Take the ling point V = (very old, short). This ling pair finds its place in the 4th quadrant given in the ling plane in figure 2.10.

Consider the ling pair W(old, tall).

This ling pair W(old, tall) find its place in the first quadrant in figure 2.10.

Consider the ling pair X(young, tall).

This ling pair find its position in the second quadrant of the ling plane.

Finally consider the ling pair Y (young, short) in the ling plane given in figure 2.10. The ling pair Y finds its place in the 3^{rd} quadrant.

This is the way the ling plane and the points on it are marked.

Now we give yet another example.

Example 2.3. Let us consider the ling variables growth and yield of paddy plants.

For the linguistic variable growth of paddy plants, the ling continuum associated with it is given by

[stunted, very tall] = P_1.

Now for the ling variable yield of the paddy plants we get the associated ling continuum

P_2 = [very poor, very good].

We have two ling planes associated with these ling continuum P_1 and P_2.

The intersection of the ling continuum can be taken as origin [medium growth, average yield] if P_1 is taken as the ling horizontal axis and P_2 as the ling vertical axis.

If on the other hand P_2 is taken as the ling horizontal axis and the P_1 as the ling vertical axis then the ling origin will be [average yield, medium growth].

Thus we have two ling planes associated with P_1 and P_2 which are given in the following.

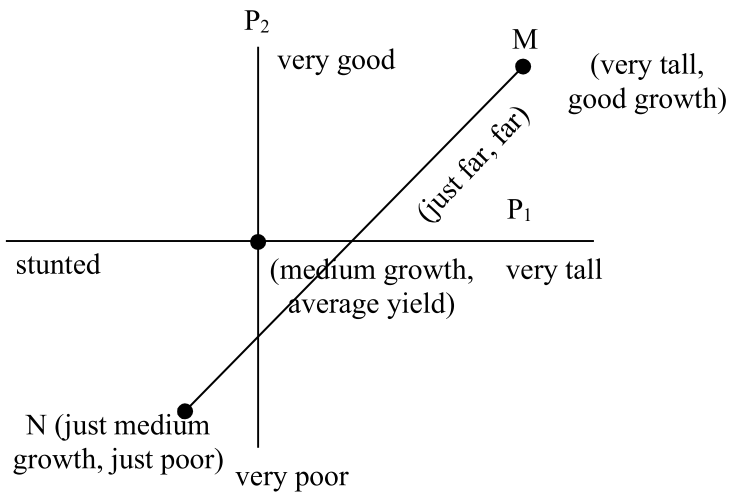


Figure 2.11

The ling plane with ling origin as (average yield, medium growth) is given in figure 2.12.

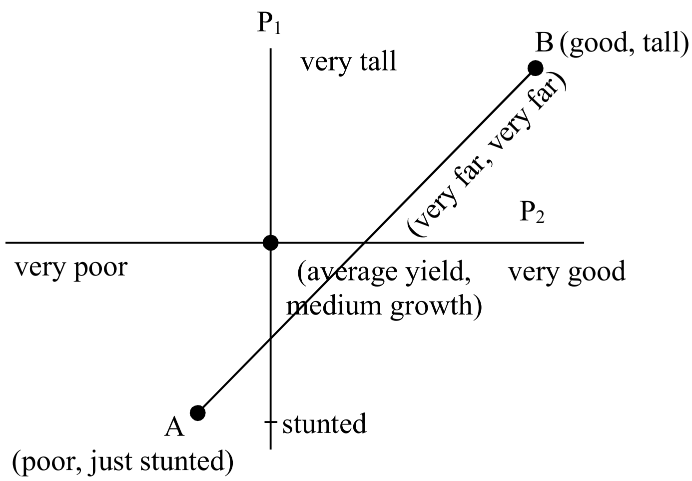


Figure 2.12

The ling pair A (poor, just stunted) finds its place in the 3rd quadrant of the ling plane given in figure 2.12.

The ling pair B (good, tall) finds its position in the first quadrant of the ling plane given in figure 2.12.

The ling distance from A to B is (very far, very far), which is again a ling pair.

In fact the ling terms in measuring the distance finds its entries from the ling continuum [nearest, farthest].

Consider two ling pair M(very tall, good growth),

M finds its place in the first quadrant of the ling plane given in figure 2.11.

Consider the ling pair

N (just medium growth, just poor yield).

The ling pair N finds its place in the 3rd quadrant of the ling plane given in figure 2.11.

The ling distance between M and N is given by (just far, far).

Now having seen examples of ling planes with four quadrant just like real plane we can find distances but they are only ling pair both the terms in these ling pair take its ling values from the ling continuum [nearest, farthest].

However finding areas etc using these ling planes is not possible.

These ling planes help one to give very naturally in ling terms the position of ling points and the ling distance between ling points.

Some of the ling distance are directed.

Further we feel these examples will help us define these concepts abstractly.

First we have define modified ling continuum.

Let us consider the ling variable age of people.

Suppose L = [youngest, oldest] is the ling continuum associated with the ling variable age of people.

The usual ling line represent in all these books are

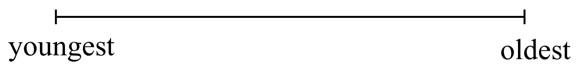


Figure 2.13

This is totally ordered by ≥. The modified ling line or continuum is one which is analogous to the real continuum is given by

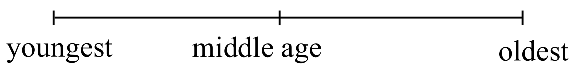


Figure 2.14

We call this as the modified ling continuum. This is also totally ordered.

Now having seen examples of both ling continuum and modified ling continuum we proceed to describe the ling plane and the modified or extended ling plane with an example.

Suppose we consider the ling variable height of people. The ling continuum associated with the ling variable height is given by [shortest, tallest] = P_1.

Now if we consider the ling variable age of people the ling continuum associated with it is given by

$$[\text{youngest, oldest}] = P_2,$$

The ling plane associated with these two ling continuum P_1 and P_2 is given by the following.

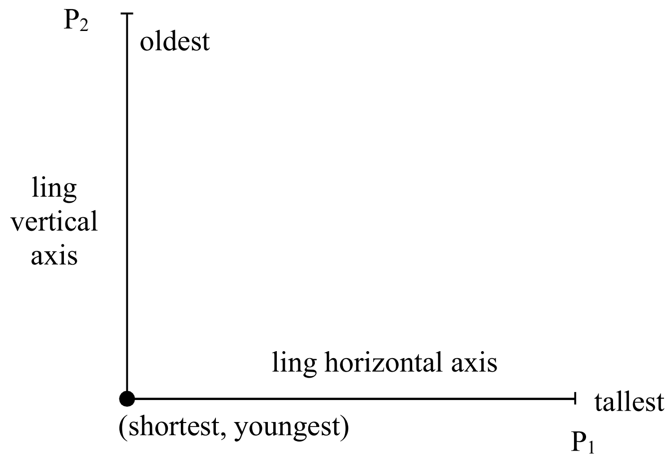
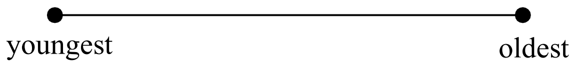


Figure 2.15

Taking the ling continuum [shortest, tallest] along the horizontal axis and the ling continuum [youngest, oldest] along the ling vertical axis we get only the first quadrant with ling origin (shortest, youngest) we get the ling plane described in figure 2.15. We use only the ling line.



and

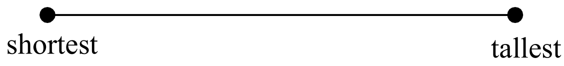


Figure 2.16

Now suppose we want to represent the ling pairs short and middle age that is P(short, middle age) is represented in the following figure 2.17.

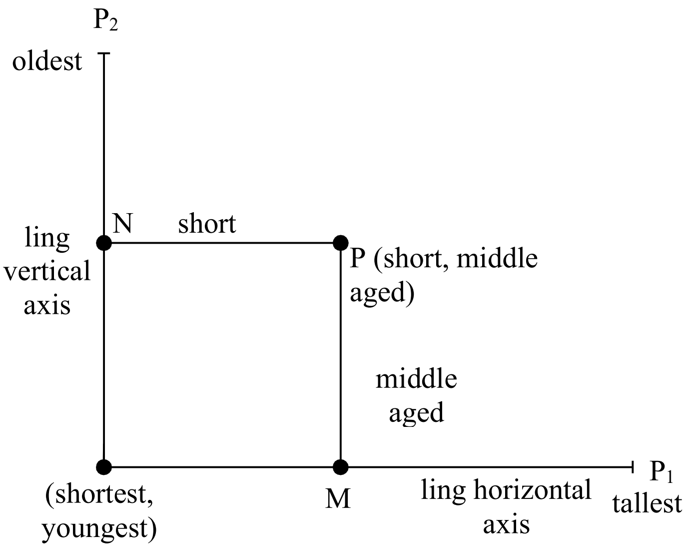


Figure 2.17

If we draw the perpendicular from the ling pair

P (short, middle aged)

as PM perpendicular to the ling horizontal axis and PN perpendicular to the vertical axis.

PM represents middle aged PN represents short.

Both of them are only in the first quadrant as we have only one quadrant in the ling plane formed by the ling continuums P_1 and P_2.

Now by considering the modified linguistic lines given in the following figure 2.18.

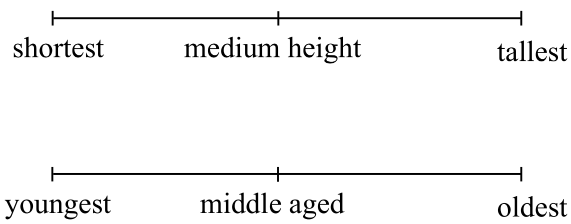


Figure 2.18

We now using these modified ling lines form the modified ling plane which has the four quadrant and the ling origin in this case is (medium height, middle aged).

The modified ling plane is given in Figure 2.19.

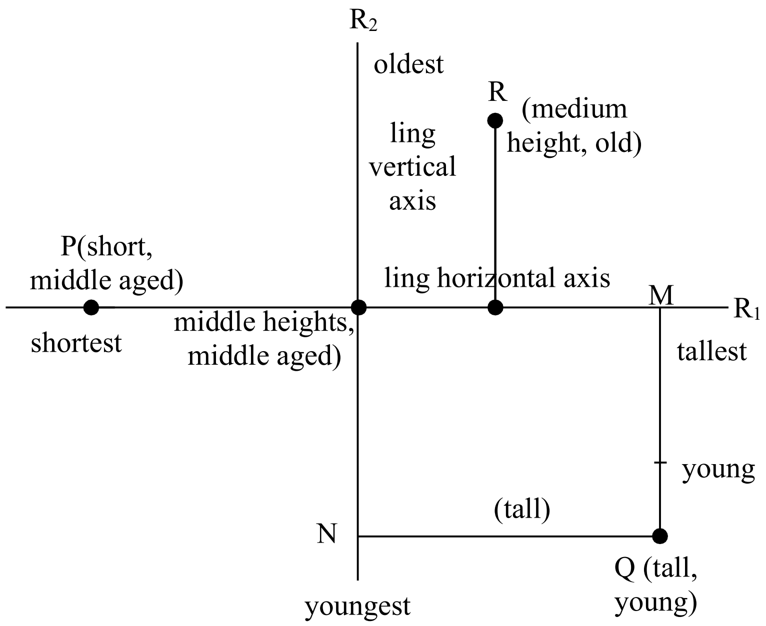


Figure 2.19

The ling point P(short, middle aged) is in the extended plane is given in figure 2.17.

The ling point P lies in the 2^{nd} quadrant on the ling horizontal axis.

Thus once we use modified ling plane the position of the ling pairs also are very differently placed

Now we can take Q (tall, young) in the modified ling plane which is in the 4^{th} quadrant and let QN be the perpendicular from Q to the vertical axis and QM be the perpendicular from Q to the horizontal axis QN corresponds to the height tall and QM the ling value young.

We see P and Q are not comparable and we also cannot find the ling distance between P and Q as the middle aged man who is short becoming tall neither the young person who is tall can become short is against nature or logic or reasoning.

So there exist no ling distance between P and Q as comparison of them is impossible.

Consider the ling pair R (medium height, old). We will represent R on this modified ling plane.

R finds its place in the 1st quadrant, however we cannot connect or find the distance between R and Q or R and P.

For a middle aged man will by no means grow to become tall as by the time he reaches his middle age he becomes fully grown. He can only reach his to old agein due course of time. So unlike in real plane if we have any 3 points on the real plane we can form a triangle whether related or not. But in case of ling pairs (points such relation or connecting them is not possible). Only when there is logical reasoning we can connect them.

First we provide the limitations of modified semi ling planes and how they are constructed first by some examples.

Example 2.4 Let us consider the ling variable age of people. The ling continuum associated with it is given by

$$P_1 = [\text{youngest, oldest}].$$

Consider the real variable age of people the real subcontinuum or subinterval associated with it is given by $Q_1 = [0,100]$. The representation of them as lines is given by figure 2.20.

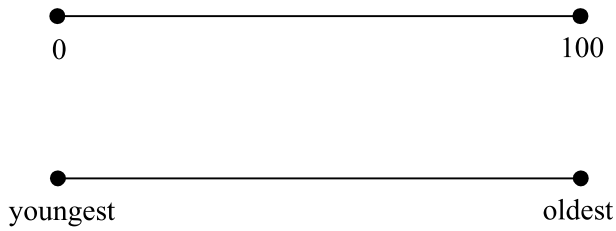
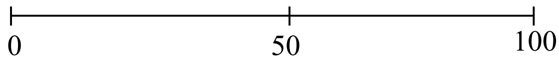


Figure 2.20

The representation as modified lines are given by the following figure 2.21



and

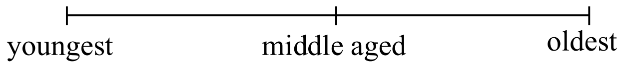


Figure 2.21

Now we give the corresponding (1, 1) semi ling plane modified (1, 1) semi ling plane.

The figure is given by the following figures 2.22 & 2.23.

We also see the (1, 1) semi ling plane has only one quadrant whereas the modified (1, –1) semi ling plane has four quadrants.

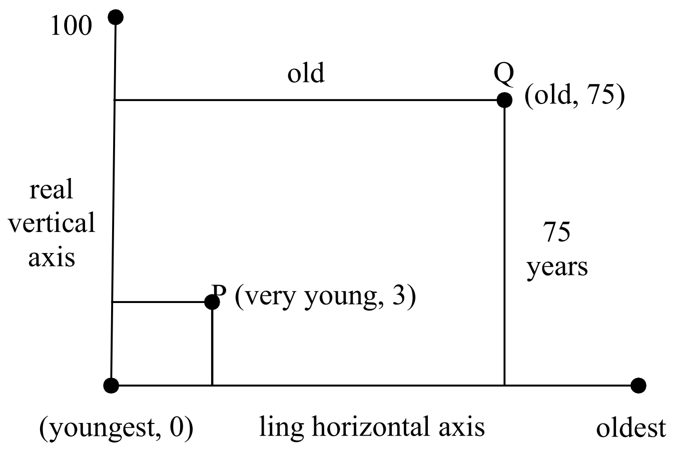


Figure 2.22

The (1, 1) semi ling origin of the (1, 1) semi ling plane is (youngest, 0).

Now we give the modified (1, –1) semi ling plane in the following Figure 2.23.

The origin for the modified (1, –1) semi ling plane given in Figure 2.23 is (middle age, 50).

Consider the (1 – 1) semi ling pair P(very young, 3) represented in the (1 – 1) semi ling plane as well as in the modified (1, 1) semi ling plane.

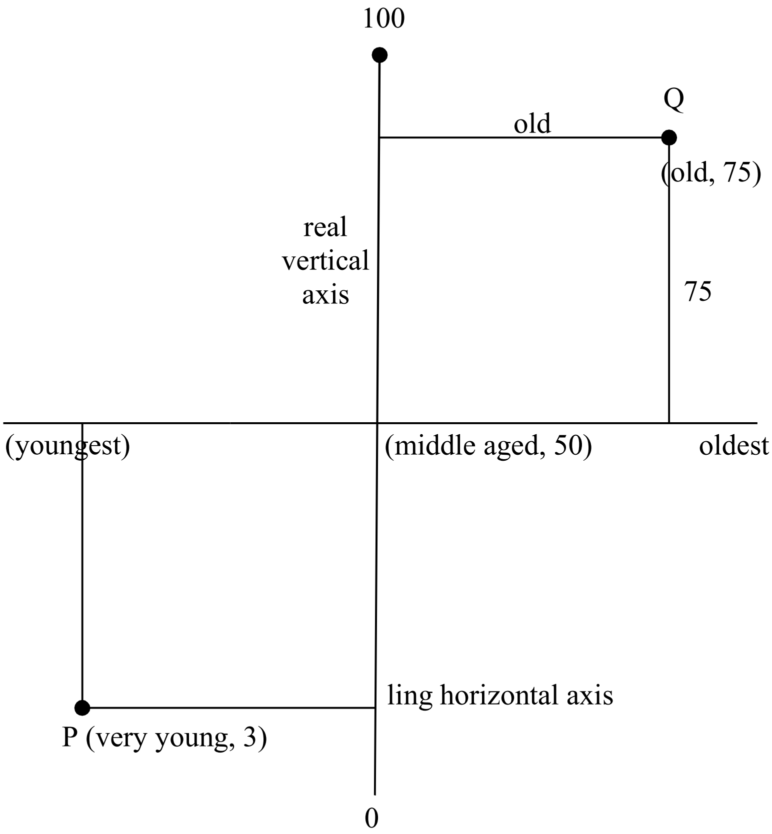


Figure 2.23

In the (1, 1) semi ling plane the semi ling pair P is very close to the ling origin, whereas in the modified (1, –1) semi ling plane the semi ling pair falls in the 3rd quadrant.

Consider the semi ling pairs Q(old, 75) we fix this semi ling pair in both the (1, 1) semi ling plane and the modified (1, –1) semi ling plane.

The semi ling pair Q (old, 75) finds its position in the first quadrant.

This is the way semi ling planes and modified semi ling planes are drawn and represented in a unique way.

Now having seen we will discuss when one can find distances between semi ling pairs in (1, 1) semi ling plane and distances between modified (1, –11) semi ling pairs.

