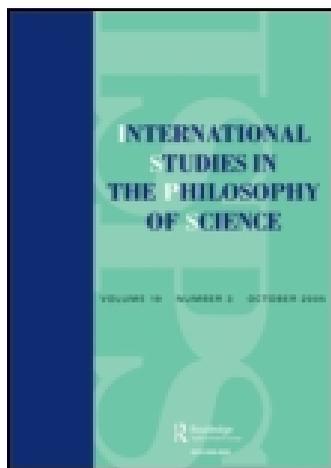


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# Dialectical Contradictions and Classical Formal Logic

Inoue Kazumi

*A dialectical contradiction can be appropriately described within the framework of classical formal logic. It is in harmony with the law of noncontradiction. According to our definition, two theories make up a dialectical contradiction if each of them is consistent and their union is inconsistent. It can happen that each of these two theories has an intended model. A number of examples of this are to be found in the history of science.*

## 1. Introduction: Hegel and Classical Formal Logic

Recently, Nectarios G. Limnatis claimed:

It is a striking fact that despite the immense and steadily growing Hegel discussion, dialectic is not frequently addressed in a systematic and comprehensive way in the English-speaking world. To the best of my knowledge, there has been no large-scale examination of Hegel's conception of dialectic in English in the past two decades. (Limnatis 2010, 3)

Indeed, in the twentieth century there was a period when G. W. F. Hegel's dialectical logic was totally ignored by the majority of logicians. In the biography of A. N. Prior (1962), the works of Petrus Abaelardus, Boethius, Cicero, J. S. Mill, Ockham, and Petrus Hispanus, among others, are listed, but none of Hegel's is. No comment is made on Hegel's logic by Józef Bocheński (1956). Neither can we find the name of Hegel in the index of William Kneale and Martha Kneale (1962). Kurt Leidecker testifies that Heinrich Scholz, the author of *Abriss der Geschichte der Logik*, considered Hegel 'a calamity' (Scholz 1961, x). This was the case not only within the area of formal logic. Logical positivists and early analytic philosophers adopted the same attitude, mostly under the influence of Bertrand Russell, who, together with G. E. Moore, 'rebelled against both Kant and Hegel' and 'began to believe everything the Hegelians disbelieved' (Russell 1959, 42, 48). It was only after several decades that some analytic philosophers began to take an interest in Hegel, mainly prompted by

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Wilfrid Sellars (see Redding 2013). In the domain of formal logic, an exceptional significant work that confronted Hegel's dialectic, though neither large-scale nor in English, was Stanisław Jaśkowski (1999), first published in 1948. We will be concerned with it later (section 14).

Note that Limnatis added 'in the past two decades'. Surely, he must have taken into consideration Graham Priest, who published *In Contradiction* in 1987. A long passage from Hegel's *Enzyklopädie* is cited on its first page. The author makes comments on Immanuel Kant and Hegel, and some of them are orthodox. It is not long, however, before Priest begins preaching his own doctrine. He claims Hegel contended 'that our concepts are contradictory, that there are true contradictions' (Priest 1987, 4). Naturally, this idea of true contradictions is totally unacceptable to classical formal logic. Priest calls it *dialetheism*. According to his definition, a *dialetheia* is any true statement of the form:  $\alpha$  and it is not the case that  $\alpha$ . Indeed, the main aim of this book is to show that 'Hegel was right: our concepts, or some of them anyway, are inconsistent, and produce dialetheias' (Priest 1987, 4). Since, in classical logic, every sentence of the language equally follows from a contradiction, it is natural that dialetheism is usually accompanied by *paraconsistent logic*, which invalidates this principle of explosion.

It seems rather strange to me that all the logicians cited above, including Priest, have the same arbitrary presupposition. They take it for granted that classical formal logic is completely inadequate for Hegel's system of dialectic. This is wrong, in my view. I maintain, among other things, that the idea of dialectical contradictions, put forward by Kant and Hegel, can be properly described within the framework of first-order logic, including first-order model theory. There is no need to resort to any type of nonclassical logic at least for this purpose.

In the following sections, we will examine the relation between Kant's theory of antinomies and Hegel's system of dialectic (section 2) and recognize that Kant gave an accurate description of an antinomy or, in our terms, a dialectical contradiction (section 3), which is not against the law of noncontradiction (section 4). We give the definition of a dialectical contradiction in the framework of first-order logic (section 7) and see how Hegel concerned himself with it (section 8), making a brief comment on so-called objective contradictions. We criticize Hegel's idea of dialectical triads (section 10) and Karl R. Popper's arguments against dialectic (section 11). A few examples of dialectical contradictions in the history of science will be given in sections 12 and 13.

## 2. Kant and Hegel

Friedrich Engels (1878, paragraph 12) said, '[T]o study dialectics in the works of Kant would be a uselessly laborious and little-remunerative task'. Some philosophers take a different view. Russell, for example, claims that Hegel's 'dialectic proceeds wholly by way of [Kant's] antinomies' and concludes, '[A]lthough [Hegel] often criticized Kant, his system could never have arisen if Kant's had not existed' (Russell 1946, 735, 757).<sup>1</sup> Hegel's own words will give the best clue as to whose comment was fair.

In fact, Hegel was full of praise for his precursor when he referred to antinomies in his lectures. He commented: 'In modern times it was primarily Kant who reminded people of dialectic again and let it stand anew on its dignity in fact by means of the perfection of the so-called antinomies of reason' (Hegel 1830, section 81 A1).

What is an antinomy? We will soon see what Kant actually said about it, but Hegel is surprisingly faithful to Kant when he gives the following description of an antinomy: 'assertion of two *opposite* propositions on the *same* object, indeed in such a way that each of them has to be asserted with equal necessity' (Hegel 1830, section 48).<sup>2</sup> Hegel openly admits that Kant's idea of antinomies in metaphysical cosmology lies at the root of his system of dialectic.

The point to note is that antinomies occur not only in the four particular objects taken from cosmology but rather in *all* objects of all sorts, in *all* ideas, notions, and concepts. To know this and to recognize objects of this nature play an essential part in philosophical observation; in fact, this nature constitutes what is to be defined as the *dialectical* factor in logic later. (Hegel 1830, section 48)

Special attention should be paid to his exposition on *quantity* where Hegel (1830, section 100) mentions 'The antinomy of space, time, or matter'. His description strongly suggests that Kant's theory of antinomies have been integrated into Hegel's own system of dialectic.

Still, there is one thing to remember. Hegel criticized Kant for recognizing only this limited number of antinomies: 'It can also be observed that the lack of further research into antinomy more than anything else made Kant enumerate only *four* antinomies' (Hegel 1830, section 48). Was Hegel right?

### 3. Kant: Antinomies

'The Antinomy of Pure Reason' is the core of 'Transcendental Dialectic' in the *Critique of Pure Reason*. Indeed, Paul Guyer and Alan W. Wood (1998, 16) testify, 'Kant originally thought that all of the errors of metaphysics could be diagnosed in the form of these antinomies'. Now I would like to ask rather a silly question: how many antinomies did Kant give there?

