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JUDGED KNOWLEDGE AND AMBIGUITY AVERSION

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Abstract

Competence has been recently proposed as an explanation for the degree of ambiguity aversion. Using general knowledge questions we presented subjects with simple lotteries in which they could bet on an event and against the same event. We show that the sum of certainty equivalents for both bets depends on the judged knowledge of the class of events. We also elicited the decision weights for events and complementary events. We found a similar effect of knowledge on the sum of decision weights.

Keywords

Ambiguity, Competence, Knowledge, Decision Making, Uncertainty

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1. Introduction

Ever since Ellsberg (1961) presented his famous paradox researchers have been interested in understanding and modelling ambiguity. In one of the Ellsberg paradoxes, decision makers can choose between one urn containing equal numbers of white and yellow balls and a second urn containing white and yellow balls in unknown proportions. The second urn will be referred to as the ambiguous urn. The decision maker can choose which colour will win him a given amount of money (the other colour will leave him at his current wealth level). Standard subjective expected utility theory (Savage 1954) suggests that the decision maker should weakly prefer the ambiguous urn. It is well known that, contrary to SEU, many decision makers choose the unambiguous urn.

During the last years empirical investigations have replicated Ellsberg's thought experiments with real subjects, extensively varying the parameters of the paradox. Another stream of work has tried to put forward new preference theories or has tried to extend SEU to model attitudes towards ambiguity. See Camerer, Weber (1991) for an overview of empirical and theoretical work. In this paper we will investigate <u>why</u> subjects are averse towards ambiguity.

Ambiguity can be defined as the uncertainy about probability created by missing information that is relevant and could be known (Frisch, Baron 1988). This kind of ambiguity is present in most real world decision situations. Investing in a plant in Russia, buying a stock of a newly listed company, or betting on having a grant proposal accepted are three examples. Especially in real world examples, the judged competence of a decision maker or his knowledge of the decision situation could be related to the degree of ambiguity aversion: high (low) competence or knowledge implies a feeling of little (a lot of) missing information. Heath, Tversky (1991) demonstrated that competence is indeed related to ambiguity aversion.

In a series of experiments Heath and Tversky first asked subjects for probabilities of natural events. Then they made them choose between betting on the event and betting on a chance device with the same subjective probability as the event. Subjects generally preferred to bet on the event (chance device) if they knew a lot (little) about the event, i.e. if they felt competent (incompetent). In their fifth experiment Heath and Tversky elicited certainty equivalents for a bet on an event and a bet against the event. They demonstrated that the sum of certainty equivalents was less (more) than the sum for chance devices in low (high) competence situations. They suggest that competence "allows people to claim credit when they are right, and in absence exposes people to blame when they are wrong." (Heath, Tversky 1991, p.22)

We replicate and extend their fifth experiment. Using a choice setting we elicit certainty equivalents <u>and</u> decision weights for simple event lotteries. On the basis of individual and group data we will investigate if knowledge and ambiguity aversion are related. We do not directly investigate the credit-blame explanation mentioned in the last paragraph (but see Taylor 1991 for a first attempt).

The goal of our study is to contribute to understanding the relation between knowledge and ambiguity, which is essential for the study of ambiguity effects in real economic settings. Experiments using urns show that high ambiguity on probabilities in general implies high ambiguity effects. In a lot of real world decision situations one will also find a high degree of ambiguity. Yet, depending on the high (low) knowledge of the decision maker no (a large) ambiguity effect can be expected.

The paper is organized as follows. In section 2 we will present hypotheses and the experimental design. Results regarding subjects' knowledge will be presented in section 3. The relation of knowledge to the sum of certainty equivalents will be discussed in section 4 and to the sum of decision weights in section 5. In section 6 the results for certainty equivalents and decision weights will be compared.

2 Hypotheses and Experimental Design

2.1 Hypotheses

In our experiment subjects evaluated simple event lotteries. An event lottery is a two-outcome lottery in which subjects would either get a high outcome (10.- DM, about US-$ 6.-) or a low outcome (0.- DM) depending on the binary event. To bet on the event means that the high outcome is paid if the event occurs. To bet against the event (or the event's complement) means that the high outcome is paid if the event does not occur.

Let p(Event) be the subjective probability of an event and u the decision maker's von Neumann Morgenstern utility function on the interval [0 DM, 10 DM]. A certainty equivalent of a lottery is the amount of money, for which the decision maker is indifferent between the amount and the lottery, i.e. u(certainty equivalent) = p(event) u(10 DM). The sum of certainty equivalents for betting on an event, denoted by event, and against the event, denoted by compl. event for complement event, is given by:

$$\text{SUM(event)} = u^{-1}(p(\text{event})) + u^{-1}(p(\text{compl.-event})).$$

For a risk neutral decision maker SUM(event) is equal to ten, for a risk averse decision maker the sum is less than ten and for risk seekers the sum is greater than ten (for events that are not certain or impossible). For all utility functions with decreasing absolute risk aversion the difference 10-SUM(event) is largest for p(event) = .5[1].

According to SEU the number SUM(event) should depend only on the risk attitude and on p(event). SEU does not allow for uncertainty in one's subjective probability judgments. Therefore, ambiguity should not influence SUM(event). Following Heath, Tversky (1991) we hypothesize that there is an ambiguity effect in SUM(event) and that this effect depends on the perceived knowledge of the event. Hypothesis 1 will be referred to as the "Knowledge-CE" hypothesis.

Hypothesis 1:

> For ambiguous events SUM(event) depends on the judged knowledge of the event. High knowledge implies a larger SUM(event) than low knowledge.

Urns generally serve as an example for chance devices. We will consider an urn containing 100 red and white balls where the decision maker wins 10 DM if a white ball is drawn and nothing if a red ball is drawn. According to SEU p(event) can be elicted by asking the decision maker for the number of white balls in the urn to make him indifferent between the urn and the event lottery. According to SEU the sum of p(event) and p(compl.-event), denoted by PROB(event), should add up to one. Earlier studies have shown that subjects show ambiguity aversion. We are interested if this ambiguity effect is reflected in PROB(event).

