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Colyvan's Dilemma: Inconsistency, Theoretic Virtues, and Scientific Practice

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Abstract

Mark Colyvan formulates a puzzle about belief in inconsistent entities. As a scientific realist, Colyvan refers to salient instances of inconsistencies in our best science and demonstrates how an indispensability argument may justify belief in an inconsistent entity. Colyvan's indispensability argument presents a two-horned dilemma: either scientific realists are committed to the possibility of warranted belief in inconsistent objects, or we have a *reductio ad absurdum*, bringing realism into a crisis. Firstly, this paper follows Graham Priest by opposing the received characterisation of inconsistent belief as a kind of epistemic hell. Secondly, I challenge the Quinean naturalism that underpins Colyvan's indispensability argument. Then, I reformulate Colyvan's argument with a fallible naturalism, better equipped to account for certain problem candidates for inconsistent entities. Finally, I contend that—even if indispensable—an inconsistent entity poses no problem for the scientific realist, who can have justified belief in inconsistent entities.

1. Introduction

1.1 Indispensability Arguments

Scientific realists look to our best scientific theories to determine what should be taken to exist. Following Mark Colyvan, this stance can be understood as the view that we are justified in believing in all, and only, those entities indispensable to our best

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science.[1] This premise is the foundation for indispensability arguments, which have been used to justify belief in entities that are not observable, particularly those entities like electrons, which some anti-realists remain agnostic about.[2] Mathematical entities are also posited in the theorems of our best science; this has controversially motivated indispensability arguments for mathematical realism, notably, by Quine, Putnam, and Colyvan.[3]

In light of the significant inconsistent entities that have featured in our best science at certain historical periods—such as the infinitesimal posited by the early calculus and Bohr's model of the hydrogen atom—an indispensability argument ('IndA') can be made for belief in inconsistent entities, in our best science (what I will call 'IndA_{io}').[4] Moreover, Colyvan points towards contemporary examples of inconsistency in modern physics, such as the theory of waves in the open ocean and the inconsistencies between relativity and quantum mechanics, to explore whether we can have justification for believing in an inconsistent state of affairs.[5] We can reformulate IndA_{io} as follows:

> (P1) We have justified belief in all and only the entities that are indispensable to our best scientific theories.
>
> (P2) Inconsistent entities are indispensable to our best scientific theories.
>
> (C) We have justified belief in inconsistent entities.

1.2 The Pressure of the Puzzle

Colyvan's conclusion may seem absurd: it seems to move towards a stranger and more uncomfortable dialetheic territory (where dialetheism refers to the position that there are some contradictions that are true). However, for Colyvan, we may treat the argument as a *reductio ad absurdum* of the premise. Importantly, Colyvan emphasises that scientific realists (like himself) must either (i) welcome a scientific realism that is willing to accept that we may be justified in believing an inconsistent state of affairs, or (ii) accept that it could never be rational to make such an affirmation and, consequently, give up on scientific realism.

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[1] Colyvan, Mark (2008) 'The Ontological Commitments of Inconsistent Theories', *Philosophical Studies* **141**, 115.

[2] Most notably, the constructive empiricist Bas van Fraassen (1980) in *The Scientific Image*, Clarendon Press, 204. Also see Antunes, Henrique (2018) 'On Existence, Inconsistency, and Indispensability', *Principia: An International Journal of Epistemology*, **22**, 8–30.

[3] Quine, W V (1986) *Theories and Things*, Harvard University Press, 148–55; Putnam, Hilary (1971/2013) *Philosophy of Logic*, Routledge; Colyvan, Mark (2001), *The Indispensability of Mathematics*, Oxford University Press.

[4] For the purposes of this paper, these may be entities that are described as inconsistent internally within the theory, or entities that are described inconsistently by two theories of our best science.

[5] Colyvan, 'Ontological Commitments', 116.

The dilemma that follows from the *reductio* is, therefore, unsurprising. *Reductio* arguments imply that we ought to reject a given premise because it entails an untenable conclusion. Typically, *reductio* arguments demonstrate, firstly, that the premise entails a contradiction and secondly, that from the Law of Non-Contradiction (LNC), the premise is untenable. However, Colyvan's dilemma is precisely the kind of puzzle that demands a re-evaluation of the authority of LNC. It forces us to evaluate whether there is, in fact, a case where we *can* have a justified belief in a contradictory state of affairs (dialetheia).

1.3 Roadmap for the Paper

In this paper, I will respond to two arguments contending the absurdity of a belief in a contradictory state of affairs. The first is the logical problem of 'explosion' and the second is an inductive argument for consistency as a necessary or law-like feature of the world. Against these two compelling cases, I defend the possibility of a justified belief in contradictory states of affairs. I call this thesis the *Provisional Lemma for Justified Inconsistent Belief* (PLJIB): we can have justified belief in inconsistent entities where the virtues of our best (but inconsistent) science trump the virtue of consistency. The PLJIB challenges the LNC and defends the possibility of justified inconsistent belief beneath the framework of Colyvan's $IndA_{io}$.