Just after the presentation of antinomies, Kant (1787, 490) remarks, 'These sophistical assertions consist of many enough attempts to solve four natural and unavoidable problems of reason; there can be just so many of them, neither more, nor less'. I suspect that this description, among other things, convinced people that he put forward four antinomies but no more. What Kant really asserted here is, however, simply that there are four antinomies of *pure reason*. Nowhere in the *Critique of Pure Reason* does Kant make a denial of the existence of *other* sorts of antinomies.

According to Kant, an antinomy consists of a *thesis* and an *antithesis* which contradict one another and, at the same time, each of them is 'without contradiction in itself . . . only unfortunately the opposite has equally valid and necessary grounds for asserting on its side' (Kant 1787, 449). Anyone who glances through this book will immediately notice that the author invented a peculiar layout to set forth the antinomies, which ingeniously depicts their logical framework. Still we must admit that the line of argument

Kant presented there is not easy to follow. For one thing, Kant often uses *reductio ad absurdum*, but fails to make it clear what axiom or theorem contradicts the denial of the proposition to be proved. In this respect, Kant is utterly unlike Spinoza. At the beginning of each part of his *Ethics*, which is actually titled *Ethica, ordine geometrico demonstrata*, Spinoza gives the full list of his definitions and axioms.<sup>3</sup> Kant never does. At each stage of a demonstration, Spinoza makes it clear what definition, axiom or theorem he is using. Again, Kant never does. At any rate, Kant's proofs of the theses and the antitheses cannot be said to be flawless or rigorous. Yet, in spite of all that, I claim that we can find some clues as to what style of inference Kant had in mind. His remark on the fourth antinomy deserves our attention in this respect: 'However a strange contrast can be seen in this antinomy: from just the same argument from which the existence of a primary being is concluded in the thesis, its nonexistence is concluded in the antithesis, indeed with the same degree of clarity' (Kant 1787, 487). If we take this literally, Kant is suggesting that the proofs of the thesis and the antithesis of this antinomy take the form of  $\varphi_1, \dots, \varphi_n \vdash \psi$  and  $\varphi_1, \dots, \varphi_n \vdash \bar{\psi}$ , respectively. According to classical logic, we could immediately infer from this that  $\{\varphi_1, \dots, \varphi_n\}$  is inconsistent. But before jumping to conclusions, let us see what Kant then commented: 'By the way, the manner of proof in both [the thesis and the antithesis] just fits the common man's reason, which often falls into disagreement with itself as a result of considering its object from two different viewpoints (*Standpunkte*)' (Kant 1787, 489). Kant mentioned this only in passing. In my view, however, it is one of the most significant statements on dialectical contradictions in the history of philosophy. It puts forward two ideas. First, Kant recognizes that not only pure reason but also *the common man's reason* (*die gemeine Menschenvernunft*) often makes a peculiar pair of inferences whereby two contradictory conclusions are drawn at the same time. Second, he says that these two conflicting conclusions are inevitable consequences of observation of the same object from *two different viewpoints*. Not only that, but Kant is ready to give an illustrative example of conflicting inferences that the common man's reason often makes:

Mr de Mairan regarded the dispute between two famous astronomers that arose from a similar difficulty in choosing the viewpoint as an interesting enough phenomenon . . . . Actually one concluded that the *moon revolves on its own axis* because it constantly turns the same side to face the earth; the other concluded that the *moon does not revolve on its own axis* because it constantly turns the same side to face the earth. Both conclusions were right; you will observe what motion the moon makes according to what viewpoint you take. (Kant 1787, 489)

Here Kant is providing another instance of conflicting arguments. Of course, it cannot be called an antinomy of pure reason. In fact, Kant (1787, 449) insists: 'A dialectical theorem of pure reason must therefore be distinguishable from all sophistical propositions [as] it does not concern an arbitrary question that is only raised just for the fun of it'. We have to admit that merely 'sophistical' pair of observations on the motion of the moon certainly cannot make up an antinomy of pure reason. At the same time, Kant recognizes that it has a particular structure similar to one of his antinomies of pure reason. I am convinced that Kant had no objection to calling it an antinomy of some sort, and would like to name it *de Mairan's antinomy*. Moreover,

according to Kant, the common man's reason often faces antinomies of this same nature, which I will give the generic name, *antinomies of the common man's reason*. In this way, we can conclude that Kant, contrary to Hegel's comment, acknowledged the existence of quite a large number of antinomies.

With respect to de Mairan's antinomy, Kant says that a proposition and its negation may both appear to be true if we observe the same range of objects from different viewpoints. I would like, in the first place, to point out that this idea is *not* against Aristotle's law of noncontradiction.

#### 4. Aristotle: The Law of Noncontradiction

In his *Metaphysics*, Aristotle put forward several different renderings of the law of noncontradiction. First, he gives the *ontological* rendering.<sup>4</sup>

Clearly, then, it is a principle of this kind that is the most certain of all principles. Let us next state *what* this principle is. 'It is impossible for the same attribute at once to belong and not to belong to the same thing and in the same relation'; and we must add any further qualifications that may be necessary to meet logical objections. (Aristotle 1989, 1005b)

The sentence in quotation marks in the English translation above may be called the law of noncontradiction in the *unrestricted form*. Aristotle foresaw that 'logical objections' would be raised to it, and accepted the necessity of some qualifications. A proviso of a similar nature should be added also to the *logical* rendering of the law: 'That the most certain of all beliefs is that opposite statements are not both true at the same time' (Aristotle 1989, 1011b).<sup>5</sup> Anyway we have to find out what these 'qualifications' should be. My proposal is to add 'based on observation from the same viewpoint' or the like, adopting Kant's term 'viewpoint'.<sup>6</sup> The logical rendering of the law in the *restricted form* would then be, for example: *A sentence and its negation cannot turn out to be both true when observation is made from one and the same viewpoint.*

Now, it is clear that any antinomy of the common man's reason, discussed in section 3 above, does not violate this law in the restricted form because the antinomy just implies that a sentence appears to be true from one *viewpoint* and its negation does from another. So there seems to be no problem at all. Actually, we would like to go still further and lay down the law exactly in terms of formal logic, but this will be carried out later in section 6.

By the way, from the law of noncontradiction in the restricted form presented above (together with the definition of a *theory* that will be given soon) a corollary follows right away: *Sentences that are judged to be true from one and the same viewpoint make up a consistent theory.* We will utilize this corollary on several occasions.

#### 5. First-order Languages

It is often said that the *Elements* begins with the definition: 'A point is that which has no part'. Strictly speaking, this is wrong. Euclid actually wrote, 'Σημεῖόν ἐστιν, οὐ μέρος οὐθέν'. I think, however, that many people would rather like to skip such

words as ‘what amounts to’, ‘something equivalent to’, and the like. In consequence, we often speak as if the *Elements* were written in modern English. Countless examples are to be found in international news reports today.