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[1]) For p = .5 we get, SUM(event) = u^{-1} (.5) + u^{-1} (.5). For p(event) = .5 + d, SUM(event) = u^{-1} (.5 + d) + u^{-1} (.5 - d). As u is concave, u^{-1} is convex, therefore u^{-1} (.5 + d) - u^{-1} (.5) > u^{-1} (.5) - u^{-1} (.5 - d). Rearranging yields the result.

Schmeidler (1989) presented an axiomaticaly based theory for decision making under ambiguity. In his models the decision weights, called capacities, do not need to add up to one. The simple indifference judgment explained in the last paragraph elicits p(event) as the (possibly non-additive) capacity or decision weight for the event (see Mangelsdorff, Weber 1991 for further details). In the same spirit as in hypothesis 1 we think that there exists an ambiguity effect in PROB(event) and that the effect is larger for events people have low knowledge about.

Hypothesis 2:

> For ambiguous events PROB(event) depends on the judged knowledge of the event. High knowledge implies a larger PROB(event) than low knowledge.

2.2 Experimental Design

During the experiment we considered two types of events: Anwsers to general knowledge questions and other events. General knowledge questions were taken from four domains:

German stocks, denote by G-St
US stocks, denoted by U-St
German geography, denoted by G-Geo
US geography, denoted by U-Geo.

For example, an event lottery for a German stock (G-St) question was as follows:

> "If the RWE stock price was lower than 370.00 DM on January, 21 1991 you win 10 DM; otherwise nothing."[2,3]

All general knowledge events were designed so that the chance of winning or losing the lottery were roughly equal. All the

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[2]) The experiment was conducted on January 31, 1991.

[3]) RWE is the largest German electricity company.

questions are listed in an Appendix.

The experiment was run as a questionnaire (in German). Sixty-five students in an introductory business class voluntarily participated in the experiment. They were randomly divided in two Groups A (31 students) and B (34 students).

The experiment had several stages. First, subjects in both groups were asked to judge their knowledge for the four general areas. Subjects estimated the knowledge on a five-point scale, ranked the areas according to knowledge and estimated the knowledge using a direct ratio procedure. In this procedure the area with the least knowledge was given 10 points and the other areas were given multiples of 10 depending on the judged knowledge (see von Winterfeldt, Edwards 1986).

Subjects belonging to Group A then gave certainty equivalents for 12 event lotteries (referred to as White in Figure 1). These 12 lotteries consisted of two lotteries in each of the four knowledge domains (a total of 8 lotteries) and 4 other lotteries. The other lotteries were based on a coin landing heads, a die coming up with a 1 or 2, a thumb-tack landing pin-up, and a red slip of paper being drawn out of an envelope with an unknown number of red and white slips. Group B also gave certainty equivalents for 12 event lotteries (referred to as Black in Figure 1). Each lotteries in Black was derived from betting against an event that was considered in White, i.e. the events in Black were the complements of the events in White[4]. The sequences of corresponding lotteries were iden-

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[4]) For the general knowledge questions the events were defined as a distance being less or a stock price being lower (respectively, more or larger) than a certain number. See Heath, Tversky (1991) for a similar procedure. The possibility that the distance or the price was exactly equal to the certain amount was not considered. Subjects who asked for this possibility were told that exactly equal was not considered.

tical in White and Black.

In the Green and Blue part of the questionnaire subjects were asked for the decision weights p, which make them indifferent between the event lottery and a chance device which had a probability to win equal to the decision weight p. The events in Blue were complements of the events in Green. In Green (and Blue) subjects were asked to evaluate 9 event lotteries. Those lotteries consist of one general knowledge question from each of the four domains and two questions from the first part of the experiment (Black or White) on US and German stocks[5]. For the other three events subjects evaluated the coin and the thumb-tack again and, as a new event, a spoon landing with its bowl facing up. Figure 1 explains the design.

Group A	Group B
Knowledge Judgment	Knowledge Judgment
White	Black
Black	White
Green	Blue
Blue	Green

Fig. 1: Experimental Design

The introduction of the questionnaire was read aloud to the participants. After each set of questions (White, Black, Green and Blue) the experimenter collected the set of answers. Subjects took about 90 minutes to fill out the questionnaire and were paid 15 DM for participating.

[5]) We repeated U-St and G-St questions because we thought the knowledge difference was likely to be largest between Germany and US, not between stocks and geography.

3 Data and Knowledge Judgments

Out of 31 Group A subjects three questionnaires had to be discarded, leaving 28 usable responses[6]. For Group B we could use 29 out of 34 questionnaires. Two subjects had incomplete questionnaires and three violated dominance.

Certainty equivalents for the coin event and the die event allow us to estimate each subject's utility function. Using these questions we could determine utilities for p = 1/3, .5 and 2/3. Unfortunately we could only use the data from 23 out of 57 subjects to estimate utility functions. Due to our bad formulation of the die questions, 18 subjects gave identical certainty equivalents for a die yielding 1 or 2 and a die not yielding 1 and 2, thus obviously misinterpreting the events. Others violated dominance, i.e. gave higher certainty equivalents for lotteries with lower (or equal) chances of winning. Of the 23 consistent subjects, 11 were risk neutral, 4 were risk neutral on most event lotteries, 7 were risk averse and 1 was risk seeking. Taking only the two coin lotteries into account (using the full sample) 32 were risk neutral, 11 risk averse, 2 risk seeking and 12 unclear. Thus, risk seeking behavior did not play a major role in our study.

Out of the 57 subjects 55 (Group A: 27, Group B: 28) gave knowledge judgments on the four domains. Our further evaluation will mainly depend on the elicited rankings of knowledge. Having four areas there are 24 possible rankings. Subjects gave the six different rankings shown in Table 1.

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[6]) One questionnaire was incomplete and two subjects violated dominance, i.e. they said a lottery which would pay them 10 DM as a maximum was worth more than 10 DM.

Rank 1	Rank 2	Rank 3	Rank 4	Number of subjects
G-Geo	U-Geo	G-St	U-St	34
U-Geo	G-Geo	G-St	U-St	5
G-Geo	G-St	U-Geo	U-St	11
G-Geo	G-St	U-St	U-Geo	1
G-St	G-Geo	U-Geo	U-St	2
G-St	G-Geo	U-St	U-Geo	2

Tab. 1: Knowledge Rankings

Table 1 shows that 39 (=34+5) subjects judged their knowledge of geography to be higher than their knowledge of stocks and 16 (=11+1+2+2) judged their knowledge of Germany to be higher than their knowledge of US.

