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Iteration and Infinite Regress in Walter Chatton's Metaphysics

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Under the pressure of a foreign military campaign, soldiers will sometimes improvise weapons and armor for themselves with an alacrity unmatched by military engineers back home whose motivational level is affected by their more peaceful surroundings. So too for the soldier in medieval theological battles; sometimes in the heat of discussion there is innovation in argumentation that logicians outside the conflict never have time to catch up to.

For the most part, medieval logicians stayed up with the theologians. Many logical and semantic theories are well discussed by logicians in the Middle Ages and are in turn well used by philosophers, theologians, etc. To take an obvious example, categorical syllogistic was constantly discussed and developed as an explicit object of theory, and was also constantly used to display the validity of reasoning on every imaginable subject. Similarly, the idea of consequences in the late medieval period was systematized into rules and cases, and one often sees such rules actually referred to and applied in theological debates. To take two narrower examples, William of Ockham gives us a theory of truth conditions in his account of personal supposition, but also uses this theory as part of his rejection of universals existing outside the mind, and again, his treatment of connotation theory and his application of it against realist metaphysicians is legend.

By contrast, some logical theories are extremely well developed but are, strange to say, seldom applied, as far as we know. An obvious example would be *obligationes*. These elaborate rule systems for detecting and maintaining logical consistency were worked out, sometimes in remarkable detail, by diverse thinkers from the thirteenth to the fifteenth centuries; yet we still do not have a clear idea what the theory was for, that is, how it really applied to actual reasoning, if indeed it did.

I want to draw attention to a third category, as presaged in my military example above: namely, those argument strategies and patterns of reasoning which, although relied upon in difficult situations by philosophers and theologians, are nevertheless not explicitly discussed in logical theory; that is, patterns on whose logic medieval philosophers rely, but whose logic is, so to speak, unknown, or at least is relatively unknown. One argument strategy often used even today, but seldom explicitly analyzed, involves appeals to infinite regress. Philosophers have made arguments about the eternity of the world, about the existence of God, and the nature of language which have an appeal to infinite regress at their core, and yet apart from some attempts to distinguish vicious and nonvicious regresses, philosophers have seldom taken time to evaluate the epistemic weight of such appeals.

Walter Chatton used a complex and interesting argument strategy involving infinite regress, together with a kind of operational iteration. He uses this strategy in a couple of tight spots in doing his metaphysics, but the strategy receives no explicit development by him outside these contexts, nor, as far as I know, was it developed by anyone else. But he is clearly quite proud of this method; he relies on this iteration technique in two very different philosophical contexts, concerning issues very dear to his heart. I will examine these two applications of his iteration rule in reverse-chronological order, because the later one, in his c.1324–28 *Lectura*[1] discussion of the anti-razor, is in fact the clearer and more elaborate of the two. I will then apply what we learn there to an earlier instance of the same reasoning, exhibited in very condensed fashion in his c.1322 *Reportatio*[2] discussion of future contingents. Overall I will show that Chatton fairly successfully uses this strategy against an Ockhamist semantic analysis, but that he rather defends himself from this very same strategy, again successfully, in his discussion of future contingents. I intend to keep the philosophical background of the larger theological issues pared down the minimum needed for illuminating his iteration strategy. I wish to focus on the general, abstract, and common features in the reasoning itself, not the issues in which the reasoning is embedded.

The Anti-Razor

Our first case study comes from Chatton's defense of his anti-razor against certain Ockham-style objections.[3] Chatton's anti-razor is a principle for

determining ontological commitment. Say we have a true propositior whose truth we want to use to help establish an ontology. Briefly, Chatto anti-razor procedure is to ask how many instances of what kinds of thi (*res*) would be required for the truth of *p*. We determine the answer to t basic question by thought experiment. If proposition *p* is about Plato Socrates, say, then we know they must exist to make it true, but to go dee we must also imagine that *only* Plato and Socrates exist, and then ask this bare situation is consistent with the falsehood of *p*?" If it is, then P and Socrates alone are not enough to make *p* true, and we know more thi are required in our ontology. Consequently, we must then posit whate metaphysical items sensibly fill this ontological gap we have detected, in general, if *n* entities aren't enough to make a sentence *p* true, we n posit an *n* + 1th entity, etc., until we have enough things such that they inconsistent with the falsehood of *p*. (I find it useful to think of Chatt anti-razor as a kind of *a priori* version of Mill's joint method; by contro experiment we discover what things are necessary and sufficient to cau proposition's truth.)