Now, I go one step further. When we deal with sentences of a scientific theory that are actually written in some natural language, we will mainly treat their translation in one of our first-order languages, instead. So, we may speak as if the *Elements* were written in a certain first-order language.<sup>7</sup>

Naturally, some philosophical texts are exceptions to this. Francesco Berto (2007, 19) insists, ‘It has been widely recognized that dialectics assumes as its starting point ordinary language’. I agree with him but only partly. I rather find that some of Hegel’s philosophical ideas depend on the characteristics of a particular natural language, namely German. This is apparent from his comments on such words as ‘aufheben’, ‘gewesen’, ‘Existenz’, ‘Urteil’, etc. (see, for example Hegel [1816] 1986, I 113f). What is more, Hegel makes full use of the ambiguity of the natural language when he spins his speculative stories. For instance, he claims, ‘[A] proposition also creates an expectation of some difference between the subject and the predicate’ (Hegel 1830, section 115), and for this reason criticizes the law of identity, ‘A is A’. We know that the copula ‘be [*sein*]’ means sometimes ‘ $\in$ ’, sometimes ‘ $\subset$ ’, and sometimes ‘ $=$ ’. Hegel occasionally points out such ambiguities, but more often abuses them. We could clear up a substantial part of Hegel’s exposition by rendering it into a first-order language, but I do not dare attempt that here.

## 6. Elementary Model Theory

When I say ‘language’, ‘logic’, ‘theory’, etc. the modifier ‘first-order’ will often be implicit in these terms. I assume that we have a definite system  $\mathcal{L}$  of first-order syntax with equality. By adding any *nonlogical vocabulary*, say  $L$ , to  $\mathcal{L}$ , we obtain a *language*, which we will call by the same name  $L$ . In the metalanguage, ‘ $\Leftrightarrow$ ’ is short for ‘if and only if’, and  $\bar{\varphi}$  is the negation of  $\varphi$ .

An  $L$ -structure  $\mathcal{A}$  consists of the domain of discourse, namely  $\text{dom}(\mathcal{A})$ , and a system of interpretation of the nonlogical vocabulary  $L$ . If  $\varphi_0$  is an atomic sentence of  $L$ ,  $\mathcal{A}$  gives  $\varphi_0$  its truth-value. The semantics of Boolean connectives and quantification are as well known. For any sentence  $\varphi$  of  $L$ , if  $\varphi$  is *true* in the  $L$ -structure  $\mathcal{A}$ , we will say that  $\mathcal{A}$  is a *model* of  $\varphi$ , or  $\mathcal{A} \models \varphi$ .

We call any set of sentences of  $L$  a *theory* of  $L$ .<sup>8</sup> For practical purposes, we will later give the definition of a *significant* theory. If  $\mathcal{A} \models \psi$  holds for any sentence  $\psi$  such that  $\psi \in T$ , we say that  $\mathcal{A}$  is a *model* of  $T$ , or  $\mathcal{A} \models T$ . If  $\varphi$  is a sentence of  $L$ , and  $\mathcal{X} \models \varphi$  holds for any  $L$ -structure  $\mathcal{X}$ ,  $\varphi$  is called *valid*. If  $\bar{\varphi}$  is valid,  $\varphi$  is called *impossible*. If  $\varphi$  is neither valid nor impossible, we call  $\varphi$  *contingent*.

First-order logic guarantees that  $\varphi \vdash \psi \Leftrightarrow \varphi \models \psi$  and  $T \vdash \psi \Leftrightarrow T \models \psi$ , where  $\varphi$  and  $\psi$  are any sentences and  $T$  is any theory. A theory is *satisfiable* (or has a *model*) if and only if it is *consistent*. I take advantage of these relations, and will simply say ‘ $T \models \varphi$ ’ short for ‘ $T \vdash \varphi$  and  $T \models \varphi$ ’, ‘consistent’ short for ‘consistent and satisfiable’, etc.

Let  $T = \{\varphi_1, \dots, \varphi_n\}$ . Then  $\varphi_1, \dots, \varphi_n$  are called the *axioms* of  $T$ , but we do not care whether they are deductively independent of each other. Generally speaking, in model theory we are not concerned with the deductive structure of a theory.

Some issues have been raised about the nature of a model of a theory in empirical science. I cannot afford to discuss this problem here, and simply take it for granted that model theory is applicable to empirical science as well as to mathematics.<sup>9</sup>

Still there is another problem. Some philosophers doubt whether *negation* used in modern logic can rightly denote Aristotle's negation. Paul Redding, at the suggestions of Laurence R. Horn (1989) and Patrick Grim (2004), makes a comment about 'Aristotle's vision':

Denying a predicate of a subject cannot be thought of asserting 'not p' where 'p' is the content expressed in affirming that predicate of the subject. This is a consequence of Aristotle's basing his logic on *terms* rather than *propositions*, such that it is terms and not 'propositions' that are the primary targets of negation. (Redding 2007, 205)

In my view, first-order logic is a worthy successor of an essential part of Aristotelian logic. Here we should remember how the negation of an atomic sentence is treated in model theory. Assume that ' $Fa$ ' is an atomic sentence of  $L$ ,  $\mathcal{A}$  is an  $L$ -structure,  $\mathbf{a} \in \text{dom}(\mathcal{A})$ ,  $\mathbf{F} \subseteq \text{dom}(\mathcal{A})$ , ' $a$ ' designates  $\mathbf{a}$ , and ' $F$ ' designates  $\mathbf{F}$ . Then,  $\mathcal{A} \models Fa \Leftrightarrow \mathbf{a} \in \mathbf{F}$ ;  $\mathcal{A} \models \neg Fa \Leftrightarrow \mathbf{a} \notin \mathbf{F} \Leftrightarrow \mathbf{a} \in \mathbf{F}^C$ , where  $\mathbf{F}^C$  is the *complement* of  $\mathbf{F}$  with respect to  $\text{dom}(\mathcal{A})$ . I think this is enough to show that first-order logic, containing first-order model theory, is able to represent the function of negation in term logic.

By the way, it is to be noted that Hegel distinguished between being *complementary* and being *contradictory* (see for example Hegel 1830, section 119). If the predicate ' $G$ ' in  $L$  designates a subset  $\mathbf{G}$  of  $\text{dom}(\mathcal{A})$  such that  $\mathbf{G} = \mathbf{F}^C$ , then ' $Fa$ ' and ' $Ga$ ' are *complementary*. On the other hand, if  $\mathbf{H} \subset \mathbf{F}^C$  and ' $H$ ' designates  $\mathbf{H}$ , then ' $Fa$ ' and ' $Ha$ ' are *incompatible* or *contradictory*.  $\mathbf{G}$  is uniquely determined by  $\mathbf{F}$ , but  $\mathbf{H}$  is not. Obviously,  $\mathcal{A} \models \forall x (Hx \rightarrow \neg Fx)$ . In a chapter on Hegel's *Phenomenology*, Robert B. Brandom (2002, 179; see also 223) rightly says, '*not-p* is the minimal incompatible with  $p$ . It is entailed by everything materially incompatible with  $p$ '.

Now finally, we are ready to present the law of noncontradiction within the framework of first-order model theory. It is a formulation of the law in the restricted form: *A sentence and its negation cannot both be true in the same structure*. It is worth noting that Jaśkowski (1999, 36–37) proposed to render this law as 'Two contradictory sentences are not both true if the words occurring in those sentences have the same meanings'. Needless to say, a structure uniquely determines the meaning of each nonlogical symbol in the language.