We now consider the (1, 1) semi ling pairs in

$P_1 \times Q_1 =$

$\{(p, q) / p \in P_1 = [\text{youngest, oldest}] \text{ and } q \in [0, 100] = Q_1\}$

Clearly $P_1 \times Q_1$ is a totally ordered semi ling pair.

Clearly $P_1 \times Q_1$ is a partially ordered set

if (p, q) and (r, s) $\times P_1 \times Q_1$ (p, r $\in P_1$ and q, s $\in Q_1$).

We say

$$(p, q) \geq (r, s) \text{ if and only if } p \geq r \text{ and } q \geq s.$$

Once $p > r$ automatically $q \geq s$ happens.

In this case we can say $P_1 \times Q_1$ is a totally ordered (1, 1) semi ling set under $\geq$.

We can find the ling semi distance only in case of a pair of semi ling points which are always comparable.

Here take the semi ling points

(young, 10) = x and y = (middle aged, 48) = y.

Clearly y ≥ x. The semi ling distance

$d_l(x, y) = d_l$((young, 10), (middle age, 48)) = (far, far).

The

$$x = (\text{young}, 0) \xleftarrow{(\text{far, far})} y\ (\text{middle aged}, 48)$$

as y ≥ x.

Figure 2.24

Consider the two semi ling points

a = (middle aged, 40) and b = (old, 75) $\times P_1 \times Q_1$. Cleary b ≥ a.

Since in this case the both the ling variable as well as the real variable pertain to age and once the ling term old is taken we have to mark only them in the age group [60 – 75] or so.

So in this case $P_1 \times Q_1$ happens to be a totally ordered semi ling pair.

Next we consider the semi ling set S × R where S corresponds to the ling variable height of persons and R corresponds to the real variable age of the people in the real subcontinuum [0, 100].

We give both the (1, 1) semi ling plane as well as modified (1 – 1) semi ling plane.

The ling continuum associated with the ling variable height is given by [shortest, tallest] = 5 and that of the real variable age we have real sub continuum [0, 100] = R.

We first give the (1, 1) semi ling plane with the horizontal axis as the ling interval is taken pertaining to height and the vertical axis the age R = [0, 100].

The figure is given in the following.

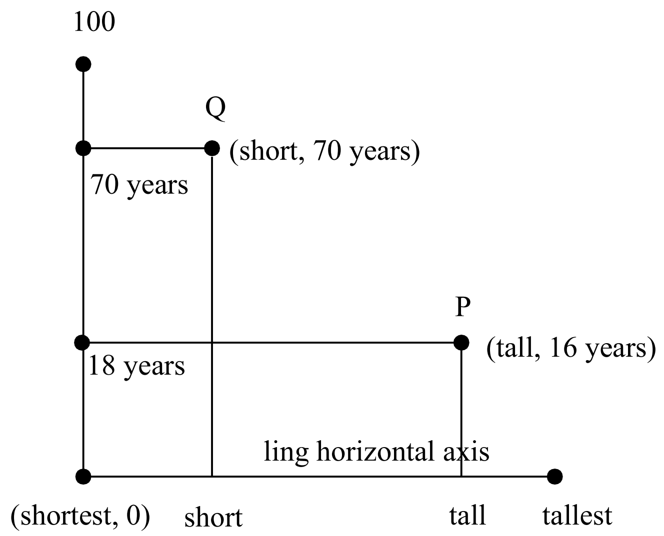


Figure 2.25

The semi ling origin of this (1 – 1) semi ling plane is (shortest, 0).

The modified (1 – 1) semi ling plane for the ling variable height and the real variable age is given by the following figure

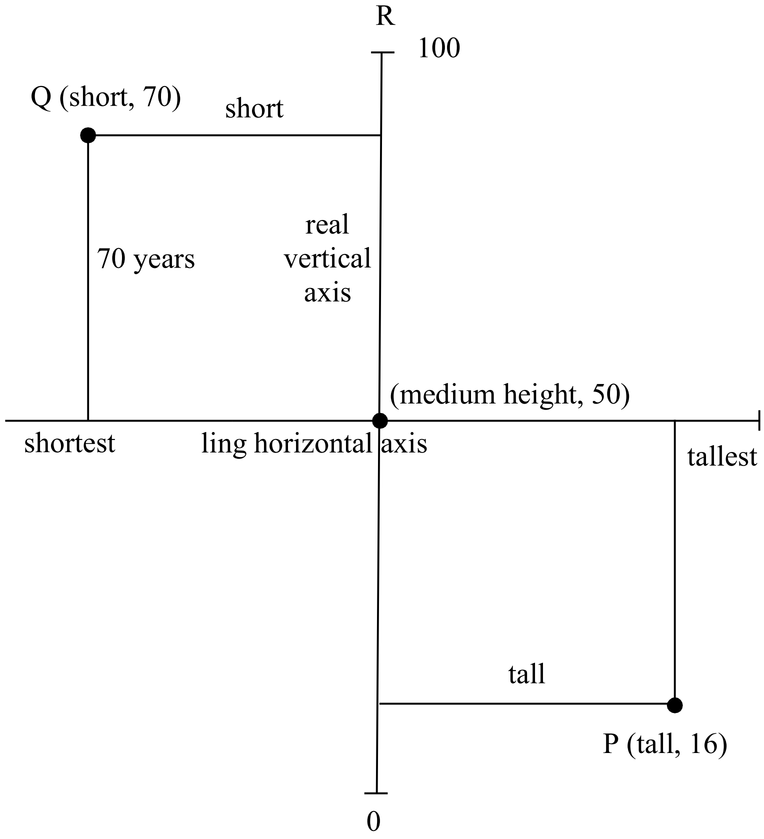


Figure 2.26

The modified real subcontinuum and the ling continuum is as follows

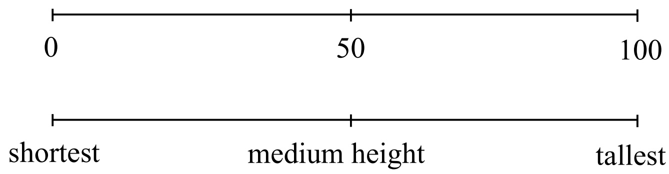


Figure 2.27

So the modified semi ling origins is given by

(medium height, 50).

Consider the (1, 1) semi ling pairs

P = (tall, 16) and Q = (short, 70 years),

in both the (1 1) semi ling plane as well as in the modified (1 –1) – semi ling plane.

These semi ling points are represented in the two planes given in the figure 2.25 and 2.26.

Clearly we see P and Q cannot be compared so we cannot give the semi ling distance between P and Q.

Now we give the representation of P and Q on the modified (1 – 1) semi ling plane given in figure 2.27. The (1 – 1) semi ling point finds its place in the forth semi ling quadrant Q the (1 – 1) semi ling point find its place in the 2nd quadrant.

Clearly P and Q are not comparable or orderable as (tall, 16) and (short, 70)

tall > short and

16 < 70 so no comparison is possible. Thus in this case.

$S \times R = \{(s, r) / s \in S \text{ and } r \in R\}$ not a totally ordered set as P and $Q \times S \times R$ but they are not comparable.

Take x = (tall, 80 years) and y = (short, 16 years) in $S \times R$.

We see x ≥ y so x and y are comparable hence S × R is only a partially ordered set.

Thus finding the semi ling distance for ling pairs in S × R is possible only if they are comparable otherwise no semi ling distance exists. The semi ling distance

$$d_l(x, y) = d_l((\text{short, 16 years}), (\text{tall, 80 years}))$$

$$= (\text{very far, very far}).$$

In this way we are not always in a position to find semi ling distances between any pair of semi ling points. It depends also on the semi ling plane under consideration.

Now suppose we consider the (1, 1) semi ling plane using the real variable height and the ling variable height of people.

The real subinterval of sub continuum for height is given by [1, 7] in feet and that of the ling variable for height of people is given in the ling continuum [shortest, tallest].

The ling line of the ling continuum M = [shortest, tallest] and the real line of the real continuum of N = [1, 7] is given in the following figure.

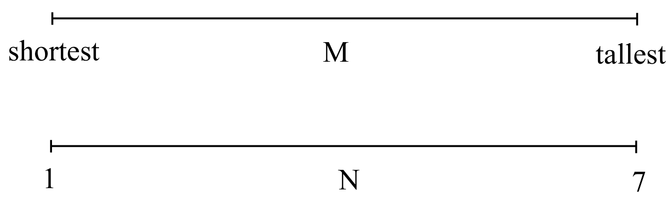


Figure 2.28

The (1, 1) semi ling plane with (shortest, 1) as origin is given by the following figure 2.28.

It is clear horizontal axis is the ling axis of M and that the real subcontinuum is taken as the vertical axis.

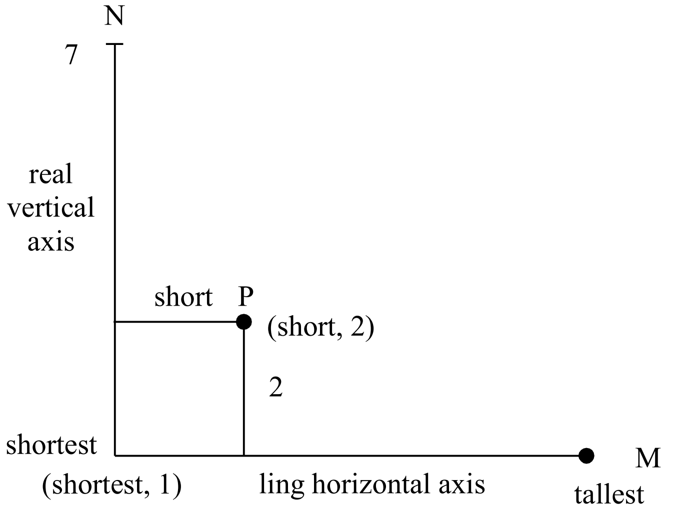


Figure 2.29

Suppose P (short, 2) is the (1 – 1) semi ling pair. P is represented in the figure 2.31.

Now we give the modified ling continuum and the modified real subcontinuum in the following.

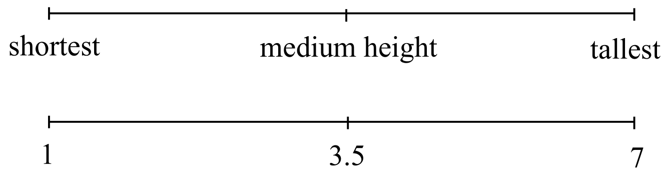


Figure 2.30

Now we for the modified (1, 1) semi ling plane take the origin as (medium height, 3.5) and the horizontal axis is the ling axis and that of the vertical is the real subcontinuum. The figure is as follows.

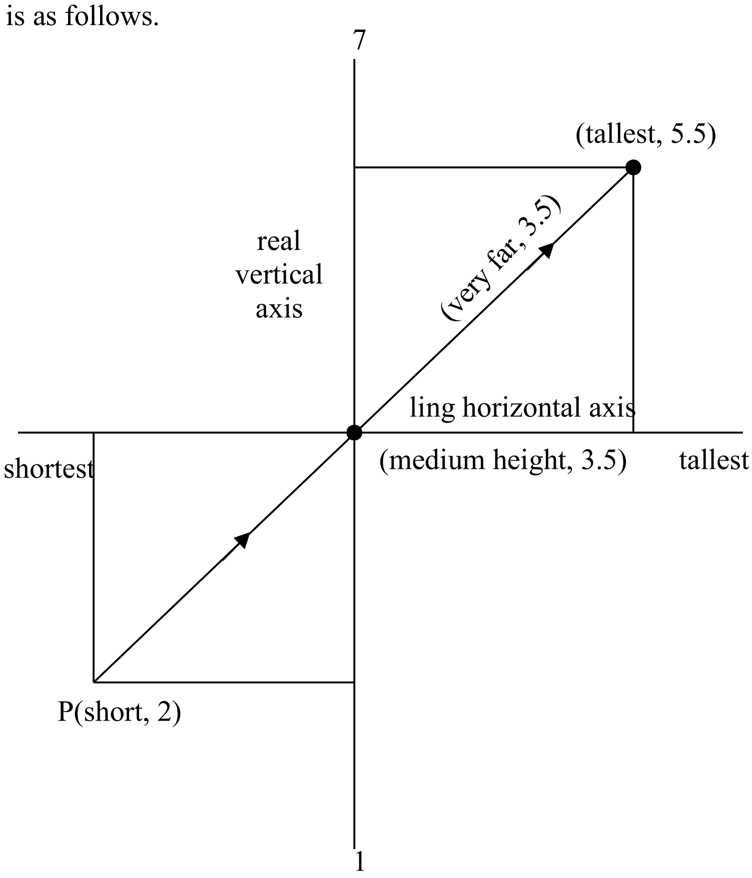


Figure 2.31

We represent the semi ling point P (short, 2) in this modified semi ling plane. This semi ling point lies on the 3^{rd} quadrant.

Consider the semi ling point Q(tall, 5.5), represent it on the semi ling plane. This semi ling point finds its place in the 1^{st} quadrant.

We can find the semi ling distance from P to Q is (very far, 3.5). However there is no meaning in finding the semi ling distance from Q to P for such things cannot occur.

With these limitations one cannot find area of triangles or quadrilaterals or so on which the classical geometry occur. As these modified ling planes and modified semi ling planes work more on the logic for ling continuum provides for every x a ling term there is a negation for it.

However for the midterm we do not have the negation. For it is like 0 of the real plane but it is the middle value of the ling continuum. We have discussed about these drawbacks in the general case of multidimensional ling spaces.

However we have not discussed modified ling planes or modified semi ling planes for the classical analytical geometry cannot be developed and also realizing analytical geometry as one which makes geometry more an algebraic thing cannot be very useful in our opinion for the linguistic set up. But this needs to be researched more.

In the following we suggest some problems for the reader to solve so that they can follow these new concepts. The starred problems are difficult to be solved.

SUGGESTED PROBLEM

1. Compare the semi ling spaces with only one quadrant with semi ling spaces with four quadrants.
2. What is the modified form of the ling continuum I = [worst, best]? Represent mid ling point of I.
3. Which of the ling continuum is easy to work modified ling continuum or the ling continuum? Justify your claim.
4. Give examples of modified ling continuums and represent the mid point or center point of them.
5. Prove all ling variables cannot give way to modified ling continuum.
6. Give examples of (1, 1) semi ling planes.
7. Bring out the difference between classical real planes and the (1,1) semi ling planes.
8. What are ling planes?
   a) How are they different from (1, 1) semi ling planes?
   b) Give examples of ling planes and compare them with real planes.
9. Give examples of modified ling lines related with some ling variable.
10. Give examples of modified real line of subcontinuums. How are these different from the classical real line?

11. Find the (1, 1) semi ling planes related with the ling variable age of people. Suppose the ling continuum be [youngest, oldest] and the reals sub continuum [0, 100].

    a) Is the (1, 1) semi ling space different from modified (1, –1) semi ling space?

    b) Give the ling planes relating the ling variables age and weight of people.

    c) Obtain the modified ling planes with ling continuum [youngest, oldest] and
    [lowest, highest].

    d) How are the ling planes mentioned in (b) and (c) different?

12. Suppose we have (1, 1) semi ling plane with the ling continuum R_1 = [youngest, oldest] for the ling variable age and [2, 80] = R_2 the real sub continuum related with weight of people.

    i) Is the semi ling set $R_1 \times R_2$ be a totally ordered set or a partially ordered set?

    ii) Can we find semi ling distances between any two semi ling pairs in $R_1 \times R_2$?

    iii) Suppose instead of taking R_2 as a real subcontinuum if we take T_1 as the ling continuum [lowest, highest]; what are the conclusions derived?

a) Is $R_l \times T_l$ a ling plane in which any ling pair is related?

b) Is $R_l \times T_l$ only a partially ordered set?

c) Give the figure of $R_l \times T_l$ and find the ling distance any two point which has ling distance defined.

d) Let x = [old, medium] and

y = [middle age, heavy weight] $\in R_l \times S_l$.

Does d_l (x,y) exist? Substantiate your claim.

e) What is the ling origin of the modified ling plane?

13. Enlist the special features enjoyed by modified ling planes.

14. Compare the real plane with the modified ling plane!

15. What are the specialties enjoyed by the ling origin of the modified ling plane?

16. Show that modified ling planes cannot be constructed in general for all ling variables.

17. Give an example of a modified ling plane in which any two ling points (pairs) are comparable.

18. Which of them, modified ling planes or real planes enjoy richer and meaningful structure?

19. Give some applications of modified ling planes in real world situations/problems.

20. Prove or disprove classical be two dimension space using analytical geometry concepts cannot be applied for any real world problems.

21*. Which of the planes ling planes or modified ling planes enjoy higher adaptability?

22*. Construct a semi ling plane using indeterminate as a ling. variable and [0, 1] the fuzzy interval as the real variable.

    a) Obtain both the planes.

    b) Construct a variable V, which is indeterminate and make a ling. variable plane with the ling. variable as one axis and the indeterminate values as the other axis.

23. Let us consider the ling variables performance of employees of a company, and the ling variable production of the company is given by S_1 = [poor, high], the ling continuum. Let the ling continuum [worst, best] = S_2 be associated with the ling variable performance of the employees.

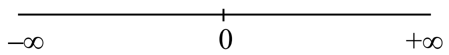
    i) Give the ling plane using $S_1 \times S_2$.
    ii) What is the origin of the modified ling plane?
    iii) Can we say $S_1 \times S_2$ is a totally ordered set?
    iv) Will $S_1 \times S_2$ be a partially ordered set?

Chapter Three

LINGUISTIC MULTI-DIMENSIONAL SPACES

Now as this is a very basic book on linguistic coordinate geometry we try our level best to recall the very basics of analytical geometry or coordinate geometry.

Everyone is well aware of the fact one dimension (plane) is a line; real line is a continuum from (–∞, ∞).



Real line

Figure 3.1

The analogue of real line is ling line given by

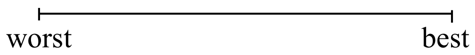


Figure 3.2

where worst is the least element and best is the greatest element in the study of performance of students in the class room (in these case), the ling. line or continuum will vary from ling. variables. Several such ling. continuum exists.

The following factors are very very important if one is interested in studying the basic of ling analytical geometry.

i) The real line or the continuum; $(-\infty, \infty)$ the interval is unique. We have in the numerical system one and only one real line, one rational line and one integer line and nothing more.

But we have several number of ling lines or ling space of dimension one.

Consider the ling variable age of the people then the ling set associated with it is [youngest, oldest].

The one dimensional ling space or the ling line associated with this is given in figure 3.3.