An informal example of how Chatton applied his anti-razor will hel clarify all this. Chatton believed that we had to posit the existence of cer kinds of real relations, in particular relations of causality, as distinct Ari telian accidents inhering in individual substances.[4] Thus, if this light r caused by the sun, then we must certainly posit the ray and the sun, Chatton believed the situation could only be fully explained by the exist of two other entities: (1) an active relational entity we could call *produ* (=Latin *actio*), which inheres in the sun and "points to" the ray as its pro and (2) and passive relational entity *being produced* (=*passio*), which inh in the light ray and "points to" the sun as its producer. Now the anti-r is used to support this kind of realist ontology as follows. Imagine nothing exists except the sun and a light ray, and that we have before u proposition "This light ray is from the sun."[5] There is nothing which g antees that these two distinct so-called absolute entities, ray and sun, the right relationship so that the proposition "This light ray is from sun" is true; that is, with only the ray and the sun existing, no part of r speaks to the "producer/produced-relation" that the proposition asser these two absolute entities, ray and sun. Since the two-element onto {ray, sun} is consistent with the falsehood of the proposition "This ligh is from the sun," this ontology is in general insufficient to guarantee its t

However, if we add to this ontology two respective accidents, one of *production* and one of *being produced*, as described above, then the ray and the sun would be related in such a way that the proposition would be true. Having completed this thought experiment, we deduce that, in the real world, where the proposition "This light ray is from the sun" is sometimes in fact true of certain light rays, there must in fact be such respective accidents (partially) causing the truth of the proposition. The intuitive center of Chatton's anti-razor is that whatever makes a difference to truth must be real, and since in our experiment respective accidents make a difference to truth, they must be real.

An interesting Ockham-style objection to this application of Chatton's ontological principle would say that propositions sometimes require more than just *things* (*res*) to make them true; sometimes, for example, they require instead that certain *conditions* be met. To put it briefly, Ockham certainly agreed with Chatton that while there must *be* in actuality all that is necessary to account for the truth of actually true propositions, he objected that, nevertheless, not everything that propositions require for their truth is therefore some *thing* in one of Aristotle's ten categories (i.e., a *res*). For example, Ockham objected that we do not need to posit an Aristotelian accident *motion* in order to explain the truth of "Object *a* moved," rather, we only need to posit the object *a* and the following three conditions:[6]

(i) *a* was in a place and now is in another;
(ii) this change happened continuously and successively;
(iii) this change happened without any intervening rest on the part of *a*.

Ockham insists it is not the positing of more *things* (*res*) that clarifies the meaning of "motion" here, but rather the positing of more conditions on moving object *a*, and as conditions are not to be reified as *things*, we have clarified the truth conditions of the proposition without expanding our ontology. An ontology of one thing, together with these three conditions, does the same thing as a Chattonian ontology of two things.

Chatton must have had just such an Ockhamist objection in mind when he defended his anti-razor principle in the *Lectura*, for in that place he gives the following complex argument against the Ockhamist objection.

> The following method ought to be used against these objections and against all other similarly derived objections. Whenever a new, added

condition is designated by an objector as required for some original proposition *p* to be true, we ought simply to accept the condition, whether the proposition expressing it is affirmative or negative. We then ought to ask what things are required for the proposition expressing the condition to be true. Either: (1) the proposition requires *n* things such that it is inconsistent with the existence of these *n* things, equally present without another thing, that *p* be false, or, (2) not *n* things, but only fewer than *n* things are required . . . If the first alternative is the case, then I have the plurality I proposed. After all, the [anti-razor] already requires *n* things such that it is inconsistent with the existence of these *n* things, consistently present without another thing, that *p* be false; [therefore the anti-razor holds;] therefore, it is required to posit the *thing*, not just the proposed condition, to account for the truth of the proposition. If the second option is given, then I argue in this way: since fewer than *n* things, howsoever they are present without another, are consistent with the falsehood of the proposition, it follows that these things so present are not sufficient to account for the truth of it, and, consequently, besides the [first] added condition, it is required to posit yet another condition.