## 7. Dialectical Contradiction

Remember the corollary given at the end of section 4. The two astronomers of de Mairan's antinomy must therefore have formulated two distinct consistent theories based on observations from two different viewpoints.<sup>10</sup> Let those theories be  $S_1$  and  $S_2$  in  $L$ , and let the sentences  $\chi$  and  $\psi$  in  $L$  stand for 'The moon constantly turns

the same side to face the earth' and 'The moon revolves on its own axis', respectively. According to de Mairan or Kant,  $S_1 \cup \chi \vdash \psi$  and  $S_2 \cup \chi \vdash \bar{\psi}$ .<sup>11</sup> Put  $U_1 = S_1 \cup \chi$  and  $U_2 = S_2 \cup \chi$ . Then,  $U_1 \vdash \psi$  and  $U_2 \vdash \bar{\psi}$ . Naturally, this means at the same time that  $U_1 \vDash \psi$  and  $U_2 \vDash \bar{\psi}$ . I would like to draw attention to this particular relation between two theories.

In fact, a similar relation has already been studied in model theory. Assume that  $L_1$  and  $L_2$  are languages, and  $T_1$  and  $T_2$  are theories of  $L_1$  and  $L_2$ , respectively. Model theory has a theorem that says: if  $T_1 \cup T_2$  is inconsistent, then there is a sentence  $\varphi$  of  $L_1 \cap L_2$  such that  $T_1 \vDash \varphi$  and  $T_2 \vDash \bar{\varphi}$ , where  $T_1$  and  $T_2$  are said to be *separated* by  $\varphi$  (see Chang and Keisler 1990, 88–95; Hodges 1997, 148). Interestingly, an antinomy of the common man's reason, as well as Kant's fourth antinomy of pure reason, implies this same logical relation. Attaching a condition that both theories are consistent, I will call this relation a *dialectical contradiction*.

**Definition.** Assume that  $L_1$  and  $L_2$  are first-order languages,  $T_1$  and  $T_2$  are consistent theories of  $L_1$  and  $L_2$ , respectively. If there is a sentence  $\varphi$  of  $L_1 \cap L_2$  such that  $T_1 \vdash \varphi$  and  $T_2 \vdash \bar{\varphi}$ , we say that  $\varphi$  *separates*  $T_2$  from  $T_1$  and

- (i) that  $\{T_1, T_2\}$  is a dialectical contradiction, or
- (ii) that there is a dialectical contradiction between  $T_1$  and  $T_2$ , or
- (iii) that  $T_1$  and  $T_2$  are dialectically contradictory.

In other words, a dialectical contradiction is a relation between two theories  $T_1$  and  $T_2$  such that  $T_1 \cup T_2$  is inconsistent but  $T_1$  and  $T_2$  are each consistent. Assuming that there is an area of science to be called *dialectic*, the idea of a dialectical contradiction should play a central role there. Note that we defined a dialectical contradiction in the first instance as a *syntactical* relation. But  $T_1 \vdash \varphi$  and  $T_2 \vdash \bar{\varphi}$  immediately imply  $T_1 \vDash \varphi$  and  $T_2 \vDash \bar{\varphi}$ . Since  $T_1$  and  $T_2$  are consistent, there will be such  $L$ -structures  $\mathcal{A}_1$  and  $\mathcal{A}_2$  that  $\mathcal{A}_1 \vDash T_1$ ,  $\mathcal{A}_2 \vDash T_2$ , and in consequence,  $\mathcal{A}_1 \vDash \varphi$ ,  $\mathcal{A}_2 \vDash \bar{\varphi}$ . Each of  $\mathcal{A}_1$  and  $\mathcal{A}_2$  has its own domain and interpretation, which make either  $\varphi$  or  $\bar{\varphi}$ , *exclusively*, true. Obviously, both  $\varphi$  and  $\bar{\varphi}$  are contingent.

Kant should be credited with originating modern dialectic. His description of de Mairan's antinomy, as we call it, gives a clear illustration of the logical form of a dialectical contradiction, which is free from any metaphysical speculation. In my view, it was one of Kant's greatest contributions to philosophy. If we look back on Kant's first three antinomies of pure reason, we will see that they are, in essence, also reducible to some dialectical contradictions. We do not go into details of this here.

Unfortunately, a couple of things prevented Kant from developing this idea of dialectical contradictions further. For one thing, he was anxious about the law of noncontradiction. For fear of violating it, he even made some attempts to 'solve' his antinomies (see Kant 1787, 504ff.), which were, from my perspective, of no use. For another, Kant's major objective was, needless to say, a critique of pure reason. He was well aware that there are many antinomies that have nothing to do with pure reason, but they were not his primary concern. Thus, it was left to Hegel to pursue this study further.

## 8. Hegel: Understanding and Reason

Klaus Düsing (2010, 97) says that Hegel's dialectic 'is best understood when one also considers the phases of its formation and the arguments behind it'. For Hegel, the ultimate goal was the *Absolute*; dialectic was the means of attaining this goal by a purely rational process, not by postulating an intellectual intuition. Karl Marx and Engels took a special interest in the method, dialectic, but never in the goal, the Absolute. The scope of my own interest is similar, or even narrower.

Hegel's great achievement is the recognition of the ubiquitous and constructive nature of dialectical contradictions. He states, '[T]here is *nothing whatsoever* in which a contradiction, that is, opposite determinations cannot and must not be shown' (Hegel 1830, section 89). Note that Hegel refers to a dialectical contradiction simply as a *contradiction* (*Widerspruch*).

As we saw above, Kant gave a precise description of a dialectical contradiction, calling it an antinomy. It does not make sense to ask whether the moon *actually* revolves on its own axis or not. That depends on our frame of reference. In some frames of reference, the moon revolves, in others not. This is not against the law of noncontradiction. The same contingent sentence may be affirmed from some perspectives and denied from others.

One of Hegel's basic ideas is, in our own words, that we should make observations not from a single viewpoint but from multiple viewpoints. We adopted Kant's term 'viewpoints'. Hegel did not. Instead, he makes use of the ambiguous word, *determination(s)* (*Bestimmung/Bestimmungen*). He would say 'adhere to one determination' rather than 'make observation only from a single viewpoint', and 'comprehends the unity of opposing determinations' rather than 'make observations from conflicting viewpoints'. In this respect, what is of fundamental importance to Hegel is the distinction between *understanding* (*Verstand*) and *reason* (*Vernunft*). Kant already contrasted these two. Hegel also draws a sharp distinction between them, but in a way quite different from Kant's. According to Hegel, understanding plays an elementary role in cognitive activity and is capable of providing only a very limited range of perspectives. He explains, 'The thinking that brings out only *finite* determinations and moves within such a range is called *understanding* (in the more precise sense of the word)' (Hegel 1830, section 25).