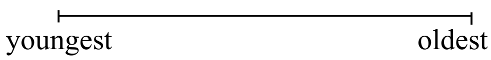


Figure 3.3

Suppose the ling variable is the height of people.

The ling set / interval / continuum associated with this ling variable is height [shortest, tallest].

The ling line associated with it is

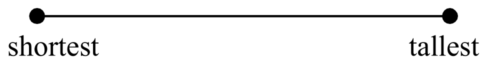


Figure 3.4

The ling set / interval / continuum associated with the ling variable weight of people is given by the ling continuum [lightest, heaviest].

The ling line associated with it is

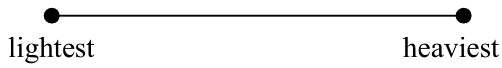


Figure 3.5

Now we give the ling set / interval associated with the ling variable yield of plants is given by the following ling continuum [low, very high] given by the line

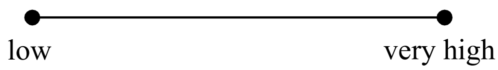


Figure 3.6

Now we see there are many ling lines or ling one dimensional space unlike the real line which is unique.

We make the following observations.

All ling variables in general do not have its ling set to be a ling interval or continuum. So we cannot say every ling variable can be associated with a ling line.

We now prove the following.

i) Every ling subinterval of the ling interval will also be a ling line which we define as ling subline. This is in keeping with the real line.

We will illustrate this by some examples.

The ling intervals [bad, good] $\subseteq$ [worst, best] is called as a ling subinterval of [worst, best], which can also be called as a ling sub line.

We can have several such ling subintervals of [worst, best].

The representation of the ling subinterval

S_1 = [bad, good] $\subseteq$ [worst, best] is represented by the following figure.

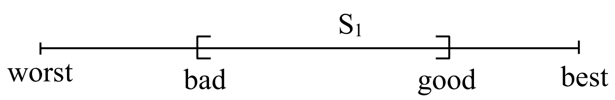


Figure 3.7

Consider the ling subinterval

S_2 = [fair, good] $\subseteq$ [worst, best] S_2 is a ling subinterval. The figure associated with S_2 is as follows.

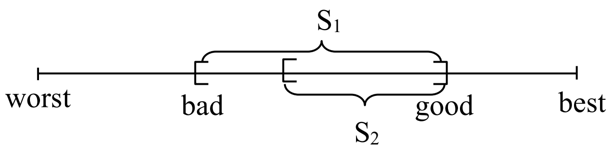


Figure 3.8

We see

S_2 = [fair, good] $\subseteq$ [bad, good] = $S_1 \subseteq$ [worst, best].

Further $S_1 \cap S_2$ = [fair, good] $\cap$ [bad, good]

= [fair, good] = S_2

This is a ling subinterval of a special form.

Consider the ling subinterval

S_3 = [bad, just fair] $\subseteq$ [worst, best].

The figure associated with the ling subinterval S_3

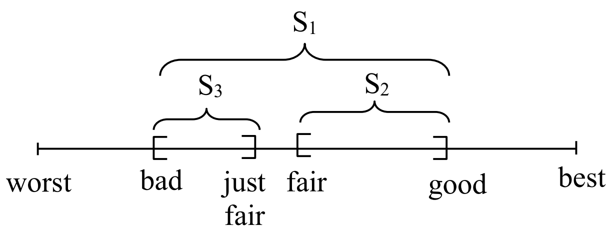


Figure 3.9

Clearly $S_3 \subseteq S_1 \subseteq$ [worst, best] … (1)

$S_2 \subseteq S_1 \subseteq$ [worst, best] …(2)

However $S_3 \cap S_2 = \{\phi\}$ empty ling interval

$S_1 \cap S_2 = S_2$ and

$S_1 \cap S_3 = S_3$.

But $S_2 \cup S_3 \neq S_1$, however $S_1 \cup S_2 = S_1$ and $S_1 \cup S_3 = S_1$. These results hold good because of (1) and (2).

Now consider the ling subintervals S_4 = [very bad, fair] and S_5 = [bad, just fair] contained in [worst, best].

The figure 3.9 represents the ling subintervals of

[worst, best]

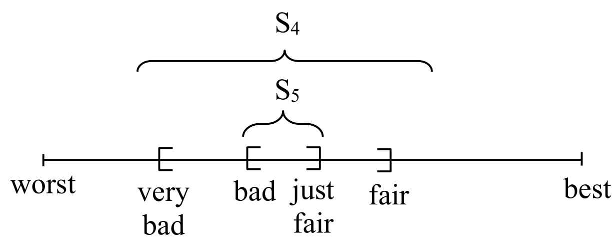


Figure 3.10

Clearly $S_5 \subseteq S_4$ and $S_5 \cap S_4 = S_5$; so S_5 is also a ling subinterval of $S_4 \subseteq$ [worst, best].

Consider the ling subintervals S_6 = [very bad, very good] and S_7 = [good, best] be two ling subintervals of [worst, best].

Figure 3.11 represents the position of the ling subintervals S_6 and S_7 on [worst, best].

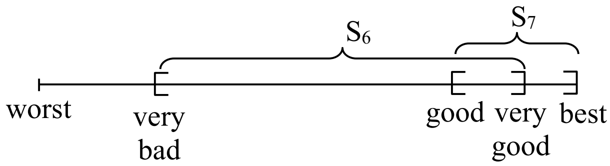


Figure 3.11

We find out $S_6 \cap S_7$ = [good, very good] and

$S_6 \cup S_7$ = [very bad, very good]

= [very bad, best]

$S_6 \cap S_7$ = [good, very good]

$\subseteq S_6$

as well as $S_6 \cap S_7 \subseteq S_7$.

These two ling subintervals overlap on each other.

Consider two ling subintervals S_8 and S_9 where

S_8 = [worst, bad] and S_9 = [good, very good] of

[worst, best] are given by the following figure 3.12.

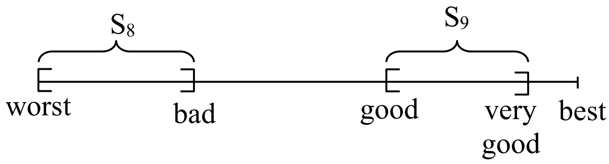


Figure 3.12

Clearly $S_8 \cap S_9$ = [worst, bad] $\cap$ [good, very good]

= $\{\phi\}$.

Now $S_8 \cup S_9$ = [worst, bad] $\cup$ [good, very good]

$\subseteq$ [worst, best] …(a)

Clearly $S_8 \cup S_9$ do not contribute to a ling subinterval but only union of the two ling subintervals represented by (a).

In view of all these we have the following results analogous to real line and subintervals of real line.

Let us now obtain some results on the ling subintervals of a linguistic interval.

S_u = {collection of all ling subintervals of a ling interval}.

If I_1 and I_2 are two ling subintervals of S_u.

i) Then we have $I_1 \cap I_2 \in S_u$.

ii) However if $I_1, I_2 \in S_u$ $I_1 \cup I_2$ may not be a ling subinterval of the form [x, y] such that [x,y] is a ling continuum but only union of two ling subintervals which cannot be written as another ling interval. $[S_u, \cap\}$ is a ling semigroup of intervals.

It can be easily verified that S_u under $\cup$ is not even closed.

We will prove this situation by an example.

Example 3.1. Let S = [shortest, tallest] be the ling continuum associated with the ling variable height of people.

Let $S_u = \{[a, b] / a, b \in S\}$ be the collection of all ling subintervals of the ling interval S. Clearly S_u includes singleton ling terms as intervals.

The ling subintervals can be overlapping, disjoint or can be contained in some other subinterval. It can also have only one ling term in common. This is described by the following figure 3.13.

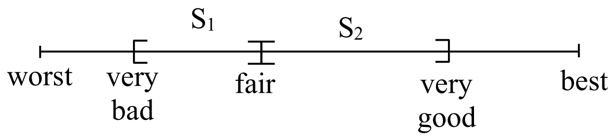


Figure 3.13

where S_1 = [very bad, fair] $\subseteq$ [worst, best] and

S_2 = [fair, very good] $\subseteq$ [worst, best].

We see $S_1 \cap S_2$ = {fair}.

It is pertinent to keep on record that when we say intervals we also include the singleton elements as intervals.

Clearly under the union operation the collection S of ling subintervals are not even closed.

However under both the operations min and max the collection of ling intervals are closed.

We will first illustrate this situation by some examples.

Example 3.2. Let S_u = {collection of all ling subintervals including single points belonging to the ling set or continuum [youngest, oldest]} where [youngest, tallest] is the ling continuum associated with the ling variable age of persons.

Let us consider x = [young, middle aged] and

y = [just young, old] be two ling subintervals belonging to S_u.

x ∩ y = [young, middle aged] ∩ [just young, old]

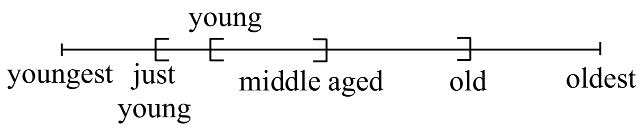


Figure 3.14

x ∩ y = [young, middle aged] = x ...(1)

x ∪ y = [young, middle aged] ∪ [just young, old]

= y …(2)

Now min{x, y}

= min{[young, middle aged], [just young, old]}

= [min {young, just young},

min{middle age, old}]

= [just young, middle age] ≠ x or y …(3)

Consider max {x, y}

= max{[young, middle aged], [just young, old]}

= [max{young, just young}, max{middle aged, old}]

≠ x or y ...(4)

Thus all the four equations are distinct.

One can prove the following results and are left as exercise to the reader.

Theorem 3.1. *Let S be a ling continuum associated with some ling variable.*

S_u = {collection of all linguistic intervals from S the ling continuum}

{S_u, min} is a ling interval semigroup.

Proof. Left for the reader to prove the following steps.

If $x = [a_i, a_j]$ and $y = [a_k, a_l] \in S_u$. $i \leq j$, $k \leq l$.

then $\min\{x, y\} = \min\{[a_i, a_j]. [a_k, a_l]\}$

$= [\min\{a_i, a_k\}, \min\{a_j, a_l\}] \in S_u$

is the way min operator works on ling subintervals.

Prove i) if $x, y \in S_u$; $\min\{x, y\} \in S_u$ (closure axiom)

ii) if $x, y, z \in S_u$ prove

$\min\{\min\{x, y\} z\} = \min\{x, \min\{y, z\}\}$

(associative law).

Then {S_u, min} is a ling subinterval semigroup.

Now we prove {S_u, max} is a ling subinterval semigroup.

Theorem 3.2. *Let S be a ling continuum associated with some ling variable.* S_u *= {collection of all ling subintervals of S};*

{S_u*, max} is a ling subinterval semigroup.*

Proof. Define the max operation on S_u as follows.

For x = $[a_i, a_j]$ and y = $[a_k, a_l]$ ($a_i \leq a_j$ and $a_k \leq a_l$) be two ling subintervals from S_u.

$\max\{[a_i, a_j], [a_k, a_l]\} = [\max\{a_i, a_k\}, \max\{a_j, a_l\}]$ is in S_u.

Prove i) $\max\{x, y\} \in S_u$

ii) If $x, y, z \in S_u$, then prove $\max\{x, \max\{y, z\}\}$

$= \max\{\max\{x, y, z\}$.

Clearly as (i) and (ii) are satisfied $\{S_u, \max\}$ is a ling subinterval semigroup.

Now for us to define analogous to real plane we have two types of ling planes. i) semi ling planes and ii) ling planes.

We have worked with straight lines ling planes and semi ling planes and just for sake of completeness we recall them very briefly.

We have introduced the notion of semi ling plane and ling plane both are known as semi ling space or ling space of order two respectively.

We have for each of these 2 such semi ling space (or ling space) by interchanging the vertical axis with the horizontal axis and vice versa in semi ling space (or ling space).

We have explained a lot about them in chapter II.

Now a semi ling space with one ling variable another a real variable which is dependent or equivalent of the ling variable.

We provide some examples of them. For instance with ling variable age we have the associated real variable varies in real interval [0, 100] where 0 corresponds to the ling term youngest and 100 corresponding to the ling term oldest. So we can have [0, 100] along the horizontal axis or vertical axis depending the choice of axis for the linguistic continuum [youngest, oldest].

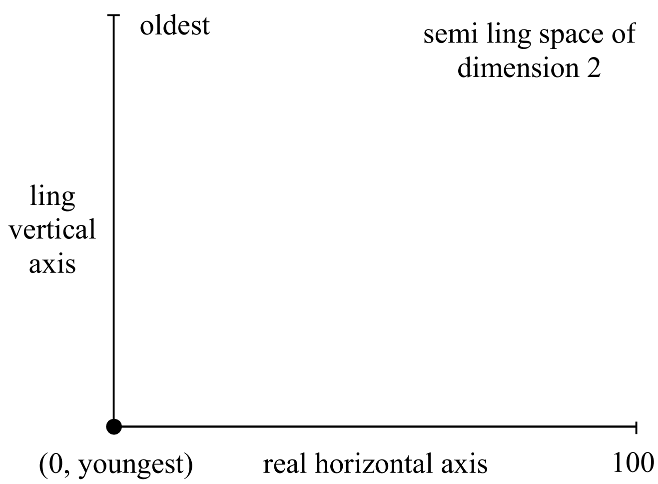


Figure 3.15

Semi ling space of dimension 2

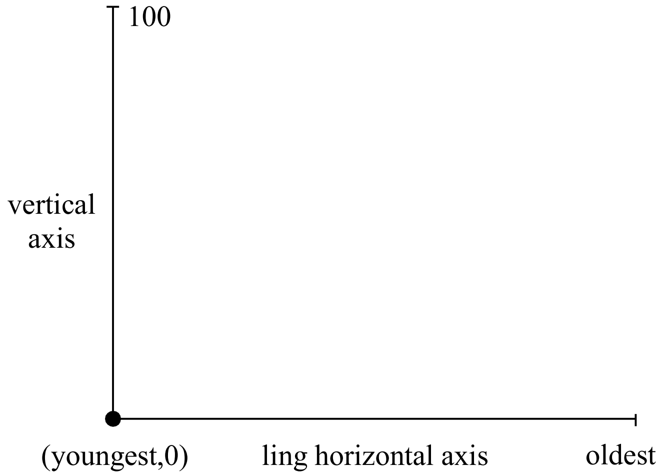


Figure 3.16

We have worked with these.

Now we proceed onto describe ling spaces of dimension two. To build ling spaces of dimension two we should have two ling variables which are connected on common feature or target or concept which we investigate.

This will be more clear if we describe first by some examples.

Consider the concept as human being, we can with each person associate the height of a person and weight of that same person. So we have two ling variables which are connected like height and age for a young or very young will be from middle aged man and so on.

Likewise we can have the two linguistic variables age and weight for the common concept of people or animals and so on.

For the ling variable age can vary with the ling weight in the ling continuum [lowest, highest]. [youngest, oldest] is the ling continuum associated with the ling variable age. So for persons the two ling variables age and weight are related for people.

Next the ling variables production and profit / loss of a company are related.

So for any company the ling variable production is related with the ling variable profit or loss. The ling sets associated with both of these ling variables are linguistic continuum.

Just like this for any paddy plants in a field the ling variable yield is related with the ling variable height of persons and so on.

Now we have seen for certain concepts / attributes / objects / living beings we have two ling variables which are related. In fact they should be only a ling continuum for the ling space of dimension two exist.

These ling continuum will have a least element and the greatest element. The least elements of both the ling continuum forms the ling origin.

With these in mind we give the representation of ling spaces of dimension two which are nothing but ling planes.

Now let us consider the linguistic variables age and height of peoples. These two linguistic variables are related or associated. In fact both of them give ling sets which are ling continuum.

The ling continuum associated with the ling variable age is [youngest, oldest].

The ling continuum height is associated with the ling continuum [shortest, tallest]; both these ling. variables are only increasing order with time. In fact we can observe the ling. variables age and weight are such that always increasing were as weight can be both increasing or decreasing with time and age.

The associated ling plane is given in the following.

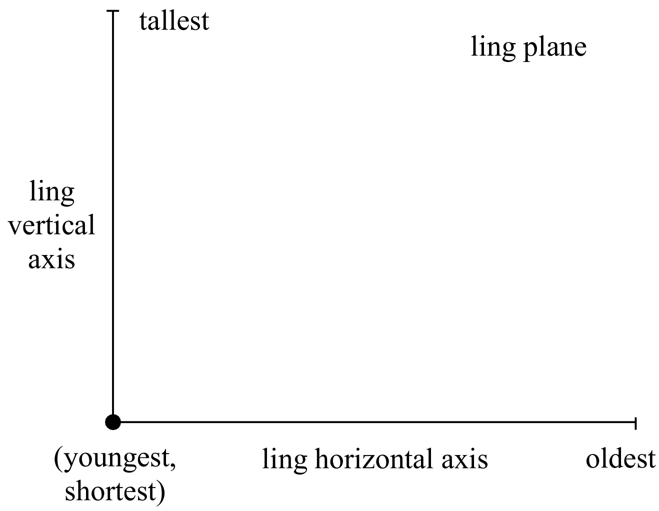


Figure 3.17

For the figure 3.17 we have taken for the ling horizontal axis the ling continuum [youngest, oldest] related with the ling variable age and along the ling vertical axis we have taken the ling continuum [shortest, tallest] associated with the ling variable height. Thus the ling origin is (youngest, shortest).

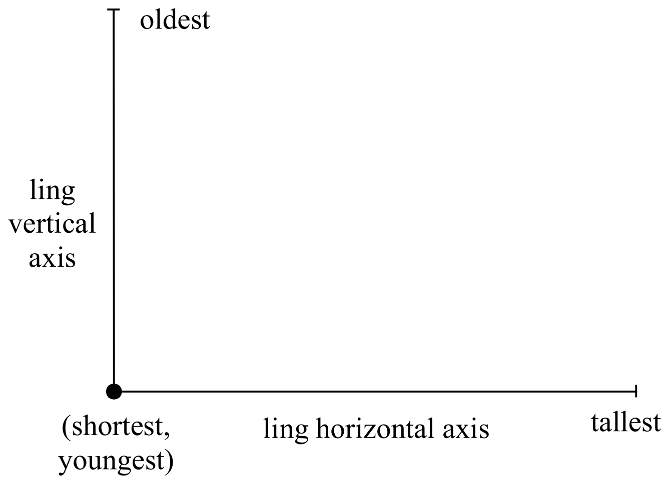


Figure 3.18

It is not mandatory the age should be taken along the ling horizontal axis it can also be taken as ling vertical axis in which case the origin will vary from

[youngest, shortest] to [shortest, youngest], refer Figure 3.17 and figure 3.18 respectively.

