# Team Reasoning and a Rank-Based Function of Team's Interests\*

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#### Abstract

Orthodox game theory is sometimes criticized for its failure to single out intuitively compelling solutions in certain types of interpersonal interactions. The theory of team reasoning provides a resolution in some such cases by suggesting a shift in decision-makers' mode of reasoning from individualistic to reasoning as members of a team. The existing literature in this field discusses a number of properties for a formalized representation of team's interests to satisfy: Pareto efficiency, successful coordination of individuals' actions and the notion of mutual advantage among the members of a team. For an explicit function of team's goals a reference is sometimes made to the maximization of the average of individuals' personal payoffs, which meets the Pareto efficiency and (in many cases) coordination criteria, but at times fails with respect to the notion of mutual advantage. It also relies on making interpersonal comparisons of payoffs which goes beyond the standard assumptions of the expected utility theory that make numerical representations of individuals' preferences possible. In this paper we propose an alternative, rank-based function of team's interests that does not rely on interpersonal comparisons of payoffs, incorporates the notion of mutual advantage and satisfies the weak Pareto efficiency and (in many cases) coordination criteria. We discuss its predictions using a number of examples and suggest a few possibilities for further research in this field.

#### 1 Introduction

The standard rational choice theory is sometimes criticized for its inability to single out what at times appears to be the only obvious choice to make in games with multiple Nash equilibria. An example is the simple Hi-Lo game, in

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which two players independently and simultaneously choose one from a pair of available options: Hi or Lo. If both choose Hi, they get a payoff of 2 each. If both choose Lo, they get a payoff of 1 each. If one chooses Hi while the other chooses Lo, they both get 0. The game is illustrated in Figure 1, where one of the players chooses between two options identified by rows and the other — by columns. The numbers in each cell represent payoffs to the row and the column players respectively.

	Hi	Lo
Hi	2,2	0,0
Lo	0,0	1, 1

Figure 1: The Hi-Lo game

The standard theory predicts that rational players will choose strategies that together constitute a Nash equilibrium, in which each player's strategy is a best response to the strategies chosen by all other players. Here it is for both players to play Hi or for both to play Lo: (Hi, Hi) and  $(Lo, Lo)^1$ . Yet (Lo, Lo) does not intuitively strike as a rational outcome in this game. It is true that if one player expected the other to play Lo, then choosing Lo would be his or her best response to the other player's choice. In other words, choosing Lo would be the rational thing to do. However, it would be odd if anyone formed an expectation that a rational individual would play Lo in the first place. Experimental results support this by revealing that over 90% of the time people do opt for Hi in this game<sup>2</sup>.

This prompted the emergence of the theory of team reasoning which suggests that certain features of the context in which interdependent decisions are made may trigger a shift in peoples' mode of reasoning from individualistic best-response reasoning to reasoning as members of a team where a group of individuals acts together in the attainment of some common goal<sup>3</sup>. By identifying this goal with the maximization of the average of decision-makers' personal payoffs the theory can be operationalized to render Hi to be the only rational choice in the Hi-Lo game for anyone who reasons as a member of a team. Similarly, this allows to explain cooperation in the widely discussed Prisoner's Dilemma and the Stag Hunt games.

<sup>&</sup>lt;sup>1</sup>These are Nash equilibria in pure strategies. There is a third equilibrium in mixed strategies, in which players randomize between the two available options with certain probabilities. Here and in the rest of this paper we focus solely on equilibria in pure strategies.

<sup>&</sup>lt;sup>2</sup>See Bardsley et al. (2010) who, among a number of other games, report experimental results from two versions of the Hi-Lo game where the outcome (Hi, Hi) yields a payoff of 10 while the outcome (Lo, Lo) yields a payoff of 9 or 1 to both players.

<sup>&</sup>lt;sup>3</sup>For early developments of this theory see Sugden (1993, 2000, 2003) and Bacharach (1999, 2006). For some of the more recent work see Gold and Sugden (2007a,b), Sugden (2011) and Gold (2012).

While we agree with the idea that peoples' mode of reasoning may sometimes undergo a shift from individualistic to reasoning as members of a team, we criticize the identification of the team's interests with the maximization of the average of individuals' personal payoffs. We do this for two reasons. First, it relies on making interpersonal comparisons of individual players' payoffs, which goes beyond the standard assumptions of the expected utility theory. Second, it may advocate a complete self-sacrifice of some individuals for the benefit of others or possibly one sole member of a team. In this paper we present an alternative function for representing team's interests that does not rely on making interpersonal comparisons of payoffs and makes participation in team play conditional on it bringing about at least some benefit to every member of the team.

The rest of this paper is structured as follows. In section 2 we discuss the theory of team reasoning in more detail and present how it is sometimes operationalized to render Hi to be the only rational choice in the Hi-Lo game and explain cooperation in the Prisoner's Dilemma and the Stag Hunt games. We also explain that the notion of team reasoning cannot be represented by a transformation of individuals' personal payoffs in games. In section 3 we discuss why the maximization of the average of individuals' personal payoffs may not be a good representation of team's interests. In section 4 we present an alternative rank-based function and illustrate its predictions using a variety of examples<sup>4</sup>. With section 5 we conclude and suggest a few possible directions for further research.

# 2 Team Reasoning

When a person reasons individualistically, he or she focuses on the question "what it is that I should do in order to best promote my interests?". The answer to this question identifies a strategy that is associated with the highest expected personal payoff to the individual, given his or her beliefs about the actions of others. This is what is meant by individualistic best-response reasoning underlying the identification of Nash equilibria in games. When a person reasons as a member of a team, on the other hand, he or she focuses on the question "what it is that we should do in order to best promote our interests?". The answer to this question identifies a set of strategies — one for each of the interacting individuals out=1—that leads to the attainment of the best possible outcome for the group of individuals acting together as a team. As explained by Gold and Sugden (2007a) 'when an individual reasons as a member of a team,

<sup>&</sup>lt;sup>4</sup>By function we mean what the rational choice theory refers to as a choice function that takes the set of the available actions to an individual, the structure of a game and the individual's beliefs about others' behaviour as inputs and produces a set of (rational) actions as an output.

<sup>&</sup>lt;sup>5</sup>Strictly speaking this need not necessarily be the case, since not all individuals in a given strategic interaction may be reasoning as members of a team. There are variants of the theory of team reasoning that consider such scenarios. For an overview see Gold and Sugden (2007a).

she considers which combination of actions by members of the team would best promote the team's objective, and then performs her part of that combination'.

Among the existing literature on the theory of team reasoning that attempts to propose an explicit function for representing the interests of a team a reference is sometimes made to the maximization of the average of individuals' personal payoffs. This suggestion can be found in Bacharach (1999, 2006) and, in later theoretical developments, it was made by Smerilli (2012). In empirical studies it was adopted by Colman et al. (2008, 2014). It is not the case that everybody operationalizes the theory of team reasoning using this function. Sugden (1993, 2000, 2003) as well as Gold and Sugden (2007a,b) do not endorse any explicit function as a possible candidate and Gold (2012) suggests that team's interests do not have to be represented by the maximization of the average of individuals' personal payoffs. Crawford et al. (2008) and Bardsley et al. (2010) provide an interpretation of team reasoning as a search for a decision rule that would resolve certain types of coordination problems in a mutually beneficial way, but do not propose a specific function as well. Sugden (2011, 2015) suggests that a function representing team's interests should incorporate the notion of mutual advantage among all members of a team. We will adopt the latter idea in our construction of the rank-based function of team's interests in section 4 below. However, from the literature on the theory of team reasoning that is known to us, it either makes a reference to the maximization of average payoffs or does not endorse any specific function explicitly and instead focuses on a number of more general properties for such a function to satisfy (e.g. successful coordination of individuals' actions or the notion of mutual benefit among the members of a team).

One reason why the maximization of the average of individuals' payoffs is attractive is that it ensures Pareto efficiency of any outcome that is selected by a team<sup>6</sup>. It is easy to see that the outcome that best promotes this objective in the Hi-Lo game is (Hi, Hi). Thus, when an individual reasons as a member of a team, he or she identifies (Hi, Hi) as the uniquely optimal outcome for the team and individually chooses Hi— his or her part in the attainment of this outcome.

