科学哲学 47-1（2014）

# The Constitution of Space and Time in the *Aufbau* Viewed from a Kantian Perspective

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**Abstract**

The foremost aim of this paper is to realize the fourth part of the *Aufbau*. This part, which provides an actual phenomenalistic constitution system, is interpretable from a Kantian perspective (§§1–4). But Carnap plotted to overcome Kant's old style of philosophy as well. We review this aspect of his constitution, focusing on space (§§7–13) and time (§§5–6), especially.

## 1[1] Introduction

Michael Friedman (1987, pp.89f.) famously defended the *Aufbau*[2] against Quine's criticism (1950, p.40), saying that Carnap's strategy had more affinity with Kantianism (Friedman 1987, p.98)[3]. Indeed, Carnap showed a strong empathy to Kant's philosophy:

(1)[4] By categories, we mean the forms of synthesis of the manifold of intuition into the unity of an object. [… The] manifold of intuition is called "the given" in [our] constitution theory […]. […And t]he synthesis of the manifold into the unity of an object is[, in our theory,] regarded as "the constitution of an object from the given" (Carnap 1928, §83).

"Categories," "synthesis," and "the manifold"—these terms of Kant's were inherited by Carnap as well. Meanwhile, he attempted even modernization of Kant's philosophy, according to Friedman:

(2) [Carnap's] project is not strictly Kantian [...]. For Kant himself, merely formal logic is quite inadequate for the constitution of objectivity, and we need to supplement it with a "transcendental logic" that makes essential reference to intuition: the "pure intuitions" of space and time. Now, in the context of the much more powerful conception of formal logic bequeathed to him by Frege and Russel, Carnap finds such an independent appeal to the "forms of intuition" quite unnecessary [...] (Friedman 1987, pp.98–99).

The bequest of Frege and Russel made Carnap think Kant's framework of space and time unnecessary.

Did Carnap succeed in this strategy? Hereafter, we review Carnap's course of thought exclusively from this perspective[5]. This inquiry provides, on the one hand, a systematic study[6] of the *Aufbau* Friedman never attempted. On the other hand, it leads to a new perspective filling the gap between a traditional philosophy once completed by Kant and a modern analytic philosophy opened up by Carnap.

I strongly hope this study is read by researchers of Kant as well, since herein could be a modernization or formalization of Kant's epistemology. The central figure carrying it out henceforward is nobody but Carnap. Did he fail or succeed? I want readers to make sure of it by themselves.

Carnap's strategy was, as far as this article is concerned, to abstract the Kantian notion of space and time from our personal, primitive experiences, using his original concept of similarity. By doing so, he thought, the Kantian "pure intuitions" of space and time could be removed. But as we shall see, Carnap never succeeded in it: Kant's notion of space and time must be presupposed even in his formalization, because without the notion, the elementary experiences are never provided (§6, §13). Was, then, Carnap's attempt in the *Aufbau* completely frustrated? No. It still remains a great precursor and role model of a modernization of traditional epistemology. And I myself think it could be applied to the explication of analyticity as well if we improve the disputable concept of similarity (§14).

## 2 The Manifold

Now, let us embark on the modernization of Kant's epistemology. What we take as such was adequately stated in the preceding citation (=(1)). Kant dealt with it, notoriously, in naïve psychological terms, according to which the manifold is integrated into a unity of an object, and then recognition is formed from the unity (cf. Kant 1787, B137).

Carnap's formalization of Kant's theory began with the clarification of this naïve terminology, so to speak. First, he worked on that of a typically Kantian term, *the manifold*[7].

Köhler & Wertheimer's Gestalt theory and Schlick's analysis seem to have influenced him at that time. As a result, he reached the following view:

(3) [The given] are the personal experiences[8] themselves in their totality and closed unity (Carnap 1928, §67).

That is, the manifold is personal (private) and never decomposed into atoms.

Carnap put this notion of the manifold at the bottom of his system, calling it *an autopsychological basis*[9].

## 3 Egocentricity

By taking the manifold as a basis, however, Carnap once broke with Kant (Carnap 1928, §§64–65). It is because, in so doing, his picture became fully *subjective*. Then, how on earth can our recognition be objective? This question eventually brought him back to a Kantian picture, which provided him the distinctive notion of form/content (Carnap 1928, §66). Hereby, the subjectivity of the manifold was attributed to its content alone, to which the *form* was applied successively. This is how the manifold turned objective.

Additionally Carnap introduced the core of Kant's theory, *the transcendental apperception*[10], as the bearer of the form. It was stated by Kant with a Cartesian twist in the following way:

(4) "I think" must be capable of accompanying all representations of mine (Kant 1787, B131–132).

Carnap favored this picture, calling it *egocentricity*[11].

## 4 The Recollection of Similarity

The universal ego gives a form to private experiences. The form is nothing but what Carnap called "category" above (cf. (1)). But what exactly is the form?