Remember again the corollary of the law of noncontradiction. Understanding makes observation on some part or phase of the world from a fixed viewpoint, and consequently formulate a consistent theory, say  $T_1$ . This is what Hegel calls the *phase of understanding*. Hegel fully appreciates the function of understanding, saying '[W]ithout understanding there is no firmness or determinacy' (Hegel 1830, section 80 A). Reason, in contrast, studies the same part or phase of the world from *multiple* viewpoints and, in this way, provides a variety of perspectives. In consequence, another consistent theory, say  $T_2$ , among other things, will be developed in addition to  $T_1$ . Hegel compares understanding with reason and criticizes understanding for its 'one-sidedness (*Einseitigkeit*)' (Hegel 1830, section 81). Understanding always performs such *abstraction* that only an aspect observable from a single definite

viewpoint is acknowledged and all others are ignored. Therefore, Hegel (1830, section 89) says, '[T]he abstraction of understanding constitutes coercive adherence to one determination and an effort to obscure and avoid consciousness of the other determination'. Understanding never breaks the law of noncontradiction even in the unrestricted form. Accordingly, Hegel (1830, section 115) says that the law of noncontradiction 'is nothing but the law of *abstract understanding*'. On the other side of the coin, understanding, by nature, does not know how to deal with dialectical contradictions. Hegel says that some sort of metaphysics is actually full of dialectical contradictions, which 'notwithstanding escape the notice of [understanding]' (Hegel 1830, section 130).

$T_1$  and  $T_2$ , mentioned above, can be dialectically contradictory. Assume that they happen to be. Then, there is a contingent sentence  $\varphi$ , which separates  $T_2$  from  $T_1$ :  $T_1 \vdash \varphi$  and  $T_2 \vdash \bar{\varphi}$ . Reason, in contrast to understanding, concerns itself with a dialectical contradiction. When it does, it notices, in the first instance, that the law of noncontradiction in the unrestricted form is broken, and takes a *negative* attitude as Kant did towards his antinomies. Certainly,  $T_1 \cup T_2$  or  $\{\varphi, \bar{\varphi}\}$  is inconsistent. Hegel (1830, section 89) comments, 'If such a contradiction is recognized, it is common to conclude, "Therefore this object is *nothing*"'. In fact, we will see that Popper, more than a hundred years later, made quite a similar claim. Hegel calls such a situation the *dialectical phase* or the *phase of negative reason*. (Often  $T_2$  is simply called the *negation* of  $T_1$ .) Comparing and contrasting  $T_1$  with  $T_2$ , or  $\varphi$  with  $\bar{\varphi}$ , Hegel remarks that the former has made '*the transition to [its] opposite*' (Hegel 1830, section 81). He emphasizes that cognitive activity is destined to move through this stage, and says, 'The dialectical [phase] therefore forms the moving spirit (*die bewegende Seele*) of scientific progress' (Hegel 1830, section 81). By the way, it is well known that Engels (1883, II) regarded 'the transition to its opposite' as one of the main 'laws' of dialectic.

In the next phase, reason turns positive. It confirms that  $T_1$  and  $T_2$  are each consistent in spite of the dialectical contradiction between them. Each can work well without violating the law of noncontradiction in the restricted form. The apparent conflict is the result of observation of the same part or phase of the world from different viewpoints. (It is interesting to see that Wandschneider 2010, 37 remarks, '[T]he dialectical contradiction is . . . actually only an *apparent* contradiction. While the reciprocally overturning predications *appear* to contradict each other, they actually relate to *different aspects* of the argument'.) In the illustration above,  $\bar{\varphi}$  can be true as well as  $\varphi$ . (It is not right to regard their truth as subjective.  $\varphi$  is objectively true in some structures, and  $\bar{\varphi}$  is objectively true in others.) So Hegel (1830, section 82) confirms, 'Dialectic has a *positive* result'. This third phase is called the *speculative phase* or the *phase of positive reason*. Hegel (1830, section 82) says, 'The speculative stage or the stage of positive reason comprehends the unity of opposing determinations'. We can, in this phase, utilize both of the two conflicting viewpoints, as we will see later.

We have ignored many of Hegel's important ideas, 'category', 'mediation', 'the negation of negation', etc., which are usually regarded as essential to his dialectic. It should be noted, however, that we are not attempting to reconstruct Hegel's philosophy. Indeed, we will soon renounce another Hegelian notion, 'triad'.

There is one other thing. It has been claimed that Hegel's logic consists in ontology and he developed the idea of 'objective contradictions,' which Marx and Engels inherited. In fact, the first volume of *The Science of Logic* is titled 'The Objective Logic.' In a 'Remark' in it, Hegel cites Zeno's paradoxical conclusion that motion is impossible, and comments, '[F]rom that it does not follow that ... there is no motion, but rather follows that motion is *existent (daseiend)* contradiction itself' (Hegel [1816] 1986, II 76). Assuming that the motion of an arrow embodies a contradiction, it might be claimed that our definition of dialectical contradiction cannot adequately cover the whole of Hegel's concept of a contradiction.

Against such a claim, I would insist that anything recognizable is describable. We know how to *describe* Zeno's arrow paradox. Hegel describes a body in motion, in general, as: '[A]t one and the same moment it is here, and not here' (Hegel [1816] 1986, II 76). Once an objective contradiction is described in one way or another, I am sure, we obtain something very similar to an antinomy, that is, a dialectical contradiction. In fact, you could, if you like, regard Kant's antinomies of pure reason as objective contradictions inherent in the *universe*. As remarked above, both thesis and antithesis of an antinomy can be objectively true.

## 9. Intended Model

We will apply model theory to theories in all scientific disciplines. *Any* set of sentences of a language is, by definition, equally called a *theory*. Some of them are consistent, others not. If a theory is consistent, it must have a variety of models, but many of them will not seem to have any practical application. The concept of the *intended model* of a theory will help us to deal with such matters.

Suppose that we are interested in a particular domain **A**. We prepare a language *L*, and construct an *L*-structure  $\mathcal{A}$ , whose domain is **A**. If we succeed in developing a theory *T* in *L* such that  $\mathcal{A} \models T$ ,  $\mathcal{A}$  is called the *intended* (or *standard*) *model* of *T*.<sup>12</sup> Certainly it is true that mathematicians, on the one hand, sometimes formulate a theory with no previous knowledge of its models and, on the other hand, sometimes study a model that was not originally intended, namely a *nonstandard model*. But, at least, the existence of an intended model provides sufficient, if not necessary, evidence of *practicality*. So, let us call a theory *significant* if it has an intended model. Of course, this is a term of a *pragmatic* nature rather than *semantic*. From now on, we will be mainly concerned with significant theories and their intended models.

Now the question will arise whether two significant theories can be dialectically contradictory. Certainly they can. In fact, plenty of examples will be found in the history of science (see sections 12 and 13).

## 10. The Dialectical Triad

The phases of understanding, negative reason, and positive reason make up the Hegelian *dialectical triad*. According to Hegel, they are 'factors of every logical entity' (Hegel

1830, section 79). Afterwards, however, philosophers began to call this triad by different names. For instance, Popper says:

Dialectic (in the modern sense, i.e. especially in the sense in which Hegel used the term) is a theory which maintains that something — more specially, human thought — develops in a way characterized by what is called the dialectic triad: *thesis*, *antithesis*, and *synthesis*. (Popper 1989, 313)

Since Hegel himself never referred to the three phases or stages by such names, Popper's description contains a factual error, but I will not go into this problem of terminology.

I admit that human cognition often shows a pattern of development just like this so-called dialectical triad. The history of science is full of stories of how seriously scholars worried when they faced a dialectical contradiction between two significant theories. They were confronted with an apparent syntactical conflict. Such a situation may be aptly called the dialectical phase. Actually, in advance of this stage, there must have been a period when a single classic theory reigned supreme, which would embody the phase of understanding. Then finally, the speculative phase begins, where the two dialectically contradictory theories are both accepted by the academic community. From a semantical perspective, there is no conflict. It may even seem suitable to call this third phase by the non-Hegelian term *synthesis*.