4 Certainty Equivalents

We first analyze average certainty equivalents for the 12 questions in the White and Black sets. Then we will take the individual knowledge judgements into account (section 4.1). Finally we will present some individual data (section 4.2).

4.1 Testing Hypothesis 1

Averages of SUM(event) for Groups A and B including the standard deviations for Groups A and B are given in Table 2.

	Group A	Group B	Groups A and B	s
Coin	9.14	9.00	9.07	(2.69)
Die	8.46	8.05	8.25[7]	(3.02)
Average	8.80	8.53	8.66	
Envelope	7.43	5.79	6.60	(3.79)
Thumb-tack	8.71	9.24	8.98	(3.22)

[7]) Note that the data for the dice are subject to the violations of dominance. These violations might cancel out, however, we will not make much use of these data.

U-Geo 1	10.11	8.37	9.22	(4.58)
U-Geo 2	8.80	7.59	8.19	(4.15)
G-Geo 1	9.88	8.47	9.16	(3.92)
G-Geo 2	9.43	8.52	8.97	(4.21)
U-St 1	9.29	6.18	7.70	(3.82)
U-St 2	9.04	5.47	7.22	(4.08)
G-St 1	9.46	6.02	7.71	(4.09)
G-St 2	8.41	7.43	7.91	(3.93)
Average Geo	9.56	8.24	8.88	
Average St	9.05	6.28	7.64	
Average U	9.31	6.90	8.08	
Average G	9.30	7.61	8.44	

Tab. 2: Average Sum of Certainty Equivalents (SUM-values) for 12 Lotteries of White and Black Set of Questions

Table 2 shows that subjects are risk averse on average (for coin and die). The event "envelope", most similar to an Ellsberg urn, has a significantly lower SUM(envelope) than the coin ($t = 4.08$, $p < .5\%$) and the die ($t = 2.58$, $p < 1\%$). The SUM-value of 9 out of 10 ambiguous lotteries is smaller than the SUM for the coin, a fact that can not be explained by SEU. Remember, that SUM gets larger if p and (1-p) become less equal. The average SUM for the geography questions (8.88 DM) is larger than the average SUM for stocks (7.64 DM), ($t = 1.98$, $p < 5\%$), reflecting a knowledge effect which we investigate in more detail below. The difference in SUM between US questions (8.08 DM) and German questions (8.44 DM) is not significant ($t = .62$). The standard deviations in Table 2 show that certainty equivalent judgments are most diverse for general knowledge questions and least diverse for nonambiguous events.

Since subjects were randomly assigned to Group A and B we would expect the data to look the similar for both groups. However, overall the sum of certainty equivalents for Group A

(9.01 DM) is significantly larger (t = 2.45, p < 1%) than the sum for Group B (7.51 DM). The difference is present in envelope and general knowledge questions, indicating an apparent difference in ambiguity aversion. Individual data for Group A and B offered no compelling reason for the difference in ambiguity aversion. However, we can not offer a conclusive explanation for this (arti?)fact.

The main test of Knowledge-CE hypothesis (hypothesis 1) is presented in Table 3.

Knowledge	Average Sum of Certainty Equivalents	
Rank 1	9.35 (3.57)	9.24
Rank 2	9.14 (3.91)	
Rank 3	7.54 (3.20)	7.58
Rank 4	7.61 (3.63)	

Tab. 3: Average SUM(event) Depending on Judged Knowledge

To derive Table 3 for each person the sum of certainty equivalents was taken for those questions which belonged to the area the subject hinself judged highest, second highest, third highest and least knowledgeable. The middle column gives the average of these numbers across subjects. The data support hypothesis 1: The average for rank 1 and 2 combined (9.24 DM) is significantly larger than the average of rank 3 and 4 combined (7.58 DM), (t = 2.70, p < .5%). For both Group A and B the average for rank 1 and 2 combined is larger than the average of rank 3 and 4 combined. The gap between rank 1 and 2 vs. rank 3 and 4 is also reflected in the quantitative knowledge judgments. Adding the points elicited by the direct ratio procedure and averaging, we get: rank 4 = 10 points, rank 3 = 22 points, rank 2 = 77 points, and rank 1 = 111 points. Thus, the large gap in knowledge betwen ranks 2 and 3 seems to be reflected in the large gap in SUM-values. We can conclude that the aversion towards ambiguity depends on the judged knowledge of the area.

Comparing events of high and low competence areas with chance events, Heath, Tversky (1991) found that subjects' sums of certainty equivalents were larger (smaller) for high (low) competence events than for chance events. We replicated their findings. The average sum is 9.07 DM for the coin, the average sum of rank 1 and 2 is larger (9.24 DM) ($t = .89$, p insignificant), and the average sum for rank 3 and 4 (7.58 DM) is significantly smaller ($t = 2.25$, $p < 5\%$).

Up to this point the sum of certainty equivalents was calculated for each subject. We also calculated SUM-values using between-subjects data comparing certainty equivalents for event lotteries for Group A (White set) with those for complement events for Group B (Black set). This analysis is a replication of the Heath and Tversky data. Using this design we were not able to take into account a subject's judged knowledge.

The analysis shows the same results as the within subject analysis. Subjects were more ambiguity averse for the envelope bets, they paid significantly less for the envelope than for the coin ($t = 3.96$, $p < .5\%$) and for the dice ($t = 2.14$, $p < 5\%$), and they paid significantly more for geography questions than for stock questions ($t = 1.93$, $p < 5\%$).

4.2 Further Analysis of Individual Data

In section 4.1 we have tested hypothesis 1. In this section we will try to gain some insight in individual decision making. We will first investigate how certainty equivalents for events and event complements are related for each subject.

