The repugnant conclusion can be avoided with moral intuitions intact: A lesson in order.

April 18, 2024

Abstract

The repugnant conclusion poses a conundrum in population ethics that has evaded satisfactory solution for four decades. In this article, I show that the repugnant conclusion can be avoided without sacrificing key moral intuitions. This is achieved using non-Archimedean orders, which admit the possibility of pairs of goods for which no amount of one is better than a single unit of the other. I show that with minimal assumptions, not only are such goods sensible, they are compulsory. I show that utilitarianism and expected utility theory in their canonical forms are not in general suitable in this setting, and using these tools naively can lead to ethical errors that are arbitrarily serious. This is due to the fact that non-Archimedean orders cannot in general be represented on Archimedean fields such as the reals. I use fuzzy sets to show that there needn't be a clear boundary separating goods that are not Archimedean equivalent.

1 Introduction

The repugnant conclusion is the idea that for any finite population of lives of arbitrarily high quality, there is a larger population consisting of lives that are barely worth living that is ethically preferable. It has been argued that this conclusion follows if one accepts that a small reduction in the quality of life for a given population may be 'compensated' by adding more people. It is claimed that this reasoning can be applied recursively to make ethical improvements at each step, ending with a larger population of lives that are each barely worth living.[1, 2, 3]

Many attempts have been made to avoid the repugnant conclusion, but they all appear to require accepting ethical positions that are often perceived to be as counterintuitive or undesirable as the repugnant conclusion itself. For a summary of such attempts and their issues, see [3]. This state of affairs has inspired a number of 'impossibility theorems' that purport to prove there is no theory of population ethics satisfying an intuitively desirable set of axioms that includes avoiding the repugnant conclusion.[4, 5, 6, 7, 8]

In this article, I introduce an appropriate mathematical structure for population ethics - ordered real vector spaces. I show that any totally ordered real vector space of dimension > 1 is non-Archimedean. Roughly speaking, this implies that there must be a pair of goods x, y such that y is better than any number of x. In the literature, y is sometimes referred to as 'superior', or 'lexically superior', to x. The conclusion is that not only are such goods sensible, they are compulsory.

Next, I show that non-Archimedean total orders on real vector spaces cannot be represented on the real numbers, with the immediate consequence that utilitarianism and expected-utility theory in their canonical forms are not generally suitable in population ethics. This also renders moot arguments that implicitly or explicitly rely on using real numbers to represent orders in population ethics.

I demonstrate that the repugnant conclusion can be avoided with moral intuitions intact using non-Archimedean orders. This result does not rely on any particular interpretation of concepts that may not have a precise defintion, such as life quality or a life barely worth living. The key innovation compared to previous work is an appropriate handling of orders on infinite sets. Finally, I use a fuzzy set construction to allow for the possibility that there may not be a precise apparent border between non-Archimedean equivalent goods.

2 The mathematics of population ethics

Definition 1. Partial/total order. Consider the following properties of a binary relation $\geq$ on a set S that hold $\forall x, y, z \in S$,

$$x \geq x \qquad \text{Reflexive} \tag{2.1}$$

$$y \geq x \text{ and } x \geq y \Rightarrow y = x \qquad \text{Antisymmetric} \tag{2.2}$$

$$z \geq y \text{ and } y \geq x \Rightarrow z \geq x \qquad \text{Transitive} \tag{2.3}$$

$$x \geq y \text{ or } y \geq x \qquad \text{Total.} \tag{2.4}$$

The relation $\geq$ is called a partial order if it satisfies 2.1-2.3, and a total order if it satisfies 2.1-2.4.

Let V be a real vector space, that may be finite- or infinite-dimensional. Let Q be a basis for V. We will think of each element of Q as corresponding to a life year with different levels of quality. For any $n \geq 0$ and $q \in Q$, nq will correspond to n life years of quality q.

Definition 2. Ordered real vector space. An ordered real vector space is a pair $(V, \geq)$ where V is a real vector space and $\geq$ is a partial order on V, such that $\forall x, y, z \in V, \lambda > 0$,

$$y \geq x \Rightarrow y + z \geq x + z \tag{2.5}$$

$$y \geq x \Rightarrow \lambda y \geq \lambda x. \tag{2.6}$$

$(V, \geq)$ will be called a totally ordered real vector space if $\geq$ is a total order.