Consider now the Prisoner's Dilemma game illustrated in Figure 2A, in which two players independently and simultaneously decide whether to cooperate (play C) or defect (play D). This game has a unique Nash equilibrium: (D, D). This is because, irrespective of what the other player is going to do, it is always better to play D from an individual's personal point of view. Individualistic reasoning thus leads to a socially suboptimal, Pareto inefficient outcome, since the outcome (D, D) yields lower payoffs to both players than does the outcome (C, C). This is the main reason why this game is so widely discussed in social sciences.

Experimental results suggest that in a one-shot version of the Prisoner's

<sup>&</sup>lt;sup>6</sup>An outcome is Pareto efficient if there exists no other outcome in which somebody's payoff could be increased without making anyone else worse off. In the Hi-Lo game the outcome (Lo, Lo) is Pareto inefficient since both players are better off in the outcome (Hi, Hi) which is the only Pareto efficient outcome in this game.

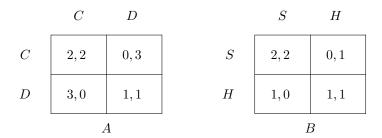


Figure 2: The Prisoner's Dilemma (A) and the Stag Hunt (B) games

Dilemma game (i.e. when it is played only once) people tend to cooperate about 50% of the time<sup>7</sup>. Notice that the outcome (C, C) uniquely maximizes the average of individuals' personal payoffs. As such, the theory of team reasoning operationalized as above suggests that in the Prisoner's Dilemma game some people may be reasoning as members of a team while others reason individualistically.

In a similar way the theory can explain cooperation in the Stag Hunt game illustrated in Figure 2B. This game has two Nash equilibria: (S,S) and (H,H). However, hunting hare (playing H) guarantees a payoff of 1 irrespective of what the other player does, whereas the attainment of the high payoff from hunting stag (playing S) crucially depends on the cooperation of the other party. Here experimental results suggest that in a one-shot version of this game people tend to choose S slightly more than 60% of the time<sup>8</sup>. We will return to discussing this result in more detail in Section 4 below.

#### Team Reasoning vs. Transformation of Personal Payoffs

An important point stressed by many decision and game theorists is that the payoff structures of games have to fully capture everything that is motivationally important in individuals' evaluations of the possible outcomes of those games. For example, imagine that two people are playing the Prisoner's Dilemma game in terms of monetary payoffs as illustrated in Figure 3A. Suppose that the row player is a pure altruist when it comes to decisions involving money — i.e. he or she always wants to maximize the other player's monetary gain. Suppose also that the column player always prefers to maximize his or her personal monetary payoff, but is extremely averse to inequitable distributions of gains among individuals. For simplicity assume that an outcome resulting in unequal monetary

<sup>&</sup>lt;sup>7</sup>However, cooperation tends to decrease with repetition (i.e. when the same game is played a number of times). See Ledyard (1995) for a survey of experimental results from *public goods* games, which involve more than two players but otherwise are very similar in their structure to the two-player Prisoner's Dilemma.

 $<sup>^8</sup>$ The proportion of people choosing S changes with repetition: in some cases it increases while in others it decreases. This seems to (at least partially) depend on the specific payoff structure of the played Stag Hunt game — the extent of risk involved in playing S and the extent of risklessness in playing H. See Battalio et al. (2001).

gains for the two players is just as good for him or her as gaining nothing. The correct representation of the true motivations of these individuals transforms the Prisoner's Dilemma game in terms of monetary payoffs into the game illustrated in Figure 3B. This transformed game has a unique Nash equilibrium (C,C) in which both players rationally cooperate following individualistic best-response reasoning in the attainment of their personal goals.

	C	D		C	D
C	£2, £2	£0, £3	C	2,2	3,0
D	£3, £0	£1, £1	D	0,0	1,1
		4		1	 3

Figure 3: The Prisoner's Dilemma game in terms of  $\pounds$  (A) and its transformation using payoffs that represent the true motivations of both players (B)

We agree with the idea that the payoff structures of games have to accurately capture the true motivations of interacting individuals. However, it is important to note that a possible shift in a decision-maker's mode of reasoning from individualistic to reasoning as a member of a team cannot be captured by a transformation of that individual's personal payoffs in the same way as it was done in the case of altruism and inequity-aversion above. To see this consider again the Hi-Lo game illustrated in Figure 1. Suppose that the row player reasons as a member of a team and adopts the maximization of the average of individuals' personal payoffs as the team's objective. Replacing his or her personal payoff numbers with averages of the two players' payoffs in each outcome does not transform the original Hi-Lo game into anything different. This is because the personal payoffs of the two players already match the average of their personal payoffs in each outcome of this game to begin with. As a result, the transformed game would still have two Nash equilibria and two rational outcomes — exactly what the theory of team reasoning was developed to contest.

The difference between an individualistic mode of reasoning and reasoning as a member of a team lies not in how an individual personally values each outcome of a game, but in the way he or she reasons when choosing among the available actions. When a person reasons individualistically, he or she chooses an action based on his or her belief about what the other player is going to do and what outcome — and personal payoff — the chosen action would subsequently yield. When a person reasons as a member of a team, on the other hand, he or she first identifies an outcome of the game that best fulfills the team's objective and then performs his or her part in the attainment of that outcome. In other words, somebody who reasons individualistically chooses an action that maximizes his

or her expected personal payoff, whereas somebody who reasons as a member of a team chooses an action that is associated with an outcome that best fulfills the team's objective.

This underlines two key assumptions on which our approach is based. First, we assume that individuals' personal payoffs represent their true motivations in games. In addition to this we assume that the payoff structures of games are commonly known by all the interacting decision-makers when they reason about what courses of action to take. The latter point rules out the possibility that a mere shift in an individual's mode of reasoning — in addition to changing the way that individual reasons — changes the way he or she personally values each outcome in the considered games. If that were not the case, these interactions would cease to be games of complete information about each other's payoffs and would take us further away from the orthodox game theory and the type of games we analyze in this paper.

Another important point to note is that reasoning as a member of a team does not imply and is not implied by the sharing of the attained payoffs among the members of a team. In other words, the attained payoffs are not transferable from one player to another. If players are able to (or indeed are going to) share their personal gains, this has to be reflected in the payoff structures of the played games to start with in order to capture the players' true motivations. In this light, equal sharing of combined payoffs in the Hi-Lo game would leave the payoff structure of the original game unchanged. It would, however, change the payoff structure of any game that contained outcomes yielding unequal distributions of payoffs among players (as is the case, for example, in the Prisoner's Dilemma and the Stag Hunt games).

# Team Reasoning and the Team's Interests: Two Separate Questions

The theory of team reasoning, as introduced above, has to answer two separate but equally important questions. First, it needs to specify clear and testable circumstances under which individuals' mode of reasoning may undergo a shift from individualistic to reasoning as members of a team. Second, in cases when individuals do reason as members of a team, it needs to specify what they take the team's interests to be upon deciding on what courses of action to take. In this paper we predominantly focus on the latter question and turn to it next.

# 3 Team's Interests: Not the Average of Personal Payoffs

As mentioned above, among the works on the theory of team reasoning to date that attempt to propose an explicit function for representing the interests of a team a reference is sometimes made to the maximization of the average of individual decision-makers' personal payoffs. We criticize this suggestion and believe that, if people do reason as members of a team in certain situations, they are unlikely to adopt the maximization of average payoffs as a guide when thinking about what it is that they should do. Our criticism is based on two points.

First, the maximization of the average of personal payoffs may, in certain situations, advocate a complete sacrifice of some individuals' personal interests for the benefit of others or possibly one sole member of a team — a consequence which we find intuitively problematic. To see this, consider a slightly amended version of the Prisoner's Dilemma game illustrated in Figure 4A. The only difference from its original version is the slightly higher payoff to the row player from defection when the column player cooperates. In this game, the maximization of the average of players' personal payoffs would identify the team's objective with the attainment of the outcome (D,C). It would thus prescribe a complete sacrifice of the column player's personal interests with the row player reaping all the benefits from team play.

	C	D		L	R
C	2,2	0,3	U	10, 1	0,0
D	5,0	1,1	D	4,4	1,9
A			'	1	3

Figure 4: The amended Prisoner's Dilemma (A) and the Chicken (B) games

For a slightly different example consider the game illustrated in Figure 4B, which is a particular version of the game known as the Chicken. This game has two Nash equilibria: (U,L) and (D,R). Here the maximization of average payoffs suggests that team's interests would be best fulfilled with the attainment of the outcome (U,L). In this case it does not prescribe a complete self-sacrifice to the column player — the outcome (U,L) is not the worst possible outcome for him or her in this game — but it does not advance the column player's personal interests anywhere far from just that.