In that case, I accept the proposition that expresses this additional condition and I ask what things are required to account for *its* being true. Either as many things, present in a certain way without a new thing, as are consistent with *p*'s being false, or as many things as are inconsistent with *p*'s being false. If the second answer is given, then I have the plurality I originally proposed, since in order that it be true, this latest proposition requires that the condition be true, and the condition requires that just as many things be posited [as the anti-razor originally claimed were required]. But therefore the proposition requires that just as many things be posited [as the anti-razor originally claimed were required]. If the first answer is given, I then add that other condition, form its proposition, and ask about it, as previously, and so on to infinity.

Whosoever labors . . . in adducing reasons why the anti-razor is false, let it be objected against him through the method sketched above, . . . since if the interlocutor should not want to object against his own position through this method, then he proceeds insufficiently, even by his own standards.[7]

The full import of this quotation is not immediately clear, but Chatton employs a very clever argument strategy using self-referential iteration

and infinite regress. We will be aided in our analysis if we use as scaffolding a more formal and precise step-by-step presentation of the anti-razor method. As we construct this general and abstract description of the anti-razor below, it will be useful to keep in mind the concrete example of the ray and the sun previously discussed.

Given a true proposition p and an original stock of n entities, $a_1, a_2, \ldots a_n$, the anti-razor sets out a two-stage meditation, forcing us to posit the existence of suitable $n+1$th thing, call it a_{n+1}, to explain the truth of p:[8]

(i) It must be that the original n entities, $a_1, a_2, \ldots a_n$, are required for the truth of p, and yet it must be that the existence of these n entities, $a_1, a_2, \ldots a_n$ is consistent with the falsehood of p. That is, $a_1, a_2, \ldots a_n$ alone are necessary but not sufficient for the truth of p. Then . . .

(ii) . . . we must ask whether with the presence of a suitable a_{n+1} thing it is still consistent to say that p is false. If it is, then we have not yet filled the truth gap, so to speak, since we have not yet explained p's truth, and so, obviously we would have to come up with another entity a_{n+2}, add it to the mix, and start again. If, however, it is inconsistent that the a_{n+1} thing exist and yet p is false, fithen p obviously requires this a_{n+1} thing for its truth, and this a_{n+1} thing is enough. Hence, since p is in fact true, this a_{n+1} thing must exist.

Chatton's Rule of Iterated Analysis

Let this be the general method of the anti-razor. Now we can ask: What is Chatton's general response in the previous paragraph to the Ockhamist objection—namely, the objection that the insufficiency the anti-razor detects is not always to be remedied by positing even more *things*, but rather, by sometimes instead by positing more *conditions* which need to be met by the things we already recognize?

In the long quotation, Chatton seems to be asking us to apply the anti-razor method to the propositional content of the very condition the objector insists on adding. Let us try to do this in detail using our previous example of the ray and the sun.[9] Let the machinery of the anti-razor be assumed as above. Now, take the proposition "This light ray comes from the sun," and suppose that it is in fact true. As we have already seen, Chatton would say

the truth of this proposition requires that a light ray exist, that the sun exist, and that besides these two absolute entities, one relative accident of *production* and one of *being produced* exist in the sun and the ray, respectively.

Now an Ockhamist objector can challenge the whole basis of this analysis in a manner similar to Ockham's challenge to realist theories of motion, by saying that the consistency of the existence of the light ray and the sun with the falsehood of "This light ray is from the sun" shows, not that other *things* must exist, but that there are in reality more *conditions* on the truth of "This light ray is from the sun," which conditions have not yet been met. What kind of conditions? To take one example, we might say that God could make a ray, the sun, and the two respective accidents, and yet could make "This light ray is from the sun" still be false because he refuses to co-act with the causal power of the sun for producing this ray. Continuing with this counter-example, we might say that what would be needed to bring about the truth of "This light ray is from the sun," is not that there are other things, but instead a further condition is met, namely, that God cooperate, and co-act with the causality in the sun to let this ray be from it. The point of the objection is that sometimes not only *things* but also *conditions* must be posited for the truth of propositions.

