

Vagueness poses a serious problem for classical logic and semantics. Borderline predications appear to be neither true nor false, the Law of Excluded Middle and other classical theorems are brought into doubt, and the sorites paradox can lead us to question various inference rules. A great many philosophical and logical questions arise in connection with the phenomenon, making the topic of interest to philosophers and both philosophically and mathematically inclined logicians.

A vast array of logics have at one time or another been considered for the modelling of reasoning within vague language: there have been advocates of various fuzzy logics, intuitionistic logics, supervaluationist logics, paraconsistent logics and many others besides. In each case there is scope for disagreement over formal details and over philosophical justifications of their applicability to vague language. This volume takes up some of the issues in relation to a range of different possible logics of vagueness, containing some papers that focus on philosophical aspects of formal models of vagueness and others that concentrate on the mathematical details.

One philosophically important theory of vagueness is supervaluationism. According to this theory, a sentence is true if and only if it is true on all precisifications of the language, which allows for borderline predications that are neither true nor false and yet preserves much of the core of classical logic. There has been some debate about the notion of validity to be employed within this framework, with *global validity* being by far the most widely adopted. In his paper in this volume, **Pablo Cobreros** offers and examines a promising new characterisation of validity—*regional validity*—where what counts is necessary preservation of truth relative to precisifications, and he argues that it solves various problems concerning higher-order vagueness.

Aspects of the supervaluational framework are also employed in **Stewart Shapiro**'s contribution to this volume. He is concerned to provide a model theory for vague predicates that is sensitive to the way that the extensions of predicates shift across different contexts, with speakers often having the freedom to go either way in the classification of borderline cases. Backing up the system developed in his monograph *Vagueness in Context* (OUP 2006) he here expands upon some types of reasoning and answers some criticisms of his views.

**Elia Zardini's** paper in this volume proposes a different type logic for vague language—tolerant logic—in which restrictions are placed on the transitivity of logical consequence. The paper provides a lattice-theoretical framework for such a logic and shows how it might vindicate a “naïve theory of vagueness” according to which there are no sharp cut-off points between positive and negative cases of a vague predicate.

At the centre of **Ariel Cohen's** paper is the use of Default Logic to formalise a notion of indiscernibility by default. This, it is argued, can be taken as analysing a notion of indiscriminability that can explain the appeal of the main premise of the sorites paradox, for, though this premise is false, consecutive members of a sorites series can still count as indiscriminable. Cohen then shows how some important theories of vagueness can employ his ideas.

**Peter Verdée** and **Stephan van der Waart van Gulik's** paper applies the apparatus of Adaptive Logic in a generic manner to various logics of vagueness. Adaptive logics are non-monotonic logics that flexibly block selected rules of inference if a given kind of abnormality (in this case, vagueness) is encountered in the set of premises. The authors isolate sufficient conditions for applying the standard format of adaptive logics to a given logic of vagueness and show that the criteria are satisfied by common logics of vagueness, including t-norm fuzzy logics, super- and subvaluationist logics, and clarity logic.

The aim of **Thomas Vetterlein's** paper is to give an alternative characterization for two important fuzzy logics, namely Łukasiewicz infinite-valued logic and Hájek's basic fuzzy logic BL, which would provide a less ad hoc motivation than the usual exposition in terms of operations on the real unit interval. He shows that the algebraic structures corresponding to these two logics, i.e., MV-algebras and BL-algebras, arise in a natural way from Boolean and Heyting algebras by taking quotients by certain equivalence relations that represent ambiguity.

Finally, **Peter Milne's** paper deals with the problem of assigning probabilities to imprecisely specified events, with Pawlak's rough sets employed as a simple model of imprecision. Two ways of assigning probabilities are discussed: one of them considers upper and lower probabilities corresponding to the upper and lower approximations of rough sets, while the other, supported by a simple betting argument, regards rough-set approximations as conditional events.

All authors also discuss the motivational significance of the results they present, and each of the papers casts new light on some aspect of logical modelling of vagueness. We, as guest editors of this special issue, hope that

the papers will be of interest to both mathematically and philosophically oriented readers. We would like to thank the authors and the referees for all their hard work.

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