Now having given a brief account of ling spaces of dimension two we proceed onto describe ling spaces of dimension 3 or (2, 1) semi ling spaces of dimension 3 or (1, 2) semi ling spaces of dimension three.

In case of (1, 2) semi ling spaces of dimension three we take 1 real variable and two ling variables. For instance age and height of person are taken as ling variables we associate with age the real and ling continuum say [0, 100] and [0, 100] and [youngest, oldest] respectively and for the ling variable height we have the related ling continuum [shortest, tallest].

The 3 dimension (1, 2) semi ling space is described below.

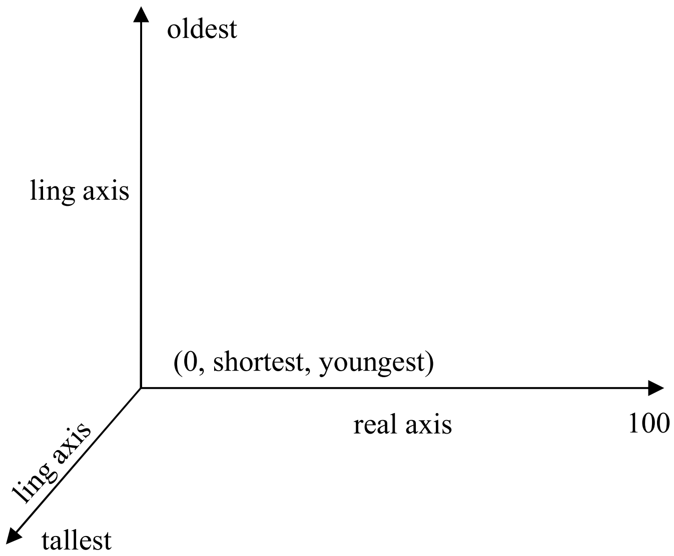


Figure 3.19

We can have 6 such frames of reference and then respective ling origins will be

(shortest, 0, youngest), (youngest, shortest, 0), (0, youngest, shortest), (shortest, youngest, 0) and (youngest, 0, shortest).

The (1, 2) semi ling space of dimension 3 with (shortest, 0, youngest) as origin is given in figure 3.20.

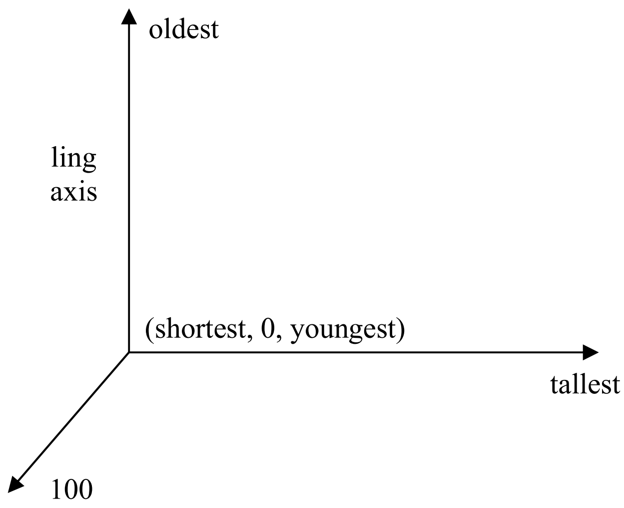


Figure 3.20

Now for the (1, 2) semi ling space of dimension 3 with origin (youngest, shortest, 0) is given in figure 3.20

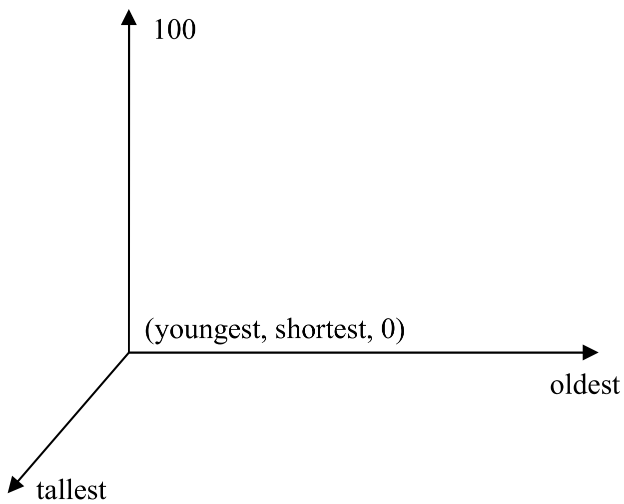


Figure 3.21

Now for (1, 2) semi ling space of dimension 3 with (0, youngest, shortest) is given in figure 3.22.

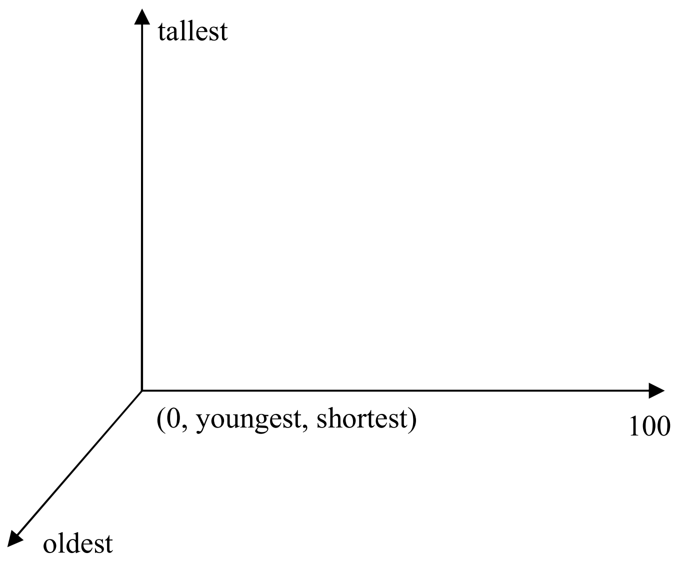


Figure 3.22

Now for (1,2) semi ling space of dimension 3 with origin (shortest, youngest, 0) is as follows.

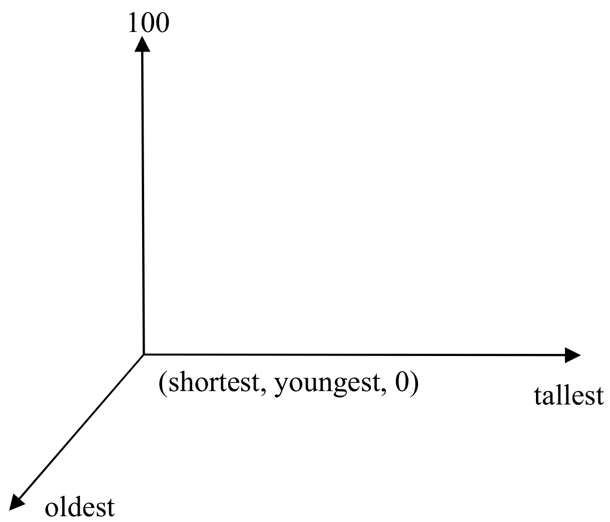


Figure 3.23

Now for the (1, 2) semi ling frame of dimension 3 with origin (youngest, 0, shortest) is given by the figure.

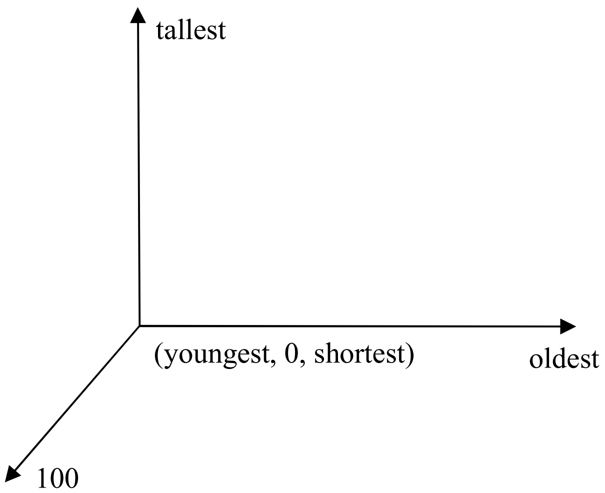


Figure 3.24

Now having seen 6 different frames resulting for a (1, 2) semi ling space we now proceed onto describe the (2, 1) semi ling space where for any ling variable two of the concepts are real variables and one of them is ling variables.

For instance take a person ling variable age and real variables for age and height. As 2 are real variables and only one is the ling variable. This also gives rise to 6 semi ling frame associated with (2,1) semi ling spaces of dimension 3.

The ling continuum associated with age be

[youngest, oldest] and the real continuum for age is [1, 100] and for height is [1 foot, 7 feet].

The (2, 1) semi ling origin are (0, 1, youngest), (1, youngest, 0), (1, 0, youngest), (0, youngest, 1), (youngest, 0, 1) and (youngest, 1, 0) given by the following figures. All of them are distinct.

For the (2, 1) semi ling space of dimension 3 with ling origin (0, 1, youngest) is given in figure.

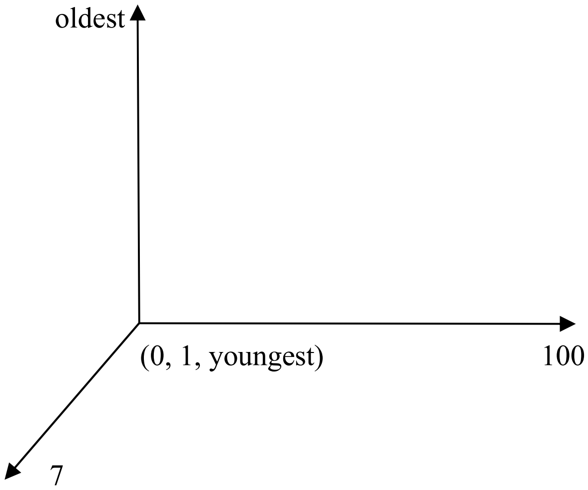


Figure 3.25

The (2,1) semi ling space of dimension 3 with ling origin (1, youngest, 0) is given by the following figure 3.26.

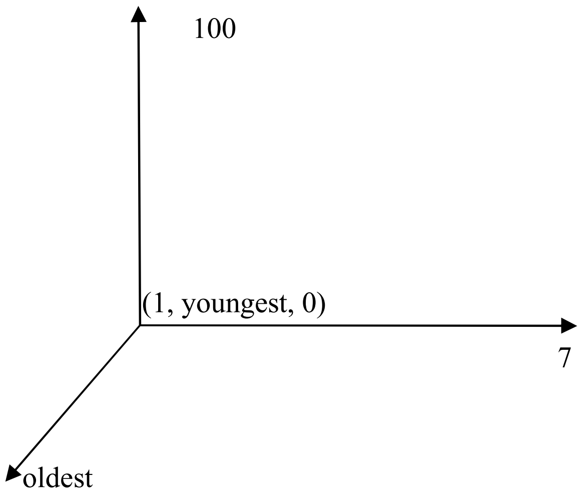


Figure 3.26

The (2, 1) semi ling space of dimension 3 with origin (1, 0, youngest) is given by figure 3.27.

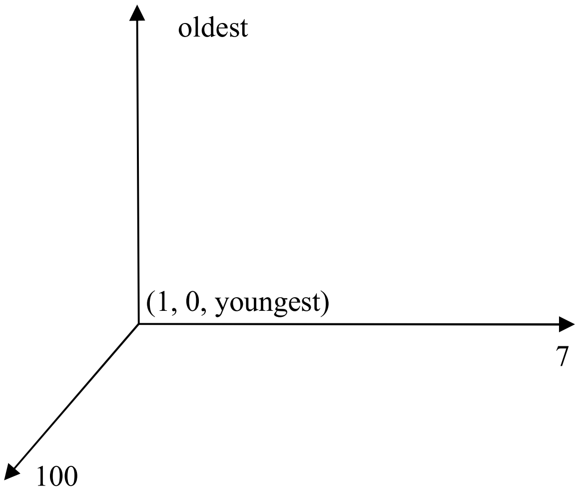


Figure 3.27

The (2, 1) semi ling space of dimension 3 with origin (0, youngest, 1) is given by figure 3.28 is as follows.

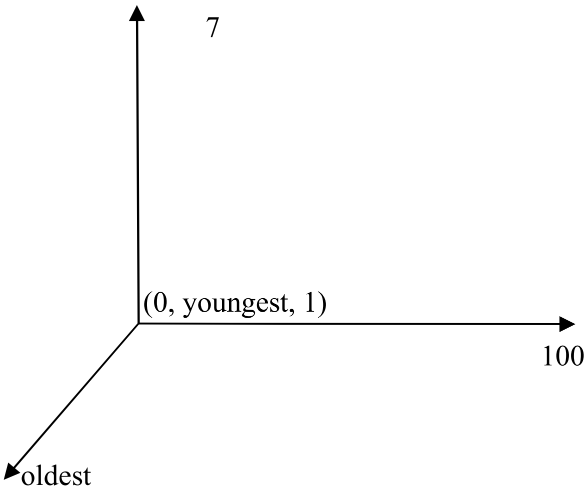


Figure 3.28

Now we give the (2, 1) semi ling space of dimension three with (youngest, 0, 1) as its origin is given by figure 3.29.

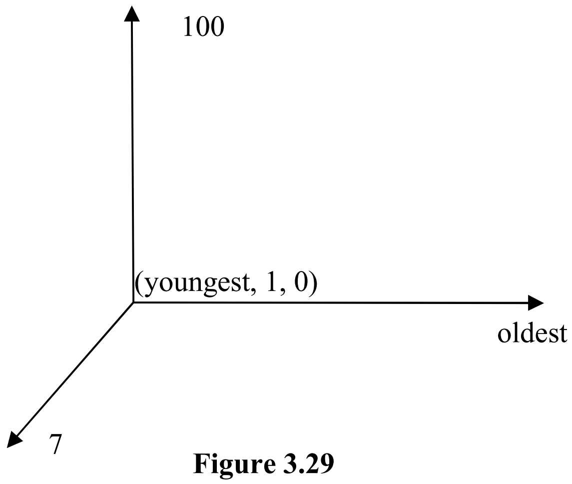


Figure 3.29

Now we give the (2, 1) semi ling space of dimension three with (youngest, 1, 0) is given in figure 3.30.

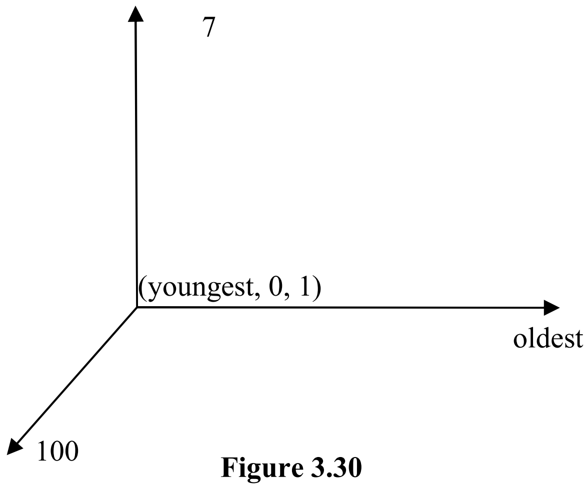


Figure 3.30

Having see all the 6 distinct (2, 1) semi ling spaces of dimension 3 is given in figure 3.25 to figure 3.30.

Next we give the 6 possible ling spaces of dimension three. Consider the 3 ling variables age, weight and height of persons. All the three concepts are related and they all give way to ling continuums.

The ling set / continuum of the ling variable age is given by [youngest, oldest].

The ling set / continuum of the ling variable weight is given by the ling continuum [lowest, highest].

For the ling variable height we have the ling continuum given by [shortest, tallest].

For the 3 dimension ling spaces we have the following 6 origins given by

(youngest, lowest, shortest), (youngest, shortest, lowest), (shortest, lowest, youngest), (shortest, youngest, lowest), (lowest, shortest, youngest) and (lowest, youngest, shortest).

The ling three dimension space with origin

(youngest, lowest, shortest) is given by the figure 3.31.

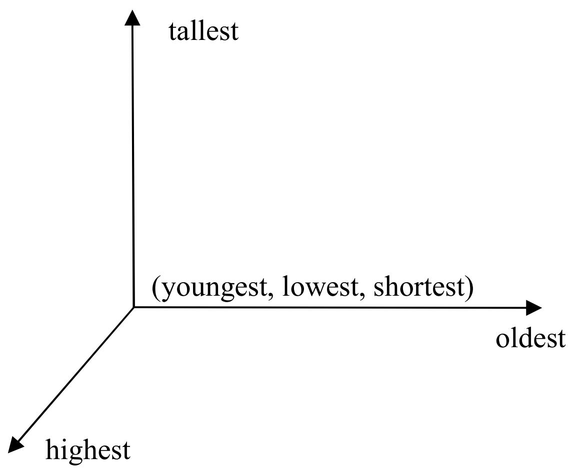


Figure 3.31

The ling 3 dimension of space with origin (youngest, shortest, lowest) is given by the following figure 3.32.

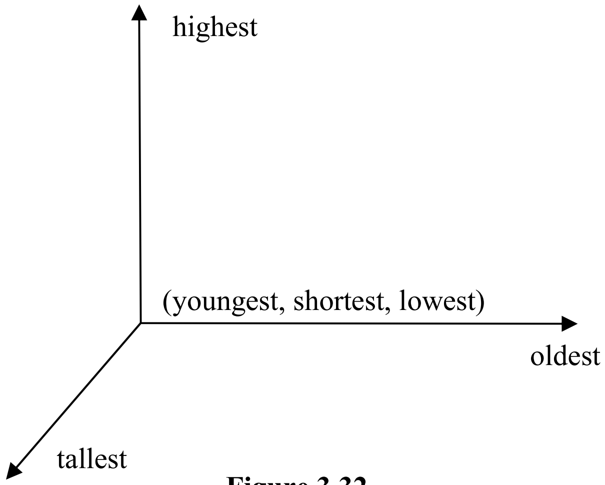


Figure 3.32

The ling 3 dimension space with origin (shortest, lowest, youngest) is given by the following figure 3.33.