This captures the idea for any two populations, adding some other population or λ-fold replication does not affect the ordering.

Note that the structure of an ordered real vector space would not be expedient in e.g. consumer choice theory. One can imagine a pair of goods x and z (tea bags and milk) that complement each other, along with another pair of goods y and z (orange juice and milk) that don't, in a way that could lead to violations of 2.5. In the population ethics setting, however, it is useful for any inter-personal or inter-population complementarities to be subsumed under quality of life. This can be achieved by e.g. interpreting a basis vector as a year of life of a given quality spent in a virtual reality machine that is indistinguishable from reality. Then, for example, a year of life enhanced by the company of one's family needn't require vectors representing the lives of family members. In this way, any complementarities between lives can be captured through an appropriate interpretation of the basis vectors. Further, this interpretation maps any possible population with any profile of life qualities onto an element of the vector space. Putting a total order on this space can then allow any population ethics question in principle to be answered.

Scale invariance 2.6 precludes the possibility of goods that have varying rates of marginal returns. So, for example, in the consumer choice setting, one can imagine two goods y and x (cake and rice) such that one unit of y is preferred to x, but there is some $\lambda > 1$ such that λ units of x is preferred to λ units of y. In the population ethics setting, however, it is reasonable to take it as axiomatic that if life quality q_2 is better than life quality q_1, then λ life years of quality q_2 is better than λ life years of quality q_1.

Note further that translation invariance 2.5 allows us to interpret a positive number of lives as a gain, and a negative number of lives as a loss, relative to some status quo. Informally speaking, for any prospective loss we wish to consider, we may imagine a population large enough that such a loss is possible, and assign it the element 0. This is possible because the ordering is preserved by translations. More formally, we may consider the affine space that V is associated to.

Finally, the antisymmetry property of partial orders 2.2 precludes indifference between two distinct elements. This requirement can be relaxed. Consider two distinct elements x, y such that $y \geq x$ and $x \geq y$. We will say that $x \simeq y$. Then 2.5 and 2.6 imply that $\simeq$ is preserved by scaling and translation. x, y can be taken to be basis vectors, and the equivalence relation $\simeq$ can be used to 'collapse' x and y into each other so that the order is once again antisymmetric. Elements of an ordered real vector space may therefore be thought of as canonical representatives of a class of vectors that are equivalent under $\simeq$.

Definition 3. Proper cone. A proper cone C in a real vector space V is a subset $C \subseteq V$ such that,

$$C + C \subseteq C \tag{2.7}$$

$$\lambda C \subseteq C \ \forall \lambda > 0 \tag{2.8}$$

$$C \cap -C = \{0\}. \tag{2.9}$$

Here, addition and multiplication for subsets of a vector space are defined by $S_1 + S_2 = \{s_1 + s_2 \mid s_1 \in S_1, s_2 \in S_2\}$ and $\lambda S = \{\lambda s \mid s \in S\}$. Proper cones must have their vertex at 0. Examples of non-proper and proper cones in $\mathbb{R}^3$ are shown in Figure 1.

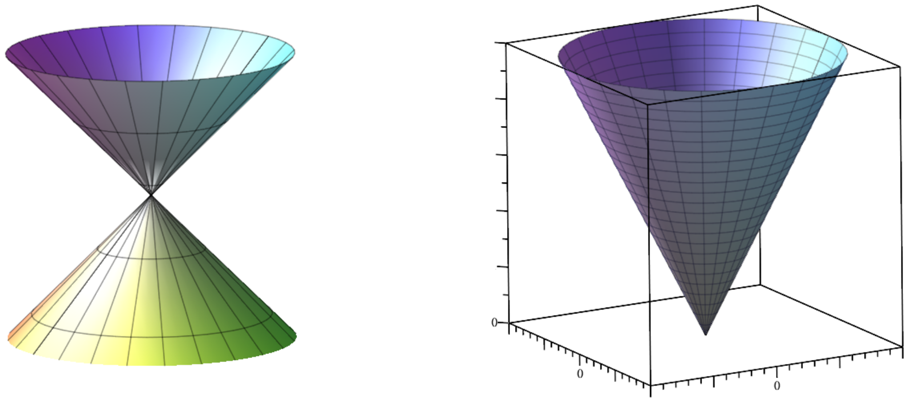

Figure 1: **Left:** A non-proper cone in $\mathbb{R}^3$. **Right:** A proper cone in $\mathbb{R}^3$.