In light of some problem examples, this thesis is not sufficient to resolve Colyvan's dilemma. Building on this provisional defence of Colyvan's conclusion stated above, I will defend the view that we ought *only* to believe in the entities of our best science, that we are justified in interpreting *realistically*. That is, some of our best scientific theories are not good candidates for realistic interpretation. Therefore, whilst some of our best science is inconsistent, such theories usually ought not to be interpreted realistically. However, I also demonstrate that a realistic interpretation is appropriate in some inconsistent cases. In this paper, I focus on two examples of inconsistent theories that have featured in our best science to show that although not all scientific practice is aimed towards truth it may nonetheless be evaluated against some other (perhaps empirical) goal. Therefore, (P1) of Colyvan's $IndA_{io}$ ought to be altered in light of my Resolving Thesis (RT): the realist ought only to believe in the entities of our best science *which we are justified in interpreting realistically*.

In §2, I provide an account of the theoretical virtues, which play two roles in this essay. Firstly, defending the PLJIB, the account reveals that the scientific practice of theory construction (and assessment) categorises consistency as merely one virtue among others, rather than as a necessary condition for what could be considered our best science. Additionally, (as discussed later in §5), I contend that we can begin our assessment of whether a theory ought to be interpreted realistically or merely instrumentally by appealing to the scientific virtues.

In §3 and §4, I evaluate two objections to the PLJIB, one logical and one *a posteriori*. In §5, I focus upon (P1) of Colyvan's $IndA_{io}$, and the principles of naturalism and

confirmational holism that underpin Colyvan's argument. I contend that Quinean naturalism plays an enthymematic role leaving the $IndA_{io}$ open to worrying counterexamples.[6] In §6, I reformulate (P1) of $IndA_{io}$ in light of the limitations of Quinean naturalism and test the new indispensability argument against two of the strongest salient historical candidates for justified belief in an inconsistent entity. I conclude that given the strong support for the PLJIB and RT, one should be open to belief in inconsistency (though not in a great number of cases).

2. The Theoretical Virtues and the Virtue of Consistency

2.1 Scientific Virtues

The PLJIB holds that we can have a justified belief in an inconsistent state of affairs. Before I consider two objections to the PLJIB, it is helpful to understand how our best science can be inconsistent and to provide an account of the theoretical virtues (which we will draw upon in §5). The assessment and confirmation of scientific theories are guided by—and make reference to—certain qualities. These qualities are conceptually understood as theoretical virtues. The adoption of the concept of 'virtues' appropriately signifies that the criteria of assessment are not categorical rules nor conditions that a theory must fulfil, but rather goals that can stand independently of, and even in tension with one another. McMullin offers a helpful account of theoretical virtues in 'The Virtues of a Good Theory'.[7] McMullin's account includes empirical fit, explanatory power, internal coherence, simplicity, optimality, consistency, consilience, fertility, and durability.[8] If science (as a common enterprise) is aimed at success, it seems that success is to produce theories that optimise the scientific virtues.

As emphasised by van Fraassen and affirmed by Maddy, the debate between scientific realists and anti-realists turns upon a different account of the '*telos* of scientific activity'.[9] Scientific practice is organised and aimed at achieving certain goals; if the assessment and confirmation of scientific theories are evaluated with reference to the theoretical virtues then we ought to expect that the *telos* of scientific practice would be inextricably connected to the theoretical virtues. Generally, scientific realists characterise the goal of scientific activity as giving a true description of the world, whilst anti-realists are concerned with the instrumental goals of scientific activity. This debate, perhaps unsurprisingly, manifests in a different account of the scientific virtues, their relation to each other, and whether they are truth-conducive.[10] For instance, constructive empiricists (members of a significant, scientific anti-realist school of thought) consider empirical fit to be the primary virtue of scientific practice

[6] An enthymeme is a concealed premise not explicitly stated in an argument.
[7] McMullin.
[8] McMullin.
[9] Maddy, Penelope (2022) *A Plea for Natural Philosophy: And Other Essays*, Oxford University Press, 50; van Fraassen, Bastiaan C (2015) 'Naturalism in Epistemology', in Richard N Williams and Daniel N Robinson, eds, *Scientism: The New Orthodoxy*, Bloomsbury Academic, 70.
[10] That is, virtues that are conducive to producing true, or approximately true, theories.

(cast within the semantic view of theories that formulates it as 'empirical adequacy').[11] For constructive empiricists the other virtues are subservient; not necessarily truth-conducive, but justified in their theoretical utility.[12]

2.2 Consistency as a Virtue

If consistency is just one virtue among others, an inconsistent theory's virtues might combine to such a level of excellence that the virtue of consistency is trumped and the theory becomes our best science. At a first glance, the positioning of consistency as a theoretical virtue, therefore, has a troubling implication. This implication subverts the received view: that inconsistent beliefs would place us in an 'epistemic hell'.[13] This received view has consistency in a ruling position over the other theoretical virtues. Consistency is often treated as a necessary condition for a coherent system of beliefs,[14] not as merely a virtue. However, this received view is perhaps an overzealous application of Aristotle's LNC in his search for essential, universal, and primitive principles on which to base his *scientia*.[15] A robust defence of the PLJIB requires one to acknowledge the compelling arguments that have been formulated in defence of the LNC. Many of these arguments have been untangled and rejected in Priest, along with Colyvan and Bueno,[16] but I will consider two of particular significance for this paper: one logical and one *a posteriori*.