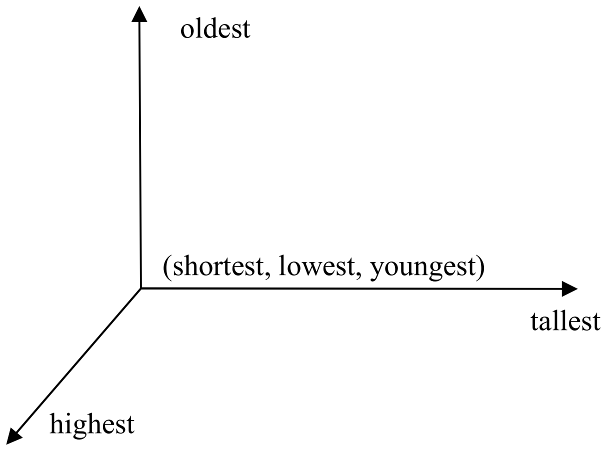


Figure 3.33

The 3 dimension ling space with origin (shortest, youngest, lowest) is given by the following figure 3.34.

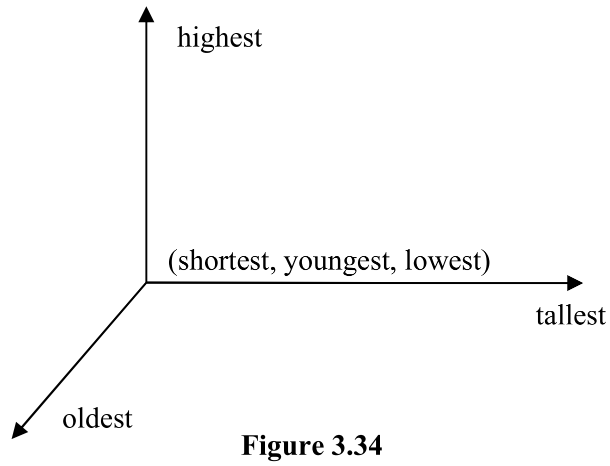


Figure 3.34

The 3 dimensional ling space with (lowest, shortest, youngest) as the ling origin is given by the following figure 3.35.

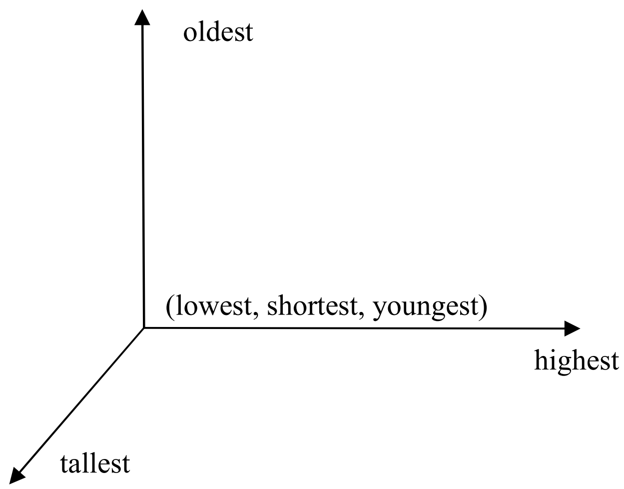


Figure 3.35

The 3 dimensional ling space with ling origin (lowest, youngest, shortest) is given by the following figure 3.36.

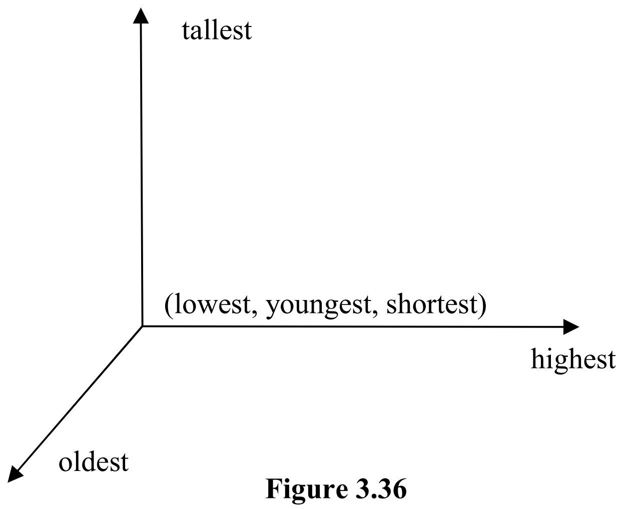


Figure 3.36

Thus we have 6 distinct ling 3 dimensional spaces. It is important to note all the three variables are associated with people in general.

Now having see all possible ling frames are spaces of dimension three as well as (2, 1) semi ling spaces / frames of dimension three and (1, 2) semi ling spaces of dimension three we proceed onto describe points, straight lines etc. in these frames.

We will first illustrate this situation by some examples.

Example 3.3. Let us consider the (1, 2) semi ling 3 dimensional space associated with the ling variable age and height of people. The ling continuum associated with ling variables height and age of people are given by [youngest, oldest] and [shortest, tallest] respectively.

Now taking the real variable as height in the real subcontinuum [1,7] measured in feet. We represent this in figure 3.37 given in the following.

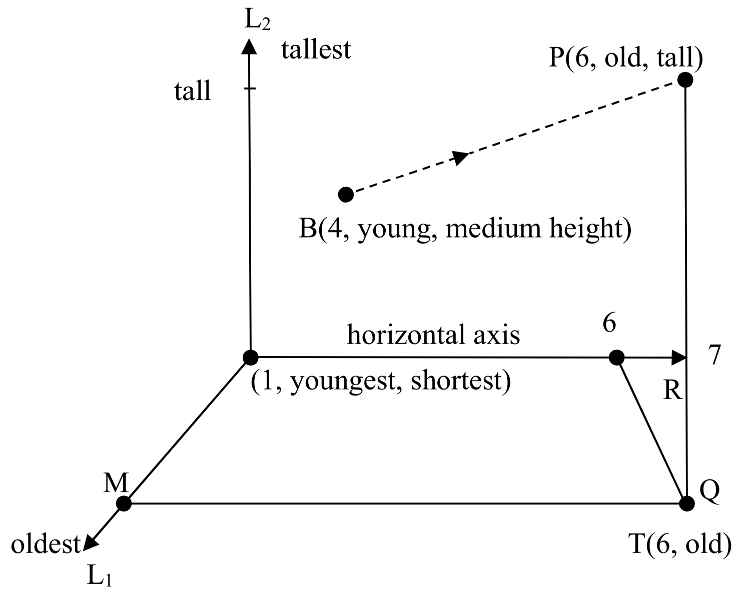


Figure 3.37

With origin as (1, youngest, shortest). Consider the semi ling triple P(6, old, tall). We represent each point on this 3 dimensional ling space as triples.

The triple highly depends on the position of these continuums. From P draw a line perpendicular to the semi ling plane ROL_1 and let T be the foot of the perpendicular from P to ROL_1 plane TM is the ling perpendicular to the ling axis OL_1 and TN is the line perpendicular to the real axis OR.

This is the way representation is done in 3 dimensional (1, 2) semi ling space. It is in fact analogous to our usual or classical 3-dimensional space.

It is pertinent to record that depending on the choice of the axis the positioning of the coordinates will also vary.

As in case of classical analytical geometry finding distance between two (1, 2) - semi ling triples is not the same for the distance is also a (1, 2) - semi ling triple, may exist or may not exist.

Consider two semi ling triples given by P(6, old, tall) and B (4, young, medium height). The semi ling distance between P and Q is denoted by

d(B, P) = d_l((4, young, medium height) (6, old, tall)

= (2, very far, far).

However, we do not have a distance concept d(P, B) for we have distance from 6 to 4 height as heights logically, a old cannot reach or become young so also a tall person cannot become medium height.

If we change the order of axis the origin will change and correspondingly the order of the distance between points will change.

We will give one or two illustrations by changing the (1, 2) semi linguistic origin for the one given in example 3.3.

Suppose the origin is taken as (youngest, 1, shortest) then the corresponding change of B and P marked or given in figure 3.38 is from d_1(B, P) = (4, young, medium height) is changed to B_1 (young, 4, medium height).

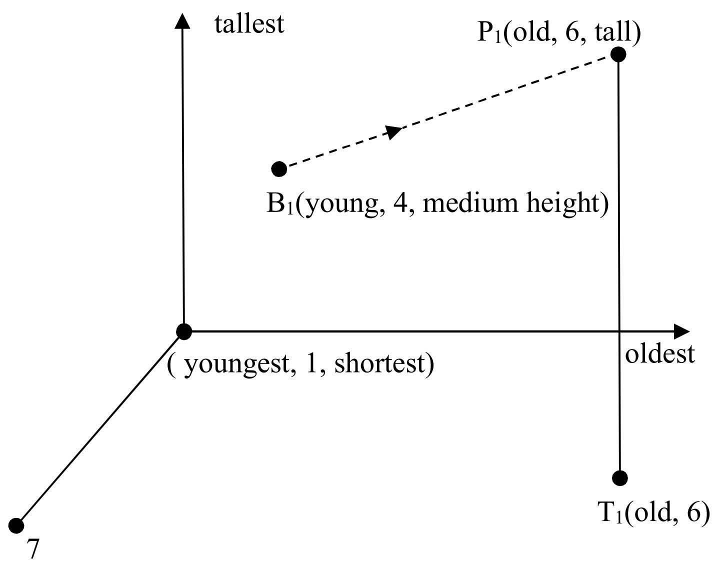


Figure 3.38

Similarly the semi linguistic triplet

P(6, old, tall) changes to P_1 (old, 6, tall).

Thus the corresponding semi ling distance is given by

d_l (P_1, B_1) = d_l ((young, 4, medium height), (old, 6, tall))

= (very far, 2, far).

Likewise if we take the (1, 2) semi ling space with origin (youngest, shortest, 1).

Now the semi ling triplets will change which is represented in figure 3.39.

P(6, old, tall) is changed for this new origin as

P_2(old, tall, 6) and that of B (4, young, medium height) is changed for this new origin as B_2(young, medium height, 4).

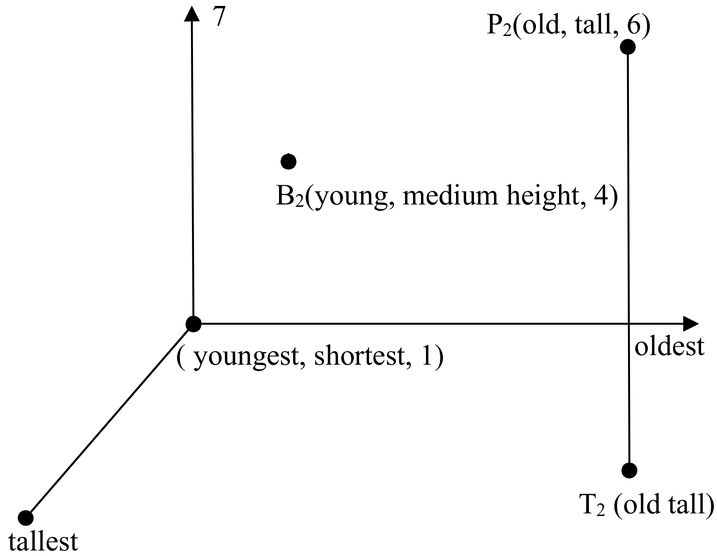


Figure 3.39

Now the semi ling distance between the two semi ling triplets B_2 and P_2 is given by

d_l (B_2, P_2) = d_l ((old, tall, 6), (young, medium height, 4))

= (very far, far, 2).

This is again a semi ling triplet by the ling variables are ling distance given by the ling continuum [nearest, farthest].

Thus by changing the semi ling origin we see a change takes place in the semi ling triplets formed by them.

Now we proceed onto give examples of (2, 1) semi ling 3 dimensional spaces.

Suppose take the ling variables age of people, height of people and real variable age of people. The ling continuum associated with the ling variable age of people is given by [youngest, oldest].

The ling continuum associated with the ling variables height of people is given by [shortest, tallest] and that of the real subinterval associated the age of people is given by [0, 100]. Now for the (2, 1) semi ling space we can have 6 types of origins. We take a few of them for our study (0, youngest, shortest), (youngest, 0, shortest) and (shortest, youngest, 0),

First we describe the ling 3 dimensional space by the following figure with the (2, 1) semi ling origin as (0, youngest, shortest).

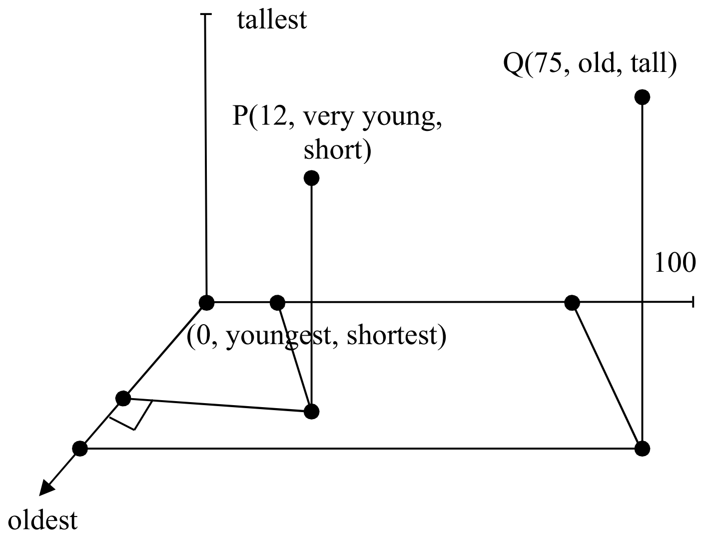


Figure 3.40

Let P(12, very young, short) and Q(75, old, tall) be two semi ling triplets in the semi ling space with semi ling origin (0, youngest, shortest).

The semi ling distance between P and Q given in figure is as follows

d_l(Q, P) = d_l ((75, old, tall), (12, very young, short))

= (63, very far, very far)

Thus, this is the deviation from usual or classical analytical geometry where distance between any two points in the three dimensional real plane will be a real value.

Suppose A = (2, 4, 9) and B = (9, 5, 7) then

$$d(A, B) = \sqrt{(2-9)^2 - (4-5)^2 + (9-7)^2}$$

$$= \sqrt{49+1+4}$$

$$= \sqrt{54}$$

$$= 3\sqrt{6}\,.$$

But the semi ling distance cannot be got as a value infact it will also be a semi ling triplet or to be a more precise it will also be a only a (2, 1) semi ling triplet.

In fact in this (2, 1) semi ling frame or (1, 2) semi ling frame we do not study arbitrarily distance between any two semi-ling points. In fact they are more a meaningful concept in comparison with the classical analytical geometry.

For instance, in 3 dimension real space. Our task may be to find the length of the sides of a triangle given the vertices of triangle.

Say A(–3, 81, 95), B(49, 78, –65) and C(24, –1, 48).

No doubt in no time with the aid of the calculator all the 3 sides; distances AB, BC and AC will given by them.

But in practice where is the place of –3 in the or –65 or – 1, can they be represented or realized non abstractly for no negative value can be seen or shown in reality, in this scenario they will not be in any position to explain it.

In fact we can give some fictions arguments to support our claim. But fixing them in the space we like is close to an impossibility. How to place in the 3 dimensional space we live all these negative ones cannot be fixed so cannot be answered satisfactorily.

So to imagine this triangle is space is not possible though we concretely plot them on paper which is only three dimensional. To achieve perfectness in mathematics we go on sacrifice several things. However the (2, 1) semi ling space or the (1, 2) semi ling space are not that abstract. When we say or define a (2, 1) or (1, 2) semi ling spaces they are not like our 3-dimensional classical real space in fact it is highly dependent on the ling variables which we work with and not like the universal real 3-dimensional space on which every concept is fitted.

Hence based on this we see semi ling spaces are entirely dependent on the ling variable under consideration.

Thus depending on the variable we have different or many (2, 1), (1, 2) semi ling spaces of dimension three entirely depending on the ling variable under consideration.

Let us take the (1, 2) semi ling space of dimension three related to the ling variable performance of students, performance of teachers and the marks 0 to 100 of the students is taken. It is a (1, 2) semi ling space for the ling variable performance of students in the ling continuum [worst, best] and that of the ling variable performance of teacher which is given by the ling continuum [very bad, very good].

The (1, 2) semi ling space with centre (0, worst, very bad) is as follows.

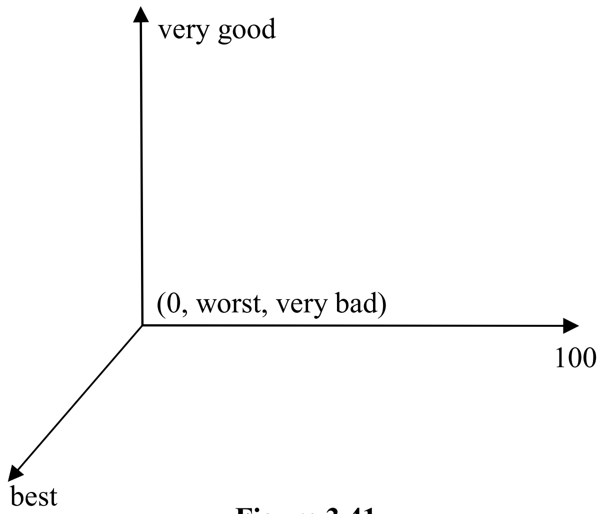


Figure 3.41

Suppose we have 3 students whose performance, teacher contribution and marks are given by (1, 2) semi ling triplets. Let S_1 = (70, good, fair) be the performance aspects of students S_1.

Let S_2 = (20, very bad, very bad) and S_3 = (90, very good, good) be the (1, 2) semi ling triplets associated with three students, S_1, S_2 and S_3.

We can find the distance from S_1 to S_2, S_1 to S_3 and S_2 to S_3 which is represented in the following

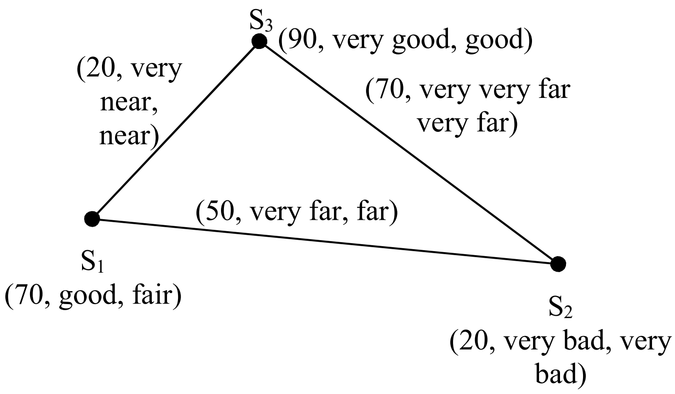


Figure 3.42

S_2 to reach S_1 is (50, very far, far)

S_2 to reach S_3 (70, very very far, very far)

S_1 to reach S_3 (20 very near, near).