There is a $1-1$ correspondence between proper cones C in a real vector space V, and ordered real vector spaces $(V, \geq)$, given by defining $y \geq x$ iff $(y - x) \in C$. C will be called the positive cone of $(V, \geq)$, and elements in C will be called positive.

Definition 4. Maximal proper cone. A proper cone C in a real vector space V is called maximal if there is no proper cone $C' \subseteq V$ such that $C \subset C'$.

Definition 5. Archimedean equivalence. Two elements $x, y \in V$ of an ordered real vector space $(V, \geq)$ will be said to be Archimedean equivalent if there exists $n, m \in \mathbb{N}$ such that

$$n|x| \geq |y| \tag{2.10}$$

$$m|y| \geq |x|, \tag{2.11}$$

where $|x| := \max(x, -x)$. I will use the notation $y \gg x$ if $y \geq x$ and x, y

are not Archimedean equivalent. $(V, \geq)$ will be called Archimedean if all its elements are Archimedean equivalent.

Proposition 1. Let $x, y \in V$ be elements of an ordered real vector space $(V, \geq)$. If $y \gg x$, then $ny \gg x \; \forall n > 0$.

Proof. From the definition 5 of Archimedean equivalence, we have $|y| \geq m|x| \; \forall m \in \mathbb{N}$. Assume that there exists $n > 0, m \in \mathbb{N}$ such that $m|x| \geq |ny|$. Then $\frac{m}{n}|x| \geq |y|$, which is a contradiction. $\square$

This implies that if one unit of y is better than x and y is not Archimedean equivalent to x, then any strictly positive number of units of y is better than x.

Proposition 2. There is a $1-1$ correspondence between maximal proper cones in a real vector space V, and totally ordered real vector spaces $(V, \geq)$.

Proof. Let $(V, \geq)$ be an ordered real vector space, where $\geq$ is defined by $y \geq x$ iff $(y - x) \in C$, and C is a maximal proper cone in V. Assume that x and y are two elements that are not related, i.e. for which $\geq$ is not defined. This implies that $(y - x), (x - y) \notin C$. Then the convex hull of the set $C \cup \{\lambda(y - x) \mid \lambda > 0\}$ is a proper cone with C as a proper subset, which is a contradiction. Every totally ordered vector space on V has a maximal proper cone C consisting of all elements ≥ 0. From 2, this total order is such that $y \geq x$ iff $(y - x) \in C$. $\square$

Proposition 3. Every totally ordered real vector space of dimension > 1 is non-Archimedean.

Proof. Every totally ordered real vector space $(V, \geq)$ of dimension > 1 with proper cone C has a two-dimensional subspace V_2 with an induced order given by the maximal positive cone $C_2 = C \cap V_2$. We can choose basis vectors e_1, e_2 for V_2 such that C_2 is of the form

$$\{a_1 e_1 + a_2 e_2 \mid (a_1 \geq 0 \text{ and } a_2 \geq 0) \text{ or } (a_1 \leq 0 \text{ and } a_2 > 0)\}. \tag{2.12}$$

This positive cone can be visualised in the graph on the right hand side of Figure 2. It is clear that there exists $y \in V_2$ such that $y \neq 0$ and $(y - ne_1) \geq 0 \; \forall n \in \mathbb{N}$ (for example, $y = e_2$). This implies that there is no totally ordered real vector space of dimension > 1 that is Archimedean. $\square$