3 First Objection to PLJIB

3.1 Ex Contradictione Quodlibet

Classical logic is *explosive*: any proposition can be derived from a contradiction ($A \wedge \neg A$). It seems—at least *prima facie*—that the realist cannot coherently commit to both PLJIB and the restrictive injunction of (P1) in IndA_{io} to believe *only* in those entities

[11] van Fraassen construes empirical fit in this way: van Fraassen, Bas C (1980) *The Scientific Image*, Clarendon Press.

[12] van Fraassen, 'Naturalism in Epistemology', 87. For a helpful summary of the distinction between the epistemic and pragmatic use of theoretical virtues see Otávio Bueno and Scott A Shalkowski: Bueno, Otávio and Scott A Shalkowski (2015) 'Modalism and Theoretical Virtues: Toward an Epistemology of Modality', *Philosophical Studies* **172**, 674.

[13] Bueno, Otávio (2006) 'Why Inconsistency Isn't Hell: Making Room for Inconsistency in Science', in Erik J Olsson, ed, *Knowledge and Inquiry: Essays on the Pragmatism of Isaac Levi*, Cambridge University Press, 70**.**

[14] Bueno, 70.

[15] Aristotle (1994) *Posterior Analytics*, Jonathan Barnes, trans, Oxford University Press, 73a24–7; and for further elaboration see Anstey: Anstey, Peter (2022) 'Principles in Early Modern Philosophy and Science', in Dana Jalobeanu and Charles T Wolfe, eds, *Encyclopedia of Early Modern Philosophy and the Sciences*, Springer.

[16] Priest, Graham (1998) 'What is so Bad About Contradictions?', *Journal of Philosophy* **95**; Bueno, Otávio and Mark Colyvan (2004) 'Logical Non-Apriorism and the "Law" of Non-Contradiction', in Graham Priest, JC Beall, and Bradley Armour-Garb, eds, *The Law of Non-Contradiction: New Philosophical Essays*, Oxford University Press.

indispensable to our best science. C I Lewis' proof demonstrates that the hypothesis of a contradiction entails some arbitrary proposition B:

1. A ∧ ¬A
2. A (Follows from (1) by and elimination).
3. A v B (Follows from (2) by or introduction).
4. ¬A (Follows from (1) by and elimination).
5. B (Follows from (3) and (4) by disjunctive syllogism).

This is known as *ex contradictione quodlibet* (ECQ) and its content is often presented as:

$$A,\neg A \vDash B$$

Under classical logic, the PLJIB appears untenable for the realist. Nonetheless, there are two plausible responses for the realist to avoid 'explosion', both of which I will outline briefly.

3.2 Inferential Quarantining and 'Chunk and Permeate'

Firstly, the realist can avoid ECQ and coherently support both the PLJIB and (P1) by making a distinction between logical entailment, and rationally justified inference practices. A relevant point, suggested by Michael (although he would disagree with the conclusions of this paper), is that theorists do not merely draw every inference that can be logically deduced within a theory.[17] For example, even in a consistent theory, there is no good reason to infer all the valid conjuncts of an accepted proposition. It simply isn't helpful for a theorist to develop valid inferences of the sort:

$$A \vDash A \wedge A,$$

$$A \vDash A \wedge A \wedge A, \text{ or}$$

$$A \vDash A \wedge A \wedge A \wedge A$$

ad infinitum.

Given inconsistent propositions, we may be rationally justified in avoiding certain kinds of inferences that, although logically implied by the theory, are of the form set out in ECQ.

Though the connection has not been made by either Priest or Michael, in my assessment, this resistance to explosive inferences has been formalised into what

[17] Michael, Michaelis (2013) 'Facing Inconsistency: Theories and Our Relations to Them', *Episteme* **10**; Michael, Michaelis (2016) 'On a "Most Telling" Argument for Paraconsistent Logic', *Synthese* **193**, 3347–62.

Priest calls an 'inferential strategy',[18] namely, his 'Chunk and Permeate'.[19] For Priest, theorists dealing with an inconsistent theory can operate by separating the theory into multiple, discrete 'chunks' that are internally consistent. That is, theorists inferentially 'quarantine' the would-be-conjuncts of a would-be-contradiction into a number of chunks; each chunk is self-consistent.[20] A limited amount of information can then permeate from one chunk to another, meaning the consistent chunks are not explosive (i.e., they do not entail every proposition). Following a Chunk and Permeate strategy, Colyvan may be able to rationally justify an inconsistent belief by avoiding certain inferences that would trivialise the theory.