So we can also say the distance from

S_1 to S_2 is (50, very far, far)

the distance from S_2 to S_3 is (70, very very far, very far)

and that of the distance from S_1 to S_3 is (20, very near, near).

This sort of studies is very helpful to find the distance linguistically between two points. When we give the result as a result we can think of making how much one should develop in each of the factors.

But if over all distance is given one may not know in which component they should develop it is a 3 values the component which has to be improved.

So one can easily find the distance between two (1, 2) semi linguistic points.

Now we give one example of finding distance between (2, 1) semi linguistic points. We consider the (2, 1) semi ling space where the height and age of people are taken as the subintervals of the real line whereas the age is taken as the linguistic variable and the ling continuum associated with age is [youngest, oldest].

Now we can have 6 distinct origins we just indicate the six origins and the real sub interval for age is [0, 100] and that of height is [1, 7] measured in years and feet respectively.

The (2, 1) semi ling space of 3-dimension is given by the following figure.

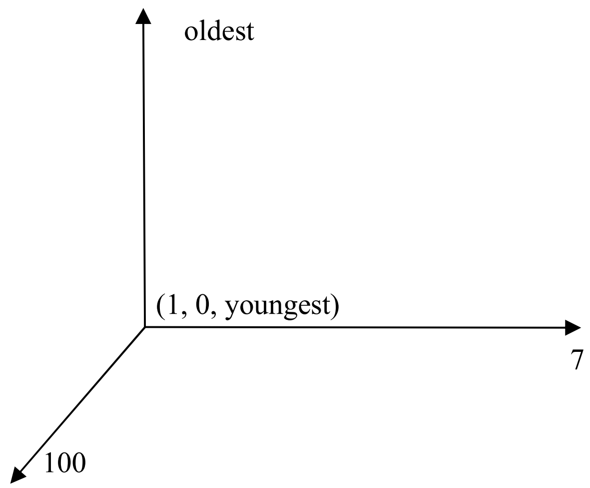


Figure 3.43

The six possible origins are

(1, 0, youngest), (1, youngest, 0), (0, 1, youngest), (0, youngest, 1), (youngest, 0, 1) and (youngest, 1, 0).

Now we give (2, 1) semi ling points (triplets) on the (2, 1) semi ling space with (1, 0, youngest) as origin.

Let P(2, 2, very young) and Q(5, 45, middle aged) be two (2, 1) semi ling spaces of triplets.

The semi ling distance between P and Q is

d_l(P, Q) = d_l (2, 2, very youngest) as origin.

Let P(2, 2, very young) and Q(5, 45 middle aged) be two (2, 1) semi ling spaces of triplets. The semi ling distance between P and Q is

d_l(P, Q) = d_l((2, 2, very young), (6,45, middle aged))

= (4, 43, very far).

Thus if we want to measure the distance between the two (2, 1) semi-ling triplets P and Q, we say to reach very young to young is very far.

Also to grow 4 feet more for just 2 feet height baby is very far. For the two year baby to go to age of 45 years is also equally very far by we mathematically say 43 more years one has to spend linguistically if we want we can say a two year toddler to reach 45 years (middle age) is very far.

Now we find the ling. distance between 3 points in (2, 1) semi line space given by A = (3, 9, young), B = (4, 14, youth) and C = (5.5, 36, middle aged).

We can find meaningful distance once from A $\to$ B, B $\to$ C and A $\to$ C. In no other way ling. distance can be found.

This method is to find how long or how distant is to reach from A to B, B to C and A to C.

The (2, 1) semi ling distance between, the 3 triplets A to B, A to C, and B to C is calculated as follows.

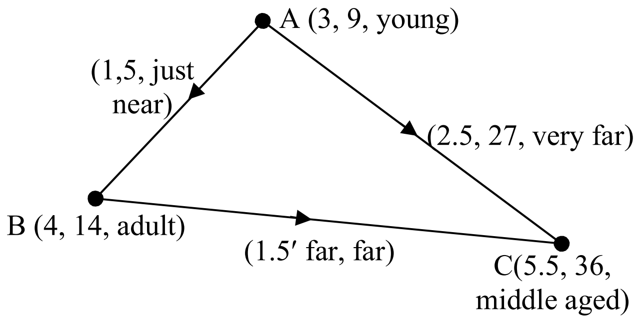


Figure 3.44

Now we proceed onto describe ling space of dimension three. We see these ling triplets are associated with concept which has 3 ling variables associated with it.

We provide first examples of the same.

Suppose we consider persons or people of a country or state or whole world. We can associate with them three ling variables, age, height and weight. All the three ling variables pave way to ling continuums. Ling continuum associated with the ling variable age is S_1 = [youngest, oldest]. The ling continuum associated with the ling variable height is S_2 = [shortest, tallest] and the ling continuum associated with the ling variable weight is S_3 = [lightest, heaviest].

Now having got the ling continuum we all the three forms a ling lines and they can be used to construct a ling 3 dimensional space which is given in the following figure 3.45.

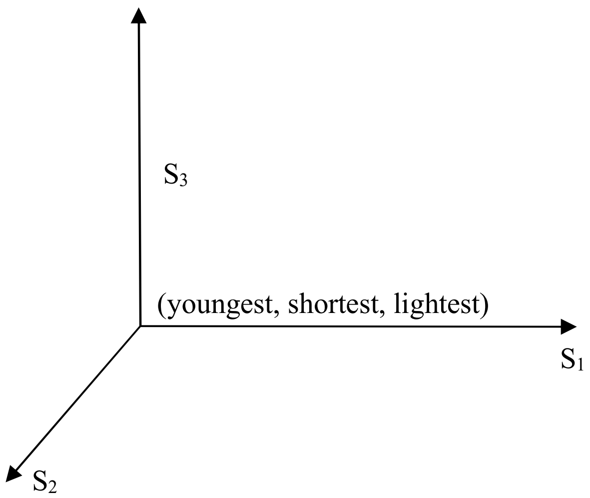


Figure 3.45

We can have six such ling 3 dimensional spaces associated with the following ling origins.

(youngest, shortest, lightest), (youngest, lightest, shortest), (shortest, youngest, lightest), (shortest, lightest, youngest), (lightest, youngest, shortest) and (lightest, shortest, youngest).

Suppose we have a ling three dimensional space has the origin to be (youngest, shortest, lightest).

Now suppose we have two linguistic triplets given by

A (young, medium height, medium weight) and

B (very old, tall, medium weight).

We want to find the linguistic distance from A to B.

Only from A to B is meaningful for a very old person cannot go to young age. Now

d_l(A, B) = d_l((young, medium height, medium weight), (very old, tall, medium weight))

= (very far, just near, ϕ).

We see for the young person to become very old he / she should go very far, that is the distance from young $\rightarrow$ very old is very far.

Similarly for that person to grow from medium height to tall is just near. That is the distance from medium height $\rightarrow$ tall is just near.

Finally the weight of both the young person and very old person is the same so the distance from them is ϕ or nil or no difference.

Now having seen how the distance is calculated in case of two linguistic triplets.

Now we give how a ling triplets of 3 points look like and how distance is measured between any two ling triplets.

Let A (very old, tall, medium weight), B(very young, short, light) and C(middle aged, very tall, heavy) be the three ling pairs of the ling space with (youngest, shortest, lightest) as its origin.

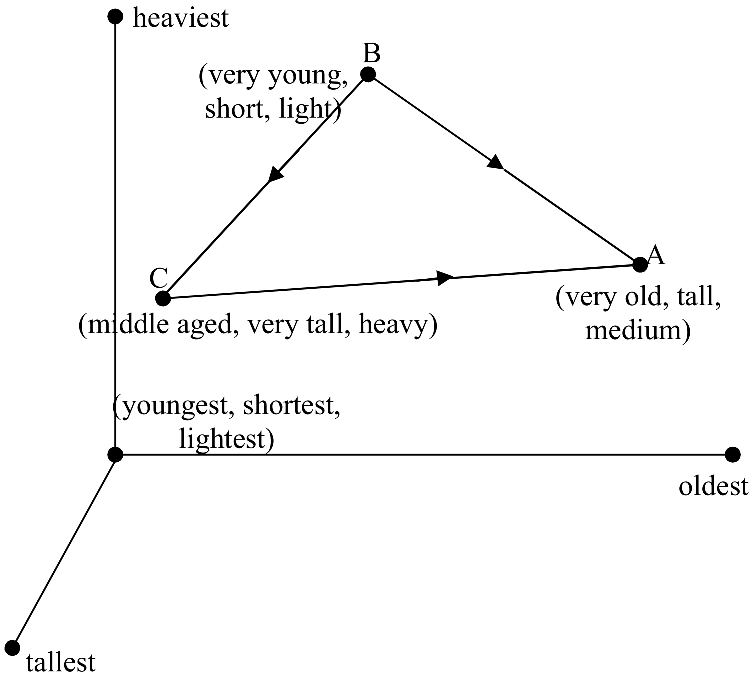


Figure 3.46

Now we can only go from

B → A   B → C and C → A. We have given the associated ling triplets; and all these triplets find their place in the ling continuum [nearest, farthest]. The ling distance from C to A is (far, ϕ, far) (ϕ is put as the middle aged man is very tall where as the very old is tall so no change can take place). The ling distances from B to A is (very very far, very far, very very far) and that of B to C is (very far, very far, very very far).

Now we explain the ling distance from very tall → tall.

We observe that we cannot go from very tall to tall as one person cannot loose his height but his weight can be lost. Age also cannot be lost so.

We as a convention put the ling distance from very tall → tall is only as empty.

This is the way distance is found however for us to work on a network and have to find ling distance how to manage them for that sake we will provide an example with four ling triplets from the ling space of dimension three.

Let A (youngest, shortest, lightest),

B (oldest, tall, medium weight),

C (old, very tall, just heavy) and

D (middle aged, very tall, heaviest) be the four linguistic triples in the ling three dimensional space with origin as (youngest, shortest, lightest).

We give the ling distance between pairs of points.

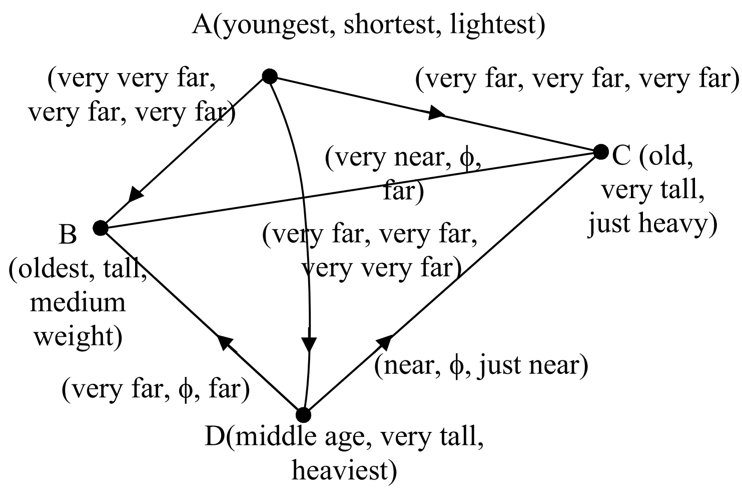


Figure 3.47

This is way we get the ling distance.

We can visualize the ling triplets connected by the ling distance as a linguistic directed graph.

We can also realize the diagram as ling weighted directed network.

In fact we can say the ling weighted directed graph further all of them are complete as ling graphs. Their edge weights also are linguistic triplets with ling values always from the ling continuum [nearest, farthest] $\cup$ $\{\phi\}$.

The limitations of ling three dimensional spaces is that

i) The ling distances are always ling triplets from the linguistic continuum; $\{\phi\}$ $\cup$ [nearest, farthest] unlike in classical analytical geometry or in three dimensional real spaces the distance a between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by
$d(x_1, y_1) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
we only take the positive value.

But for A(old, tall, medium weight) and

B(young, medium height, medium weight) is

$d(\overrightarrow{B, A})$ = ((very far, ϕ, ϕ).

We do not mark as A to B for linguistically it becomes meaningless. Clearly it is a triplet.

Suppose M (youngest, shortest, lightest) and

N(middle aged, very tall heavy)

then we can have the distance measured only from M to N (and cannot imagine $N \to M$ logically $N \to M$ is absurd as a middle aged man cannot become youngest also very tall cannot reduce in humans to shortest).

This observation is quiet important in ling study.

So $M \to N$ the ling distance is (very far, very far, very far)

ii) We cannot as in case of classical analytical geometry find the area (in case of ling space of dimension two) or volume in case of ling space of dimension 3 and so on.

iii) Another important factor to be kept in mind is that when forming the 3 dimensional space the ling variables are contributed by a concept. In the example which we have discussed people in general has 3 ling variables associated with them viz. age, weight and height.

Now it may so happen that we may have more than 3 attributes associated with some variable. So if we assume any 4 dimensional semi ling space, we can have the following 4 types (3, 1) semi ling space of dimension four (1, 3) semi ling space of dimension four (2, 2) this is true in case of ling space of dimension four.

We will illustrate this situation by one example.

Example 3.4. Let us consider the ling variable height of person and three real variables height, age and weight of persons. The ling set / continuum associated with the ling variable height of a person is the ling variable height of a person is given by [shortest, tallest]. The real subcontinuum associated with height is [1, 7] in feet. The real subcontinuum or subinterval associated with weight of a person is [2, 80] kilograms. The real subcontinuum associated with age of people is [0, 100]. So this semi ling space of dimension four is denoted as (3, 1) semi ling space of dimension four.

This space will of dimension four, 3 of them real and one being linguistic. The (3, 1) semi ling space of dimension four can have 24 number of origins.

In this case given by

{(0, 1, 2, shortest), (1, 0, 2, shortest), (0, 1, shortest, 2), (0, 2, 1, shortest), (shortest, 1, 2, 0), (1, 0, shortest, 2), (shortest, 2, 1, 0), (0, shortest, 2, 1), (2, shortest, 0, 1), (2, 1, 0, shortest), (0, 2, shortest, 1), (0, shortest, 1, 2), (2, 1, shortest, 0), (shortest, 1, 0, 2), (1, shortest, 2, 0), (shortest, 0, 2, 1), (1, 2, 0, shortest), (2, 0, 1, shortest), (1, 2, shortest, 0), (shortest, 0, 1, 2), (shortest, 2, 0, 1), (1, shortest, 0, 2), (2 shortest, 1, 0) and (2, 0, shortest, 1)}.

Now we call these semi ling spaces of dimension four as (3, 1) semi ling spaces of dimension four.

We can have (1, 3) semi ling space of dimension four where only one is a real variable and rest 3 are ling variables. Thus we can have the height to be a real variable given by the real subinterval or continuum given by [1, 7] measures in feet and height, weight and age being the three ling variables given by the ling continuums [shortest, tallest], [lowest, highest] and [youngest, oldest] respectively.

Thus this (3, 1) semi ling space of dimension four also has 24 semi ling origins we mention a few of them (1, lowest, youngest, shortest), (shortest, 1, youngest, lowest), (lowest, shortest, youngest, 1), (lowest, 1, shortest, youngest) and so on.

The other semi ling space of dimension four is a (2, 2) semi ling space of dimension four. In this case 2 of them are real variables resulting in real subcontinuums or subintervals and two of them are ling variables resulting with a ling continuum for those ling variables. We can have height and age of the person to be real variables with associated real subcontinuum or subintervals given by [1,7] and [0, 100] respectively weight and age are the ling variables associated with ling continuum / interval / set given by [lowest, highest] and [youngest, oldest] respectively.

Thus the (2, 2) semi ling space of dimension can have 24 distinct semi ling origins for the (2, 2) semi ling space we have

(youngest, lowest, 1, 0) or (lowest, 1, 0, youngest)

or (youngest, 0, 1, lowest) and so on as semi ling origins.

In fact we have 24 such (2, 2) semi ling space of dimension four as they are formed in an arbitrary way.

Now if the origin is fixed we can discuss about the other ling origins. However we find the semi ling distance of any two (2, 2) semi ling points which 4 quadruples or 4-tuples.

However for the peoples we are not in a position to built a ling space of dimension four.

For these 3 semi ling spaces of dimension four we provide the concept of distance between two semi ling 4 tuples.

Consider a (1, 3) semi ling space of dimension four. Let A(4, medium weight, young, short) and

B(6, heavy weight, middle age, tall) be any

two semi ling 4-tuples of (3, 1) semi ling space of dimension four.

The semi ling distance from A and B is given by

d_l(A, B) = d_l((4, medium weight, young, short),

(6, heavy weight, middle age tall))

= (2, far, very far, very far).

More than saying the semi ling distance we can say how far is A from B or A to go B or become B.

We now find the distance between 2 semi ling quadruples in (2, 2) semi ling space of dimension four.

Let x = (young, low, 3′, 12) and y = (old, high, 6′, 68) be two (2, 2) semi ling quadruples in (2, 2) semi ling space of dimension 4.

d_l(x, y) = d_l((young, low, 3′, 12), (old, high, 6′, 68))

= (very far, very far, 3, 56).

This is the way semi ling distances are found.

We see in the quadruples the ling components takes ling values only from the ling continuum [nearest, farthest].

We can also form triangles and the direction of the sides is also dependent from which point to which point we traverse.

Now we wish to give an example of a ling space of dimension four.

Consider the weather report of a particular place. It depends on four ling variables; temperature, rainfall, wind and humidity. All the four factor can be considered as ling variables.

The ling variable rainfall can be represented by the ling continuum [no rainfall, very heavy rainfall].

The ling variable windy can be represented by the ling continuum [very slow, very strong]. The ling variable humidity is represented by the ling continuum [lowest, highest].

The ling variable temperature is measured from

[very cool, very hot].

We find all the four ling variables associated with weather give way a ling continuum.

So we can construct a ling space of dimension four.

One of the ling origin for this ling space can be

(very cool, lowest, very slow, no rainfall).

However it is pertinent to keep on record that there are 24 such ling origins for this ling space of dimension four.

Another important note which we wish to keep on record is that unlike the ling variables age or height of persons which constantly increases and takes a peak value after which is neither increase nor decrease.