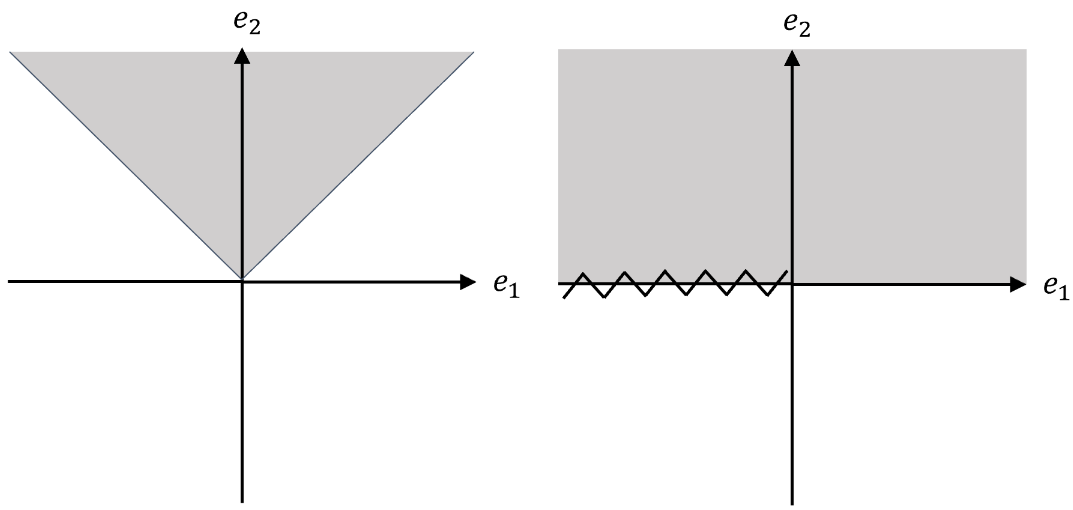

Figure 2: **Left:** A non-maximal proper cone in $\mathbb{R}^2$. **Right:** A maximal proper cone in $\mathbb{R}^2$. The zigzag line indicates that the strictly negative half-line in the e_1 direction is not included in the shaded cone.

Looking at the positive cone on the right hand side of Figure 2, it should be clear that if the vertical component of vector y is greater than or equal to the vertical component of vector x, then $y \geq x$. Therefore e_2 is the greatest unit vector in V_2, and e_1 is the least unit vector in V_2 that is positive. Moreover, while no two distinct vectors (populations) are equivalent to each other, there are populations that are arbitrarily close in the sense that their difference can be as near to zero as one desires. It is thus possible to choose a basis of a totally ordered real vector space appropriately so that any pair of basis vectors are as 'near' or 'distant' as required in terms of the quality of life years that they represent. In the (e_1, e_2) basis, we have that $(a_1, a_2) \geq (b_1, b_2)$ iff $a_2 > b_2$ or ($a_2 = b_2$ and $a_1 \geq b_1$), which is just the reverse lexicographic ordering. For any positive vector v that is not a scalar multiple of e_1, $v \gg e_1$. All other vectors excluding multiples of e_1 are Archimedean equivalent to each other.

Proposition 4. The lexicographic order on $\mathbb{R}^2$ cannot be represented on $\mathbb{R}$.

Proof. Let $\geq$ denote the lexicographic order on $\mathbb{R}^2$. Assume there is a function $u : \mathbb{R}^2 \to \mathbb{R}$ such that $u(y) \geq u(x) \Leftrightarrow y \geq x$. Consider the map

$$f : \alpha \to [u((\alpha, 0)), u((\alpha, 1))]. \tag{2.13}$$

The Archimedean property of the real numbers can be used to show that every non-empty interval in the real numbers contains a rational number. I will use ϕ to denote a function that selects a rational number from a non-empty interval given as its argument. The function $\phi \circ f : \mathbb{R} \to \mathbb{Q}$ is an injection, since for $\alpha \neq \beta$ we have that either $u((\alpha, 0)) > u((\beta, 1))$, or $u((\beta, 0)) > u((\alpha, 1))$.