However, avoidance strategies leave Colyvan open to criticism. Even if Colyvan refuses to explicitly make certain inferences implied from his theory, a critic of Colyvan's position could make that inference. Take, for example, a debate between a realist, committed to classical logic (who takes some contradictory propositions to be true) and an anti-realist. The anti-realist would point out that the realist's position is untenable, referring to ECQ and simply asking the realist if their position logically entails every proposition. A realist must concede that their theory entails explosion and can consequently only justify this position by affirming that they refuse to believe everything their theory logically entails. With this resolution likely being unsatisfying for a realist, I will now turn to a more promising approach.

3.3 Paraconsistent Logic

Another approach—as pointed out by Colyvan—is to give up on classical logic and take on a paraconsistent logic.[21] A logic is paraconsistent if, and only if, its logical consequence relation is not explosive. Whilst Quine insisted that to change the logic was to 'change the subject',[22] a serious commitment to naturalism suggests we ought to abandon classical logic *before* we give up on our best science. If our classical logic cannot accommodate belief in an inconsistent entity indispensable to our best science, naturalists would be better served by taking on a logic that serves their project better. In fact, as has been argued by Priest, misperceptions about the aggrandised status and history of classical logic are rife and, on the other hand, contemporary research into, and developments in, paraconsistent logics have blossomed.[23] Each logic encapsulates a substantial metaphysical and/or semantic theory and, perhaps, we ought to give up

[18] Priest, Graham, Koji Tanaka, and Zach Weber (2022) 'Paraconsistent Logic', in Edward N Zalta, ed, *The Stanford Encyclopedia of Philosophy*, Metaphysics Research Lab, Stanford University.

[19] Brown, Bryson and Graham Priest (2004) 'Chunk and Permeate, a Paraconsistent Inference Strategy. Part I: The Infinitesimal Calculus', *Journal of Philosophical Logic* **33**; Brown, M Bryson and Graham Priest (2015) 'Chunk and Permeate II: Bohr's Hydrogen Atom', *European Journal for Philosophy of Science* **5**; Benham, Richard, Chris Mortensen, and Graham Priest (2014) 'Chunk and Permeate III: The Dirac Delta Function', *Synthese* **191**.

[20] Priest, Graham (2006) 'What Is Philosophy?', *Philosophy* **81**, 206.

[21] Colyvan, 'Ontological Commitments'.

[22] Priest, 'What Is So Bad About Contradictions?', 416.

[23] Priest, Tanaka, and Weber; Priest, 'What Is So Bad About Contradictions?'; Priest, Graham (2005) *Doubt Truth to be a Liar*, Oxford University Press.

on the classical assumption that truth and falsity in an interpretation are exclusive and exhaustive.[24]

As noted above, paraconsistent logic is defined by the minimum requirement—to not have an explosive consequence relation.[25] The scientific realist, however, requires a far more discriminating logic than this minimum condition for paraconsistency. The realist requires a condition that will fit their purpose to believe in *all* and *only* those entities indispensable to our best science. Like Colyvan, I will not specify the paraconsistent logic suitable for this purpose; investigations of this sort are outside of this paper's scope. Nevertheless, I emphasise that adopting a fit-for-purpose paraconsistent logic removes the explosive consequence of belief in an inconsistent state of affairs under classical logic. If this can be done, we have a response to the logical problem of ECQ.[26]

4. Second Objection to the PLJIB: *A Posteriori* Justification

4.1 Case for a Consistent World and the Problem of Occlusion

Aside from the logical problem for the PLJIB, it is important to recognise an *a posteriori* case for justifying the received commitment to the LNC. One might infer that the empirical evidence for consistency in the world gives tenable grounds for consistency as a necessary feature of the world.[27] If the world is consistent per se, there could be no warrant for belief in an inconsistent state of affairs. Of course, this inference may have some important issues. First, consider Priest's commentary on the 'problem of occlusion' raised by Beall.[28] Beall challenges the strong empirical evidence for consistency in our perceivable world by introducing the concern as to whether our cognitive or perceptual mechanisms preclude the possibility of perceiving an inconsistent state of affairs. Beall wonders whether we are wearing, (to put it metaphorically) consistency lenses that pick out a given worldly state of affairs—either A, or ¬A—and never both. However, as Priest rightly recognises, there doesn't seem to be any empirical evidence for such a cognitive or perceptual-consistency-conforming feature.[29] Moreover, there is evidence that we do in fact have the capacity to perceive an inconsistent state of affairs, as evoked by illusory representations of impossible figures.

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[24] Priest, 'What Is So Bad About Contradictions?', 415–16.

[25] Priest, Tanaka, and Weber.

[26] There are costs to taking on paraconsistent logic. However, whilst a paraconsistent logic may have its weaknesses, and in some respects be less intuitive, I contend that this price may be worth paying in order to defend scientific realism.

[27] The evidence indicates that we never perceive an actual inconsistency in the world. Although we might perceive an inconsistency, like that which will be introduced in Figure 1, there is other good empirical evidence that this perception is merely an illusion. For further discussion, see Priest: Priest, *Doubt Truth to Be a Liar*, 60–70.

[28] Priest, *Doubt Truth to Be a Liar*, 60–70. Beall himself doesn't take on this terminological tag: Beall, JC (2000) 'Is the Observable World Consistent?', *Australasian Journal of Philosophy* **78**.