Carnap adopted only one thing as such: *the recollection of similarity*[12]. With the manifold (the content) newly named *elementary experiences*[13], now the basis of his system was decided:

(5) *Phenomenalistic constitution system*[14]
(i) Basic relation[15] : the recollection of similarity, i.e., Er(x,y) (or xEry)[16].
(ii) Basic element[17] : elementary experiences, i.e., erl$=\{x_1, x_2,\ldots\}$.

The more fundamental of the two was the recollection of similarity; elementary experiences were no more than the elements of its field[18], i.e., erl=fldEr.

## 5 The Constitution of Time

Now we came to the starting point of the *Aufbau*. From this basis, other objects are *constituted*[19]. In the constitution, we deal with that of space and of time above all.

As for time, however, Carnap constituted it with the basic relation, Er, alone. The ground for it was that he thought Er to parallel the order of time. He says:

(6) When [we recall x and find it similar to y], the memory image of the earlier, i.e., x, must be compared with y. Therefore, this recognition process is not symmetric; the way x appears when we compare it with y is different from that when we compare y with x conversely (Carnap 1928, §78).

The recognition of similarity is not symmetric, not reversible. In this respect, it resembles the course of time. It should be noted that the time in question is our *inner state of time*. Precisely here, Kant's theory steps into our picture:

(7) [T]ime decides the relationship of representations in our inner states (Kant 1787, B50).

But did this parallelism work so well? It seems difficult at first sight, because temporal order is fundamentally *linear*[20], i.e., transitive[21] and trichotomous[22].

Similarity lacks, in the first place, transitivity. Suppose Tom and Mike are similar, and so are Mike and John. Yet it is not always the case that Tom and

John are similar[23]. But regarding this defect, Carnap provided a relief measure, the *power relation*[24] symbolized as $Er_{po}$. Its definition is as follows.

(8)[25] $Er_1(x,y) \longleftrightarrow_{def.} Er(x,y)$
$Er^2(x,y) \longleftrightarrow_{def.} \exists z_1[Er^1(x,z_1) \wedge Er(z_1,y)] \longleftrightarrow \exists z_1[Er(x,z_1) \wedge Er(z_1,y)]$
$Er^3(x,y) \longleftrightarrow_{def.} \exists z_2[Er^2(x,z_2) \wedge Er(z_2,y)] \longleftrightarrow \exists z_2[\exists z_1\{Er^1(x,z_1) \wedge Er(z_1,z_2)\} \wedge Er(z_2,y)]$
$\ldots Er^{n+1}(x,y) \longleftrightarrow_{def.} \exists z_n[Er^n(x,z_n) \wedge Er(z_n,y)]$

Here $Er^{k+1}$ is defined so that it complements the lack of transitivity (of $Er^k$). Carnap added extensions further to these; that is, $Er^1 =_{def.} \{<x,y> | Er(x,y)\}$[26], $Er^2 =_{def.} \{<x,y> | Er^2(x,y)\}$ etc. Lastly, these extensions are connected into a union: $Er^0 \cup Er^1 \cup \ldots \cup Er^n = \cup_{i=0}^{n} Er^i$. This union is called *a chain*[27] with the symbol $Er_{po}$.

$Er_{po}$ is the attribute of this class: $Er_{po} = \{<x,y> | Er_{po}(x,y)\}$. It is certainly transitive in distinction from Er; for example, if $Tom Er_{po} Mike$ ($<Tom,Mike> \in Er^1 \subset Er_{po}$) and $Mike Er_{po} John$ ($<Mike,John> \in Er^1 \subset Er_{po}$), then $Tom Er_{po} John$ ($<Tom,John> \in Er^2 \subset Er_{po}$).

This is how Carnap followed up the lack of transitivity. But another defect remained. Trichotomy did not hold even of $Er_{po}$. There could be a pair, xi and $x_j$, of which none of $x_i Er_{po} x_j$, $x_i = x_j$, and $xj Er_{po} xi$ holds; simply put, they could be not similar at all. Nevertheless, Carnap optimistically expected that this defect would be overcome as his constitution develops (1928, §120)[28].

## 6 Goodman's criticism

This is the constitution of time by Carnap. Probably, the author can make any excuse for the technical defect of this kind. Was the constitution then successfully made? Nelson Goodman, who wrote the best commentary (1951, V), objected:

(9) [I]t is questionable whether [Carnap's arguments on the recollection of similarity] make possible a satisfactory constitution of temporal order [...]. Carnap's argument [...] would seem to assume [against his will] that memory images and afterimages [which are temporally specified in advance] are epistemologically as fundamental as [the recollection of similarity] (Goodman 1951, p.132).

"[M]emory images and afterimages"[29] mean the same thing: past experiences. Regarding them, Goodman insists: to recognize the past experiences *as such*, i.e., to *specify* them in time, we need more than the recollection of similarity.

This makes sense, practically. Consider the case where we recognize a similarity between two past experiences. How exactly could we know one is temporally precedent to the other? It is impossible unless we specify them in time *in advance*. For this very reason, we cannot but say Carnap's argument is quite unsatisfactory.

## 7 The Other Point at Issue: Space

This is how Carnap's theory of time is criticized. It did not supersede Kant's theory, either[30]. Then, what about space? Let us continue our inquiry.