This in turn implies that $f(\alpha) \cap f(\beta) = \emptyset$, and $\phi(f(\alpha)) \neq \phi(f(\beta))$ for $\alpha \neq \beta$. Then the cardinality of the rationals must be greater than or equal to the cardinality of the reals, which is a contradiction. □

Proposition 4 has the immediate consequence that there does not exist a real-valued function of the expected utility form that can represent an ordering of probability measures over an ordered real vector space $(V, \geq)$ of dimension > 1. This is because there doesn't even exist a real-valued function that can represent an ordering on the subset of such probability measures that are degenerate on a single event (i.e. sure outcomes in V). The axioms of von-Neumann-Morgenstern utility theorem are not satisfied.[9] Note that this is not due to any 'irrationality' on our part regarding our evaluation of uncertain outcomes. The failure occurs when only considering outcomes that happen with probability 1, and is due to the fact there aren't enough real numbers to represent the order.

This has profound implications for the application of utilitarianism and expected utility theory in population ethics, and any setting involving non-Archimedean ordered real vector spaces or lexicographic orders. Any line of reasoning that implicitly or explicitly attempts to represent such orders using real numbers is unsound. If we proceeded ignoring the above, it would be possible to come to ethical conclusions that are **arbitrarily wrong**. This can occur, for example, by mistakenly taking two goods to be Archimedean equivalent when they are not.

The conflict is with the continuity/Archimedean axiom of the von-Neumann-Morgenstern utility theorem. If this axiom is dropped, an ordering of probability measures may be represented with multi-dimensional lexicographic expected utilities.[10, 11]

If one insists on representing such orders unidimensionally, then proper care should be taken that the structure is sufficiently rich to accommodate the ordering. This can fail to be the case for the naturals, reals, ordinals, hyperreals etc, because they are not big enough in either the ascending or descending directions.

Our only assumptions so far are the axioms for an ordered real vector space 2. However, we are forced to conclude that there are pairs of goods that are not Archimedean equivalent. Although non-Archimedean life qualities have previously been suggested as a way of avoiding the repugnant conclusion, it has been claimed that if such a pair of life qualities exists, then other extremely counterintuitive results follow. In particular, that there necessarily must exist a pair of quality levels that are only marginally different from each other that are not Archimedean equivalent. I will next show that this conclusion is false, after introducing some preliminaries on ordinal numbers.

3 An informal introduction to ordinal numbers

Alice likes apples and oranges. She always prefers a larger number of apples/oranges to a smaller number. However, she likes apples more than oranges, to the extent that there is no number of oranges that she would prefer to even a single apple.

Let us try to represent her preferences using numbers. We will assign the number 1 to the bundle consisting of 1 orange, the number 2 to 2 oranges, etc. Then the $\geq$ operator on the natural numbers represents Alice's preferences over oranges.

However, if we limit ourselves to the natural numbers, we immediately run into a problem. The bundle consisting of a single apple cannot be assigned a natural number in a way that respects Alice's preferences. Whichever number it is assigned, there is always a larger natural number available, which would incorrectly imply that there is some number of oranges Alice would prefer to one apple.

In a sense, we have 'run out' of numbers that we can use to order the available bundles. This is a general issue encountered when naively attempting to define total orders on infinite sets. The theory of ordinal numbers was constructed to handle this more than a century ago by Cantor, and developed further by von Neumann and others.[12, 13] We give a brief whistlestop tour of this theory here.

There is nothing stopping us from simply defining an abstract number, which we will call ω, and extending the relation $\geq$ by assigning ω to be greater than any natural number. ω can then be used to represent the bundle consisting of one apple for Alice.

We can take this further and consider bundles consisting of some number of both apples and oranges. Remembering that Alice always prefers more fruit to less, we may define a new element denoted by $\omega + 1$ that is greater than ω, corresponding to 1 apple and 1 orange. This can be repeated for

$\omega + 2$, $\omega + 3$ etc. For 2 apples, we symbolically assign the element $\omega 2$.