Now, in his *Lectura* text, Chatton attacks such an objection this way. Take the new condition the objector claims is necessary, in this case the condition *that God co-acts so that this ray is from the sun*, and make the condition into a proposition, thus: "God co-acts so that this ray is from the sun." Since "God co-acts so that this ray is from the sun" is just a proposition, we can simply apply the anti-razor to it and see what happens. That is, we ask, what kind and how many *things* must exist in order for the truth of the new proposition "God co-acts so that this ray is from the sun"?

Now, there are two possible answers to this last question. Option 1: "God co-acts so that this ray is from the sun" requires us to posit the same number of things as the anti-razor would say the original proposition "This ray is from the sun" itself requires. That is, one possibility is that "God co-acts so that this ray is from the sun" requires that four things exist (sun, ray, two respective accidents), which is just as many as the anti-razor said were required. Option 2: The truth of "God co-acts so that this ray is from the sun" requires that fewer than four things exist, presumably just the two things outside of the dispute, that is, the ray and the sun. (We do not consider that "God co-acts

so that this ray is from the sun" could require *more* than four things, since the objector is obviously a nominalist, and would not introduce a condition that *expanded* our ontology beyond even Chatton's requirements!)

But under either option the objector has a problem. If the first option holds, then four things exist, and Chatton and his anti-razor were right all along anyway, since application of his anti-razor showed that, indeed, we had to posit four things. If the second option holds, then even with the new condition added, the ray and the sun are still insufficient for the truth of "God co-acts so that this ray is from the sun." That this is so is shown this way. Everyone agrees that two things, the ray and the sun, are insufficient for the truth of "This ray is from the sun'"; they only disagree on how to fill the gap. But since "This ray is from the sun" is an embedded dictum in "God co-acts so that this ray is from the sun," clearly "God co-acts so that this ray is from the sun" requires at least as many things for its truth as the dictum "This ray is from the sun" does. Hence, if two things were insufficient for the truth of "This ray is from the sun," obviously two things are also insufficient for the truth of the new proposition, "God co-acts so that this ray is from the sun."

So, *on his own principles*, even the objector would have to agree that, on Option 2, the insufficiency we detected at the first level is pushed up to this new, higher-level proposition. Now, the objector holds that insufficiency for propositional truth in this case requires posting, not more things, but rather more conditions, so by his own lights, the insufficiency of these two things for the truth of "God co-acts so that this ray is from the sun" requires us to posit the existence of still *another* condition, this time a condition on the proposition "God co-acts so that this ray is from the sun." It is difficult to say what this new condition would be, but let us try, for example, the condition *that God wills that God co-act so that this ray is from the sun*. Some such new condition is clearly necessary to fill the insufficiency that still exists, and, so to speak, make up the ontological gap.

Now we can again propositionalize this new condition *that God wills that God co-act so that this ray is from the sun*, just as we did previously with *that God co-act so that this ray is from the sun*, to yield the new, even more complex proposition "God wills that God co-act so that this ray is from the sun." We then proceed exactly as before, and ask: What is required for the truth of "God wills that God co-act so that this ray is from the sun"? Either as many things as the anti-razor says, or fewer. If as many as, Chatton was

right all along; if fewer, then the objector's own strategy forces us to posit still *another* condition, which new condition we can propositionalize as before, etc.

Now either this process proceeds to infinity, with each new condition in its turn requiring we posit yet another condition to explain the previous proposition's truth—and in that case we have an explanatory regress, since the truth of "This ray is from the sun" never finally gets explained—or else at some level we jump off this infinity train. But the only station through which we can exit is Option 1 or its equivalent, that is, the only way to break the regress is to admit that Chatton was right to begin with: more than two *things* are needed for the truth of "This ray is from the sun." But then of course, the entire nominalist line of objection was for naught.