Space is concerned with our visual field in particular. Hence we need more detailed information of the manifold. But unfortunately, it is not found in the *Aufbau* (cf. Kleinknecht 1980, p.23 note1). Nevertheless, by reference to other researches (cf. Leitgeb 2007, p.190, Goodman 1951, p.141), we can provide it:

(10) Now I see a red spot in the upper left place of my visual field and a blue spot in the lower right place.

This is an example of the manifold. How is space constituted from this coarse, raw experience? This is our concern below.

## 8 Color Spot

For the discussion, let us first segment the preceding example:

(11) <now, <red, the upper left>, <blue, the lower left>>

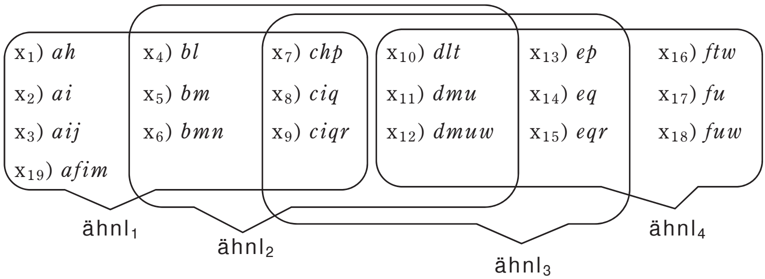
Although these expressions are merely for simplicity, it is quite interesting that these ordered pairs[31] show a similar structure to *protocol sentences*[32].

In this formulation, we realize that an elementary experience consists of two parts: specification of time like "now," and a color spot like "<red, the upper left>." The *color spot* is so important in the following discussion, which was introduced by Goodman[33] and defined as a pair of a *color* like "red" and a *place* like "the upper left." Clearly, this latter factor is concerned with our present interest: space.

Our experience could have different color spots at one time like "<red, the upper left>" and "<blue, the lower left>" in (11)[34]. So the elementary experience is fundamentally *a colorfully spotted plane* (*two-dimensional visual field*).

We are largely indebted to Goodman for this interpretation (1951, p.141). Following him, we can arrange elementary experiences in the following way:

(12)[35]




Here, to each color-spot, e.g., "<red, the upper left>," one alphabet, e.g., "a" is assigned. And "$x_i$" stands for one elementary experience, on the right side of which its content is shown like "*ah*."

## 9 Similarity Circle

However, the most outstanding above is surely the balloons. They stand for *the similarity circles based on Ae*[36]. This is an *abstraction* concept in Carnap's system[37] and defined as follows:

(13)[38] $\text{ähnl}_i$ is a similarity circle[39]

$$\longleftrightarrow_{\text{def.}} \forall x \forall y[((x \in \text{ähnl}_i \land y \in \text{ähnl}_i) \rightarrow xAey) \land ((x \in \text{ähnl}_i \land y \notin \text{ähnl}_i) \rightarrow \neg(xAey))]$$

This formula says, "In a similarity circle, every element is similar to *any* of the other members." Take another look at (12). Certainly, in one balloon, every element is similar to any of the other members. But what exactly does it mean? What is "similarity" in the first place?

In (13) just stated, *Ae* stands for *part similarity*[40], which is reduced to Er:

(14) For any x and y, xAey $\longleftrightarrow_{def.}$ xEry ∨ yErx ∨ x=y (Carnap 1928, §110)[41]

But there is room to discuss the notion of Er further. Actually, Er is definable here in a stricter manner than before (§5):

(15) For any x and y, xEry $\longleftrightarrow_{def.}$ at least one alphabet of x is adjacent to at least one alphabet of y (cf. Goodman 1951, p.127)[42].

See, e.g., $ähnl_1$ (cf. (13)). Certainly, x1Erx$_3$ holds since a in $x_1$ is the same as (thus, adjacent to) *a* in $x_3$; again, $x_1Erx_9$ holds since *h* in $x_1$ is adjacent to *i* in $x_9$. This is how we realize every element is similar to *any* of the other members in a similarity circle.

## 10 Quality Class

The constitution of space is made by abstracting places from the similarity circles. But places are still inside color spots, which consist of elementary experiences in similarity circles. So next, we must take color spots out of similarity circles, and then, abstract the places.

As we see in (13), each color spot is already arranged neatly. For example, $\{x_4, x_5, x_6\}$ brings *b* into relief. Interestingly enough, these arrangements occupy *vertical* spaces alone, where similarity circles overlap each other. Carnap called them *essential overlaps*[43] (cf. Goodman 1951, pp.134f.).

Thus, if we can take out essential overlaps, it soon leads to the abstraction of color spots. On this procedure, Carnap had two obstacles in mind[44]. One is the case where a class not fitting into the vertical space is wrongly chosen. For example, $\{x_{19}, x_5, x_6, x_{11}, x_{12}\}$ is possibly chosen to abstract *m*. However, in the present situation, it is not favorable.

The other is the case where a subclass of an essential overlap is wrongly chosen. For example, $\{x_{17}, x_{18}\}$ is possibly chosen to abstract *u*. But in the present situation, it is not favorable.