In spite of all that, I do claim that we should not attach much importance to such a triad. Contrary to popular belief, the triad is not essential for dialectic. Dialectic should be characterized as the study of dialectical contradictions but not of dialectical triads. Whenever Hegel talks about a dialectical triad, it is found that what really matters is a dialectical contradiction.

Triadic structures appear everywhere in Hegel's writing. He says that the solar system as well as the state (*der Staat*) is 'a system of three syllogisms' (see Hegel 1830, section 198). A syllogism consists of *three* sentences and contains *three* terms; logic itself consists of *three* parts; and so forth. Triads are characteristic of Hegel's own speculation rather than dialectic. Noticeably, Kant had an extraordinary attachment to ternary structures.<sup>13</sup> So did Hegel. Probably, it was associated with the Christian doctrine of the *Trinity*. A triad is an *idée fixe* of Hegel's. He seems to have forced dialectic into this mould. As a consequence, for instance, when *becoming* (*Werden*) makes the transition to *determinate being* (*Dasein*), we have to jump suddenly from one triad to another.

I admit that 'the transition to its opposite' sometimes seems to be an apt description when a dialectical contradiction takes form. In fact, I will use this expression occasionally in subsequent sections. Still this does not mean that every 'transition to its opposite' is worth studying. The worst example is *being* and *nothing* which are said to be the same (see Hegel [1816] 1986, I 83; 1830, section 88). Unfortunately, this strange identification has attracted and enchanted many philosophers. Certainly it is not difficult to make up a specious argument about it. One type often found refers to *complementarity* mentioned in section 6 (e.g. Wandschneider 2010). Now I give a greatly simplified version just for the fun of it.

Let our domain of discourse be  $A$ . You can define any subset of  $A$  as you like. When you define a subset  $F$  of  $A$ , you are at the same time defining the *complement* of  $F$  with respect to  $A$ , namely  $F^C$ . In other words, the determination of  $F$  divides  $A$  into two complementary subsets,  $F$  and  $F^C$ . Now consider such an extreme case that  $F$  coincides with  $A$ . Obviously,  $F = A \Leftrightarrow F^C = \emptyset$ . Generally speaking,  $x \in A \Leftrightarrow x$  exists (in the domain  $A$ ); if ' $F$ ' designates  $F$  and  $F = A$ , ' $F$ ' embodies no determination except existence. On the other hand,  $x \in \emptyset \Leftrightarrow x$  does not exist. Thus, the definition of existence amounts to the definition of nonexistence.

## 11. Popper: Criticism of Dialectic

Popper (1989) severely criticized dialectic both of Hegel and of Marx. More than half a century after its publication in 1940, his article is still regarded as one of the most influential critiques of dialectic. Sean Sayers (1992, 1) testifies: '[Popper's] criticisms of dialectic are accepted by many analytical philosophers who are sympathetic to Marxism, and particularly by analytical Marxists' (see also Groisman 2007).

Popper (1989, 316) claims, in the first instance, '[Dialectician's] assertion amounts to an attack upon the so-called "law of contradiction"'. In terms of the difference between viewpoints I showed, in section 4, that Kant's idea of antinomies is in accordance with the law of noncontradiction in the restricted form. Here I will prove the same thing about dialectical contradictions in terms of model theory. Actually, we only need to go over what was said in section 7.

Let  $\varphi$  be a contingent sentence in  $L$ . Popper maintains that according to the law of noncontradiction, it is impossible that both  $\varphi$  and  $\bar{\varphi}$  are true. Model theory tells us that whenever we talk about the truth-value of a contingent sentence we have to make it clear what *structure* we are referring to. Let  $\mathcal{A}_1$  and  $\mathcal{A}_2$  be the models of  $\varphi$  and  $\bar{\varphi}$ , respectively; that is to say,  $\mathcal{A}_1 \models \varphi$  and  $\mathcal{A}_2 \models \bar{\varphi}$ . In other words,  $\varphi$  is true in  $\mathcal{A}_1$ , and  $\bar{\varphi}$  is true in  $\mathcal{A}_2$ , which is, as shown in section 6, completely in accordance with the law of noncontradiction in the restricted form. I think this can be a simple but satisfactory answer to Popper.  $\varphi$  and  $\bar{\varphi}$  are *each true*, certainly not in the *same* structure, but in two *distinct* structures.

I surmise that model-theoretic semantics, already in its embryonic stage, was destined to be concerned with dialectical contradictions. Just think of Alfred Tarski's famous paradigm: "[I]t is snowing" is a true sentence if and only if it is snowing', which was given in 1930s well in advance of the birth of model theory (Tarski 1956, 156). But why did Tarski take this particular sample 'It is snowing' instead of any so-called *eternal sentence* such as 'The earth moves around the sun'? Apparently, he took an obvious example that is *sometimes* true and *sometimes* false. He must have wanted to show that the truth-value of a contingent sentence depends on the domain of discourse as well as the interpretation of the nonlogical vocabulary.

Interestingly, Berry Groisman (2007) gives the following example in his criticism of Popper. He argues that the sentence 'The sun is shining now' can be both true and false at the same moment. It may happen, he points out, that 'the Sun is shining in Madrid, Florence, Cairo, Kiev, but it is also *not* shining in Tokyo, Sidney, London, Amsterdam,

etc.' (Groisman 2007, 7). I think Groisman is perfectly right about that. Different domains may give different truth-value to the same contingent sentence.

Yet, critics of dialectic may try another tactic: the *principle of explosion* (ex contradictione sequitur quodlibet). In fact, Popper goes on:

[I]f one were to accept contradictions then one would have to give up any kind of scientific activity: it would mean a complete breakdown of science ... for from a couple of contradictory statements any statement whatever can be validly inferred. (Popper 1989, 317)

Suppose that  $T_1$  and  $T_2$  are consistent theories in  $L$  and  $T_1 \vdash \varphi$ ,  $T_2 \vdash \bar{\varphi}$ . Assuming that  $\perp$  is any impossible sentence in  $L$ , Popper is right in saying that  $\varphi, \bar{\varphi} \vdash \perp$  or that  $\{\varphi, \bar{\varphi}\} \vdash \perp$ . But, so what? It has no effect on  $T_1$  or  $T_2$ .  $\perp$  is not a theorem of  $T_1$ ; nor of  $T_2$ . Neither theory breaks down.

## 12. Examples in Physical Science

Assume that  $T$  is a significant theory generally accepted by an academic community. A rebel group puts forward such a new theory  $U_0$  that  $T$  and  $U_0$  are dialectically contradictory. Actually,  $U_0$  may merely be a set of a few hypotheses. As time goes on, however,  $U_0$  develops into a greater theory  $U$ , which finally gains the acceptance of the community. (Note that we follow the convention adopted in section 5.) Now, suppose we try to get a bird's-eye view of dialectical contradictions in the history of science, where  $T$ ,  $U_0$  and  $U$  are among the subjects. Then, we can outline the main features by talking only about the dialectical contradiction between  $T$  and  $U$  without mentioning  $U_0$ . At least this is the way we are going to work here.