Our suggestion is that, if some people's mode of reasoning does undergo a shift from individualistic to reasoning as members of a team in the amended Prisoner's Dilemma and the Chicken games, the team's objective would be the attainment of the outcomes (C,C) and (D,L) respectively. More importantly, we suggest that no individual would willingly subscribe to team play if there was no personal gain for him or her from doing so, which precludes the possibility of a self-sacrifice. Our intuition is shared by Sugden (2011) who suggests that a team's goal should be, in one way or another, beneficial to every member of the team.

This is not to say that self-sacrifice does not exist. It is evident that we

are often willing to sacrifice our personal material gains for the well-being of our loved ones. However, we suggest that such motivational factors need to be fully captured by individuals' personal payoffs associated with the possible outcomes in games before the different modes of reasoning are considered. And if team reasoning is a mode of reasoning that individual decision-makers may adopt in one-shot interactions with potentially complete strangers — cases that we consider in this paper — self-sacrifice of individuals is unlikely.

The essence of this argument rests on an idea somewhat similar to the notion of the separateness of persons used to criticize utilitarianism. Utilitarianism, which focuses on the maximization of the aggregate well-being of a society while ignoring the distribution of the attained well-being among the individual members of that society, is said to be insufficiently sensitive to the separateness of individuals and the advancement of their personal well-being. In a similar fashion it can be argued that any aggregative function of team's interests that fails to take into account the distribution of payoffs among the members of a team is susceptible to being rejected for failing to respect the separateness of interacting players. So long as there is space to make an objection on such grounds, any individual decision-maker could reasonably protest against taking part in the prescribed team play when their personal gains from doing so are insufficiently addressed<sup>9</sup>.

An important implication of the first point of our criticism is that we remain committed to the idea that all that matters for individual decision-makers are their personal motivations in the considered games. In this sense, the use of the term "team" in the interpretation of the theory of team reasoning in this paper is meant to be very loose. Most importantly, it is not meant to carry any psychological connotations that may be present in various other interpretations of team work and team members' duties (e.g. in sport, work groups, neighbourhoods, faculties, etc.) which may lead to individuals' abandonment of their personal aspirations (even when these aspirations take into account the well-being of others). In certain types of interpersonal interactions, however, the decision-makers' personal interests may be advanced further relative to where they would end up at if everyone followed individualistic best-response reasoning. This advancement of personal interests is made possible — as we will attempt to show in the next section — by a shift in the decision-makers' mode of reasoning from individualistic to reasoning as members of a team.

The second point of our criticism is based on the fact that the use of the average function relies on making interpersonal comparisons of the interacting individuals' payoffs. However, the standard assumptions of the expected utility theory that make numerical representations of individuals' motivations possible by themselves do not allow such comparisons to be made<sup>10</sup>. To illustrate what this means, consider again the Prisoner's Dilemma game presented in

<sup>&</sup>lt;sup>9</sup>See Rawls (1971) and Nozick (1974) for criticism of utilitarianism. Although we use an analogy to their notion of the separateness of persons, our claim here is not based on grounds of moral normativity.

 $<sup>^{10}</sup>$ In the literature on the theory of team reasoning this has also been pointed out by Sugden (2000).

Figures 2(A) and 3(A). The numerical representation of the row player's preferences allows us to say that he or she prefers the outcome (C,C) to the outcome (D,D). It also allows us to say that he or she prefers the outcome (C,C) to the outcome (C,D) by a greater extent than he or she prefers the outcome (D,C) to the outcome (C,C). However, it does not allow us to claim that the row player "enjoys" the benefits of the outcome (C,C) by as much as does the column player.

Although in this paper we do not discuss the technical arguments of why it is so, any payoff function that numerically represents a decision-maker's preferences or motivations is unique only up to positive affine transformations<sup>11</sup>. What follows from this is that the payoff structure of the game illustrated in Figure 5(B) represents exactly the same motivations of the interacting individuals as the one in Figure  $5(A)^{12}$ .

	C	D		C	D
C	2, 2	0,3	C	6, 3	0,4
D	3,0	1,1	D	9,1	3, 2
A			'	1	3

Figure 5: The original Prisoner's Dilemma game (A) and its representation using positive affine transformations of the row and the column players' payoffs (B)

If the theory of team reasoning makes use of an aggregative payoff function to represent the interests of a team — such as the maximization of the average or the sum of individuals' personal payoffs — it ceases to be a mere extension of the framework used by the standard rational choice theory and needs to suggest how the required interpersonal comparisons of payoffs are possible. An alternative is to drop the idea that team's goals are best represented by such aggregative functions and in the next section we propose a representation of team's interests based on the latter approach. To put this point differently, we hope to show that team play is possible in certain types of interpersonal interactions even without invoking interpersonal comparisons of the interacting players' payoffs.

<sup>&</sup>lt;sup>11</sup>This means that if u is a payoff function representing an individual's personal motivations, then so is function u' = au + c where a > 0 and c are constants. For a detailed discussion of why this is so see, for example, Luce and Raiffa (1957, ch. 2).

 $<sup>^{12}</sup>$ The column player's payoffs in Figure 5(B) were transformed by adding 1 to every number representing his or her payoffs in the original representation of the Prisoner's Dilemma game in Figure 5(A) while the row player's payoffs were transformed by multiplying each number by a factor of 3.

#### 4 Team's Interests: A Rank-Based Function

In this section we propose an alternative representation of team's interests that fits better with our intuition about a number of conditions that need to be satisfied in order for individual decision-makers to take part in team play. We start with the proposition of two properties for a candidate function of team's interests to have:

- 1. A team's objective has to be the attainment of an outcome that is beneficial to every member of the team.
- 2. Decision-makers' payoffs are not interpersonally comparable, but each individual's preferential ranking of the possible outcomes of a game is commonly known by all players. The derivation of a team's objective should be done without invoking interpersonal comparisons of players' payoffs.

The first property captures the idea that there has to be something "in it" for a potential member of a team from team play in order for that individual to agree to participate in the attainment of the team's goal. As mentioned earlier, we share the intuition behind this property with Sugden (2011) who suggests that team play has to be recognized as being mutually advantageous by all those partaking in it. Of course, we will need to define what it means for something to be regarded as mutually advantageous as well as how and relative to what that advantage is measured. The second property is a corollary of the standard assumption of game theory for games of complete information about the interacting players' payoffs. It is also in line with the standard axioms of rationality that make numerical representations of decision-makers' preferences possible but do not automatically grant their interpersonal comparability.

With these properties in mind, we suggest a team's objective to be the maximal advancement of mutual benefit to the members of the team. There are two ways to motivate our proposed function of team's interests. One is axiomatic and is based on presenting a set of more fine-grained properties for a function of team's interests to satisfy. The other describes a plausible reasoning process that we believe rational decision-makers may often engage in when facing particular types of games. We start with the axiomatic approach first by proposing three axioms to characterize the interests of a team.

**Axiom 1 (Weak Pareto optimality):** If an outcome x of a game is strictly preferentially dominated by some other outcome y for all players, then the outcome x is not chosen by the team.

This axiom ensures Pareto efficiency of the selected outcome in a weak sense<sup>13</sup>. We believe this to be an essential feature of any candidate function for representing team's interests. Since team-reasoning individuals evaluate all outcomes

<sup>&</sup>lt;sup>13</sup> An outcome is Pareto efficient in a weak sense if there exists no other outcome in which every player is better off in terms of their personal payoffs. The set of Pareto efficient outcomes in a weak sense is a subset of all Pareto efficient outcomes, since the latter requires there to be no other outcome in which somebody's payoff could be increased without making anyone else worse off.

of a game and identify the attainment of a subset of these as an objective for a team, it would be odd if they picked an outcome that, from every player's personal point of view, was worse than some other available alternative <sup>14</sup>.

Before presenting the second axiom we need to introduce a few additional terms. Let the smallest personal payoff that a player can attain from choosing a particular strategy in a game be called that player's personal security payoff associated with the strategy in question. Recall the Stag Hunt game illustrated in Figure 2(B). The personal security payoff associated with hunting stag for either player is 0, since it is the minimal personal payoff that can be attained by playing S. Similarly, the personal security payoff associated with hunting hare (playing H) for either player is 1. Given this and any game, a player can always choose to play a strategy that is associated with the highest personal security payoff. This, in turn, guarantees the player the attainment of a payoff that is at least as high as the security payoff in question, irrespective of what the other players are going to do. In the case of the Stag Hunt game, the maximal payoff that any player can guarantee him or her self in this way is 1, which is the personal security payoff associated with hunting hare. This is usually referred to as the player's personal maximin payoff level in the game and the corresponding strategy — the maximin strategy.