[Insert Figure 1 around here]

Fig. 1: Certainty Equivalents for Events and Compl.-Events for Coin

In Figure 1 each dot represents one subject. The x-coordinate shows the certainty equivalent the subject has given in the White question set and the y-coordinate shows the certainty equivalent in the Black question set. For the coin lottery quite a number of subjects gave a certainty equivalent of 5 DM. Every dot below the 10 DM budget line symbolizes a subject who is risk averse or ambiguity averse. If subjects have unequal probabilities or decision weights dots in a diagram should be more towards (10,0) or towards (0,10).

[Insert Figure 2 around here]

Fig. 2: Certainty Equivalents for Events and Compl.-Events for Envelope, U-Geo 1, and U-St 2

Figure 2 shows similar diagrams for the Ellsberg alike event (envelope) as well as for the events with the highest (U-Geo 1) and lowest (U-St 2) sum of certainty equivalents. The dots for the envelope clearly lie below the 10 DM budget line, pretty much around the diagonal indicating equal decision weights and showing ambiguity aversion or risk aversion. Comparing coin and envelope questions points for the envelope are closer to (0,0) indicating ambiguity aversion. For U-St 2, betting on the price of EXXON, subjects exhibit a different behavior. Still the SUM(EXXON) is small (7.22 DM), but the dots are much more scattered: Some people want to stay away from both sides of the bet - thus offering low certainty equivalents - whereas few others want to take both sides. We did not find a relation between individual knowledge judgment and location in the diagram. For U-Geo 1, betting on the distance between New York and Los Angeles, subjects show very diverse behavior. Some subjects even seem to be sure that the event occurs.

Further insight into the distribution of individual sum of certainty equivalents can be gained by plotting the distribution of these sums. Figure 3 and 4 will present those cumulative distributions also called profiles. For every DM amount x on the x-axis one profile will give the number of subjects whose certainty equivalent for an event lottery plus certainty equivalent for the corresponding compl.-event lottery was at least equal to x.

[Insert Figure 3 around here]

Fig. 3: Cumulative Distributions for SUM(event), for Coin, Envelope, and Thumb-tack

The profile for the coin event again shows that some subjects were risk averse, the majority were risk neutral, and only 3 were risk seeking. For the thumb-tack more subjects have SUM-values smaller and larger than 10. The size of the ambiguity effect is clearly represented in the profile for the envelope. The distribution is more dispersed and shifted to the left, reflecting the many subjects whose sum are less than 10. There seems to be no substantial differences for DM values above 10 DM. Note, that for these events knowledge should not play any role.

[Insert Figure 4 around here]

Fig. 4: Cumulative Distributions for SUM(event), for General Knowledge Questions

Figure 4 contains as a boundary the profiles for coin and envelope. The shaded areas contain the four profiles for geography and stocks respectively. Geography and stock events lie between this boundary for DM values below 10 DM. The fact that geography lies above stock shows a smaller ambiguity effect in geography. For DM values above 10 DM a considerable number of subjects gave a higher SUM value for geography and stocks questions than for the coin. The envelope appears to be the pure ambiguity case with no specific knowledge available. An increase in knowledge shifts the SUM profiles to the right, reflecting the reduction in ambiguity aversion and the increase in certainty equivalents.

5 Decision Weight Judgments

5.1 Testing Hypothesis 2

We will present the analysis for PROB(event), i.e. the sum of decision weights for event and compl.-event, in the same way as the analysis for SUM(event). Recall that four events (coin, thumb-tack, U-St 1, G-St 1) were identical for both types of analysis. The average PROB(event) for Groups A and B as well as standard deviations are given in Table 4.

	Group A	Group B	Groups A and B	s
Coin	99.6	97.2	98.4	(7.5)
Thumb-tack	96.7	96.0	96.4	(25.9)
Spoon	98.8	98.8	98.8	(15.6)
U-Geo 3	98.8	80.5	89.5	(25.1)
G-Geo 3	107.3	94.7	100.9	(29.3)
U-St 1	94.0	89.2	91.5	(28.9)
U-St 3	95.3	78.3	86.6	(25.7)
G-St 1	95.4	84.0	89.6	(25.8)
G-St 3	87.3	85.0	86.1	(28.5)
Average Geo	103.3	87.6	95.2	
Average St	93.0	84.1	88.5	
Average U	96.0	82.7	89.2	
Average G	96.7	87.9	92.2	

Tab. 4: Average Sum of Decision Weights (PROB-Values) for 9 Lotteries of Green and Blue Set of Questions (in %)

Table 4 shows that PROB(coin) is close to one. For spoon and thumb-tack we got similar results as for coin. All three values are not significantly smaller than 100%. Similar to the results for the certainty equivalents the average sum of decision weights is significantly larger for geography (95.2%) than for stocks (88.2%), ($t = 1.73$, $p < 5\%$). Both values are significantly smaller than 100%, (Geo: $t = 1.77$, $p < 5\%$, St: $t = 4.16$, $p < 1\%$). German questions do not have a significantly higher PROB-value than US questions ($t = .85$, $p > 5\%$). Again, there is a puzzling difference between Group A (97.0%) and Group B (89.3%) ($t = 2.33$, $p < 5\%$), due to the difference in the six general knowledge questions.

Describing the design we noted that subjects might have thought that a stock price or a distance was exactly equal to the number given in the definition of the event. Then the subadditivity should be smaller for larger numbers defining the event. Comparing G-St 1 with G-St 3 and U-Geo 3 with G-Geo 3 shows that the size of numbers can not explain the degree of subadditivity.

Table 4 supports hypothesis 2 as the knowledge of geography on average is judged higher than the knowledge on stocks, and decision weights are higher too. The main test of hypothesis 2 is given in Table 5.

Knowledge	Average Sum of Decision Weights	
Rank 1	98.6 (28.8)	96.0
Rank 2	93.4 (24.6)	
Rank 3	85.2 (24.7)	87.4
Rank 4	89.6 (24.0)	

Tab. 5: Average PROB(event) Depending on Judged Knowledge (in %)

Average PROB(event)-values in Table 5 were derived similarly to the average SUM(event)-values in Table 3. The data clearly support hypothesis 2: The average for rank 1 and 2 (96.0%) is significantly larger than the average of rank 3 and 4 (87.4%), ($t = 2.08$, $p < 5\%$). Again, the difference in PROB-values for rank 1 and 2 vs. rank 3 and 4 also shows up for Group A and B. Thus, decision weights depend on the judged knowledge of subjects. Contrary to the (insignificant) results in Table 3, the PROB(event)-values for events with rank 1 or 2 is smaller than the value of PROB(coin), not larger. We will elaborate more on this point in section 6.