To avoid the first obstacle, Carnap laid down the following regulation.

(16)[45] $\forall\gamma[\{(\gamma \text{ is a similarity circle}) \land \exists x(x \in \alpha \land x \in \gamma)\} \rightarrow (\alpha \subset \gamma)]$

"α" stands for "a quality class for one color spot," which we call a *color-spot* *class* hereafter (cf. Goodman 1951, p.140). Now, (16) says, "If some members of α belong to a similarity circle, α as a whole must be included in the circle." By this regulation, $\{x_{19}, x_5, x_6, x_{11}, x_{12}\}$ is excluded. Take this class as α; $x_{19} \in$ ähnl$_1$, but $\{x_{19}, x_5, x_6, x_{11}, x_{12}\} \not\subset$ ähnl$_1$.

To avoid the second obstacle, Carnap laid down the following regulation:

(17)[46] $\forall x[(x \notin \alpha) \rightarrow \exists \delta\{(\delta \text{ is a similarity circle}) \wedge (\alpha \subset \delta) \wedge (x \notin \delta)\}]$

Roughly speaking, this means: "For any x outside α, there must be a bigger circle (=δ) to which x does not belong, either." By this regulation, $\{x_{17}, x_{18}\}$ is excluded. Take this class as α; $x_{16} \notin \{x_{17}, x_{18}\}$, but there is no similarity circle which $\{x_{17}, x_{18}\}$ is wholly included in, and x16 does not belong to.

By these regulations, the abstraction of color spots seems to be made smoothly. But practically, the first regulation (=(16)) was too strong.

Have a look at $x_{19}$ in (12). Although this is not illustrated, ähnl$_1$ and ähnl$_4$ overlap at $x_{19}$. In other words, a member ($=x_{19}$) of $\{x_1, x_2, x_3, x_{19}\}$, which is a subset ofähnl$_1$ and promising for the abstraction of a, belongs to ähnl$_4$ as well. But $\{x_1, x_2, x_3, x_{19}\}$ is not wholly included in ähnl$_4$. So it violates regulation (16).

Carnap called this kind of accident *an accidental overlap*[47] (cf. Goodman 1951, pp.134f.). To keep $\{x_1, x_2, x_3, x_{19}\}$ promising for a, he then turned his eyes to the *number* of the members belonging to a similarity circle. As for $\{x_1, x_2, x_3, x_{19}\}$, the number of its members belonging to ähnl$_4$ is only one ($=x_{19}$), while that belonging to ähnl$_1$ is all of the four. Distinguishing these two cases, Carnap laid down the bar of *half*, through which the accidental overlap is avoided. With this modification, the color-spot class is defined as follows:

(18) (α is a color-spot class)
$\longleftrightarrow_{\text{def.}} \forall \gamma[\{(\gamma \text{ is a similarity circle}) \wedge (\frac{|\alpha \cap \gamma|}{|\alpha|} > \frac{1}{2})\} \rightarrow (\alpha \subset \gamma)] \wedge \forall x[(x \notin \alpha) \rightarrow \exists \delta\{(\delta \text{ is a similarity circle}) \wedge (\alpha \subset \delta) \wedge (x \notin \delta)\}]$[48] (Carnap 1928, §112).

The first conjunct on the right side (="∀γ...") is the modified version of (16), and the second (="∀x...") is the same as (17). Hereafter, we symbolize the class of all color-spot classes, i.e., $\{\alpha_1, \ldots, \alpha_n\}$, as *qual*[49].

## 11 Similarity between color-spot classes

Now then, suppose that we obtain some color-spot classes, $\alpha_1$, $\alpha_2$, …, from (18). The next step is to partition them into "similar" groups. For this purpose, we must define the "similarity" concept between color-spot classes in advance:

(19) For any $\alpha_i$ and $\alpha_j$, $\alpha_i Aq \alpha_j \longleftrightarrow_{def.} \forall x \forall y[(x \in \alpha_i \wedge y \in \alpha_j) \rightarrow xAey]$[50]

Definiendem *Aq is the similarity between color-spot classes*[51]. Taking Def. (14) of Ae into account, simply this formula reduces Aq to Er, which was defined earlier (=(15)). But oddly enough, Carnap introduced a *new* similarity concept here again:

(20)[52] Let x be $<t_1, <c_1, p_1>>$, and y be $<t_2, <c_2, p_2>>$[53]. Then, xAey $\longleftrightarrow_{def.}$ $[(c_1 \text{ is similar to } c_2) \wedge (p_1 = p_2)] \vee [(c_1 = c_2) \wedge (p1 \text{ is near (similar to) } p_2)]$.