To recapitulate briefly and more formally, the general structure of this objection and Chatton's reply is as follows:

(1) Assume the machinery of the anti-razor for the sake of objection.
(2) Objection: the consistency of the existence of $a_1, a_2, \ldots a_n$, with the falsehood of p shows, not that we must posit a new thing a_{n+1}, but rather a new condition on the truth of p, call it Cp.
(3) Form the proposition expressing the new proposed necessary condition Cp, written $\pi(Cp)$. Ask: What must exist in order for $\pi(Cp)$ itself to be true?
(4) Now, there are two possible answers. Option 1: $\pi(Cp)$ requires more than n things exist, just as Chatton's anti-razor says p did. Option 2: The truth of $\pi(Cp)$ requires that than n or fewer things exist.
(5) If Option 1, then the anti-razor was correct after all.
(6) If Option 2, then even with $C(p)$ added, entities $a_1, a_2, \ldots a_n$ without a_{n+1} are still insufficient for the truth of $\pi(Cp)$. *Proof:* By hypothesis, n or fewer things are ontologically insufficient for the truth of p, but since p is an embedded dictum in $\pi(Cp)$, clearly $\pi(Cp)$ requires at least as many things for its truth as p does. Hence, if n or fewer things are insufficient for the truth of p, obviously n or fewer things are insufficient for the truth of $\pi(Cp)$ as well. QED.
(7) The objector's general method would therefore require us to posit still *another* condition to explain the truth of proposition $\pi(Cp)$; call the new condition $C^*\pi(Cp)$. This $C^*\pi(Cp)$ is necessary to fill the insufficiency which, by (6) above, still exists for $\pi(Cp)$.

(8) But we can propositionalize $C^*\pi(Cp)$ just as we did Cp, to yield $\pi C^*\pi(Cp)$, and then proceed again as in (4)–(7) above, for the positing of $C^{**}\pi C^*\pi(Cp)$, which we can make into $\pi C^{**}\pi C^*\pi(Cp)$, etc.

(9) Now either this process proceeds to infinity, with the nth requirement of a new condition producing a true proposition of the form $\pi C^{*n+1} \ldots \pi(Cp)$—and in that case, p's truth conditions never having been finally stated, we have an explanatory regress—or else admit (4) Option 1 above, and the entire objection was for naught.

Assume for the moment that this argument strategy works. What has Chatton accomplished thereby? He has shown that one cannot fill ontological gaps with nonentities; positing conditions only generates more propositions whose truth conditions must be similarly explained by the nominalist, which explanation requires yet more conditions, and so on. Only real things are ontologically sturdy enough to fill the chinks in this sinking semantic ship. The Ockhamist analysis in which conditions make propositions true, even if it is correct, still depends upon the more basic fact that propositions about things are made true by things, which basic fact is given more proper due by the realist analysis. Hence the Ockhamist objection sheds no light upon the correctness or incorrectness of Chatton's own realist ontological analysis with his anti-razor. Put simply, Chatton has shown that the Ockhamist analysis is dependent upon a more basic realist analysis, and so is not capable of adjudicating on questions raised about that more basic level of analysis.

How though can we briefly summarize Chatton's argument strategy in plain English? I think the following statement captures what is important:

> *Chatton's rule of iterated analysis*: Given two competing analyses A and B, where we want to show that A is more fundamental, and B as less so, we can ask what happens if we self-referentially iterate B, that is, what happens when B is used to analyze its own outputs (assuming this is legitimate). If the legitimate iteration of B leads to an explanatory regress unless analysis A is used to terminate it, then clearly B at bottom depends upon A.[10]

I wish to stress that this rule of iterated analysis we have discovered is not identical with the anti-razor, nor is it a part of it. The anti-razor is a semantic theory which Chatton here defends with the rule of iterated analysis,

but, as the next section will reveal, the rule can be easily applied to very different philosophical contexts just as easily. In short, *A* and *B* can be any two analyses at all.[11]

Is Chatton's strategy a good one? I think that it is, if we add some provisos on its application and results:

(i) Clearly *B* may depend upon things other than *A*; that is, with his rule we show at most that the success or truth of analysis *A* is necessary for the success or truth of *B*, but there may of course be other factors upon which *B* depends, or upon which *A* depends. Chatton's *Lectura* discussion does not show any awareness of this issue.