ω is called the first infinite ordinal number. An arithmetic of ordinal numbers can be constructed recursively using disjoint unions of sets. A well-ordered set is a totally-ordered set in in which every subset has a least element. Given two well-ordered sets X, Y, we may define $X + Y$ as the set obtained by taking their disjoint union, and assigning every element of Y to be greater than every element of X, but otherwise preserving the ordering within X and Y. For example,

$$\{0, 1, 2\} + \{0', 1'\} := \{0, 1, 2, 0', 1'\}, \tag{3.1}$$

where sets are written so that the elements increase from left to right. We can map the set $\{0, 1, 2, 0', 1'\}$ onto $\{0, 1, 2, 3, 4\}$ while preserving the ordering by

$$0 \to 0, \tag{3.2}$$

$$1 \to 1, \tag{3.3}$$

$$2 \to 2, \tag{3.4}$$

$$0' \to 3, \tag{3.5}$$

$$1' \to 4. \tag{3.6}$$

We can go further and identify $\{0, 1, 2, 3, 4\}$ as the ordinal number 5. This is because every element of a well-ordering is uniquely determined by the set of elements that it is larger than. Similarly, ω can be identified with the set of natural numbers. In this scheme, ordinal numbers are simply canonical representatives of a set of well-orderings that are all equivalent to each other, with the different ordinal numbers providing labels for every possible distinct well-ordered set. The ordinal number associated with a well-ordered set is called its order type.

Note that the above addition operation is commutative for finite ordinal numbers, i.e. $X + Y = Y + X \; \forall X, Y < \omega$. This however, ceases to be the case when considering infinite ordinals. For example,

$$2 + \omega = \{0', 1', 0, 1, 2...\}. \tag{3.7}$$

This ordering is equivalent to ω, since we can simply relabel as follows

$$0' \to 0, \tag{3.8}$$

$$1' \to 1, \tag{3.9}$$

$$0 \to 2, \tag{3.10}$$

$$1 \to 3, \tag{3.11}$$

$$\vdots \tag{3.12}$$

On the other hand,

$$\omega + 2 = \{0, 1, 2... \omega, \omega + 1\}. \tag{3.13}$$

This is not equivalent to ω, since there are two elements in $\omega + 2$ that are greater than all the natural numbers, as opposed to none in ω. Thus $\omega + 2 \neq 2 + \omega$.

Multiplication and exponentiation operations can also be defined for ordinal numbers, but I will not develop this here. I refer the reader to some of the many excellent texts on set theory.[14, 15, 16]

4 Ordinal numbers and the repugnant conclusion

Results in [17, 18] imply that in any finite increasing sequence for which the first and last elements are not Archimedean equivalent to each other, there must be a pair of successive elements that are not Archimedean equivalent. This can used as an argument against the existence of goods that are not Archimedean equivalent, based on the idea that one should be able to construct such a sequence where each element is only marginally better than its predecessor. This would then imply that there are a pair of life qualities that are near-identical, but there is no number of the marginally worse life years that is better than just one of the marginally better life years. The argument can be extended to infinite sequences, using transitivity of Archimedean equivalence to obtain the result that in any sequence where every two successive elements are Archimedean equivalent, all elements are Archimedean equivalent.

The problem can be remedied by generalising to transfinite sequences. A transfinite sequence is a collection of set elements indexed by ordinal numbers (rather than the natural numbers). We may, for example, take an ordered real vector space that is infinite dimensional, and construct an increasing transfinite sequence of basis vectors $e_1, e_2, \ldots e_\omega$ where every pair of successive elements is Archimedean equivalent, but e_ω is not Archimedean equivalent to any e_n for $n \in \mathbb{N}$. This is possible because e_ω **does not have a predecessor**. That is, there is no natural number that has ω as its successor, because one can always find a larger natural number. Thus it is perfectly possible to have basis vectors e_1, e_ω such that e_ω can only be 'reached' from e_1 by a countably infinite number of marginal increments. If that is not enough, there are ordinal numbers that are uncountable when considered as a set, so one can construct an increasing transfinite sequence of basis vectors where at least an uncountable number of increments is required to move from some basis element to another. If one wishes to have a set of basis vectors that contains arbitrarily large ascending and descending chains, one can index them using e.g. surreal numbers.

5 Uncertainty and vagueness

The above formalism may be viewed as a theoretical framework for population ethics in circumstances of complete information, sharply demarcated boundaries and ethical preferences that are sensitive to arbitrarily small changes. However, these conditions may not exist in reality. For example, it may not be clear what constitutes a life that is barely worth living, or to give the concept a precise definition. One way of generalising to accommodate uncertainty/vagueness is to allow the positive cone to be a fuzzy set.