Again analysing accross-subject data, i.e. taking Green questions from Group A and Blue questions from Group B, supports hypothesis 2. The sum of decision weights for the geography questions (101.1%) is on average significantly larger than the sum for stock questions (85.6%), ($t = 2.23$, $p < 5\%$). Similar to the results for certainty equivalents the decision weights for the coin event lie between the geography and stock decision weights. Our data indicate an order effect for the events coin, thumb-tack and spoon. Table 4 (betweeen subjects) shows that there is no real difference in PROB-values for those events. Contrary, the within subject analysis shows that the average sum of decision weights for thumb-tack and spoon (90.4%) is smaller than PROB(coin) (97.9%) ($t = 1.56$, insignificant).

5.2 Further Analysis of Individual Data

In our first analysis we will present the distribution of individual decision weights for events and compl.-events.

[Insert Figure 5 around here]

Fig. 5: Decision Weights for Events and Compl.-Events for Spoon and U-St 3

In Figure 5 each dot represents a subject. The x-coordinate gives the decision weight for Blue derived from a question of the Blue set, the y-coordinate a weight derived from a question out of the Green set. All subjects with an additive decision weight, i.e. giving SEU consistent subjective probabilities, should be represented by a dot on the 100% line. Dots below the 100% line indicate subadditive decision weights. Those dots could represent subjects that have changed their minds, made some error in reading the questions or were ambiguity averse. The analysis across subjects shows that explanation one can not explain the data and explanation 2 is very unlikely.

Out of 58 subjects for the coin event 53 subjects gave a 50%-50% weight and 4 were very close to this point. Subjects obviously understood the task of assigning decision weights. Figure 5 contains the diagram for the spoon and the event with the lowest PROB-value (U-St 3, betting on the stock price of General Motors).

For the spoon the decision weights are scattered around the 100% line with a small tendency to lie below. For the event U-St 3 (PROB(U-St 3) = 86.6) more than half the subjects are sub-additive, with many points having located towards the lower right corner of the diagram.

Figure 6 shows the cumulative distributions for PROB(event). As in Figure 3 and 4, for every decision weight p on the x-axis a profil will give the number of subjects that have a greater or equal sum of decision weights p.

[Insert Figure 6 around here]

Fig. 6: Cumulative Distributions for PROB(event), for Coin, and German General Knowledge Questions

The profiles for the two events based on German stocks lie to the left of the profile of German geography reflecting the higher knowledge judgments in this area. For all events there is still a considerable number of subjects whose decision weights add to one.

6 Combining Certainty Equivalent and Decision Weight Judgments

For the SUM and PROB-values a clear mathematical relation could easily be derived for each event:

$$SUM(event) = u^{-1}(p(event)) + u^{-1}(p(compl.-event)),$$

where PROB(event) = p(event) + p(compl.-event). The decision weights p(.) for complementary events do not need to add up to one, reflecting the degree of ambiguity aversion. Assuming equal decision weights for event and compl.-event and a reasonable utility function, a higher ambiguity aversion, i.e. greater subadditivity of decision weights, should result in a smaller SUM-value[8] but SUM < 1 does not imply p(event) + p(compl.-event) < 1.

Figure 7 shows some average SUM-values and some average PROB-values. The points for the four general knowledge areas and for the coin question are derived from Table 2 and 4, the points for rank 1 and 2, and rank 3 and 4 are taken from Table 3 and 5.

[Insert Figure 7 around here]

Fig. 7: Average SUM and PROB-Values for Events and Sets of Events

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[8]) To derive a more formal relation between SUM and PROB-values we would have to make strong assumptions on the utility function and on the distribution of decision weights. Taking the noiseness of empirical data into account we abandoned this route.

Figure 7 clearly reflects the formal relation between SUM and PROB-values. A higher generally PROB(event) corresponds to a higher SUM(event). We ran a linear regression through the points representing the four general knowledge areas ($r^2_{adjusted}$ = .88) showing the positive relation between PROB and SUM-values. The fact that the point for rank 1 and 2 lies north west of the point for coin could be due to learning, i.e. order effects, or to the well known probability of the coin event.

Individual data comparing SUM and PROB-values can be presented for the four events where both values were elicited. Using the same type of diagram as in Figure 7, Figure 8 gives, as an example, the individual data for thumb-tack and U-St 1.

[Insert Figure 8 around here]

Fig. 8: SUM and PROB-values for Thumb-tack and U-St 1

For the thumb-tack quite a lot of subjects had PROB-values lying on or close to the 100% line. The data exhibits some risk aversion and some risk seeking. The US stock event offers a more diverse picture. There is a stronger tendency for points to be in the south east of (100%, 10 DM). For 28 subjects PROB(U-St 1) was less than 100%. Note, that subjects who have a linear utility function should all be represented by dots on the diagonal; very few are.

7 Conclusion

For applications of ambiguity research it is essential to understand what causes ambiguity aversion. Our experiment investigated if the judged knowledge of events is related to the ambiguity aversion of lotteries based on events. We found strong suppport for the hypotheses that ambiguity version is negatively related to knowledge. For events subjects know more about decision weights and certainty equivalents of lotteries are higher. So far no similar relation was proposed for risk attitude. Our data also show that individual data for ambiguous events are diverse.

The fact that high knowledge implies ambiguity neutrality (or preference) is important for the application of ambiguity research to economic problems. Formal theories to model ambiguity have to take the knowledge effect into accout. We now can explain why managers - generally judging themselves as having high knowledge - may not exhibit ambiguity aversion.

The results of our study have important implications for capital market research. Ambiguity should be reflected in those stocks or events where the majority of agents have a low knowledege. In addition the diversity of judgments in ambiguous situations should influence market behavior. The profiles for certainty equivalents could be regarded as reverse supply curves in a market setting. As profiles for ambiguous events have a larger spread and are shifted depending on the knowledge of the events we will expect these facts to influence market volume and prices.