Although incompatible with previous arguments, this new concept of similarity actually work very well, which properly partitions qual, i.e. $\{\alpha_1, \ldots, \alpha_n\}$, into equivalence classes:

(21)[54] Make a power relation of Aq, i.e., $Aq_{po}$, which becomes an equivalent relation as well[55]. Thereby, we can partition qual into $\{\{\alpha_1, \ldots\}, \{\alpha_2, \ldots\}, \ldots\}$, that is, $\{\{\alpha_i | \alpha_1 Aq_{po} \alpha_i\}, \{\alpha_j | \alpha_2 Aq_{po} \alpha_j\}, \ldots\}$[56]. This latter class is called *a partition of qual modulo $Aq_{po}$* (*a quotient set of qual modulo $Aq_{po}$*), symbolized as *qual/$Aq_{po}$* or $\{[\alpha_i]_{Aq_{po}} | \alpha_i \in \text{qual}\}$[57].

## 12 The Visual Field Place

Each member of this class, $[\alpha_i]_{Aq_{po}}$ ($\in$qual/$Aq_{po}$), is called an equivalence class[58]. Let us then take one, $[\alpha_1]_{Aq_{po}}$) ($=\{\alpha_1, \alpha_3, \ldots\} = \{\alpha_i | \alpha_1 Aq_{po} \alpha_i\}$)[59], numbering its members all over again: $\{\alpha_{11}, \alpha_{12}, \ldots\}$ ($=\{\alpha_1, \alpha_3, \ldots\}$). Its content is, taking (19) and (20) into account, supposed to be as follows (Given that the class is composed of only four elemenets):

(22) $\alpha_{11}$=the color-spot class for <red, the upper left>
$\alpha_{12}$=the color-spot class for <pink, the upper left>

$\alpha_{13}$=the color-spot class for <orange, the upper left>
$\alpha_{14}$=the color-spot class for <red, the left>

Here, $\alpha_{11}\mathrm{Aq}\alpha_{12}$ holds, because any member of $\alpha_{11}$, which has the form <t, <red, the upper left>, …>, is similar (from Def. (20)) to any member of $\alpha_{12}$, which has the form <t, <pink, the upper left>, …>, so that $\alpha_{11}$ and $\alpha_{12}$ are taken to belong to the same equivalence class, $[\alpha_1]_{\mathrm{Aq_{po}}}$, in accordance with (21).

How can we then take the spatial place "the upper left" out of this class? This is the final stage of our abstraction. Carnap focused on a certain feature here:

(23) Two color-spot classes indicating the same spatial place cannot have any elements in common[60].

For example, $\alpha_{11}$, $\alpha_{12}$, and $\alpha_{13}$ above cannot have any elements (elementary experiences) in common, because they refer to the same place, "the upper left."

This could be realized if we admit one place, even if referred to by different experiences, cannot have different colors. Surely it *can* if we admit a certain length of time in the experiences. But Carnap excluded such cases by adding a proviso "at the same time"[61] to (23)[62].

Thus, to abstract the spatial place, we should partition $\{\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}\}$ into subclasses the members of which do not have any elements in common. We introduce the following relationship to carry it out:

(24)[63] For any $\alpha_i$, $\alpha_j \in \mathrm{qual/Aq_{po}}$, $\alpha_i \mathrm{Fre} \alpha_j \longleftrightarrow_{\mathrm{def.}} \alpha_i = \alpha_j \lor \neg\exists x[x\in\alpha_i \land x\in\alpha_j]$

Using this *Fre*, we can partition $\mathrm{qual/Aq_{po}}$ into the following *k*'s:

(25) k is a certain subclass of an equivalence class in $\mathrm{qual/Aq_{po}}$ the members of which do not have any elements in common $\longleftrightarrow_{\mathrm{def.}}$ For any $\alpha_i$ and $\alpha_j \in \mathrm{qual/Aq_{po}}$, $[((\alpha_i\in k \land \alpha_j\in k) \rightarrow \alpha_i \mathrm{Fre} \alpha_j) \land ((\alpha_i\in k \land \alpha_j\notin k) \rightarrow \neg(\alpha_i \mathrm{Fre} \alpha_j))]$.

Compare this with (13) above. As is soon realized, k is taken *a similarity circle based on Fre*. And we symbolize the class of all k's as *Sim'Fre* (Carnap 1928,

§117). This k is much the same as the spatial places sought for. The following definition becomes a finish:

(26)[64] P is a place $\longleftrightarrow_{\text{def.}} \exists\alpha_i(\alpha_i \in P) \wedge \exists k(k \in Sim`Fre) \wedge P=(k-\cup(Sim`Fre-\{k\}))$

A concrete example may help to understand this definition:

(27) $x_1$=<t[65], <red, the upper left>>
$x_2$=<t, <red, the upper left>, <green, the lower right>>
$x_3$=<t, <red, the upper left>, <blue, the center>>
$x_4$=<t, <pink, the upper left>>
$x_5$=<t, <pink, the upper left>, <black, the just lower part>>
$x_6$=<t, <orange, the upper left>>
$x_7$=<t, <orange, the upper left>, <green, the just upper part>>
$x_8$=<t, <red, the left>>
$x_9$=<t, <red, the left>, <pink, the upper left>>
$x_{10}$=<t, <red, the left>, <red the upper left>>

Following the preceding notation (§8), $x_1$ can be symbolized as "*b*," $x_2$ as "*bi*," $x_3$ as "*bm*," $x_4$ as "*c*," $x_5$ as "*cu*," $x_6$ as "*d*," $x_7$ as "*dw*," $x_8$ as "*a*," $x_9$ as "*ac*," $x_{10}$ as "*ab*." These are, in accordance with (13) and (15), put into one similarity circle, while the color spot <red, the upper left> is common among $x_1$ to $x_3$ and $x_{10}$, <pink, the upper left> among $x_4$, $x_5$ and $x_9$, <orange, the upper left> between $x_6$ and $x_7$, <red, the left> among $x_8$ to $x_{10}$, respectively. Suppose that these groups are located in the essential overlaps (cf. §8). Then, color-spot classes corresponding to (22) above are constituted.