Axiom 2 (Preservation of personal security): Team play cannot leave any player worse-off than his or her personal maximin payoff level in a game.

This axiom limits a team's objective to the attainment of only those outcomes that result in players' personal payoffs being at least as good as the payoffs that they could secure themselves by playing their maximin strategies individually. As such, it defines the lower threshold points in games, below which potential members of a team would, so to speak, not "agree to go" in team play, since they can guarantee themselves a better personal payoff individually.<sup>15</sup>.

We now turn to defining individual and mutual advantage and the way these are measured. To do this we introduce the following method for assigning preferential rank values to the available outcomes in games based on each player's personal preferential ordering of those outcomes. For a particular player, the least preferred outcome in a game is assigned the preferential rank value 0. The second least preferred outcome is assigned the preferential rank value 1 and so on. A shift from one outcome to another that results in an increase of the assigned preferential rank value by 1 for some player is said to advance that player's personal interests by 1 unit.

**Individual advantage:** An outcome of a game is individually advantageous to a particular player if that player preferentially ranks this outcome above the outcome(s) associated with his or her personal *maximin* payoff level in the game. The extent of individual advantage provided by an outcome to

<sup>&</sup>lt;sup>14</sup>Bardsley et al. (2010) consider possible cases where the Pareto criterion may be abandoned in team play. We will briefly return to this in more detail in Section 5 below.

<sup>&</sup>lt;sup>15</sup>In a recent article Sugden (2015) uses the same lower threshold point in defining mutually advantageous team play.

a particular player is given by the number of units this outcome advances that player's personal interests relative to the outcome(s) associated with his or her personal *maximin* payoff level in the game.

Mutual advantage: An outcome of a game is mutually advantageous to the interacting players if each player preferentially ranks this outcome above the outcome(s) associated with his or her personal maximin payoff level in the game. The extent of mutual advantage provided by an outcome to the interacting decision-makers is given by the number of units this outcome advances all players' personal interests in parallel relative to the outcome(s) associated with each player's personal maximin payoff level in the game.

For an example imagine a two-player game where some outcome x advances the first and the second player's personal interests relative to the outcomes associated with their personal maximin payoff levels in the game by 1 and 2 units respectively. Since this outcome advances the two players' personal interests in parallel by 1 unit, it is said to provide 1 unit of mutual advantage to the interacting decision-makers. The additional unit of advancement of the second player's personal interests provided by this outcome represents individual advantage to the second player over and above the 1 unit of mutual advantage. We will illustrate this with a number of more concrete examples later in this section.

**Axiom 3 (Maximal mutual advantage):** An outcome selected by a team has to be maximally mutually advantageous.

This axiom has two implications. First, if a game contains a mutually advantageous outcome, then any outcome that is selected by a team must also be mutually advantageous. Second, if a game contains multiple mutually advantageous outcomes, then the outcome(s) selected by the team must provide maximal mutual advantage. In other words, if there is an outcome that provides more mutual benefit to the interacting decision-makers than some outcome x, then x is not chosen by the team.

Given the three axioms above we propose the following function to represent the interests of a team.

Rank-based function of team's interests: Maximize the minimum number of units by which an outcome advances individuals' personal interests among the interacting players relative to each player's threshold point — the outcome(s) associated with his or her *maximin* payoff level in the game.

It can be shown that this function satisfies the three axioms introduced earlier (for proofs see Appendix). It is not, however, the only function that does so. For an example consider a two-player game where a particular outcome x advances each of the two players' personal interests by 2 units while some other outcome y advances the first and the second player's personal interests by 2 and 3 units

respectively. The extent of mutual advantage provided by either of the two outcomes is the same — 2 units. Assuming there to be no outcomes providing a higher extent of mutual advantage in this game, the rank-based function of team's interests selects both x and y. As such, any function that would further discriminate between these two outcomes would also satisfy the introduced axioms. We believe, however, that the proposed function captures best the interacting individuals' primary motivation to satisfy their personal preferences in a mutually beneficial way without invoking additional motivational attitudes such as benevolence towards others in team play.

The second way to motivate the proposed function is to describe a plausible reasoning process that rational decision-makers may undergo when facing particular types of games that is in line with its predictions. We will do this by discussing a number of examples below. Before that we present a formal representation of the function just described.

#### **Formalization**

Let  $\Gamma$  be a normal form game defined as a triple  $(I, S_i, u_i)$  where  $I = \{1, 2, \dots, m\}$  is a finite set of m players,  $S_i$  is a set of pure strategies available to player  $i \in I$ , and  $u_i(\mathbf{s})$  is a payoff function that assigns to every player  $i \in I$  a personal payoff for each outcome in the game. An outcome of a game is defined as a strategy profile  $\mathbf{s} = (s_1, \dots, s_m)$  where  $s_i \in S_i$  is a particular pure strategy chosen by player  $i \in I$ . Let  $\Sigma$  be the set of all possible strategy profiles (i.e. outcomes) in the game  $\Gamma$  and  $\mathbf{s}^{\vee i} \in \Sigma$  be a strategy profile that is associated with player i's maximin payoff level in the game.

Each player  $i \in I$  has a personal preferential ranking of all the strategy profiles in the set  $\Sigma$ . Let  $\mathbb{Z}_{0...k}$  be a set of non-negative integers from 0 to k where  $k \leq n-1$  and n is the total number of strategy profiles available in the game  $\Gamma$ . Let  $\wp_i : \Sigma \to \mathbb{Z}_{0...k}$  be a ranking function that, for each player  $i \in I$ , assigns a personal preferential rank value to each strategy profile in the set  $\Sigma$  as follows: the strategy profile that player  $i \in I$  prefers the least in the game  $\Gamma$  is assigned the preferential rank value 0, the second least preferred strategy profile is assigned the preferential rank value 1 and so on.

The rank-based function of team's interests  $\mathcal{F}^{\tau}: \Sigma \to \Sigma^{\tau}$ , where  $\Sigma^{\tau} \subseteq \Sigma$ , is a choice function that selects a subset from the set of all possible strategy profiles of the game  $\Gamma$  such that each selected strategy profile maximizes the minimum difference, across all players, between the preferential rank value of the selected profile and the preferential rank value of the strategy profile associated with a particular player's personal  $\max m$  payoff level in the game. In other words, each element  $\mathbf{s}^{\tau} \in \Sigma^{\tau}$  is such that  $\mathbf{s}^{\tau} = \max_{\mathbf{s} \in \Sigma} \{\min_{i \in I} [\wp_i(\mathbf{s}) - \wp_i(\mathbf{s}^{\vee i})]\}$ .

#### Examples

We now turn to discussing a number of examples. First, we apply the function of team's interests to four well known two-player games with multiple Nash equilibria. We then turn to discussing the implications of the proposed function in two games with unique Nash equilibria.

#### The Hi-Lo

Recall the Hi-Lo game illustrated in Figure 1. The row and the column players' personal preferential rankings of the four outcomes are shown below with numbers representing the corresponding preferential rank values and arrows indicating outcomes associated with each player's *maximin* payoff levels in the game.

Row player: 
$$\begin{bmatrix} (Hi, Hi) & 2 \\ (Lo, Lo) & 1 \\ \Rightarrow (Hi, Lo)(Lo, Hi) & 0 \end{bmatrix}$$
 Column player: 
$$\begin{bmatrix} (Hi, Hi) & 2 \\ (Lo, Lo) & 1 \\ \Rightarrow (Hi, Lo)(Lo, Hi) & 0 \end{bmatrix}$$

Relative to the *maximin* payoff level, the outcome (Hi, Hi) advances each player's personal interests by 2 units. The outcome (Lo, Lo) does so by 1 unit. Hence, the rank-based function of team's interests identifies (Hi, Hi) as the best outcome for a team and its attainment as the team's objective. As a result, for any individual who reasons as a member of a team, (Hi, Hi) is the only rational outcome in this game.

A plausible reasoning process by which individual players may arrive at this conclusion can be described as follows. Having worked out all the best-response strategies in this game, individualistically reasoning decision-makers face a Nash equilibrium selection problem. This leaves them stuck with no further indication of what actions they ought to perform independently from each other in order to best promote their personal interests. The next question they ask themselves is "what would be best for both of us in this situation?". As soon as they do so, they start reasoning as members of a team and identify the uniquely rational outcome from the perspective of team's interests.