In further studies it will be interesting to replicate the findings above with managers or agents in markets. Investigating different real world events, like, e.g. realistic economic situations, would also be of interest. In addition the search for further causes for ambiguity should be pursued. The fact that missing information that could be know, i.e. ambiguity, is present in most real world decision situations will make the effort worthwhile.

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Appendix

List of questions used in the experiment. All gains were equal to 10 DM. All stock questions were for prices at the Frankfurt Stock Exchange, January 21, 1991.

Coin	Number up
Die	1 or 2
Thumb-tack	Pin down
Spoon	Down
Envelope	Red slip of paper drawn
G-St 1	BASF more than 200 DM
G-St 2	VW less than 340 DM
G-St 3	RWE more than 370 DM
U-St 1	IBM more than 164 DM
U-St 2	EXXON less than 70 DM
U-St 3	General Motors less than 45 DM
G-Geo 1	München - Bremen less than 600 km
G-Geo 2	Hamburg - Frankfurt less than 410 km
G-Geo 3	Stuttgart -Hannover more than 420 km
U-Geo 1	New York - Los Angeles more than 3400 km
U-Geo 2	Chicago - New Orleans less than 1200 km
U-Geo 3	Miami - Dallas more than 2000 km

Fig. 1

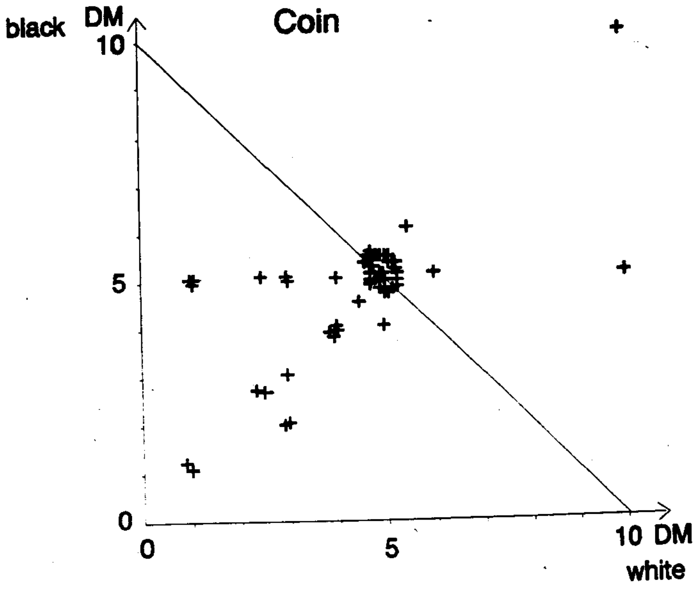


Fig. 2a

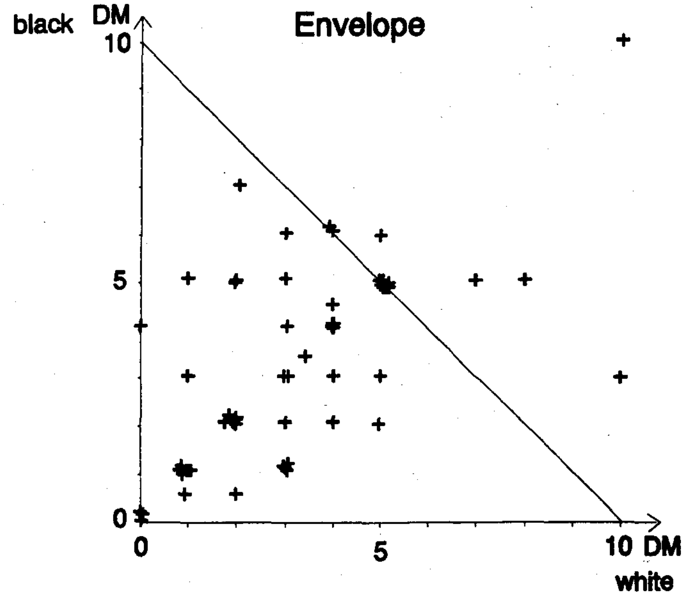


Fig. 2b

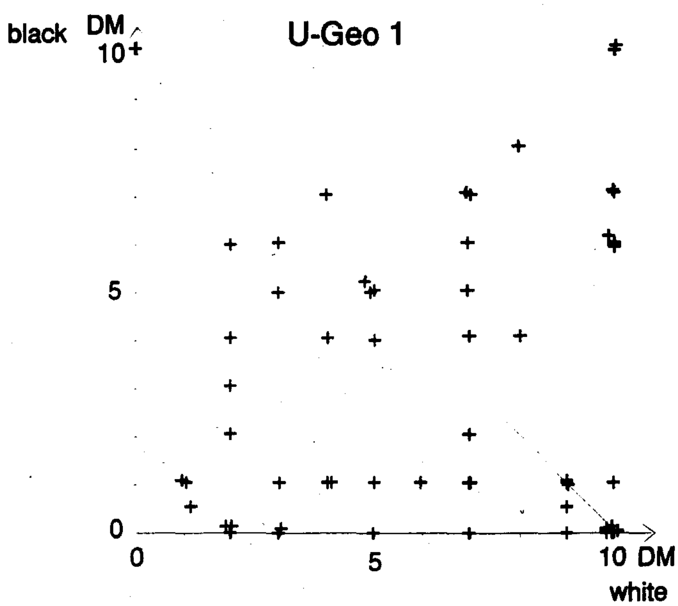


Fig. 2c

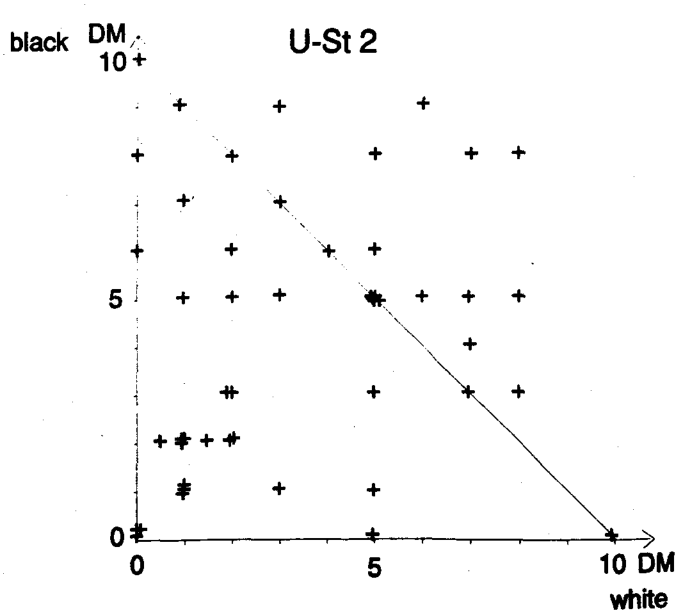


Fig. 3

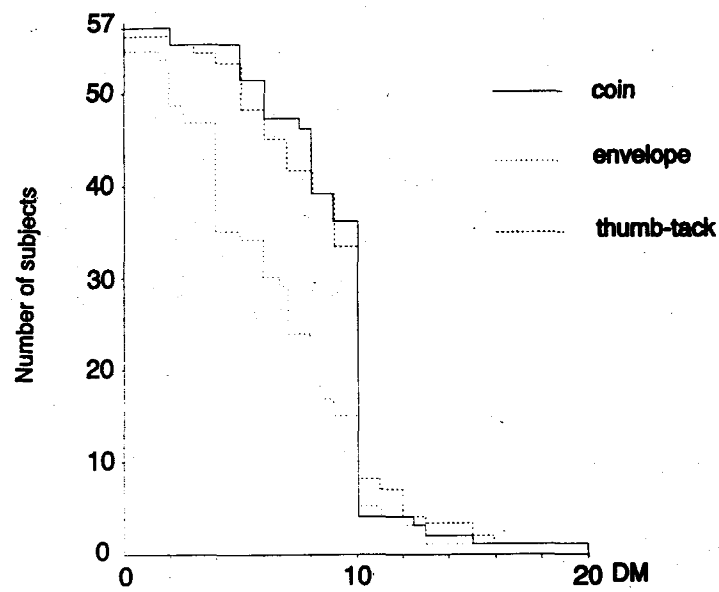

57
50
40
30
20
10
0
Number of subjects
0
10
20 DM
coin
envelope
thumb-tack

Fig. 4

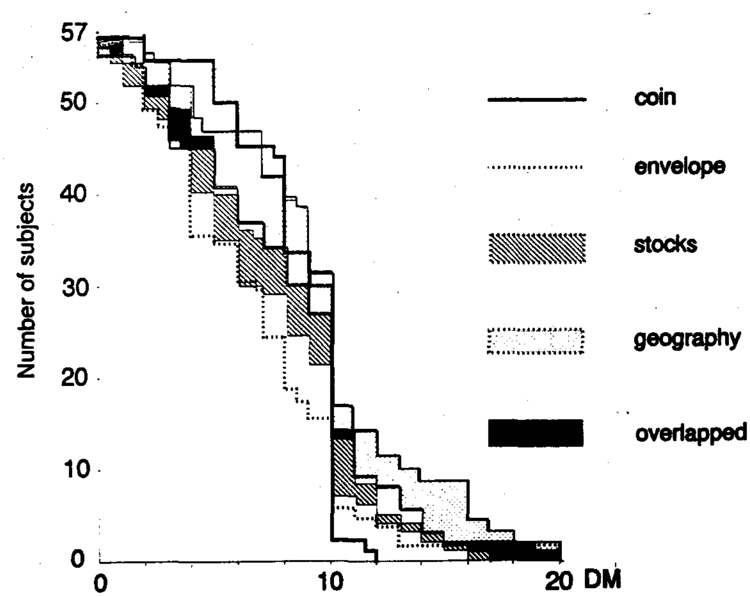

57
50
40
30
20
10
0
Number of subjects
0
10
20 DM
coin
envelope
stocks
geography
overlapped