Definition 6. Fuzzy set. A fuzzy set is map $m : V \to [0, 1]$, where V is a set. m is called the membership function.

In the current setting, the value of the membership function may be interpreted as a frequentist or Bayesian probability that a given element of the ordered real vector space $(V, \geq)$ is in the positive cone, and therefore greater than or equal to 0. That is, there really is a sharp boundary between elements that are ≥ 0 and elements that are not, but we do not know exactly where it lies. The uncertainty may originate at least partly from our inability to detect differences below a certain threshold.

Alternatively, the membership function may be interpreted as the 'degree' to which a given vector is greater than or equal to zero. In this view, the 'positive cone' need not have a sharp boundary, similar to the way there is no sharp distinction between e.g. a sunny day and a cloudy day. This interpretation is fundamentally distinct from a probabilistic interpretation, in which there is uncertainty about the answer to a well defined question. By contrast, fuzzy sets allow for the possibility of a question that is not well defined, and answers with truth values ranging between 0 (false) and 1 (true).

With either interpretation, there is no longer a precise apparent border between elements that are not Archimedean equivalent to each other. This is consonant with the intuition that there is no clear dividing line between life qualities that are qualitatively different. For example, we might hold the position that one completely blissful life is better than any number of lives that are barely worth living, without having a clear idea of where exactly the boundary lies between lives that are Archimedean equivalent to the single blissful life, and those that are not.

This induces a new preference relation $\succeq$ characterised by a decision rule with threshold $\alpha \in [0,1]$, defined by

$$y \succeq x \quad \text{if} \quad m(y-x) \geq \alpha \tag{5.1}$$

$$y \sim x \quad \text{if} \quad m(y-x) < \alpha, \tag{5.2}$$

where $y \sim x \iff (y \succeq x \text{ and } x \succeq y)$. $\succeq$ may be non-transitive. For example, m and α may be such that $\succeq$ restricted to a finite-dimensional subspace is the lexicographic semiorder.[19] This can be understood as the lexicographic ordering with imperfect discriminatory power, such that only differences above a certain threshold in a given dimension are detectable. In this case, intransitivity of the strict preference $\succ$ can occur as in examples given by Ng.[20] Intransitivity of indifference $\sim$ can also occur along the lines of the example given in seminal work by Luce [21] of an individual who strictly prefers a coffee with one sugar over a coffee with five sugars, but is pairwise indifferent between a series of intermediate coffees that differ by tiny increments in the quantity of sugar.

6 Conclusion

Applying informal logic to mathematical problems can lead to errors, particularly when dealing with infinities. It is also important to use appropriate mathematical structures for the problem at hand. In this paper, I have introduced ordered real vector spaces, transfinite sequences and fuzzy sets as suitable mathematical tools in population ethics. I have shown that in this setting, not only is the existence of non-Archimedean equivalent goods (i.e. goods for which no amount of one is better than a single unit of the other.) sensible, it is compulsory. I note that utilitarianism and expected utility theory in their canonical form fail in this setting, as does any attempt to use real numbers to represent orders in population ethics. This is because non-Archimedean orders cannot generally be represented on Archimedean fields. Generalisations of the von-Neumann-Morgenstern utility theorem may be required that drop the continuity/Archimedean axiom, leading to multi-dimensional lexicographic expected utility representations. I show that counterintuitive conclusions that have previously been thought to follow as a consequence of the existence of non-Archimedean equivalent goods, do not obtain. In particular, there need not be pairs of goods that are near identical that are non-Archimedean equivalent. This is demonstrated using an appropriate handling of orders on infinite sets. Finally, I use a fuzzy set construction to show that there needn't be a sharp boundary between pairs of goods that are non-Archimedean equivalent.

These results are general and do not rely on any particular interpretation of concepts that may not have a precise definition, such as life quality or a life barely worth living. That is, they remain true regardless of any such interpretation that is made. The main implication of this work is that the repugnant conclusion can be avoided without contradicting key moral intuitions.

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