According to the deliberative process just described, the adoption of the team mode of reasoning comes about as a result of extensive thinking about games by otherwise individualistically rational agents who are able to work out each other's best-response strategies and realize that these leave them with a Nash equilibrium selection problem. As such, it uses as its starting point the standard assumption of the orthodox game theory that the basic mode of the interacting decision-makers' mode of reasoning is individualistic best-response reasoning <sup>16</sup>.

#### The Stag Hunt

Recall the Stag Hunt game illustrated in Figure 2(B). The row and the column players' preferential rankings of outcomes are shown below.

<sup>&</sup>lt;sup>16</sup>For other possible suggestions of what may trigger shifts in individuals' mode of reasoning, some of which abandon this assumption, see, for example, Bacharach (2006), Sugden (2003) or Gold and Sugden (2007a,b).

Row player: 
$$\begin{bmatrix} (S,S) & 2 \\ \Rightarrow (H,S)(H,H) & 1 \\ (S,H) & 0 \end{bmatrix}$$
 Column player: 
$$\begin{bmatrix} (S,S) & 2 \\ \Rightarrow (S,H)(H,H) & 1 \\ (H,S) & 0 \end{bmatrix}$$

Since either player can obtain a payoff of 1 by hunting hare (playing H) irrespective of what the other player is going to do, the personal maximin payoff level is associated with the pair of outcomes (H, S) and (H, H) for the row, and (S, H) and (H, H) for the column players. Since (S, S) is the only mutually advantageous outcome relative to these threshold points, the rank-based function of team's interests identifies it as the uniquely rational solution of the game for players who reason as members of a team.

With regards to this game in particular, however, it is important to mention other solution concepts to the Nash equilibrium selection problem that are based on risk or uncertainty aversion with respect to choosing a particular action. Notice that in the Stag Hunt game it can be argued that the Nash equilibrium (H, H) is in some sense safer than the equilibrium (S, S). This is because either player is guaranteed a certain payoff from hunting hare (playing H) whereas the high payoff from hunting stag (playing S) crucially depends on the other player's choice. While we don't review the various models of risk and uncertainty aversion in this paper (some of which are based on probabilistic assessments of players' actions while others on the sizes of foregone payoffs in cases of deviations from equilibrium play) we do believe that these considerations may play a role in determining the likelihood of individuals' adoption of the team mode of reasoning. However, since existing experimental data suggests that in one-shot interactions people tend to play S slightly more than 60% of the time, we do suggest that a shift in individuals' mode of reasoning to that as members of a team in one-shot versions of this game is likely<sup>17</sup>.

#### The Chicken

Recall the Chicken game illustrated in Figure 4(B). The row and the column players' preferential rankings of outcomes are shown below.

Here, similarly as in the Stag Hunt game above, (D, L) is the only mutually advantageous outcome relative to the two players' threshold points — their personal maximin payoff levels in the game. As such, the rank-based function of team's interests identifies its attainment as the objective for the team.

The deliberative process by which individual decision-makers may arrive at this conclusion is described as follows. Thinking individualistically, both

 $<sup>\</sup>overline{\phantom{a}}^{17}$ Rankin et al. (2000) report an experiment in which different versions of the Stag Hunt game are played repeatedly, but in a way that aims to induce one-shot reasoning of participating individuals in each round (by changing labels associated with the available actions and varying sizes of payoffs resulting from different outcomes each time the game is played). In this setting virtually everyone switches to playing S over time, which seems to support our suggestion.

players identify the two Nash equilibria in this game: (U, L) and (D, R). From a personal point of view, the row player prefers the attainment of the outcome (U, L). At the same time he or she recognizes the column player's preference for the attainment of the outcome (D, R). If both players were to pursue their preferred options, they would end up with the outcome (U, R), which is the worst of all possible outcomes for both. The Nash equilibrium selection problem, as previously, leaves them stuck with no further indication of what actions they ought to perform independently from each other in order to best promote their personal interests. At this stage they ask themselves "what would be best for both of us in this situation?". As soon as they do so, they identify the outcome (D, L) as the uniquely rational solution of this game from the perspective of team's interests. Now, since the outcome (D, L) is not a Nash equilibrium itself, they may each consider unilateral deviation from team play. However, the previously recognized conflict of their personal interests associated with the two Nash equilibria prevents them from doing so, emphasizing the outcome (D, L) as the only mutually advantageous solution of the game.

#### The Dividing of a Cake

Another interesting case involving multiple Nash equilibria is the Dividing of a Cake game illustrated in Figure 6, which is a particularly simple version of the well known Nash Bargaining game. In this game two players are presented

	0	1	2	3	4
0	0,0	0, 1	0, 2	0,3	0,4
1	1,0	1, 1	1, 2	1,3	0,0
2	2,0	2, 1	2, 2	0,0	0,0
3	3,0	3, 1	0,0	0,0	0,0
4	4,0	0,0	0,0	0,0	0,0

Figure 6: The Dividing of a Cake game

with a cake that is cut into four equal-sized pieces and simultaneously place a demand for the number of pieces for themselves (from  $\theta$  to 4). If the sum of their demanded pieces does not exceed 4, they both get what they asked for. If, on the other hand, the sum exceeds 4, they both get nothing. The game has six Nash equilibria:  $(4, \theta), (3, 1), (2, 2), (1, 3), (\theta, 4)$  and an inefficient (4, 4). The row and the column players' personal rankings of outcomes are shown

 $below^{18}$ .

```
Row player: \begin{bmatrix} (4,0) & 4 \\ (3,0)(3,1) & 3 \\ (2,0)(2,1)(2,2) & 2 \\ (1,0)(1,1)(1,2)(1,3) & 1 \\ \Rightarrow (0,0)(0,1)(0,2)(0,3)(0,4) & 0 \end{bmatrix}
Column player: \begin{bmatrix} (0,4) & 4 \\ (0,3)(1,3) & 3 \\ (0,2)(1,2)(2,2) & 2 \\ (0,1)(1,1)(2,1)(3,1) & 1 \\ \Rightarrow (0,0)(1,0)(2,0)(3,0)(4,0) & 0 \end{bmatrix}
```

Here, relative to the two individuals' maximin payoff levels, the outcome (2, 2) advances each player's personal interests by 2 units. Since every other outcome either advances one of the players' personal interests by only 1 unit or is not mutually advantageous (i.e. does not advance one of the players' personal interests at all) the rank-based function of team's interests identifies the team's objective with the attainment of the outcome (2, 2). This usually appeals to most decision-makers and is supported by experimental results<sup>19</sup>.

The above result is in line with Nash's bargaining solution of this game<sup>20</sup>. In a recent paper Misyak and Chater (2014) propose a theory of virtual bargaining as an alternative mode of reasoning that individuals may adopt when choosing among the available outcomes and courses of actions in various types of games. According to the proposed model the interacting decision-makers are said to be undergoing implicit mental bargaining processes in an attempt to work out possible agreeable outcomes in a somewhat similar fashion as it is done with the rank-based function of team's interests presented here. There are, however, some differences between the two approaches.

First, the existing theories of bargaining generally rely on the existence of a unique reference outcome that obtains when individuals fail to reach an agreement following a bargaining process. In the Dividing of a Cake game this is assumed to be the outcome in which both players gain nothing. In the Prisoner's Dilemma game this is usually the Nash equilibrium (D,D)—the outcome that both players can fall back to in case of a failure of reaching an agreement with regards to anything else. In most other cases that we address in this paper, however, there is no such unique reference point. This does not pose a problem for the proposed rank-based function of team's interests, since its outputs are determined not in relation to a unique reference outcome but by considering multiple threshold points — one for each player in a game.

<sup>&</sup>lt;sup>18</sup>There are 11 outcomes in total that yield a payoff of 0 to both players. For the sake of brevity they are all represented by the outcome (0, 0) in the two rankings.

 $<sup>^{19}</sup>$ See Nydegger and Owen (1974) for an experiment in which two players are asked to divide \$1 among themselves and virtually everybody agrees on a 50%-50% split.

<sup>&</sup>lt;sup>20</sup>The fact that the predicted outcomes are the same in this particular example is a coincidence and the proposed rank-based function of team's interests is more in line with the bargaining solution presented by Kalai and Smorodinsky (1975). Although the differences and implications of the two bargaining models are interesting, we do not discuss them in more detail here. For a discussion of Nash's bargaining model see, for example, Luce and Raiffa (1957, ch. 6).