(ii) Although other things may be necessary, in addition to analysis *A*, in order to terminate *B*'s iteration, it really must be the case that *A* is strictly *necessary* to terminate *B*'s iteration. If there is another way of doing so that is independently acceptable and that does not involve *A*, then all bets are off.

(iii) The application of analysis *B* to itself must be otherwise logically and philosophically legitimate, for example, it must not make a category mistake. Chatton's discussion in *Lectura* does show awareness of this issue.

The best way to argue against a particular application of Chatton's rule of iterated analysis is, obviously, to show that it fails on one of the provisos above, or, still more directly, to show that the regress that drives the argument is not really a problem: for example, that it is not vicious, or that the regress is not infinite, but instead collapses to the finite. Perhaps such a response could be made above on behalf of the Ockhamist above.

Chatton Defends against His Own Rule

In his highly original treatment of future contingents in *Reportatio* I, d. 38, Chatton again faces this complex iteration strategy. Only on this occasion it seems that, instead of applying the strategy, he is rather defending against it, since in that text he is at pains to show that a certain seemingly infinite regress stemming from an iterated analysis in fact collapses to the finite level. Hence, in this second example, Chatton is trying to show that there is not (despite initial appearances) a case of infinite regress of the sort to

which his rule would apply, quite the opposite of what he was doing in the first example.

For complex reasons that we need not go into, Chatton's solution to the problem of future contingents requires that there be two distinct, independent analyses of what it means to be committed to the proposition "Socrates will be sitting," and in general to any future-tensed proposition of the form "*a* will be P," where *a* names a contingent thing. Our commitment to "*a* will be P" can have two particular, distinct analyses, according to Chatton. Thus "*a* will be P" can mean either:[12]

(i) "*a* will be P" is true [A future-tense proposition *is true*.]
or
(ii) "*a* is P" will be true [A present-tense proposition *will be true*.]

Although Chatton thinks true propositions of the first form lead to fatalism, he thinks those of the second form do not, and hence the distinction between these two forms can be the basis of a stable solution to the problem of future contingents.[13] That is, to avoid fatalism but still safeguard veridical prophecy and divine foreknowledge, we have to say that Analysis (i) yields a proposition which is really indeterminate in truth value, while Analysis (ii) yields a proposition which can be regarded as true. Consequently, it is absolutely critical to Chatton's solution that these two analyses are distinct and independent, and in particular, it cannot be that Analysis (ii) depends at all on Analysis (i). At the point where he should naturally make an argument for this important point, he instead offers this extremely compressed and obscure remark:

> If 'The Antichrist will come' is true according to the second mode of assertion, then if it were again asserted to be true, it would again be true in that very same mode in which it was originally asserted. The reason is that from the opposite of something the opposite conclusion follows (this dictum being understood here in a general sense).[14]

What could he mean here?

I believe what we have learned above from his rule of iterated analysis can shed light on this remark. Imagine someone opposed Chatton's claim that Analyses (i) and (ii) above are distinct and independent. She could attack his whole strategy by applying the iterated analysis rule against him, in

particular, she might try to show that Analysis (ii) in fact depends upon (i). This would of course finish off Chatton's solution to future contingents.

How could we go about using Chatton's rule against him in this way? Chatton's rule of iterated analysis suggests we iterate Analysis (ii) on its own outputs in such a way that a regress is generated which only Analysis (i) can break. Now, Analysis (ii) says that the truth of "*a* will be P" commits you to the truth of "'*a* is P' will be true." Notice that the original sentence is future-tensed, and that, after we apply Analysis (ii), the result is another future-tensed proposition. Consequently we can legitimately iterate Analysis (ii), applying it to its own original output, to get a proposition of a higher "level," so to speak: from "'*a* is P' will be true" we get "'*a* is P' is true' will be true." This could continue, and, using parentheses instead of quotation marks, we would then have, schematically:

Level 0	*a* will be P *commits you to:*
Level 1	(*a* is P) will be true *commits you to:*
Level 2	((*a* is P) is true) will be true *commits you to:*
Level 3	(((*a* is P) is true) is true) will be true *etc.*

At each new level we have used Analysis (ii) to obtain a new future contingent sentence, the truth of which is entailed by the previous level.