(28) $\alpha_{11}=\{x_1, x_2, x_3, x_{10}\}$, $\alpha_{12}=\{x_4, x_5, x_9\}$, $\alpha_{13}=\{x_6, x_7\}$, $\alpha_{14}=\{x_8, x_9, x_{10}\}$

The preceding definitions, (25) and (26), are understandable from this instance. First, recall that $\{\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}\}$ forms an equivalence class. That is, $\{\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}\}=\{\alpha_i \mid \alpha_1 Aq_{po} \alpha_i\} \in qual/Aq_{po}$. Then, we can constitute "the similarity circles based on Fre" in accordance with (25):

(29) $k_1=\{\alpha_{11}, \alpha_{12}, \alpha_{13}\}$, $k_2=\{\alpha_{13}, \alpha_{14}\}$

For a finish, the spatial places are constituted in accordance with (26):

(30) Sim‘Fre$=\{k_1, k_2\}$

(i) Let $k_1$ be k in (25). Then, $\cup$(Sim‘Fre $-\{k_1\}=\cup\{k_2\}=k_2$[66]. Therefore, $k_1-\cup$(Sim‘Fre$-\{k_1\})=k_1-k_2=\{\alpha_{11}, \alpha_{12}, \alpha_{13}\}-\{\alpha_{13}, \alpha_{14}\}=\{\alpha_{11}, \alpha_{12}\}$[67]$=$P.

(ii) Likewise, when $k_2$ is k in (25) instead, $\{\alpha_{14}\}=$P.

In the case of (i), the place "the upper left" is abstracted. In the case of (ii), the place "the left" is abstracted. Each of them is called a *visual field place*[68], which is nothing but the spatial place we have sought for.

## 13 Evaluation

This is how space was constituted in the *Aufbau*. Let us then ask: Could it supersede Kant's picture? Kant's picture here means the following:

(31) Through external senses, we represent objects as outside of us and in space. It is in the space that we recognize the form of the objects [as far as they appear to us], their size [as far as they appear to us], and their mutual relations [as far as they appear to us] (Kant 1787, B37).

In this passage, Kant defines space not as something like a coordinate, but as a fundamental framework for our recognition of external objects. It implies that even initial recognition of spatial locations, such as "right," "left," "upper," "lower," "in front of," and "behind," is unfeasible without that framework[69].

It is true that Carnap succeeded in the abstraction of space. But it never follows that Kant's framework of space is no longer necessary. The fact is the opposite. Kant's framework is indispensable even for the *Aufbau*. For see the original example of the elementary experience (=(10)); therein, the spatial locations, "the upper left" and "the lower right," are inscribed in an inerasable manner, which means: the spatial location is indispensable even for the elementary experience, since otherwise we would be lost in regard to where each color is. For this reason, we should say, the *Aufbau* never superseded Kant's philosophy; it still needs the latter framework.

## 14 Conclusion

We have seen Carnap's course of argument in the *Aufbau* from a Kantian perspective. After all, it never superseded Kant's picture. This does not mean, however, Carnap's theory was useless. Its significance remains. At the cutting edge, Hans Leitgeb (2007, 2011) has worked on its revival.

As for me, I think the *Aufbau* is more suitable to explicate *analyticity*. Analyticity cohesively concerns the concept of the objects, which is also the specialty of the constitution theory, as we have seen so far.

While details are left to another paper, there remains a few parts to be corrected in Carnap's theory. In particular, its central notion, *similarity*, is still unclear. As much as three characterizations of it are presented heretofore; that is, (7), (15), and (20). We will not be able to apply the theory of the *Aufbau* to the explication of analyticity until this ambiguity is removed.

**Notes**

1. Each section is referred to with the symbol "§." But "§" also stands for "section" of the *Aufbau*, for example.
2. As for the abbreviation of titles, see REFERENCES.
3. Pincock's survey (2009) is informative for the overview.
4. The translation of German texts (of Carnap's and of Kant's) is arbitrarily made by the author.
5. However, we do not deal with the relationship of our argument to modern physics like Einstein's relativity theory.
6. Hans Leitgeb (2007, 2011) is one of the researchers who develop the technical aspect of the *Aufbau*. But his study is not concerned with Kant's philosophy.
7. "Das Mannigfaltige" (Carnap 1928, §83).
8. "Die Erlebnisse" (Carnap 1928, §64).
9. "Eine eigenpsychische Basis" (Carnap 1928, §§63-64).
10. "Transzendentales Subjekt" (Carnap 1928, §66).
11. "Ich-Bezogenheit" (Carnap 1928, §65).
12. "Die Ähnlichkeitserinnerung" (Carnap 1928, §66).
13. "Elementarerlebnis" (Carnap 1928, §66).
14. A *constitution system* is a system according to which almost all the objects are "constituted" (cf. note19) from more basic ones. We could imagine many systems of that kind, but they are theoretically unified into the *constitution theory*, which

Carnap elaborated on in the first half of the *Aufbau* (Carnap 1928, §§25-26).