The second difference lies with the fact that Nash's bargaining model (which is presented as a possible starting point in the development of the theory of virtual bargaining) does not entirely avoid the need to make some interpersonal comparisons of individuals' payoffs. This is because Nash's solution requires the interacting decision-makers' payoff functions to be unique only up to positive linear transformations and not positive affine transformations as we assume here. This difference, however, can be overcome by adopting a different bargaining model, such as the one presented by Kalai and Smorodinsky (1975)<sup>21</sup>.

#### The Prisoner's Dilemma and the PD-SH Hybrid

Recall the original and the amended Prisoner's Dilemma games illustrated in Figures 2(A) and 4(A) respectively. The row and the column players' preferential rankings of the four outcomes and their personal maximin threshold points (which are the same in both games) are shown below.

Row player: 
$$\begin{bmatrix} (D,C) & 3\\ (C,C) & 2\\ \Rightarrow (D,D) & 1\\ (C,D) & 0 \end{bmatrix}$$
 Column player: 
$$\begin{bmatrix} (C,D) & 3\\ (C,C) & 2\\ \Rightarrow (D,D) & 1\\ (D,C) & 0 \end{bmatrix}$$

The outcomes (C, D) and (D, C) lie below one or the other player's threshold point. Hence their attainment is not viable in team play. Since (C, C) is the only outcome that advances both players' personal interests relative to their personal maximin payoff levels, it is identified as the unique solution of the game from the perspective of team's interests.

For a slightly different example consider a game in which the row player's payoffs are exactly the same as those in the Prisoner's Dilemma but the column player's — as in the Stag Hunt games. We call this the PD-SH Hybrid game and it is illustrated in Figure 7<sup>22</sup>.

	S	H
C	2, 2	0, 1
D	3,0	1,1

Figure 7: The PD-SH Hybrid game

<sup>&</sup>lt;sup>21</sup>The underlying mechanism of the rank-based function of team's interests is somewhat similar to that behind the egalitarian solution to bargaining problems presented by Kalai (1977) and Myerson (1977). The egalitarian solution is in line with the maximization of the minimum advancement of players' personal payoffs relative to a given reference point. However, since it is concerned with equal advancement of players' payoffs, the model is also reliant on the interpersonal comparability of those payoffs.

<sup>&</sup>lt;sup>22</sup>In an unpublished manuscript Kavka (1989) calls this game the Assurance Dilemma, in which he analyzes it from the perspective of individualistic best-response reasoning.

This game has a unique Nash equilibrium: (D, H). The row and the column players' preferential rankings of outcomes are shown below.

Row player: 
$$\begin{bmatrix} & (D,S) & 3 \\ & (C,S) & 2 \\ \Rightarrow & (D,H) & 1 \\ & (C,H) & 0 \end{bmatrix}$$
 Column player: 
$$\begin{bmatrix} & (C,S) & 2 \\ \Rightarrow & (C,H)(D,H) & 1 \\ & (D,S) & 0 \end{bmatrix}$$

Similarly as in the Prisoner's Dilemma, the outcomes (C, H) and (D, S) are not viable options in team play, since they lie below one or the other player's maximin threshold points. The rank-based function of team's interests selects (C, S)— the only mutually advantageous outcome in the game.

Even though both these games have a unique Nash equilibrium, the motivation to opt for a mutually beneficial outcome may prevail. This is because players are aware of the fact that, if each of them pursues his or her personal goals individualistically, they will end up at suboptimal, Pareto inefficient outcomes (in the weak sense of Pareto efficiency). According to orthodox game theorists, games such as the Prisoner's Dilemma are trivial, since they contain a unique rational outcome in terms of individualistic reasoning. Many people outside this field, however, feel that cooperation in the Prisoner's Dilemma game is not unreasonable. We believe that the major source of controversy and disagreement about this game may lie with the fact that some people tend to think about it from the perspective of team's interests.

It is possible to provide two interpretations of what happens when decision-makers' mode of reasoning undergoes a shift from individualistic to reasoning as members of a team in these cases. According to one interpretation, individuals who reason as members of a team identify the uniquely rational outcome from the perspective of team's interests with other options no longer appearing to them as rational solutions of these games. According to another interpretation and one that is in line with the deliberative processes we discussed in the preceding examples, individuals who reason as members of a team recognize the existence of two rational solutions — one in terms of individualistic reasoning and the other from the perspective of team's interests. This leaves their chosen courses of actions undetermined and rationalizable in two ways. It is only the latter interpretation, however, that turns these games into genuine dilemmas for the interacting decision-makers, since only those individuals who will identify in such cases two distinct rational solutions will be puzzled about how to proceed.

### Indeterminacy of the Rank-Based Function

It is not always the case that the proposed rank-based function yields a unique solution to a group of individuals who are reasoning as members of a team. For an example consider a slightly different version of the Chicken game illustrated in Figure 8. As in the previous case, this game has two Nash equilibria: (U, L) and (D, R). The row and the column players' preferential rankings of outcomes and their personal maximin threshold points are shown below.

	L	R
U	3, 2	0,0
D	1,1	2, 3

Figure 8: The Chicken game (version 2)

Row player: 
$$\begin{bmatrix} & (U,L) & 3 \\ & (D,R) & 2 \\ \Rightarrow & (D,L) & 1 \\ & (U,R) & 0 \end{bmatrix}$$
 Column player: 
$$\begin{bmatrix} & (D,R) & 3 \\ & (U,L) & 2 \\ \Rightarrow & (D,L) & 1 \\ & (U,R) & 0 \end{bmatrix}$$

Here the outcome (U,L) advances the row and the column players' personal interests by 2 and 1 units respectively. The outcome (D,R) does so by 1 and 2 units respectively. The minimal advancement of personal interests across the two players as well as the extent of mutual advantage that is provided by both outcomes is 1. Hence the attainment of either one of these two outcomes remains a viable goal for a team. As a result, this game poses a coordination problem for individualistically reasoning decision-makers as well as those who reason as members of a team and further methods for resolving this game may be sought. We will briefly return to this idea in section 5.

# Normative vs. Descriptive Status

It may be asked whether the theory of team reasoning operationalized using the proposed function of team's interests is a normative or a descriptive theory of choice in interdependent decision situations. The answer, in our view, is somewhat mixed and depends largely on the type of decision problem individuals face. We propose to classify games using the following four categories: [1] games with multiple Nash equilibria, [2] games with a unique inefficient Nash equilibrium (in the weak sense of Pareto efficiency) [3] games with a unique efficient Nash equilibrium (in the weak sense of Pareto efficiency) and [4] games with no Nash equilibria, where in all four categories we refer to Nash equilibria in pure strategies alone.

In the case of category [1] — games with multiple Nash equilibria — we suggest that the theory of team reasoning has a strong normative appeal and that a shift in the interacting players' mode of reasoning from individualistic to that as members of a team is likely from the descriptive point of view as well. Individualistic best-response reasoning, albeit provides a number of rational solutions, does not resolve these games fully in terms of providing a definitive course of actions for the decision-makers to take. The theory of team reasoning operationalized as above, however, suggests that best-response reasoning is not the endpoint of rational deliberation and provides a rational resolution of these games based on the notion of mutually advantageous advancement of

the interacting players' personal interests. As such, if one agrees with the proposed definition of mutual advantage and the suggested axioms to characterize the interests of a team, the rank-based function of team's interests provides a normatively compelling basis for resolving these games. Whether the switch to reasoning as a member of a team fully resolves such games, of course, depends on whether the outcome selected by the function of team's interests is unique. This, as illustrated earlier, may not always be the case and other methods for resolving such scenarios may be required. In many cases, however, a switch to reasoning as a member of a team does resolve these games definitively and we take this switch to be a natural progression in a player's rational deliberative process in an attempt to resolve such games when best-response reasoning fails to do so.

In the case of category [2] the answer is less clear. These games have a unique Nash equilibrium and, as such, individualistic best-response reasoning resolves them fully. However, having identified the uniquely optimal outcomes based on individualistic best-response reasoning, the interacting players may easily recognize their unappealing inefficiencies. This, in turn, may prompt them to identify mutually advantageous outcomes from the perspective of team's interests. From the descriptive point of view we expect this to happen often, making such cases genuine dilemmas for the interacting decision-makers due to the existence of multiple differently rationalizable resolutions of these games. From the normative point of view, however, multiple courses of actions remain possible: some based on the principles of best-response reasoning and others based on those of mutually advantageous team play. As a result, a decision-maker who has recognized both modes of reasoning in such games will have to decide on which underlying principles to base his or her choice.