Chatton claims that Level 0 and Level 1 are equivalent. But this analysis could obviously be repeated to infinity, so that Level 1 automatically generates Level 2, and it seems the truth conditions of the sentence on Level 1 might seem to await determination by what happens at Level 2, but 2 generates 3, so 2 awaits determination by 3, which generates 4, etc. Hence we seem to have an explanatory regress. How could the regress be broken? The only way would be to determine *independently* the truth conditions for any arbitrary Level $n > 0$ of the schema above. So consider, under what conditions are any of these higher sentences in the above schema true? Since at any Level $n > 0$ of this analysis we are dealing with a future-tensed proposition, to answer this question we must ask: Under what conditions is a future-tensed sentence generally true? And this is just to ask, how do we know that, for example, the sentence "(((*a* is P) is true) is true) will be true" is true? But—and here is Chatton's worry—*to ask this is really just to ask what makes a sentence of the form in Analysis (i) true.* To see this clearly it is best to approach the matter formally. Note that no matter what level we are at in the schema above we have a long proposition on the left, and

the main logical operator—the phrase "will be true"—on the right. Now regard "true" in this phrase as a predicate, P; we then have at every level a proposition function "will be P." Since a proposition is a contingent thing, the long proposition on the left can be represented by *a*, which symbol you recall can stand for any contingent thing. We see immediately that each level of the schema actually has the logical form "*a* will be P," thus:

Level *n* of Analysis (ii): $(\ldots(a \text{ is P}) \text{ is true}_1) \text{ is true}_2)\ldots)$ will be true_n

Analysis (i): *a* will be P

Hence, to ask of any Level *n* in the schema for Analysis (ii) whether it is true is just to ask whether a sentence of the form "*a* will be P" is true, which sentence is as the form in Analysis (i); hence, to terminate the regress in the schema of Analysis (ii) we are forced to resort to Analysis (i), which Chatton said leads ultimately to fatalism.

Chatton seems to face here the same stark choice we just saw him put before the Ockhamist. For in order to give the truth-conditions of any arbitrary future-tensed proposition of any Level $n > 0$ in his schema, by his own lights Chatton has only two choices: use Analysis (i) or use Analysis (ii). If he uses Analysis (i) the fatalist was right all along. If he uses Analysis (ii) he simply obtains the $n + 1$th level of the schema, and the regress continues unless Analysis (i) finally be admitted. In sum, by Chatton's own rule of iterated analysis, since Analysis (i) is needed to break the regress on the iteration of Analysis (ii), we have shown that (ii) depends upon (i). So it seems that Chatton has been sunk with his own rule here.

It is just such an objection that Chatton is trying to head off, I think, when he asserts the compressed remark with which we began this section of the essay:

> If 'The Antichrist will come' is true according to the second mode of assertion, then if it were again asserted to be true, it would again be true in that very same mode in which it was originally asserted.

The reply suggested in this quotation amounts to this. Analysis (ii) does not really create a new irreducible sentence whose determinate truth cannot be explained otherwise—rather, the product of Analysis (ii) is a new, contingent, future-tense sentence that can be understood *in just the same way as the original*, and so is reducible to the original, because the iteration outputs

of Analysis (ii) are all equivalent to the original sentence, that is, each Level n > 1 really reduces by equivalence to Level 1. And it is just here where Chatton's remark about the *ex opposito* dictum comes in—it turns out we can in fact reason *ex opposito* to collapse the infinite schema down to the finite. Without an infinite regress, the rule of iterated analysis does not apply, Chatton can have his distinction, and we can all avoid the iron hand of fate. The gist of the strategy becomes clearer if we look a bit deeper at the details.