In the preface to the second edition of the *Critique of Pure Reason*, Kant compares his own achievement in philosophy to the *Copernican revolution* in astronomy. In fact, Copernicus showed that a conversion from one viewpoint to another can achieve a significant scientific breakthrough.

Let  $T_1$  in  $L_1$  and  $T_2$  in  $L_2$  be geocentric (Ptolemaic) and heliocentric (Copernican) theories, respectively, of the motion of the sun and earth. There will be a dialectical contradiction between  $T_1$  and  $T_2$ . Let  $\varphi$  be the sentence in  $L_1 \cap L_2$  that denotes 'The sun makes a periodic motion with the period of approximately 24 hours'. Then,  $T_1 \vdash \varphi$  and  $T_2 \vdash \bar{\varphi}$ . Observations from two different viewpoints lead us to two dialectically contradictory theories.

Reductionists may insist that we should exclusively accept the heliocentric system and totally reject the geocentric. But this is an irrational attitude based on prejudice in theory as well as in practice. In theory, the general principle of relativity entitles us to choose whatever frame of reference we like. In practice, it is impossible for us to live without geocentrism. Most people will regard  $\varphi$  above as an 'eternal truth'. More than anything else, I would like to point out that the geocentric system is indispensable to astronomers in ground-based observatories. They make observations exactly in a geocentric frame of reference, and then translate their data. We should never say that  $T_1$  is wrong or incorrect. At the same time, the need for heliocentrism is beyond any doubt. As soon as *mass* or *force* is taken into account, geocentrism

becomes completely impractical. After all, we require both of the two viewpoints. We have to accept and make use of two theories that are dialectically contradictory. Science should cover both.<sup>14</sup>

At the beginning of the twentieth century, a number of epoch-making dialectical contradictions confronted physicists. One, introduced by Albert Einstein in 1905, is the dialectical contradiction between classical mechanics and the special theory of relativity. Suppose that  $T_1$  and  $T_2$  are classical and special relativistic theories of mechanics, respectively, in the language  $L$ , and  $\varphi$  is the so-called *theorem of the addition of velocities* in  $L$ . Then  $T_1 \vdash \varphi$ ,  $T_2 \vdash \bar{\varphi}$ . There is a dialectical contradiction between the two theories. Let  $\mathcal{A}_1$  and  $\mathcal{A}_2$  be the intended models of  $T_1$  and  $T_2$ , respectively. Then  $\text{dom}(\mathcal{A}_1) \subset \text{dom}(\mathcal{A}_2)$ . A (macroscopic) body can belong to  $\text{dom}(\mathcal{A}_1)$  if its velocity is far less than the velocity of light.  $T_1$ , as well as  $T_2$ , is applicable to any body that satisfies this condition.

Apparently, the development from  $T_1$  to  $T_2$  is another instance of ‘the transition to its opposite’. Something that was affirmed in  $T_1$  is now denied in  $T_2$ . At the same time, we may say that  $T_1$  is *preserved* (*aufgehoben*) in  $T_2$ . In the next section, we will see some other examples of this.

### 13. Examples in Mathematics

We saw how observations from different viewpoints lead us to a dialectical contradiction. Once a dialectical contradiction is given in itself, however, there is no need to care what combination of viewpoints, if any, has brought it about. Admittedly, the idea of different viewpoints provided motivation for dialectic, and it can work well for illustrative purposes. If we want to make our methodology as rigorous as possible, however, we should not continue to rely on it too long. Remember that we have already defined a dialectical contradiction without the help of the idea of viewpoints. Anyway, in the description of the following examples, we will make no reference to it.

In the nineteenth century, mathematicians faced the dialectical contradiction between *Euclidean* and *non-Euclidean* (namely, either *hyperbolic* or *elliptic*) systems of geometry. It was another epoch-making event in the history of science. Suppose that  $L$  is a certain first-order language,  $T_1$  and  $T_2$  are the sets of axioms of Euclidean and hyperbolic plane geometry, respectively, both written in  $L$  (see Tarski and Givant 1999). Let  $\varphi$  be the sentence of  $L$  for the *parallel postulate*. Then  $T_1 \vdash \varphi$  and  $T_2 \vdash \bar{\varphi}$ . Theoretically, this example is good enough, but it might be commented that  $\varphi$  should be called an *axiom* rather than a *theorem* of  $T_1$ . Well, then let  $\psi$  be the sentence of  $L$  for ‘The sum of the three interior angles of any triangle equals two right angles’. Evidently,  $T_1 \vdash \psi$  and  $T_2 \vdash \bar{\psi}$ . In any case, there is a dialectical contradiction between  $T_1$  and  $T_2$ . The intended model  $\mathcal{B}_1$  of  $T_1$  is obvious. As the model  $\mathcal{B}_2$  of  $T_2$  we may take, for example, the so-called Poincaré disc model. Let ‘ $Ln(x)$ ’ in  $L$  denote ‘ $x$  is a line’. ‘ $Ln(x)$ ’ will be given different interpretations by  $\mathcal{B}_1$  and  $\mathcal{B}_2$ .

It was shown in the twentieth century that the *axiom of choice* and the *continuum hypothesis* are propositions of an independent nature akin to the parallel postulate.

Mathematicians have examined dialectically contradictory theories separated by each of these propositions.

Looking back on the history of mathematics, the theories of natural numbers, integers, rationals, reals, and complex numbers are dialectically contradictory of one another. Let these theories, written in adequate first-order languages  $L_1, \dots, L_5$  (without inequality) be  $U_1, \dots, U_5$ , respectively. Some sentences that are affirmed in  $U_i$  are denied in  $U_{i+1}$  ( $1 \leq i \leq 4$ ). Again, we see ‘the transition to its opposite’.

For example, put  $\varphi = \neg \exists x (x^2 = 2)$ .  $U_3 \vdash \varphi$  and  $U_4 \vdash \bar{\varphi}$ .  $U_3$  and  $U_4$  are dialectically contradictory. Speaking in Hegel’s terms, understanding is not able to cope with  $U_4$  when it concerns itself with  $U_3$ . Hegel (1830, section 231) says that geometry ‘finally encounters *incommensurability* and *irrationality* on its way, ... where, if it wants to go further with determination, it is *driven out* over the understanding principle’. Naturally, Hegel knew that the diagonal  $d$  of a square and its side  $l$  are incommensurable and the ratio of  $d$  to  $l$  is irrational.

Let  $\mathcal{C}_1, \dots, \mathcal{C}_5$  be the intended models of  $U_1, \dots, U_5$ , respectively. Then,  $\text{dom}(\mathcal{C}_i) \subset \text{dom}(\mathcal{C}_{i+1})$ . (See, for example, Lightstone 1978, 122ff.) Historically,  $U_i$  was developed for the purpose of describing  $\mathcal{C}_i$ . Let  $\varphi$  be as above. Then  $\mathcal{C}_3 \models \varphi$  and  $\mathcal{C}_4 \models \bar{\varphi}$ .  $\varphi$  is true if ‘ $x$ ’ ranges over only  $\text{dom}(\mathcal{C}_3)$ , namely the set of rationals, but false if it ranges over  $\text{dom}(\mathcal{C}_4)$ , namely the set of reals. Note that only a sentence with quantifiers can separate  $U_{i+1}$  from  $U_i$ . If  $U_i \vdash \chi$  and no quantifiers occur in  $\chi$ , then  $U_{i+1} \vdash \chi$ . Though  $U_{i+1}$  and  $U_i$  are dialectically contradictory,  $U_i$  is *preserved* in  $U_{i+1}$  in this sense.