15. Grundbeziehung” (Carnap 1928, §61, §78).
16. Relations are symbolized as xRy or as R(x,y) depending on contexts.
17. “Grundelement” (Carnap 1928, §61, §67).
18. “Das Feld” (Carnap 1928, §34). Carnap symbolized it as “C‘Er” (1928, §109, §34) in accordance with *Principia* (Whitehead&Russel 1910, p.35). In any case, C‘Er=fldEr=domEr∪ranEr={x|∃y(xEry)∨∃z(zErx)} (cf. Enderton 1960, p.40).
19. *Constitution* is to define a less basic (so abstract) object from more basic ones by the *constitutional definition* (Carnap 1928, §35). The constitutional definition has two kinds. One is the *explicit definition*. The other is the *definition in use* or the *contextual definition* (Carnap 1928, §§38-40, Whitehead & Russel 1910, p.25, p.69). All the definitions below are the definitions in use, because they define the objects in the context of a biconditional sentence as a whole.
20. See Sugihara’s analysis, for example (1974, pp.38f.). It is naturally true of Kant’s theory (Sakai 1978, p.64, Kant 1787, B46). As for the notion of linear ordering, see Enderton’s explanation (1977, p.170).
21. For any $t_1$, $t_2$, and $t_3$, if $t_1 \prec t_2$ and $t_2 \prec t_3$, then $t_1 \prec t_3$. (“$t_i \prec t_j$” stands for “ti is temporally prior to $t_j$.”)
22. For any $t_1$ and $t_2$, exactly one of the following three holds: $t_1 \prec t_2$, $t_1 = t_2$, or $t_2 \prec t_1$.
23. This feature (intransitivity) was stated by Carnap in his logical definition of similarity (Carnap 1928, §11). But there, in contrast with Er, symmetricity and reflexivity were admitted.
24. “Potenzrelation” (Carnap 1928, §34). This concept is attributable to *Principia* (Whitehead & Russel 1910, pp.35-36).
25. We use logistic notation a bit sloppily. For example, “∀” is sometimes replaced with “for any…,” and often omitted in the case of definition especially. Again, we do not observe the distinction between the object and the meta-language, the application of Quine’s quasi-quotes, and so on.
26. Carnap distinguished *Beziehung* from *Relation* (1928, §28, §34). The latter is the extension of the former. He symbolized the former as Er, for example, and the latter as Er (Carnap 1928, §109). Then, Er={<x,y>|Er(x,y)}.
27. “Eine Kette” (Carnap, 1928, §34).
28. Carnap called trichotomy “Zusammenhang” (1928, §11).
29. Goodman sometimes says “a memory image or afterimage” (1951, p.132).
30. Not supporting the Kantian view, actually Leitgeb (2011, p.280) adopted the temporal order as a primitive term in his “*Aufbau*-like” system.
31. Carnap defined *the sensation* (*die Empfindung*) in a similar manner (1928, §93, §116), but it is a pair of an elementary experience and a quality class or color-spot (cf. Goodman 1951, p.145).

32. “Protokollsätze” (Carnap 1932, p.438).
33. Goodman’s notation is “color-spot” (1951, p.134). Carnap generally called it a quality class (1928, §81).
34. Although there are naturally far more color spots in reality, we ignore this point in the present discussion.
35. This figure is made from Goodman’s rough arrangement (1951, p.135).
36. “Die Ähnlichkeitskreise in bezug auf Ae” (Carnap 1928, §80, §111).
37. Russel’s *principle of abstraction* (1937, §210) is one of its predecessors (Carnap 1928, §97), which is originally an application of the *partition into equivalence classes* in set theory (cf. Enderton 1977, pp.55f., Leitgeb 2007, p.181). On the other hand, Carnap (1928, §§69–73) called this method *quasianalysis* (*Quasianalyse*), because abstracting a quality from an elementary experience is contradictory to his doctrine of totality (cf. (3)). Whether this self-criticism is taken seriously or not, most researchers later challenged this part exclusively. Among them were Goodman’s famous *companionship difficulty* (1951, p.123) and *difficulty of imperfect community* (1951, p.125). Since this criticism, most researchers customarily have dealt with Goodman’s argument (Kleinknecht 1980, Leitgeb 2007; 2011).
38. Carnap did not articulate this definition except in an informal style (1928, §71). We are indebted to later researchers for this definition (Leitgeb 2007, p.214, Kleinknecht 1980, p.24, Goodman 1951, p.121).
39. Instead of “ähnl$_i$,” we can use “Sim‘Ae$_i$” (Carnap 1928, §111).
40. “Teilähnlichkeit” (Carnap 1928, §77).
41. As Goodman suggested (1951, pp.132–133), in this definition, Carnap is said to have withdrawn his doctrine on the temporal order of Er (cf. §5). But now it does not matter since we have already discarded that doctrine (§6).
42. But this definition is exclusively concerned with the preceding figure (=(12)). So Carnap did not state it.
43. “Wesentliche Überdeckungen” (Carnap 1928, §80).
44. We owe the following argument to Goodman (1951, pp.135–136).
45. See the first proviso of Goodman’s (1951, p.135).
46. See the second proviso of Goodman’s (1951, p.135).
47. “Eine zufällige Überdeckung” (Carnap 1928, §80).
48. “$|\alpha|$” stands for α’s cardinality.
49. This is originally used for the class of all quality classes (Carnap 1928, §112).
50. We can obtain our formulation of (19) from Carnap’s original (1928, §114) by reference to *Principia* (Whitehead&Russel 1910, p.278).
51. In Carnap’s terminology, “die Ähnlichkeit zwischen Qualitäten” (1928, §114).
52. This was stated only in an informal style (Carnap 1928, §88, Goodman 1951, p.140).