[29] Priest, *Doubt Truth to Be a Liar*, 63.

4.2 Penrose Stairs

Priest appeals to imagistic representations of impossible states of affairs to demonstrate that we can perceive impossible states of affairs. Figure 1 is a creation by Oscar Reutersvärd, published by Roger Penrose (such stairs are now called Penrose stairs). It is important to recognise that the picture is not a veridical representation of an impossible state of affairs but merely a drawing that utilises artifice to disguise its tricks and fool us into perceiving a contradiction.

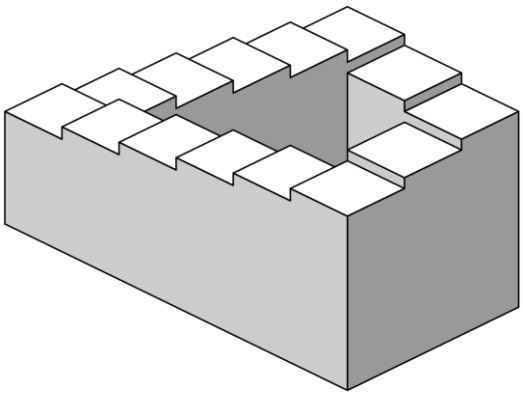

Figure 1. Penrose Stairs.

Looking at the figure, if we choose a given corner, we perceive that climbing the stairs in ascension will lead us back to the same place. In this way, we are perceiving something that is both higher than itself and not higher than itself: both A and ¬A.[30] The perceptual experience of both conjuncts gives us reason to suppose, contra Beall, that we do not have a consistency-limited perceptual or cognitive structure. This is not to say that the representation *is* an instance of a contradictory state of affairs; rather, it is only a trick upon our perception. The contradictions that Priest himself takes to be true seem all to arise in the 'unobservable' realm, including semantics, set theory, and instantaneous change.[31]

4.3 Fallibilism About a Consistent World

An *a posteriori* case for the consistency of the world is not sufficient for asserting an *a priori* LNC, it merely justifies the use of consistency as one theoretical virtue among others. It would be irrational to treat the apparent consistency of the world as an infallible thesis. On the contrary, I suggest that Naturalists ought to remain epistemically open to the possibility of evidence for inconsistent states of affairs and be ready to believe in inconsistent objects as the evidence directs. Science has

[30] Although this account could be contested, here I follow Priest's compelling interpretation: Priest, *Doubt Truth to Be a Liar*, 60.

[31] Priest, Graham, Francesco Berto, and Zach Weber (2022) 'Dialetheism', in Edward N Zalta and Uri Nodelman, eds, *The Stanford Encyclopedia of Philosophy*, Metaphysics Research Lab, Stanford University.

continually subverted historical intuitions about the world. For this reason, it would be foolish to hold tightly onto consistency as a law of nature.

Moreover, if we were to do so, we risk preventing the possibility of being open to dialetheias that may be found in the world. In this sense, Beall's concern of occlusion has weight, not as a theory about how our innate perceptual and cognitive structures filter out consistency, but as a warning that our observations are laden with the received commitment to consistency as a necessary condition of theories which, in turn, restrict the possibility of accepting, or even observing, actual inconsistencies in the world.

4.4 The Virtue of Consistency and PLJIB

Colyvan's paper gestures towards several examples where the virtues of a scientific theory have warranted acceptance despite its inconsistency.[32] The discussion of scientific virtues thus far demonstrates how such an acceptance is possible. Acceptance of an inconsistent scientific theory is rationally justified where its virtues trump the virtue of consistency. This discussion has established a coherent way the realists can bite the bullet on beliefs in inconsistent entities. Thus, the PLJIB: we are justified in believing in the inconsistent objects indispensable to our best scientific theories *where our best scientific theories' virtues trump the virtue of consistency*.

However, the PLJIB by itself is not enough to comfort the realist in the face of $IndA_{io}$. In §5, we will review $IndA_{io}$ in light of a troubling candidate for inconsistent beliefs proffered by Maddy, to demonstrate that a requirement to reformulate (P1) of $IndA_{io}$ ought to extend to our best science *that we are justified in interpreting realistically*.[33]

5. Problem Candidate for $IndA_{io}$ and the Resolving Thesis

5.1 Naturalism and Holism

In this section, I contend that (P1) of Colyvan's argument ought to be altered in light of the shortcomings of Quinean naturalism. Two stances underpin (P1): naturalism and confirmational holism.[34] Maddy begins her 2001 paper 'Naturalism: Friends and Foes' with the comical recognition that '[t]hese days, it seems there are at least as many strains of naturalism as there are self-professed naturalistic philosophers'.[35]

The strain of naturalism referred to in IndA is the strain associated with Quine. Quinean naturalism is taken to be the stance that metaphysics ought to be a kind of 'last science'.[36] Our epistemic commitments—or our 'home theory'—are given to us