In the case of category [3] individualistic best-response reasoning resolves these games fully and efficiently. As such, there is no need for team reasoning in order to resolve these games efficiently and we do not expect people to reason as members of a team from the descriptive point of view either.

Finally, in the case of category [4] similarly as was the case with category [1] we suggest the theory of team reasoning to have a strong normative appeal, since individualistic best-response reasoning provides no definitive solutions in these games. In particular scenarios — those in which the function of team's interests is able to identify uniquely rational solutions — we also expect a shift in decision-makers' mode of reasoning to provide a descriptively accurate explanation of the chosen courses of action.

#### Payoff Intervals in Individual and Mutual Advantage

The rank-based function of team's interests presented above is based on the interacting players' ordinal preferential rankings of outcomes in games and it ignores the relative preferential intensities in the players' pairwise comparisons of those outcomes. Suppose a player prefers some outcome x to the outcome y and the outcome y to the outcome y. The function presented thus far considers the player's preferential ranking of the three outcomes, but ignores the informa-

tion on whether the player prefers x to y by a greater, lesser or the same extent as he or she prefers y to z. The standard axioms of rationality, however, make the ratios of payoff intervals between different pairs of outcomes to be meaningful representations of decision-makers' relative preferential intensities. We now present a slight modification of the above approach that takes the information about such preferential intensities into account.

We introduce the following modification for determining the number of units by which a shift from one outcome to another is said to advance a particular player's personal interests. First, each player's personal payoffs associated with the available outcomes in games are normalized to assign the least preferred outcome the payoff value of 0 and the most preferred outcome the payoff value of 100. This is done by applying an appropriate positive affine transformation of personal payoffs. For example, if a particular player's preferences over four outcomes in a game are represented by payoff values 0, 1, 2 and 3, the normalization transforms these into 0, 33, 67 and 100 respectively. After this normalization, a shift from one outcome to another that results in an increase in the assigned normalized personal payoff value of 1 for some player is said to advance that player's personal interests by 1 unit. Apart from this modification, the three axioms introduced earlier, the definitions of individual and mutual advantage and the description of the rank-based function of team's interests all remain unchanged. In the next two paragraphs we adapt the formal representation of the function of team's interests from the one presented earlier to the modified version of team's interests here.

Let  $\Gamma$  be a normal form game defined as a triple  $(I, S_i, u_i^*)$  where  $I = \{1, 2, \ldots, m\}$  is a finite set of m players,  $S_i$  is a set of pure strategies available to player  $i \in I$ , and  $u_i^*(\mathbf{s})$  is a normalized payoff function that assigns to every player  $i \in I$  a personal payoff for each outcome in the game in a way that the least and the most preferred outcomes are given the values 0 and 100 respectively. An outcome of a game is defined as a strategy profile  $\mathbf{s} = (s_1, \ldots, s_m)$  where  $s_i \in S_i$  is a particular pure strategy chosen by player  $i \in I$ . Let  $\Sigma$  be the set of all possible strategy profiles (i.e. outcomes) in the game  $\Gamma$  and  $\mathbf{s}^{\vee i} \in \Sigma$  be a strategy profile that is associated with player i's maximin payoff level in the game.

The rank-based function of team's interests  $\mathcal{F}^{\tau}: \Sigma \to \Sigma^{\tau}$ , where  $\Sigma^{\tau} \subseteq \Sigma$ , is a choice function that selects a subset from the set of all possible strategy profiles of the game  $\Gamma$  such that each selected strategy profile maximizes the minimum difference, across all players, between the normalized payoff value of the selected profile and the normalized payoff value of the strategy profile associated with a particular player's personal  $\max m$  payoff level in the game. In other words, each element  $\mathbf{s}^{\tau} \in \Sigma^{\tau}$  is such that  $\mathbf{s}^{\tau} = \max_{\mathbf{s} \in \Sigma} \{\min_{i \in I} [u_i^*(\mathbf{s}) - u_i^*(\mathbf{s}^{\vee i})]\}$ .

In the case of all of the examples discussed earlier the prescriptions of both versions of the rank-based function are the same. For an example of where these differ consider the game illustrated in Figure 9(A). This game has two Nash equilibria: (U, L) and (D, R). The row and the column players' preferential rankings of outcomes are shown below with each player's normalized payoffs

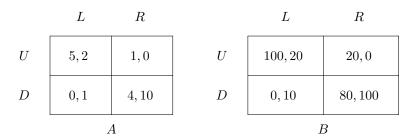


Figure 9: An example where payoff intervals matter

given in parentheses. The same game, but using the two players' normalized payoffs is illustrated in Figure 9(B).

Row player: 
$$\begin{bmatrix} & (U,L) & 3 & (100) \\ & (D,R) & 2 & (80) \\ \Rightarrow & (U,R) & 1 & (20) \\ & (D,L) & 0 & (0) \end{bmatrix}$$
 Column player: 
$$\begin{bmatrix} & (D,R) & 3 & (100) \\ & (U,L) & 2 & (20) \\ \Rightarrow & (D,L) & 1 & (10) \\ & (U,R) & 0 & (0) \end{bmatrix}$$

Using the preferential rank value numbers 0 to 3, the outcome (U, L) advances the row and the column players' personal interests by 2 and 1 units respectively. The outcome (D, R) does so by 1 and 2 units respectively. The minimal advancement of personal interests across both players as well as the extent of mutual advantage that is provided by either of the two outcomes is 1. As such, the approach that considers the preferential rank value numbers does not discriminate between the two outcomes further and the attainment of either one of these is a viable goal in team play.

It may seem, however, that the outcome  $(D,\,R)$  is in some sense better for a team than the outcome  $(U,\,L)$ . This is because the former results in the row player attaining a payoff that is almost as good as the best possible payoff to him or her in this game and the column player attaining a payoff that is far better than any of the other three possibilities. This reasoning is captured by the modified approach as follows. Using the normalized payoff values 0 to 100, the outcome  $(U,\,L)$  advances the row and the column players' personal interests by 80 and 10 units respectively. The outcome  $(D,\,R)$  does so by 60 and 90 units respectively. The minimal advancement of personal interests across players as well as the extent of mutual advantage provided by the outcome  $(U,\,L)$  is now 10 and that provided by the outcome  $(D,\,R)$  — 60. As a result, the rank-based function of team's interests in the modified approach favours the outcome  $(D,\,R)$ .

### 5 Conclusion

In this paper we argued against the operationalization of the theory of team reasoning with the use of the maximization of the average of individuals' personal payoffs as a representation of team's interests. Our arguments focused on its

reliance on making interpersonal comparisons of the interacting players' payoffs and a possible advocacy of a complete sacrifice of some individuals' personal interests for the benefit of other players. While only a subset of the existing texts on the theory of team reasoning make a reference to the maximization of average payoffs, most of the remaining literature in this field does not endorse a specific function and instead focuses on specifying a number of general conditions for a candidate function of team's interests to satisfy. We attempted to fill this gap by proposing a rank-based function of team's interests as a possible alternative that is in line with the orthodox conception of payoffs and is based on the notion of mutual advantage in team play that is compatible with the idea suggested by Sugden (2011, 2015).

While the proposed rank-based function of team's interests fits with some experimental findings from games discussed in this paper, further empirical tests will need to be constructed to test this model's empirical validity. In principle this task is possible, since the rank-based function of team's interests provides testable predictions in many types of games.

The second area requiring further research concerns cases where the proposed rank-based function of team's interests does not yield a unique solution. Bardsley et al. (2010) consider possible scenarios where the Pareto efficiency criterion may be abandoned in such cases. An example is the Enlarged Hi-Lo game illustrated in Figure 10. In this game the rank-based function identifies

	Hi 1	Hi 2	Lo
Hi 1	10, 10	0,0	0,0
Hi 2	0,0	10, 10	0,0
Lo	0,0	0,0	9,9

Figure 10: The Enlarged Hi-Lo game

team's interests with the attainment of outcomes ( $Hi\ 1$ ,  $Hi\ 1$ ) or ( $Hi\ 2$ ,  $Hi\ 2$ ) but does not discriminate further among the two. One way to proceed for a team in this case is to abandon the Pareto efficiency criterion and instead focus on the attainment of the outcome (Lo, Lo) for the sake of successful coordination of players' actions. This suggests that the function of team's interests may need to be developed further to account for such scenarios. Another possibility is to separate the question of which outcomes in a considered game are the best from the point of view of mutually advantageous team play from the question of how to coordinate players' actions having identified the best outcomes. From this point of view, the Enlarged Hi-Lo game could be seen to have three rational solutions in terms of individualistic reasoning and two rational solutions in terms of mutually advantageous team play. This would prompt the

decision-makers to seek further methods for coordinating their actions among the team-optimal outcomes first and, if no coordination was possible among these, revert back to considering suboptimal outcomes from the perspective of team's interests — such as the outcome (Lo, Lo) in the above example — to seek possible coordination among those.