However in case of weather forecast windy can go on be increasing, than decrease than increase and so on by forming a wave pattern.

Similarly the temperature of a day in weather report can increase or decrease and may form a wave pattern.

So is the humidity and rainfall. Thus unlike the linguistic variables present with people these ling variable present or associated with weather can decrease or increase or when we find the distance between two ling points we do not use the direction, it can be A to B as well as B to A so no direction is used in all semi ling graphs or ling graphs pertaining to weather report.

Consider the ling quadruples

A (cool, low, just slow, scanty rainfall) and

B(hot, medium, just fast, moderate rainfall) in the ling space of dimension four.

We find the ling distance of A and B

d_l(A, B) = d_l ((cool, low, just slow, scanty rainfall),

(hot, medium, just fast, moderate rainfall))

= (just far, near, near, very near) for in case of weather we cannot say very far and so on for if it is a rainy season we can say just slow to just fast can be near or very near likewise other concepts.

If on the other hand it is a sunny day some concepts can be just far or near. So working with them happens to be very dependent on the season or weather conditions like rainy or sunny on that day. Thus depending on the ling variable we have the related distance to have direction or not. This is a big deviation from the usual or classical distance in case of real planes.

Now we mention about the linguistic spaces of higher dimension say n. First we see that there is only one ling space of dimension n depending solely on the ling variable under consideration. As we vary the ling variable one may have also the change in the dimension but it is pertinent to keep on record if we have a ling space say of dimension n related with the fixed ling variable then for that same ling space we can have (n – 1) number of semi ling spaces of dimension n given by (n – 1, 1), (1, n – 1), (2, n – 2), (n – 2, 2), …, (n – r, r) and (r, n – r).

All these (n – 1) of them are semi ling spaces of dimension n. So for the (n – r, r) semi ling space of dimension n we have n – r real variable associated with that space and r ling variables (where $1 \leq r \leq n – 1$).

Study in this direction is both innovative and interesting; however we see the semi ling space's dimension is always fixes the ling variable under consideration. We can have several such semi ling spaces depending on the ling variable under consideration.

The dimension of both semi ling spaces and ling spaces are only or solely dependent on the ling variable.

It is important to mention that analytical geometry in general was invented to represent geometry algebraically so we do not find much of application.

Secondly after discovering the relation between geometry the concepts of analytical geometry after dimension three is impossible to get any representation.

Further one of the biggest drawback of analytical geometry is that negative number negative planes etc. which may be appreciated theoretically; however finding natural reasonable interpretation is not possible.

For one of the fundamental problems we face with negative number or negative values is that is impossible for anyone to give any form of representations of them. For even negative numbers can be only abstractly written on paper but giving them meaningful description so far is not acceptable by children and persons with nationalism. For show -5 if one asks they may place some 8 balls and say take away 5 balls them we see that -5 ball from 8 balls.

This is the way negative numbers upto $-\infty$ is introduced.

If their argument is good and correct another practical problem in understanding in negative number is if a large number of balls are taken from a set of large number of ball say some 99 balls and 89 ball are taken away then we say

$$+10 > -89 \qquad \ldots(1)$$

still in general understanding this is not easy.

Further if out of 99 balls one takes away just 10 balls then

$89 > -10$ …(2)

This situation (1) and (2) is explained as

$10 > -89$ if multiplied by -1 then > symbol is changed to less than symbol and equation (2) is obtained.

For $10 > -10$ so $-10 < 10$.

What is still a problem with school goers is when one writes $-10 > -89$ they are perturbed they felt when a large part of my belonging is taken how can one say remaining is greater than the taken portion.

But practically if you try to explain this phenomenon with objects school children are not convinced and try to argue what is taken is bigger in this case and what one possesses is smaller.

Such sort of problems a primary school maths teacher always faces and even some time in unguarded moments (some of college professors have argued with me $-n > -m$ when $n > m$; after a minute or two when remained it is the real line and the ordering relation on it accepted and said, just confused).

So only negative numbers is a problem to school children when ordering etc. is imposed on them.

However in analytical geometry we find triangles of the form.

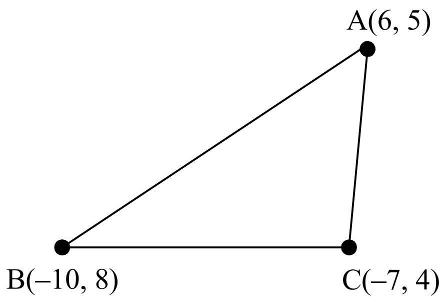


Figure 3.48

and calculate the area etc with them. However if analytical geometry was the algebraic version of geometry then which version of a triangle in geometry gave way to the triangle given in that figure 3.48.

As far as geometry is concerned only positive length was supplied but if a triangle is taken in a the two dimension space or plane (analytical plane) given by the following figure.

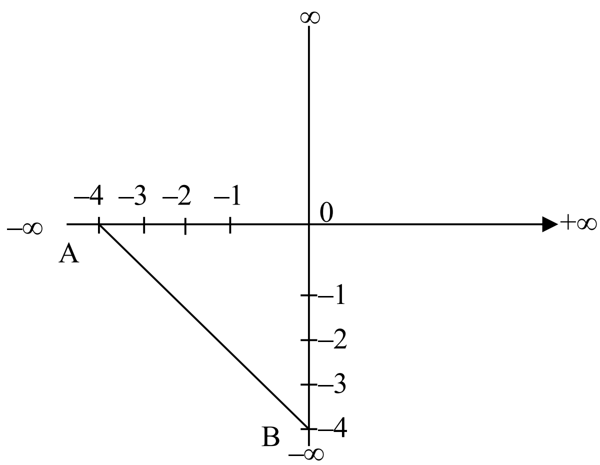


Figure 3.49

What is the geometrical triangle which gave way to this triangle AB. Such questions raised by some graduates while teaching analytical geometry and geometry one cannot give a convincing answer to them.

One observation made by authors is the algebraist in general seek for only a perfect structure. For instance the concepts like field and group stand as an evidence of this in very simple or fundamental case of abstract algebra.

When positive integers was taken immediate reaction was to make a perfect negation of it viz the negative integers was invented.

However Boolean did not fall into this trap that is why he proclaimed using philosophy or psychology which we are not sure to every statement which is true there is a false statement. So if T stands for truth then F stands for false. He extended this notion very diplomatically that if one represents true then 0 represents false. One can represent if something exists is true non existence of that thing is false.

He went onto make one more step by saying

$$T + T = T \text{ or}$$

$$1 + 1 = 1.$$

So even today Boolean algebra happens to be true and more so is the Boolean logic which happens to be the fundamental logic which is used in building all types of logics.

Infact Boolean logic lies as the base for all types of logics. So far one cannot replace it by some other logic.

In fact one can say suddenly not very happy with algebra, algebraist developed modern algebra to tackle several problems. Mostly in the beginning modern algebra basically was built using sets and set theory flourished with the advent of set theory whose basis works like the Boolean algebra basics $1 + 1 = 1$ here also union of set A with itself is A, that is $A \cup A = A$ and $1 \times 1 = 1$ so $A \cap A = A$.

So collection of subsets got from any set satisfies under the two operations $\cup$ and $\cap$ only, the fundamental structure of Boolean algebra.

One of the classical examples of Boolean algebra is powerset P(S) of a set S and $\{P(S), \cup, \cap\}$ is a Boolean algebra. So analogous to the {T, F} table given below.

+	T	F
T	T	T
F	T	F

Table 3.1

$\times$	T	F
T	T	F
F	F	F

Table 3.2

Now set theoretically if $S = \{a\}$ has only one element.

Then P(S) = {ϕ, {a}} is the powerset concept used to illustrate Boolean algebra of order 2^n when n = 1 we have P(S) = {ϕ, {a}} as order of the set S is 1 since S = {a}.

Now the table of P(S) under $\cup$ is given below.

$\cup$	ϕ	{a}
ϕ	ϕ	{a}
{a}	{a}	{a}

Table 3.3

and {P(S), $\cap$} is as follows.

$\cap$	ϕ	{a}
ϕ	ϕ	ϕ
{a}	ϕ	{a}

Table 3.4

Consider table 3.1 and table 3.3. If T is replaced by {a} and ϕ by F then the tables 3.1 and 3.3 are the same. Analogously table 3.2 and table 3.4 are identical with the above mentioned replacement.

We represent the Boolean algebra as

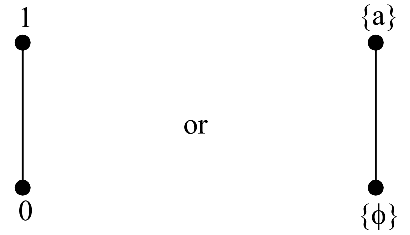


Figure 3.50

In fact this basis is the lattice theory built and all Boolean algebras are lattices however all lattices in general are not Boolean algebras.

So the principle of 1 + 1 = 1 and 1 · 1 = 1 (which is the same as ∪ replace + with ∪ and · with ∩).

Now with set theory if we have S = {a, b} then P(S) the power set of S is P(S) = {ϕ, {a}, {b}, {a, b}}.

Now {P(S), ∪} closed and the table for the same as follows.

∪	ϕ	{a}	(b)	{a, b}
ϕ	ϕ	{a}	{b}	{a, b}
{a}	{a}	{a}	{a, b}	{a, b}
{b}	{b}	{a, b}	{b}	{a, b}
{a, b}	{a, b}	{a, b}	{a, b}	{a, b}

Table 3.5

Now the table for {P(S), ∩} is given in the following

∩	ϕ	{a}	(b)	{a, b}
ϕ	ϕ	ϕ	ϕ	ϕ
{a}	ϕ	{a}	ϕ	{a}
{b}	ϕ	ϕ	{b}	{b}
{a, b}	ϕ	{a}	{b}	{a, b}

Table 3.6

Now {P(S), $\cup$, $\cap$} is a Boolean algebra of order 4 given by the following figure

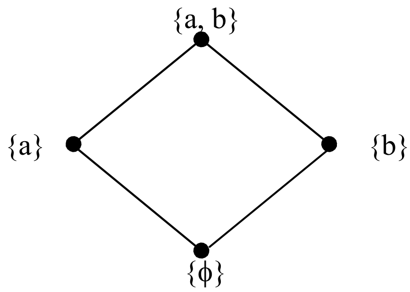


Figure 3.51

Boolean algebra of order 4 is the power set of a set of order 2. That is $4 = 2^2$.

Consider the direct product of

$$\{0, 1\} \times \{0, 1\}$$

$$= \{(0, 0), (1, 0), (0, 1), (1, 1)\}$$

$$= B$$

Table for {B, $\times$} is as follows

$\times$	(0, 0)	(1,0)	(0, 1)	(1, 1)
(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
(1, 0)	(0, 0)	(1, 0)	(0, 0)	(1, 0)
(0, 1)	(0, 0)	(0, 0)	(0, 1)	(0, 1)
(1, 1)	(0, 0)	(1, 0)	(0, 1)	(1, 1)

Table 3.7

Table for (B, +) is as follows.

+	(0, 0)	(1, 0)	(0, 1)	(1, 1)
(0, 0)	(0, 0)	(1, 0)	(0, 1)	(1, 1)
(1, 0)	(1, 0)	(1, 0)	(1, 1)	(1, 1)
(0, 1)	(0, 1)	(1, 10	(0, 1)	(1, 1)
(1, 1)	(1, 1)	(1, 1)	(1, 1)	(1, 1)

Table 3.8

We see (B, +, ×) is a Boolean algebra. The figure 3.52 associated with it is as follows.

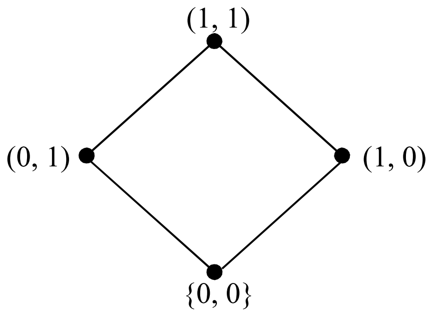


Figure 3.52

(B, +, ×) is a Boolean algebra of order four got by the direct product of a Boolean algebras of order 2.

We see {B, +, ×} is isomorphic with {P(S), ∪, ∩};

Where (S = {a, b}).

The mapping being

$(0, 0) = \phi$

$(1, 0) = \{a\}$

$(0, 1) = \{b\}$

$(1, 1) = \{a, b\}$.

Now consider the set S = {a, b, c}. The power set P(S) of S is as follows:

P(S) = {ϕ, {a}, {b}, {c}, {a, c}, {a, b}, {b, c}, {a, b, c}} be the power set of S.

The table {P(S), $\cup$} is as follows.

$\cup$	ϕ	{a}	(b)	{c}	{a,b}	{b,c}	{a, c}	{a,b,c}
ϕ	ϕ	{a}	{b}	{c}	{a,b}	{b,c}	{a,c}	{a,b,c}
{a}	{a}	{a}	{a,b}	{a,c}	{a,b}	{a,b,c}	{a,c}	{a,b,c}
{b}	{b}	{a,b}	{b}	{b,c}	{a,b}	{b,c}	{a,b,c}	{a,b,c}
{c}	{c}	{a,c}	{b,c}	{c}	{a,b,c}	{b,c}	{a,c}	{a,b,c}
{a,b}	{a,b}	{a,b}	{a,b}	{a,b,c}	{a,b}	{a,b,c}	{a,b,c}	{a,b,c}
{b,c}	{b,c}	{a,b,c}	{b,c}	{b,c}	{a,b,c}	{b,c}	{a,b,c}	{a,b,c}
{a,c}	{a,c}	{a,c}	{a,b,c}	{a,c}	{a,b,c}	{a,b,c}	{a,c}	{a,b,c}
{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}

Table 3.9

Now we give the table for {P(S), $\cap$} in the following.

∩	ϕ	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}	{a,b,c}
ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
{a}	ϕ	{a}	ϕ	ϕ	{a}	{a}	ϕ	{a}
{b}	ϕ	ϕ	{b}	ϕ	{b}	ϕ	{b}	{b}
{c}	ϕ	ϕ	ϕ	{c}	ϕ	{c}	{c}	{c}
{a,b}	ϕ	{a}	{b}	ϕ	{a,b}	{a}	{b}	{a,b}
{a,c}	ϕ	{a}	ϕ	{c}	{a}	{a,c}	{c}	{a,c}
{b,c}	ϕ	ϕ	{b}	{c}	{b}	{c}	{b,c}	{b,c}
{a,b,c}	ϕ	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}	{a,b,c}

Table 3.10

Thus we see {P(S), ∪, ∩} is a Boolean algebra of order $2^3 = 8$ as $|S| = 3$. The figure related with {P(S), ∪, ∩} is as follows.

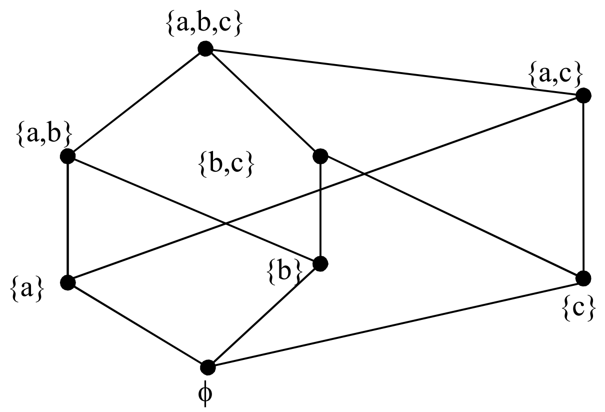


Figure 3.53

Now we know {0, 1} is a Boolean algebra of order two. Let B = {0, 1} × {0, 1} × {0, 1} = {(0, 0, 0), (0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 0), (1, 0, 1), (0, 1, 1), (1, 1, 1)}. The Table of {B +} is as follows.

+	(0,0,0)	(1,0,0)	(0,1,0)	(0,0,1)	(1,1,0)	(1,0,1)	(0,1,1)	(1,1,1)
(0,0,0)	(0,0,0)	(1,0,0)	(0,1,0)	(0,0,1)	(1,1,0)	(1,0,1)	(0,1,1)	(1,1,1)
(1,0,0)	(1,0,0)	(1,0,0)	(1,1,0)	(1,0,1)	(1,1,0)	(1,0,1)	(1,1,1)	(1,1,1)
(0,1,0)	(0,1,0)	(1,1,0)	(0,1,0)	(0,1,1)	(1,1,0)	(1,1,1)	(0,1,1)	(1,1,1)
(0,0,1)	(0,0,1)	(1,0,1)	(0,1,1)	(0,0,1)	(1,1,1)	(1,0,1)	(0,1,1)	(1,1,1)
(1,1,0)	(1,1,0)	(1,1,0)	(1,1,0)	(1,1,1)	(1,1,0)	(1,1,1)	(1,1,1)	(1,1,1)
(1,0,1)	(1,0,1)	(1,0,1)	(1,1,1)	(1,0,1)	(1,1,1)	(1,0,1)	(1,1,1)	(1,1,1)
(0,1,1)	(0,1,1)	(1,1,1)	(0,1,1)	(0,1,1)	(1,1,1)	(1,1,1)	(0,1,1)	(1,1,1)
(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)

Table 3.11

The table for (B, ×) is as follows.