[32] See §1—examples will be turned to in greater depth shortly, from §5.2.
[33] Maddy, Penelope (1992) 'Indispensability and Practice', *Journal of Philosophy* **89**.
[34] Quine himself uses a more controversial semantic holism in his indispensability argument for mathematical realism: Quine, *Theories and Things*.
[35] Maddy, Penelope (2001) 'Naturalism: Friends and Foes', *Philosophical Perspectives* **15**, 37.
[36] Colyvan, Mark (2019) 'Indispensability Arguments in the Philosophy of Mathematics', in Edward N Zalta, ed, *The Stanford Encyclopedia of Philosophy*, Metaphysics Research Lab, Stanford University.

by science; the task of philosophy is to perform a kind of 'housekeeping' role.[37] This Quinean stance arises from the naturalistic impulse to be suspicious of any kind of first philosophy (i.e., any philosophy that purports to tell us about the world without recourse to the methods of science).[38] Moreover, as recognised by Colyvan, Quinean naturalism is driven by enthusiasm and deep respect 'for scientific methodology and an acknowledgment of the undeniable success of this methodology as a way of answering fundamental questions about all nature of things'.[39] In §5.3 I will show that Quine's naturalism, taken at face value, has obscured that scientific practice is not always aimed at answering fundamental questions about the nature of things.

On the other hand, confirmational holism—also known as the Quine-Duhem Thesis—holds that theories are confirmed or disconfirmed as wholes.[40] If a theory is confirmed (by, for instance, the successful observation of a novel prediction), then all of the theory's commitments—rather than merely some of them—are confirmed holistically. For example, when classical mechanics was confirmed by the discovery of Neptune,[41] the inconsistent early calculus that the theory employed was also confirmed (we will return to an analysis of the early calculus in §5.3). Following Colyvan, (P1) is constituted by the 'all' given to us by holism and the 'only' given to us by naturalism.[42] However, in §5.3 we will endeavour to show that, *pace* Colyvan, Quinean naturalism is playing another enthymematic role in (P1). First, I will introduce a problem example employed by Maddy, and outline her misguided objection to the doctrine of holism underpinning (P1). Although Maddy rejects the doctrine of holism and the use of IndA altogether, I show that an alternative path can emerge for a scientific realist.

5.2 Theory of Waves in Open Oceans and Scientific Practice

The forceful example employed by Maddy, [43] which continues to rear its head as a problem candidate,[44] is the theory of waves in the open ocean. Scientists have found that the assumption of infinite depth in the ocean simplifies equations without any loss to the accuracy of the models. With an infinitely deep ocean, scientists can ignore the reactions of waves bouncing off the bottom of the ocean, complicating the wave function. Maddy uses this example to demonstrate that scientists do not consider confirmation of this best theory of waves, as confirmation of the assumption of infinite depth. On Maddy's assessment, our philosophical doctrines about scientific practice

[37] Michael, 'Facing Inconsistency', 356.
[38] Maddy, 'Naturalism', 39.
[39] Colyvan, 'Indispensability Arguments in the Philosophy of Mathematics'.
[40] I defend, in this paper, this weaker form of holism, rather than Quine's suggestion that 'the unit of empirical significance is the whole of science': Quine, Willard Van Orman (1976) 'Two Dogmas of Empiricism', in Sandra G Harding, ed, *Can Theories be Refuted? Essays on the Duhem-Quine Thesis*, D Reidel Publishing Company, 56.
[41] Psillos, Stathis (2005) *Scientific Realism: How Science Tracks Truth*, Routledge, 33.
[42] Colyvan, 'Indispensability Arguments in the Philosophy of Mathematics'.
[43] Maddy, 'Indispensability and Practice', 281.
[44] Caret, Colin R (2021) 'In Pursuit of the Non-Trivial', *Episteme* **18**, 289; Colyvan, 'Ontological Commitments', 117; Michael, 'Facing Inconsistency', 356.

ought to cohere with the doxastic standards of the scientific community.[45] Whilst Maddy admits that the doctrine of holism is 'logically … unassailable',[46] she encourages us to embolden our naturalism such that we defer to scientific practice as a greater precedential authority than a philosophical doctrine like holism. For Maddy, we must reject the doctrine of holism and, for that matter, the use of IndA altogether.

A closer look at the problem candidate, through the lens of theoretical virtues, reveals an alternative path for the realist that avoids rejecting the doctrine of holism. In the previous discussion of theoretical virtues, it was noted that certain theoretical virtues—and the question of whether or not they are assessed as truth-conducive—are associated with realism; others are associated with anti-realism. Moreover, scientific practice is guided by the theoretical virtues that it strives to achieve. In the present example, we can see that the scientists are not seeking to provide a veridical, explanatory account of the causal history of waves in the open ocean. It seems that the theorists are seeking to achieve instrumental goals, such as modelling currents to make predictions concerning the tidal dissipation of certain seas.[47] This was indicated by the fact that the theorists justified a significant trade-off, giving up on the virtue of consistency and explanatory power for a simpler formula, with reference to its negligible impact upon the virtue of empirical fit (*qua* predictive power). Scientific theories guided by an instrumental goal—attempting to maximise their virtues as an instrumental theory—are not good candidates for belief. As will be demonstrated below, *pace* Maddy, the problem isn't the doctrine of holism: it is the enthymematic role played by Quinean naturalism that undiscerningly extends the scope of IndA to all our best science.