Finally, the exact ways by which the interacting individuals arrive at the conclusions predicted by the rank-based function of team's interests depend on what is responsible for triggering shifts in people's mode of reasoning from individualistic to reasoning as members of a team and vice versa. While we did not discuss the various accounts addressing this question in the existing literature, we presented a possible suggestion according to which a shift in individuals' mode of reasoning may come about as a result of extended reasoning in games by otherwise individualistically rational agents. Further research into this possibility would allow to bridge the gap between the two modes of reasoning within the framework of non-cooperative game theory.

# **Appendix**

We here show that the rank-based function of team's interests presented above satisfies the three axioms introduced in section 4.

**Axiom 1 (Weak Pareto optimality):** If an outcome x of a game is strictly preferentially dominated by some other outcome y for all players, then the outcome x is not chosen by the team.

Suppose that an outcome selected by the rank-based function of team's interests does not satisfy this axiom. In other words, suppose that some outcome x that is selected by a team is strictly preferentially dominated by some other outcome y. Following the notation introduced in section 4, let the two outcomes x and y be denoted as strategy profiles  $\mathbf{s}^x, \mathbf{s}^y \in \Sigma$  respectively. Since  $\mathbf{s}^y$  strictly preferentially dominates  $\mathbf{s}^x$ , it is the case that (i) for every player  $i \in I$ ,  $\wp_i(\mathbf{s}^y) > \wp_i(\mathbf{s}^x)$  and/or  $u_i^*(\mathbf{s}^y) > u_i^*(\mathbf{s}^x)$ , where  $\wp_i(\mathbf{s})$  is player i's preferential rank value and  $u_i^*(\mathbf{s})$  is player i's normalized payoff value associated with the strategy profile  $\mathbf{s} \in \Sigma$ .

By Axiom 2, any outcome that is selected by a team is, for every player, at least as good as the outcome associated with his or her personal *maximin* payoff level in the game. This means that (ii) for every player  $i \in I$ ,  $\wp_i(\mathbf{s}^x) \ge \wp_i(\mathbf{s}^{\vee i})$  and/or  $u_i^*(\mathbf{s}^x) \ge u_i^*(\mathbf{s}^{\vee i})$ . Combining (i) and (ii) gives

$$\min_{i \in I} \left[ \wp_i(\mathbf{s}^y) - \wp_i(\mathbf{s}^{i}) \right] > \min_{i \in I} \left[ \wp_i(\mathbf{s}^x) - \wp_i(\mathbf{s}^{i}) \right]$$

and/or

$$\min_{i \in I} \left[ u_i^*(\mathbf{s}^y) - u_i^*(\mathbf{s}^{\succeq i}) \right] > \min_{i \in I} \left[ u_i^*(\mathbf{s}^x) - u_i^*(\mathbf{s}^{\succeq i}) \right]$$

Hence,

$$\mathbf{s}^{x} \neq \max_{\mathbf{s} \in \Sigma} \{ \min_{i \in I} \left[ \wp_{i}(\mathbf{s}) - \wp_{i}(\mathbf{s}^{\vee i}) \right] \}$$

and/or

$$\mathbf{s}^x \neq \max_{\mathbf{s} \in \Sigma} \{ \min_{i \in I} \left[ u_i^*(\mathbf{s}) - u_i^*(\mathbf{s}^{\leq i}) \right] \}$$

and so  $\mathbf{s}^x \notin \Sigma^{\tau}$ .

Axiom 2 (Preservation of personal security): Team play cannot leave any player worse-off than his or her personal maximin payoff level in a game.

Let  $\mathbf{s}_i^{\vee} \in S_i$  be player i's maximin (pure) strategy in a game. Let  $\mathbf{s}^{\vee} = (s_1^{\vee}, \dots, s_m^{\vee})$  be a strategy profile where each player  $i \in I$  plays his or her maximin (pure) strategy  $\mathbf{s}_i^{\vee} \in S_i$ . Every player preferentially ranks all strategy profiles that are associated with his or her maximin strategy in the game at least as high as the strategy profile(s) associated with his or her maximin payoff level in the game. As such, for every player  $i \in I$ ,  $\wp_i(\mathbf{s}^{\vee}) \geq \wp_i(\mathbf{s}^{\vee})$  and/or  $u_i^*(\mathbf{s}^{\vee}) \geq u_i^*(\mathbf{s}^{\vee})$ . Since the strategy profile  $\mathbf{s}^{\vee} \in \Sigma$  exists in every game, it cannot be the case that

$$\mathbf{s}^{x} = \max_{\mathbf{s} \in \Sigma} \{ \min_{i \in I} \left[ \wp_{i}(\mathbf{s}) - \wp_{i}(\mathbf{s}^{\leq i}) \right] \}$$

and/or

$$\mathbf{s}^{x} = \max_{\mathbf{s} \in \Sigma} \{ \min_{i \in I} \left[ u_{i}^{*}(\mathbf{s}) - u_{i}^{*}(\mathbf{s}^{\leq i}) \right] \}$$

such that, for some player  $i \in I$ ,  $\wp_i(\mathbf{s}^x) < \wp_i(\mathbf{s}^{\succeq i})$  and/or  $u_i^*(\mathbf{s}^x) < u_i^*(\mathbf{s}^{\succeq i})$ .

Axiom 3 (Maximal mutual advantage): An outcome selected by a team has to be maximally mutually advantageous.

The extent of individual advantage provided by a strategy profile  $\mathbf{s} \in \Sigma$  to a particular player  $i \in I$  is given by  $\wp_i(\mathbf{s}) - \wp_i(\mathbf{s}^{\succeq i})$  and/or  $u_i^*(\mathbf{s}) - u_i^*(\mathbf{s}^{\succeq i})$ . Let  $d(\mathbf{s})$  denote the extent of mutual advantage provided by the strategy profile  $\mathbf{s} \in \Sigma$  to the interacting players. Since the extent of mutual advantage is given by the number of units the strategy profile  $\mathbf{s} \in \Sigma$  advances all players' personal interests in parallel relative to the strategy profile(s) associated with each player's personal maximin payoff level in the game, it is the case that, for every player  $i \in I$ ,  $\wp_i(\mathbf{s}) - \wp_i(\mathbf{s}^{\succeq i}) \geq d(\mathbf{s})$  and/or  $u_i^*(\mathbf{s}) - u_i^*(\mathbf{s}^{\succeq i}) \geq d(\mathbf{s})$ .

Suppose that some strategy profile  $\mathbf{s}^x \in \Sigma$  is selected by a team that is not maximally mutually advantageous. In other words, there exists another strategy profile  $\mathbf{s}^y \in \Sigma$  such that  $d(\mathbf{s}^y) > d(\mathbf{s}^x)$ . It then follows from the above that

$$\min_{i \in I} \left[ \wp_i(\mathbf{s}^y) - \wp_i(\mathbf{s}^{i}) \right] > \min_{i \in I} \left[ \wp_i(\mathbf{s}^x) - \wp_i(\mathbf{s}^{i}) \right]$$

and/or

$$\min_{i \in I} \left[ u_i^*(\mathbf{s}^y) - u_i^*(\mathbf{s}^{\succeq i}) \right] > \min_{i \in I} \left[ u_i^*(\mathbf{s}^x) - u_i^*(\mathbf{s}^{\succeq i}) \right]$$

Hence

$$\mathbf{s}^x \neq \max_{\mathbf{s} \in \Sigma} \{ \min_{i \in I} \left[ \wp_i(\mathbf{s}) - \wp_i(\mathbf{s}^{\leq i}) \right] \}$$

and/or

$$\mathbf{s}^x \neq \max_{\mathbf{s} \in \Sigma} \{ \min_{i \in I} \left[ u_i^*(\mathbf{s}) - u_i^*(\mathbf{s}^{\leq i}) \right] \}$$

and so  $\mathbf{s}^x \notin \Sigma^{\tau}$ .

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