Fig. 5a

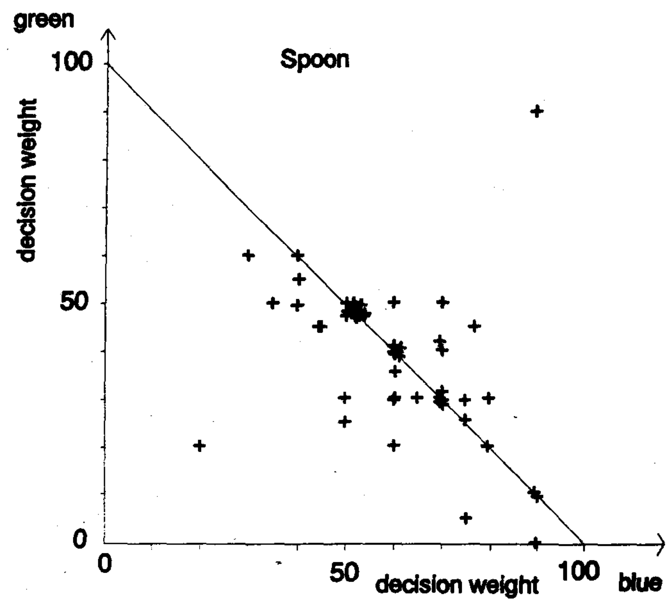

green
100
Spoon
decision weight
50
0
0
50
decision weight
100
blue

Fig. 5b

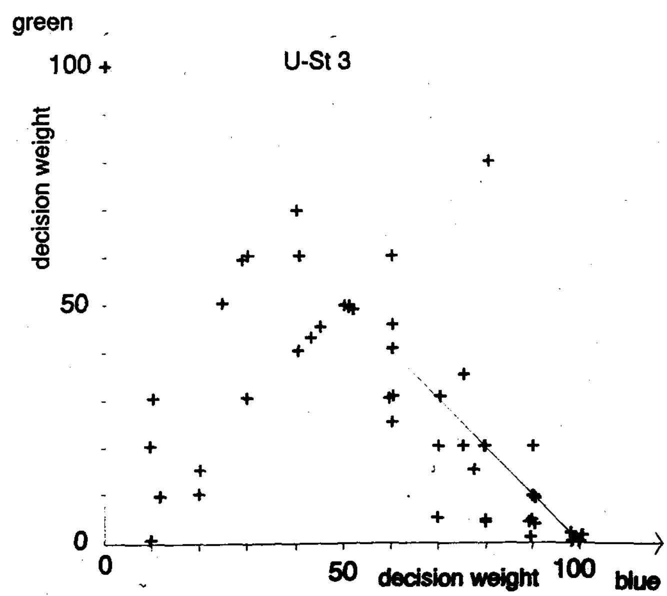
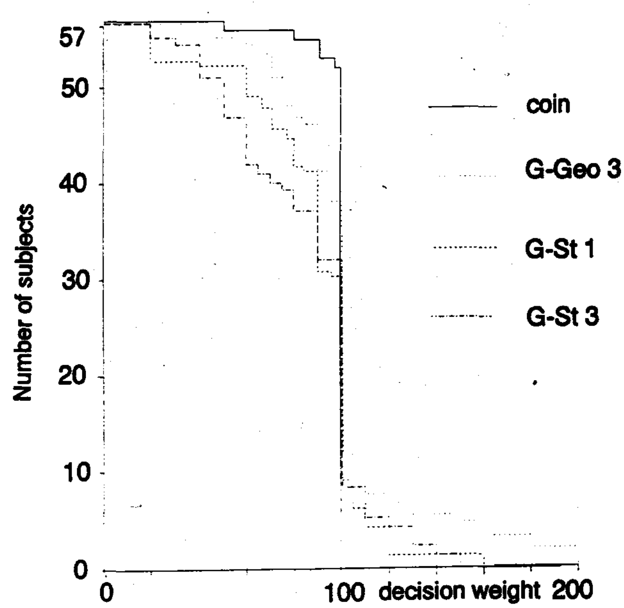
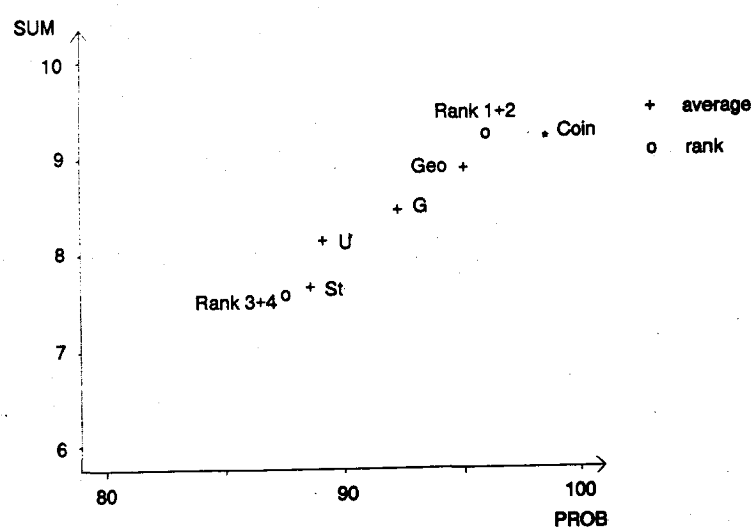

green
100
U-St 3
decision weight
50
0
0
50
decision weight
100
blue

Fig. 6



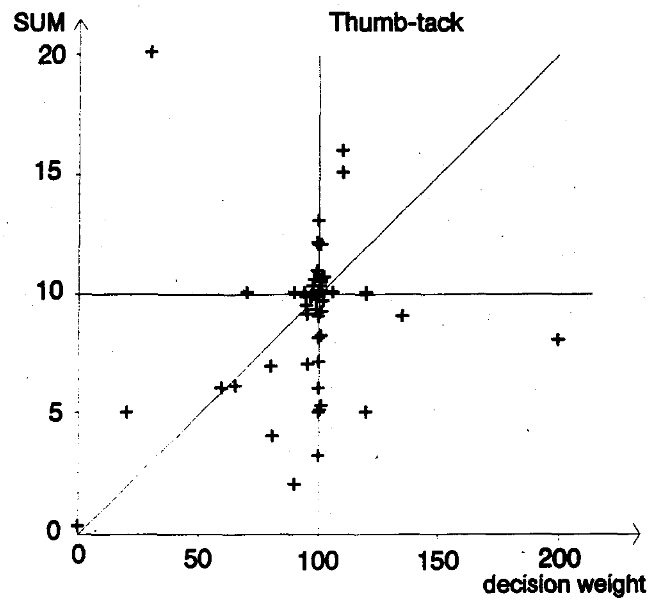
57
50
40
30
20
10
0
Number of subjects
0
100 decision weight 200
coin
G-Geo 3
G-St 1
G-St 3

Fig. 7



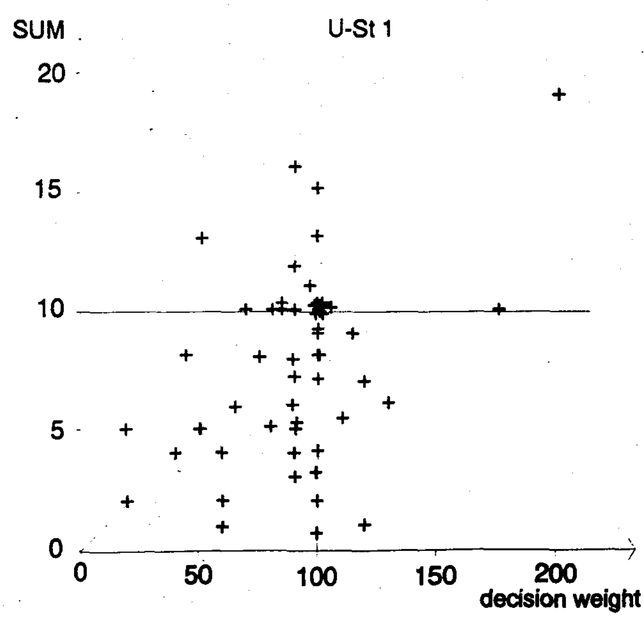
SUM
10
9
8
7
6
80
90
100
PROB
Rank 1+2
Coin
Geo
G
U
St
Rank 3+4
+ average
o rank

Fig. 8a



SUM
Thumb-tack
20
15
10
5
0
0
50
100
150
200
decision weight

Fig. 8b



SUM
U-St 1
20
15
10
5
0
0
50
100
150
200
decision weight