The sequence of theories  $U_1, \dots, U_5$  embodies the development of algebra in both theoretical and historical senses. Naturally, Hegel said nothing about the progress of algebra, but did mention the progress of philosophy:

With regard to seemingly various philosophies, the *history of philosophy* shows, on the one hand, that they are simply different stages in the development of a single philosophy and, on the other hand, that the particular *principles*, each of which lies on the basis of a system, are merely *branches* of one and the same totality. The last philosophy in chronological order is the result of all preceding philosophies and must therefore contain the principles of all of them. (Hegel 1830, section 13)

Of course, Hegel is referring to the development of philosophical systems that began with Parmenides of Elea and was completed by Hegel himself. At the same time, it will not be any the less interesting to read ‘philosophy’ in it as ‘algebra’. The last system in the example above, namely  $U_5$ , *contains* all of  $U_1$  to  $U_4$ .

#### 14. Jaśkowski: Discussive Logic

There are various systems of paraconsistent logic (see, for example, da Costa 1999; Batens et al. 2000), but in any of them the principle of explosion is not validated by the consequence relation it adopts. The first formal system of paraconsistent logic was given by Jaśkowski in 1948 (see Priest 2000, 223).

When Jaśkowski developed his *discussive logic* more than 30 years in advance of Priest’s dialetheism, he already bore in mind Hegel’s dialectic. He says, ‘[Hegel]

opposed to classical logic a new logic, termed by him dialectics, in which co-existence of two contradictory statements is possible' (Jaśkowski 1999, 35). The basic idea of Jaśkowski's discussive logic may be described in our terms as follows. Let  $T_1, \dots, T_n$  be consistent theories some of which are dialectically contradictory one another. Put  $S = T_1 \cup \dots \cup T_n$ . The theory  $S$ , which is evidently inconsistent, is called a *discussive system*.

Similar ideas were later explored by Nicholas Rescher and Brandom (1979) and Bryson Brown and Priest (2004). According to the latter,  $T_1, \dots, T_n$  are to be referred to as 'chunks'. Some, but not all, theorems of  $T_i$  can be brought to  $T_j$ . (Remember the relation between  $U_i$  and  $U_{i+1}$  mentioned in section 13. From  $U_i \vdash \varphi$  we can infer  $U_{i+1} \vdash \varphi$  if  $\varphi$  satisfies certain conditions.)

Some of Jaśkowski's comments attract our attention. He says, '[O]ne person's [mutually inconsistent] opinions are so pooled into one system although that person is not sure whether the terms occurring in his various theses are not slightly differentiated in their meanings' (Jaśkowski 1999, 43). Needless to say, if two theories are dialectically contradictory, no structure can be a model of both. Assuming that they have models with the same domain, these models cannot provide the same *interpretation*.

Interestingly, I notice that when Jaśkowski makes observations on  $S$ , namely on  $T_1 \cup \dots \cup T_n$ , actually he is examining the original consistent theories  $T_1, \dots, T_n$  instead of  $S$  itself. For example, ' $\diamond\varphi$ ' in  $S$  is, by definition, equivalent to 'there is such an  $i$  ( $1 \leq i \leq n$ ) that  $T_i \vdash \varphi$ '. For the purpose of discussing such modality, therefore, what we need is each  $T_i$  or at most  $\{T_1, \dots, T_n\}$  rather than  $S$  itself. Obviously,  $\{T_1, \dots, T_n\}$  can always take the place of  $S$  because the latter is immediately derivable from the former.

## 15. Conclusion

The presentation of antinomies forms the core of Kant's transcendental dialectic. A clear description of a dialectical contradiction was given there probably for the first time in the history of philosophy: '[T]he common man's reason . . . often falls into disagreement with itself as a result of considering its object from two different viewpoints' (Kant 1787, 489). We carried out the formal construction of this idea in first-order logic. It can be defined as a syntactical relation between two consistent theories. On the other hand, semantics, namely model theory, gives an explanation of how two dialectically contradictory theories turn out to be compatible. Their models give different interpretations of the language or assign different domains of discourse or do both. Naturally, we will be interested in a dialectical contradiction between two theories particularly when each theory has an intended model. We will find a number of such examples in every area of science. This implies that a single consistent theory cannot cover an area of science, let alone the totality of science. Thus, dialectical methodology is diametrically opposed to reductionism in the form of Wittgenstein (1922, 4.11) or Carnap (1995).

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## Notes

- [1] In spite of what was mentioned in section 1, Russell was never ignorant of Kant or Hegel. As to Kant he says: 'I was much impressed by Kant's *Metaphysische Anfangsgründe der Naturwissenschaft* and made elaborate notes on it', and as to Hegel: 'I was at this time [about 1896] a full-fledged Hegelian, and I aimed at constructing a complete dialectic of the sciences' (Russell 1959, 32).
- [2] Any emphasis in a quotation is original, throughout.
- [3] I do not imply that the *Ethics* is logically well organized. Actually, it bears only some superficial similarities to Euclid's *Elements*.
- [4] Grim (2004, 49) names this 'LNC2'.
- [5] Grim (2004, 49) names this 'LNC1'.
- [6] We may as well say 'point of view', 'standpoint', 'perspective', etc.
- [7] For the capability of first-order language to express various areas of mathematics, see Ebbinghaus, Flum, and Thomas (1984).
- [8] If we apply this definition not only to mathematics but also to science in general, it might be criticized for disregarding the so-called *semantic* or *model-theoretic view of scientific theories*. It should be remembered, however, that Suppes, who is regarded as the founder of the *semantic view*, clearly says, 'The important distinction that we shall need is that a theory is a linguistic entity consisting of a set of sentences and models are non-linguistic entities in which the theory is satisfied' (Suppes 1960, 5). See also Halvorson (2012).
- [9] I think again this idea is in accordance with Suppes, who says '[T]he meaning of the concept of model is the same in mathematics and the empirical sciences' (Suppes 1960, 4).
- [10] Actually, from our perspective, there was no need to mention the rotation of the moon for the purpose of illustrating the frame dependence of motion. Any body will be found to be at rest in some frames of reference, but not in others. Probably, the idea of absolute space prevented Kant from conceiving of such relativity.
- [11] Actually, it might happen that  $\chi \in S_1$ ,  $\psi \in S_1$ , or the like, but this would not invalidate our argument.
- [12] Often the standard model of a theory is identified with the intended model, but some authors distinguish them. See, for example, Gaifman (2004).
- [13] Kneale and Kneale (1962, 355–356) criticize Kant's apparent trichotomy in his *Critique of Pure Reason*.
- [14] It may appear that what Kuhn (1996) called a *scientific revolution* or a *paradigm shift* has something to do with what we call a dialectical contradiction. Certainly, most examples he cited there relate to some dialectical contradictions, and it seems likely that two theories are 'incommensurable' if they are dialectically contradictory. But here I refrain from discussing this problem too hastily.

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