53. See (12). "t" stands for time, "c" for a color, and "p" for a place. Strictly speaking, x must be $<t_1, <c_1, p_1>, \ldots>$ to make it general. But in the present discussion, we omit "…" for simplicity.
54. Carnap provided this definition as the constitution of the sense class (die Sinnesklasse) symbolized as "$Aeq‘Aq_{po}$" (1928, §115).
55. The relation which is reflexive, symmetric and transitive (cf. Enderton 1962, p.56). As for the power relation, see (8) above.
56. Here, we step up from a class (qual) to a class of classes ($qual/Aq_{po}$). This is nothing but constitution (Carnap 1928, §40).
57. In detail, see Enderton's explanation (1962, p.57), for example.
58. In detail, see Enderton's explanation (1962, p.57), for example.
59. "$\alpha_1$" is replaceable with other members, e.g., "$\alpha_3$."
60. This is not stated by Carnap directly (1928, §88). See also Goodman's commentary (1951, p.140).
61. "zugleich" (Carnap 1928, §88).
62. Here, we can realize what kind of experience Carnap had in mind.
63. In Carnap's notation, $\alpha i Fre \alpha_j \longleftrightarrow_{def.} \alpha_i I \alpha_j \vee \alpha_i Fr \alpha_j$ (1928, §97, §117).
64. In Carnap's notation, "$\exists \alpha_i(\alpha_i \in P)$" was "$\exists ! P$." But this does not stand for the unique existence (cf. Canrap 1928, §97, Whitehead&Russel 1910, p.229).
65. These are supposed to be experienced within a short length of time (cf. note62).
66. $\cup\{a\}=a$ (Enderton 1962, p.25).
67. Recall the calculation of the relative complement (Enderton 1962, p.27).
68. "Sehfeldstelle" (Carnap 1928, §88, §117).
69. Recall the famous argument in *Prolegomena* as well (Kant 1783, §13).

**REFERENCES (Letters in square brackets mean abbreviations)**

Carnap, R. (1928). *Der logische Aufbau der Welt*. Felix Meiner. [*Aufbau*]

———(1932). Die physikalische Sprache als Universalsprache der Wissenschaft. *Erkenntnis*, Bd.2.

Goodman, N. (1951). *The Structure of Appearance*. Harvard U.P.

Enderton, H.B. (1977). *Elements of Set Theory*. Academic Press.

Friedman, M. (1987). Carnap's *Aufbau* Reconsidered. In *Reconsidering Logical Positivism*. Cambridge U.P.

Kant, I. (1783). *Prolegomena zu einer jeden künftigen Metaphysik, die als Wissenschaft wir auftreten können*. Felix Meiner. [*Prolegomena*]

———(1787). *Kritik der reinen Vernunft*. I. KANTS WERKE hrsg. von E. Cassirer Bd.III. [*KrV*]

Kleinknecht, R. (1980). Quasianalyse und Qualitätklassen. *Ganzer Philosophische Studien*

II.
Leitgeb, H. (2007). A New Analysis of Quasianalysis, *Journal of Philosophical Logic*, 36(2).
———(2011). New Life for Carnap's *Aufbau*? *Synthese*, 180(2).
Quine, W.v.O. (1950). Two Dogmas of Empiricism. In *From a Logical Point of View*. Harvard U.P.
Pincock, C. (2009). Carnap's Logical Structure of the World, *Philosophy Compass* 4/6.
Russel, B. (1937). *The Principle of Mathematics* (2nd ed.). WW.Norton & Company, INC.
Sakai, H. (1978). A Study of Kant's theory of Scheme in KrV. In *Philosophical Investigations*. The University of Tokyo Press. (Japanese)
Sugihara, T. (1974). *The Logic of Time*. Waseda U.P. (Japanese)
Whitehead,A.N.&Russel,B. (1910). *Principia Mathematica* vol.1. Merchant Books.

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