How though can we collapse these levels to the finite by reasoning *ex opposito*? In general, reasoning *ex opposito* is just reasoning by what we call contraposition, viz., p → q therefore ~ q → ~ p, or vice versa. Now, we already have that Level 0 ↔ Level 1, and also the ascending set of implications that Level 1 → Level 2 → Level 2 → Level 3 → . . . from Analysis (ii) itself. Hence, if we could establish set of a corresponding descending implications, showing that for any $n > 1$ that (Level n → Level $n - 1$), then we would have proven the equivalences Level 0 ↔ Level 1 ↔ Level 2 ↔ Level 2 ↔ Level 3 ↔ . . . This would collapse the infinite regress.

The argument from contraposition that collapses these levels is easy to establish in full generality by mathematical induction, but tediously long to state in that form; instead I will illustrate the method by using contraposition to reduce Level 3 to Level 2:

> Take a sentence of Level 3; it has the form (((a is P) is true) is true) will be true. We want to show this entails the Level 2 sentence ((a is P) is true) will be true. The proof is by contraposition. We assume *(((a is P) is true) will be true) is false* and show *((((a is P) is true) is true) will be true) is false.*
>
> | | |
> |---|---|
> | 1. (((*a* is P) is true) will be true) is *false* | *given; this implies that* |
> | 2. ((*a* is P) is true) will *not* be true | *which implies that* |
> | 3. ((*a* is P) is *not* true) will be true | *which implies that* |
> | 4. ((*a* is *not* P) is true) will be true | |
> | 5. (*a* is *not* P) = (*a* is P) is *not* true | *law of negation; substitution in 4 yields* |
> | 6. (((*a* is P) is *not* true) is true) will be true | *which implies that* |
> | 7. (((*a* is P) is true) is *not* true) will be true | *which implies that* |
> | 8. (((*a* is P) is true) is true) will *not* be true | *which implies that* |
> | 9. ((((*a* is P) is true) is true) will be true) is *false* | QED |

Hence, Level 3 → Level 2.

Although my hypothesis explains a great deal about what Chatton has in mind, I may be wrong, of course, and this might not be the way to exposit the *ex opposito* portion of this compressed remark. After all, the reduction from Level 3 to Level 2 can be done directly and more simply by using the logical equivalence "*a* is P" is true if and only if *a* is P, and substitution into sentence at level *n*, for example, merely from

(((a is P) is true) is true) will be true

and

"*a* is P" is true if and only if *a* is P

alone it follows immediately by simple substitution that

((a is P) is true) will be true

Hence, Level 3 → Level 2. But if Chatton did not intend this reasoning as I have reconstructed it here, it is difficult to imagine how else to sort out his very obscure remark.

Conclusion

It seems Chatton's defense against his own strategy is sound here. But it is possible that an Ockhamist might adopt a somewhat similar form of reply, by trying to find added conditions which explain relational sentences but which nevertheless do not add another entity to ontology when they are propositionalized to embed the original sentence.[15] What such a condition might be I leave to the reader to consider.

Despite what I said in the Introduction about the behavior of regress arguments not being well studied, it should be noted in closing that the other aspect of Chatton's strategy, iteration of sentential operators, has definitely received great scrutiny in modern logic. Indeed, Chatton's intense interest in the logical behavior of certain sentential analyses under iteration puts one in mind of modern modal logicians and their worries about the axiom sets under which iterated modalities collapse. For example, it can be shown that under the powerful and seemingly useful assumptions of the modal system S5, one ends up with a logic that cannot sustain iterated

modalities, in the sense that any string of unary operators ~, □, ◇, in front of a formula p is S5 equivalent to one of only four basic (nonempty) modalities, $\sim\Box p$, $\Box p$, $\sim\Diamond p$, or $\Diamond p$. Thus S5 is axiom rich but theorem poor.

And again, modern philosophers have found that they must grapple with infinite regress in order to sort out their own debates. One thinks of Russell and the paradoxes of self-reference plaguing set theory early last century. It is interesting that we have now found Chatton six centuries ago combining these two powerful tools, self-referential iteration and infinite regress, into one interesting argument strategy, although this strategy itself is apparently never named or discussed explicitly by logicians.