5.3 The Limits of Quinean Naturalism

Maddy's example is helpful in showing that naturalism is playing another enthymematic role in (P1). As mentioned above, naturalism extends the scope of IndA_{io} to *all of our best science*. Quinean naturalism is asking us to be undiscerning in determining whether one ought to epistemically commit to our best science, obscuring the different and distinct aims that instigate, motivate, and direct science. Quine is perhaps right; science is our best (and perhaps only) chance of having justified beliefs about the world. However, it would be naïve to claim that all scientific practice is aimed at giving a picture of the world that we are justified in believing. The scientific methodology is called to attend to different goals which are often in tension with each other. Attending to the ontology of nature is one of these goals but so are: solving certain problems, providing us with reliable predictions, or constructing a provisional

[45] Maddy, 'Naturalism', 45.

[46] Maddy, 'Indispensability and Practice', 280.

[47] Bell, T H, Jr (1975) 'Topographically Generated Internal Waves in the Open Ocean', *Journal of Geophysical Research* **80**, 326.

theory that may be fruitful for future theories.[48] Per my RT, (P1) of $IndA_{io}$ ought to be adjusted to:

> (P1) We have justified belief in all and only the entities that are indispensable to our best scientific theories *that we are justified in interpreting realistically*.

To make this adjustment is not to become an anti-realist,[49] an anti-naturalist, or agnostic about problem cases such as that described in §5.2. All our best theories in oceanography that we are justified in interpreting realistically posit a finitely deep ocean, an entity which, from our indispensability argument, we are justified in believing in. Realists need only concede realistic interpretations of those theories that are driven, directed, or organised by goals (and virtues) that are *instrumental*. There is another concession for the realist, and that is the simple path to metaphysics. Where indispensability arguments previously allowed metaphysicians to simply defer to the entities contained in our best scientific theories, realists might engage in the interpretive practice of understanding whether it is appropriate to realistically construe a given theory. Colyvan worries about whether introducing a realistic or instrumental distinction is an *ad hoc* adjustment to the indispensability argument.[50] For the reasons stated in this subsection, the enforcement of this distinction in our indispensability argument is not *ad hoc* but adjusts our argument to a better-developed naturalistic stance that is sensitive to the orthogonal, competing aims of scientific practice.

6. Two Case Studies for the Adjustment of $IndA_{io}$

6.1 Resolving Thesis

We have now arrived at a higher standard for belief in inconsistent entities than Colyvan's initial dilemma proposed. We can reformulate Colyvan's $IndA_{io}$ to reflect RT and call it $IndA_{io}$*:

> (P1) We have justified belief in all and only the entities that are indispensable to our best scientific theories *that we are justified in interpreting realistically*.
>
> (P2) Inconsistent entities are indispensable to our best scientific theories *that we are justified in interpreting realistically*.
>
> (C) We have justified belief in inconsistent entities.

The realist ought to accept this conclusion in light of $IndA_{io}$*. On the one hand, the standard of the RT promises to deal with worrying counter-examples, and on the other hand, the PLJIB epistemically prepares us to bite the bullet on any inconsistent beliefs

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[48] Other reasons for taking on specifically inconsistent theories are developed in Bueno: Bueno, 74.

[49] A distinction between realist theories and instrumental theories is happily drawn by the realists: Brown, Bryson (1990) 'How to Be Realistic About Inconsistency in Science', *Studies in History and Philosophy of Science Part A* **21**, 281–94.

[50] Colyvan, 'Ontological Commitments', 122.

that are derived from our best science that we are justified in interpreting realistically.[51]

I will now demonstrate the desirability of the conclusion of $IndA_{io}$* with reference to two of the salient inconsistencies in our historiography of science.

6.2 Bohr's Atom and the Interpretive Assessment

Bohr's model of the atom draws together classical electrodynamics and the quantum theory of radiation.[52] One of the two ways that Bohr's model of the hydrogen atom has been characterised as inconsistent is by the account of the electron as a charged, orbiting (and therefore accelerating) particle that isn't emitting radiation.[53] The daring thesis that the particle isn't emitting radiation (derived from the quantum theory of radiation) was inconsistent with Maxwell's equations (from electrodynamics); this unsurprisingly caused suspicion from within the scientific community: 'This is nonsense! Maxwell's equations are valid under all circumstances, an electron in an orbit must radiate.'[54] Under Colyvan's $IndA_{io}$, our two best scientific theories purport to justify a belief in an inconsistent entity: an electron behaving in a manner that entails it is both emitting radiation and not emitting radiation. Under $IndA_{io}$*, we ought only to believe in the inconsistent electron if we are justified in realistically interpreting the theory.