×	(0,0,0)	(1,0,0)	(0,1,0)	(0,0,1)	(1,1,0)	(1,0,1)	(0,1,1)	(1,1,1)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(1,0,0)	(0,0,0)	(1,0,0)	(0,0,0)	(0,0,0)	(1,0,0)	1,0,0)	(0,0,0)	(1,0,0)
(0,1,0)	(0,0,0)	(0,0,0)	(0,1,0)	(0,0,0)	(0,1,0)	(0,0,0)	(0,1,0)	(0,1,0)
(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,1)	(0,0,0)	(0,0,1)	(0,0,1)	(0,0,1)
(1,1,0)	(0,0,0)	(1,0,0)	(0,1,0)	(0,0,00	(1,1,0)	(1,0,0)	(0,1,0)	(1,1,0)
(1,0,1)	(0,0,0)	(1,0,0)	(0,0,0)	(0,0,1)	(1,0,0)	(1,0,1)	(0,0,1)	(1,0,1)
(0,1,1)	(0,0,0)	(0,0,0)	(0,1,0)	(0,0,1)	(0,1,0)	(0,0,1)	(0,1,1)	(0,1,1)
(1,1,1)	(0,0,0)	(1,0,0)	(0,1,0)	(0,0,1)	(1,1,0)	(1,0,1)	(0,1,1)	(1,1,1)

Table 3.12

Now we give the figure of the Boolean algebra {B, +, ×}

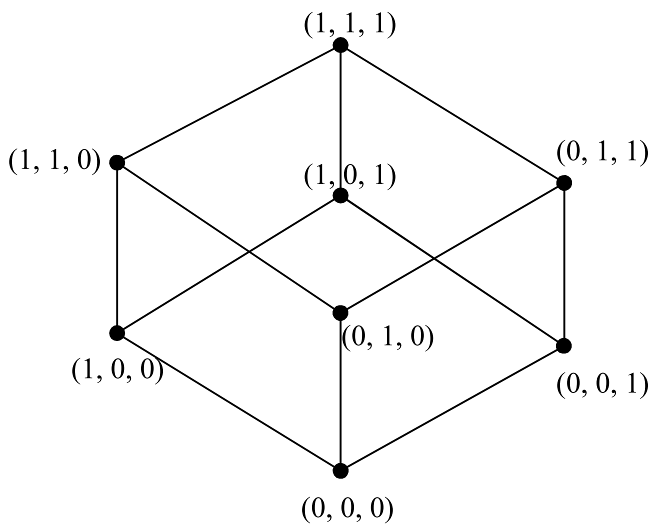


Figure 3.54

We see the figure 3.53 is the same as figure 3.54.

We have the correspondence

(a, b, c)	↔	(1, 1, 1)
{a, b}	↔	(1, 1, 0)
{a, c}	↔	(1, 0, 1)
{b, c}	↔	(0, 1, 1)
{a}	↔	(1, 0, 0)
{b}	↔	(0, 1, 0)
{c}	↔	(0, 0, 1)
{ϕ}	↔	(0, 0, 0)

Thus all Boolean algebras of order $8 = 2^3$ are isomorphic with the power set P(S) where

S = {a, b, c} or with $\{0, 1\} \times \{0, 1\} \times \{0, 1\}$.

Based on this in general we can say all Boolean algebras will be of order 2^n where n = |S| or

$$\underbrace{\{0,1\} \times \{0,1\} \times .... \times \{0,1\}}_{\text{taken n-times}}.$$

However 1 + 1 = 1 and 1 × 1 = 1 is the law following. Thus we make the conclusion that ∪ and ∩ operations on the power set can give perfect algebraic structure, however they do not have for every n ∈ R, –n ∈ R where R is the reals.

With this in mind we wish to conclude the ling spaces of any dimension under the operations ∪ and ∩ will be closed.

To this effect we will give example where we have taken a finite set and test the structure of them under ∪ and ∩.

Example 3.5. Consider the ling variable of a person we take 3 ling variables a associated with a persons viz height, weight and age. We do not for the present take the ling continuum. Further we have just discussed how the concept of analytical geometry as not gone much far than with some ways of finding volume or area or distance which can as well be done by any geometric methods or using formulas.

So with the ling variable height we take the ling set

S_1 = {tall, short}.

With the ling variable age we have the ling set S_2 = {old, young} and with the ling variable weight we have the ling set S_3 = {low, medium}.

For the linguistic triplets

$S_1 \times S_2 \times S_3$ = {(tall, old, low) = x_1, (tall, old, medium) = x_2, (tall, young, low) = x_3, x_4 = (tall, young, medium), (short, old, low) = x_5, x_6 = (short, old, medium), x_7 = (short, young, low) and x_8 = (short, young, medium)}.

Clearly order of P = $S_1 \times S_2 \times S_3$ = 8; further

short ≤ tall, young ≤ old and low ≤ medium.

Thus all the sets are ordered however we cannot order P for consider x_3 and x_5. We cannot order them. But we have well defined operation of min and max defined on P.

For instance min{x, x_2} = {(tall, old, low)} = x_1 and so on.

We provide the table for them. The table for {P, max} is as follows.

max	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_1	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2
x_2	x_2	x_2	x_2	x_2	x_2	x_2	x_2	x_2
x_3	x_1	x_2	x_3	x_4	x_3	x_2	x_3	x_2
x_4	x_2	x_2	x_4	x_4	x_2	x_2	x_4	x_4
x_5	x_1	x_2	x_3	x_2	x_5	x_6	x_5	x_6
x_6	x_2	x_2	x_2	x_2	x_6	x_6	x_6	x_6
x_7	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_8	x_2	x_2	x_2	x_4	x_6	x_6	x_8	x_8

Table 3.13

Now we give the table {P, min} in the following

max	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_1	x_1	x_1	x_3	x_1	x_5	x_5	x_7	x_7
x_2	x_1	x_2	x_3	x_4	x_5	x_5	x_7	x_8
x_3	x_3	x_3	x_3	x_3	x_7	x_7	x_7	x_7
x_4	x_1	x_4	x_3	x_4	x_7	x_8	x_7	x_8
x_5	x_5	x_5	x_7	x_7	x_5	x_5	x_7	x_7
x_6	x_5	x_5	x_7	x_8	x_5	x_6	x_7	x_8
x_7	x_7	x_7	x_7	x_7	x_7	x_7	x_7	x_7
x_8	x_7	x_8	x_7	x_8	x_8	x_8	x_7	x_8

Table 3.14

Now {P, ∪, ∩} is a ling semi ring of order 8. We give the figure of these 8 ling triplets.

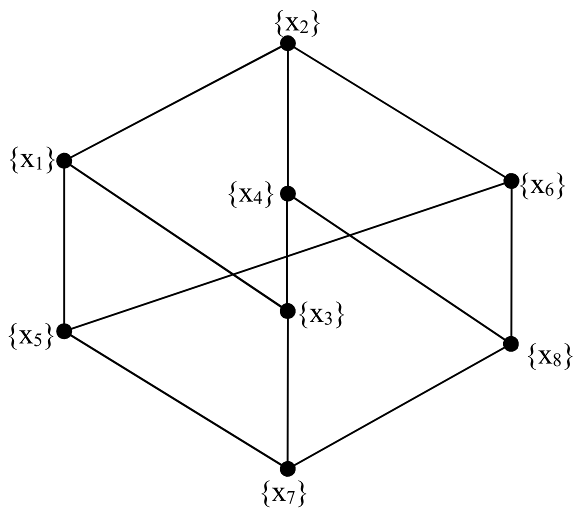


Figure 3.55

This is a ling Boolean algebra under max and min or

{P, max, min} is a ling Boolean algebra of order 8 isomorphic with $\{P(S), \cup, \cap\}$ where S = {a, b, c}.

This is not so even if one the linguistic sets S_1 or S_2 or S_3 has cardinality greater than two. Because $|S_1| = |S_2| = |S_3| = 2$ and all of them are isomorphic with

Figure 3.56

we have the Boolean algebra structure. However if even one of the S_i is of order greater than two then we will not get the Boolean algebra it will only be a lattice. For instance consider the ling sets

S_1 = {tall, short, medium height} and

S_2 = {old, young, very old} associated with the ling variables height and age of persons.

We find $S_1 \times S_2$ = {(tall, old) = x_1, x_2 = (tall, young), (tall, very old) = x_3, x_4 = (short, old), x_5 = (short, young), (short, very old) = x_6, (medium height, old) = x_7, x_8 = (medium height, young), (medium height, very old) = x_9} = S.

Now we give the table for {S, max} in the following:

max	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x_1	x_1	x_1	x_3	x_1	x_1	x_3	x_1	x_1	x_3
x_2	x_1	x_2	x_3	x_1	x_2	x_3	x_3	x_2	x_3
x_3	x_3	x_3	x_3	x_3	x_5	x_3	x_3	x_3	x_3
x_4	x_1	x_1	x_3	x_4	x_4	x_6	x_7	x_7	x_9
x_5	x_1	x_2	x_5	x_4	x_5	x_6	x_7	x_8	x_9
x_6	x_3	x_3	x_3	x_6	x_6	x_6	x_9	x_7	x_9
x_7	x_1	x_3	x_3	x_7	x_7	x_9	x_7	x_7	x_9
x_8	x_1	x_2	x_3	x_7	x_8	x_7	x_7	x_8	x_9
x_9	x_3	x_3	x_3	x_9	x_9	x_9	x_9	x_9	x_9

Table 3.15

We see {S, max} is a ling semigroup.

Now we find {S, min} and the table of {S, min} is given in the following

min	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x_1	x_1	x_2	x_1	x_4	x_5	x_4	x_7	x_8	x_7
x_2	x_2	x_2	x_3	x_5	x_5	x_5	x_8	x_8	x_8
x_3	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x_4	x_4	x_5	x_4	x_4	x_5	x_4	x_4	x_5	x_5
x_5	x_5	x_5	x_5	x_5	x_5	x_5	x_5	x_5	x_5
x_6	x_4	x_5	x_6	x_4	x_5	x_6	x_4	x_5	x_6
x_7	x_7	x_8	x_7	x_4	x_5	x_4	x_7	x_8	x_7
x_8	x_8	x_8	x_8	x_5	x_5	x_5	x_8	x_8	x_8
x_9	x_7	x_8	x_9	x_6	x_5	x_6	x_7	x_8	x_9

Table 3.16

We see {S, min} is again a ling semigroup.

Thus {S, max, min} is a ling semiring or a ling lattice.

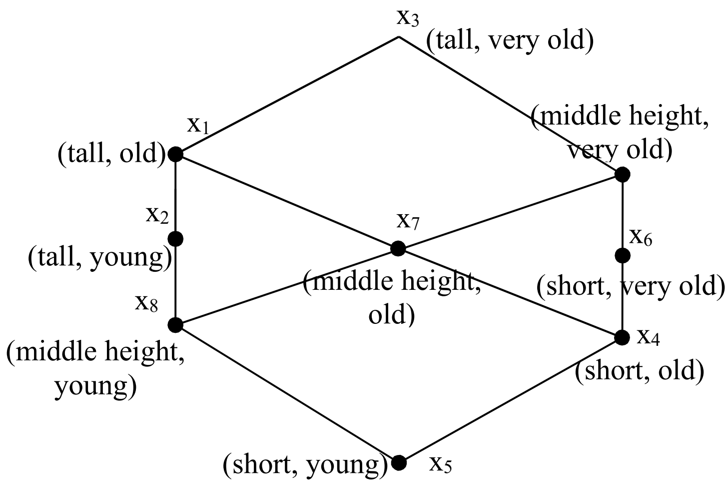


Figure 3.57

Clearly the above figure 3.57 is a ling lattice not a Boolean algebra.

For more about ling lattices refer [23].

Now we have deviated from linguistic analytical geometry of dimension 3 to lattices and Boolean algebras as we are not able to convince ourselves about these structures are the ones which gives the algebraic form of geometry.

Infact we have listed some of it short comings. However in case of ling analytical geometry we cannot define all the classical structures found in the classical coordinate geometry.

We have brought out some of the deviations from classical analytical geometry. In fact our ling analytical spaces or for short ling spaces are useful as ling directed graphs. Most of these ling graphs are directed however there are ling graphs which are not directed.

But most of these ling graphs will be either complete directed ling graphs or complete undirected ling graphs.

We have given an example. Then we saw that negative terms in reals happen to be problematic that is why Boolean never use negative terms only the notion of truth and algebraic structures false transformed as 1 and 0 respectively.

We have discussed the limitations of ling spaces of n dimension and to each ling spaces we have a different set of ling spaces. This concept also have been defined and discussed by us.

We further felt that in ling spaces of dimension n we have ling graphs to have ling row vector as the vertices and the edge weights are also labeled as ling row vectors.

Now ling variable can have ling spaces of dimension n or so. Several related possible results are also discussed. We can easily develop ling multidimensional spaces which solely depends on the ling variable.

This is very non abstract and highly logical for any one can construct ling spaces of any dimension which is dependent on the ling variable.

To improve the comfort zone the semi ling spaces which give parallel real subinterval representation also.

Ling higher n-dimensional spaces L can in some cases function superior to R^n real tuples when the vector $x = (x_1, x_2, \ldots, x_n) \in L$ where each x_i is a ling term which depicts a feature. However, if $y = (y_1, \ldots, y_n) \in R^n$ then $y_i \in R$; $(1 \leq i \leq n)$ are reals. However, depicting features by linguistic terms is much better than describing with real values for the former is non abstract found in formal / natural languages.

We suggest the following problems solving them can make the reader understand the subject with ease. However the starred problems are difficult to solve.

PROBLEMS

1. Give examples of ling variables which contribute to ling lines (continuum).

2. Show by example ling variables which do no contribute to ling continuum.

3. How is a ling line different from real line?

4. Prove every ling subinterval is again a ling continuum.

5. List out the difference between real line and ling lines.

6. Show the ling subintervals of a ling continuum under 'min' operation is a semigroup.

7. Let S be the ling continuum S_u = {collection of all ling subintervals of S}.

a. Is $\{S_u, \cap\}$ a ling semigroup of ling subintervals?

b. Is $\{S_u, \max\}$ a ling semigroup of ling subintervals.

c. Is $\{S_u, \cup\}$ a ling semigroup of ling subintervals? Justify your claim.

d. Show all the operations which are defined on S_u are distinct.

8. Give an example of (2, 1) semi ling space of dimension three.

9. Compare the example you have constructed in problem 8 with ling space of dimension 3 related with it.

10. Can the ling variables colour of eyes, age of a man, height of a man and his complexion form a 4 dimensional ling spaces?

11. Suppose we take the ling variable as properties of mango fruit.

    a. What can be the highest dimension one can have for the ling spaces constructed for this variable?

    b. Can we use semi ling spaces to analyse it? Justify your claim.

12. For the ling variable performances aspects of students in the classroom, what is maximum dimension of the semi-

ling spaces or ling spaces you will associate? Justify your claim.

13. For the problem 12 list out of the possible linguistic origins.

14. Suppose we have the ling variable yield of paddy plants.

    a. What will be the dimension of the semi ling spaces and ling spaces one needs to work with?

    b. Find all the ling origins and semi ling origins mentioned in (a).

15*. Suppose we have the ling sets {good, bad, fair} = S_1 denoting the performance aspects of workers, the product quality being {good quality, worst, average quality} = S_2 the ling set associated with ling variable profit of the company {high, very low, low, just medium} = S_3.

    i) Prove $S = S_1 \times S_2 \times S_3$ is only a ling lattice L.

    ii) Give the sketch of the ling lattice L.

    iv) Is the ling lattice L distributive or modular? Prove your claim?

    v) Can this ling lattice have a ling sublattice which is a ling Boolean algebra of order 8?

16*. Obtain all special features associated with ling spaces.

17*. Distinguish the properties enjoyed by ling spaces and semi ling spaces.

18. What is the advantage of using semi ling spaces?

19. Show that ling analytical geometry has lot of limitations.

20. Prove ling distances are more appropriate in studying real world models.

21. Give some nice applications of classical analytical geometry.

22. Does classical analytical geometry have an applications to networking? Justify your claim.

23. What type of classical analytical geometry is used in computer application? Discuss with illustrations.

24. Suppose we have to built a semi ling space and a ling space using the ling variable yield of plants related with the ling continuum [stunted, tallest] for the ling variable growth in height of plants. The ling continuum related with yield is [very bad, good].

    We can have a maximal 4 dimensional semi ling space.

    i. List out all 3 dimensional semi ling spaces.

    ii. Prove you can have only two dimensional ling space.

    iii. For S_1 = {bad yield, good yield} and

    S_2 = {short, tall, stunted, medium height} be two ling sets;

    a. Find $S_1 \times S_2 = S$.

b. Draw the figure associated with S.

c. Is the figure given in (b) a ling lattice or ling Boolean algebra?

d. Is the ling lattice a ling distributive lattice or a ling modular lattice?

e. If instead of S_1 and S_2 we replace it by P_1 and P_2 where P_1 = {bad yield, good yield} and

P_2 = {stunted, tall} $\subseteq S_2$.

i) In $P = P_1 \times P_2$, show P generates a ling Boolean algebra of order 4.

ii) Can we say as $P \subseteq S$. P is a ling sublattice of a ling lattice L?

25. Show by example ling multidimensional space is more appropriate than the usual multidimensional space.

26. Apply to real world problem the ling directed graphs in ling models.

27. Give an example of a ling directed graph.

28. Can we say only time dependent ling variables give way to directed ling graphs? Justify your claim.

29. Suppose the ling variable V is weather report.

i) Can we say the ling graph associated with it can be undirected? Substantiate your claim.

ii) Is this ling variable V time dependent?

30. Give any other example of a ling variable which is time dependent but the ling graph associated with it is not directed.

31. Prove / disprove even if ling variables does not yield to ling continuum still we can have ling graphs.

32. What are the advantages of using ling graphs in the place of classical graphs? Establish your claim by some examples.

33. Distinguish between ling graphs and classical graphs.

34. Let L be a ling variable which has a linguistic set S that is a ling continuum.

    a. Let S_u = {collection of all subintervals of S}

        i) Prove $\{S_u, \min\}$ is a ling semigroup.

        ii) Will $\{S_u, \cup\}$ is a ling semigroup? Justify your claim.

        iii) Prove $\{S_u, \max\}$ is a ling semigroup. Is it different from the ling semigroup $\{S_\mu, \min\}$? Substantiate your claim.

        iv) Prove $\{S_u, \cap\}$ is a ling semigroup provided S_u contain the ling empty interval $\{\phi\}$.

35. Give an example of a ling variable whose ling set is a totally ordered finite set.

36. Suppose one studies the height of 100 persons. Will the ling set S for these 100 persons for the ling variable height is a totally ordered set?

37. Will the ling variable complexion of people all over the world yield a ling set S which is totally ordered?

38. Will the ling set mentioned in problem (37) be a partially ordered set? Justify your claim.

39. Can one say the ling variable colour of the eyes of cats will yield a ling set P which is a totally ordered set or a partially ordered set will P be a finite or infinite set?

40*. Using a ling variable which has the concept of indeterminacy associated with it. Describe the ling plane. How is it different from usual ling planes with no indeterminacy?

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This book extends the concept of linguistic coordinate geometry using linguistic or semi-linguistic planes. In the case of coordinate planes, we are always guaranteed the distance between any two points in that plane. However, we cannot always determine the linguistic distance between any two points in the case of linguistic and semi-linguistic planes. This is the first limitation of linguistic planes and semi-linguistic planes. Several properties and limitations of dimensional linguistic planes and (1, 1) semi-linguistic planes are discussed. The book also introduces the notion of multidimensional linguistic and linguistic spaces.

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