So, how are we to determine whether we are justified in realistically interpreting the theory? As discussed, this can be assessed with reference to the telos of scientific activity, namely, which virtues scientific practice is aiming to fulfil, and the relation of these virtues to realism. I think it is wise, as naturalists, to take scientists' assessment as *prima facie* acceptable. In this case, contemporaneous physicists did not take the model to be true, even though they accepted the model as the best theory of the atom.[55] However, there is nothing principally 'first philosophical' about making such an assessment as philosophers. We can see that the model was working to account for a peculiar observation, namely, the spectral lines of the hydrogen atomic emission spectrum. Bohr understood that the best provisional theory with empirical fit with these observations needed to be inconsistent. The demands of observation called for a theory that the physicists could use instrumentally as a provisional path towards a theory of the atom that was appropriate for realistic acceptance. A brief analysis of the goals directing the theory, in conjunction with its scientific virtues, together indicates

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[51] Of course, it may be the case that none of the inconsistent theories are ones that are justified in interpreting realistically.

[52] For a brief introduction and elaboration on the implication of Bohr's model, see Bueno, Otávio and Steven French (2011) 'How Theories Represent', *British Journal for the Philosophy of Science* **62**, 866; Caret, 'In Pursuit of the Non-Trivial', 284.

[53] Vickers, Peter J (2008), 'Bohr's Theory of the Atom: Content, Closure and Consistency', presented at the 1st conference of the European Philosophy of Science Association.

[54] Vickers, quoting Max von Laue, quoted in Pais, Abraham (1991) *Niels Bohr's Times*, Oxford University Press, 154.

[55] Bueno and French, 'How Theories Represent', 867.

that realists ought to have interpreted the theory instrumentally. Therefore, under $IndA_{io}$*, there was no time at which belief in the inconsistent entity was justified.

6.3 The Early Calculus and Holism

As suggested by Colyvan himself, perhaps the strongest historical case that could be made for belief in an inconsistent object is the inconsistent infinitesimal posited by the early calculus.[56] Before Bolzano's development of the notion of a limit and the epsilon-delta technique in 1817, the early calculus (developed in the 1660s) depended upon the notion of an infinitesimal, defined as a quantity greater than 0, but smaller than any real number. This definition allows the infinitesimal to play a dual role in early calculus; for some purposes, the infinitesimal is taken to be a quantity *distinguishable* from zero whilst for other purposes the infinitesimal is taken to be *indistinguishable* from zero. If 'δ' is an infinitesimal, then in early calculus '$\delta = 0$' and '$\delta \neq 0$'.[57]

Would the inconsistent infinitesimal meet the criteria of the RT? Notably, George (Bishop) Berkeley emphasised in *The Analyst* that even Newton's fluxional calculus (that attempted to resist any reference to infinitesimals) contravened the LNC.[58] However, despite Berkeley's critique of the alleged artifice of calculus, experimental philosophers took on Newton's use of calculus which, given its inconsistency, was an extraordinarily useful theory in formulating classical mechanics. These physical theories were theoretically strong, developing over time and proving fertile in their production of novel predictions confirmed by observation (e.g., the discovery of Neptune). It is intuitively striking and strange to recognise that the doctrine of holism claims that as these theories were confirmed, the infinitesimal—an indispensable mathematical *abstracta*—was also being confirmed. Indeed, from $IndA_{io}$* there is a strong case that during the 150 years in which the infinitesimal was indispensable to our classical mechanics—a science which the practitioners were justified in interpreting realistically—there was good justification for belief in the inconsistent object of the infinitesimal.

7. Conclusion

$IndA_{io}$* produces a result that may worry the scientific realist. This is the result that for a significant period in our history there was a strong case for a justified belief in the infinitesimal: an inconsistent entity. This would have been untenable for the early moderns who held onto the LNC. However, if we are to suppose that the logical explosion of a contradiction can be defused by paraconsistent logic and that we ought to be epistemically open to finding inconsistency in the world, then a justified belief in infinitesimals might not seem so surprising. If we are justified in stepping into a

[56] Colyvan, Mark (2009) 'Applying Inconsistent Mathematics', in Otávio Bueno and Øystein Linnebo, eds, *New Waves in Philosophy of Mathematics*, Palgrave Macmillan, 161.
[57] Caret, 'In Pursuit of the Non-Trivial', 284.
[58] Berkeley, George (1734/2002) *The Analyst; or, a Discourse Addressed to an Infidel Mathematician,* David R Wilkins, ed, Trinity College Dublin, 3.

post-LNC epistemic world, justified belief in an inconsistent object is exactly something we might expect.

Colyvan's dilemma provokes a worrying puzzle for his own scientific realism. I have aimed to show that although there are compelling motivations for treating Colyvan's conclusion as absurd, we may be warranted in a belief in an inconsistent state of affairs. Moreover, in light of worrying examples, such as the idealisation in modelling waves in the open ocean, I have emphasised that the dilemma highlights the naivety of a Quinean naturalism that fails to recognise that not all scientific practice is aimed at truth, nor is treated as veridical by scientists. However, if Colyvan's paper is aimed to provoke, this paper is aimed to clarify that scientific realists should accept all and *only* realistic interpretations of our best science and should not have an iron insistence on consistency as a necessary condition of a good theory.

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