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Research Article

New Iterative Method for the Solution of Fractional Damped Burger and Fractional Sharma-Tasso-Olver Equations

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The new iterative method has been used to obtain the approximate solutions of time fractional damped Burger and time fractional Sharma-Tasso-Olver equations. Results obtained by the proposed method for different fractional-order derivatives are compared with those obtained by the fractional reduced differential transform method (FRDTM). The 2nd-order approximate solutions by the new iterative method are in good agreement with the exact solution as compared to the 5th-order solution by the FRDTM.

1. Introduction

Most of the problems arising in the physical and biological area of science are nonlinear in nature, and it is not always possible to find the exact solution of such problems. These problems become more complicated when they involve fractional derivatives and are modelled through mathematical tools from fractional calculus. Fractional partial differential equations (FPDEs) are tremendous instrument and are widely used to describe many significant phenomena and dynamic processes such as engineering, rheology, acoustic, electrical networks, and viscoelasticity [1–6]. Generally, partial differential equations (PDEs) are hard to tackle, and their fractional-order types are more complicated [7, 8]. Therefore, several analytical and approximate methods can be used for finding their approximate solutions such as Adomian decomposition [9], homotopy analysis [10], tau method [11], residual power series method [12], and optimal homotopy asymptotic method [13]. Though the study of FPDEs has been obstructed due to the absence of proficient and accurate techniques, the derivation of approximate solution of FPDEs remains a hotspot and demands to attempt some dexterous and solid plans which are of interest. Daftardar-Gejji and Jafari proposed an iterative method called the new iterative method (NIM) for finding the approximate solution of differential equations [14]. NIM does not require the need for calculation of tedious Adomian polynomials in nonlinear terms like ADM, the need for determination of a Lagrange multiplier in its algorithm like VIM, and the need for discretization like numerical methods. The proposed method handles linear and nonlinear equations in an easy and straightforward way. Recently, the method has been extended for differential equations of the fractional order [15–17].

In the present study, we have implemented NIM for finding the approximate solution of the following fractional-order damped Burger equation.

$$D_t^{\alpha} u(x,t) + u(x,t) D_x u(x,t) - D_x^2 u(x,t) + \lambda u(x,t) = 0, \quad t > 0, 0 < \alpha \le 1. \tag{1}$$

Second, consider the fractional-order Sharma-Tasso-Olver equation of the following form.

$$D_t^{\alpha} u(x,t) + a D_x u^3(x,t) + \frac{3}{2} a D_x^2 u^2(x,t) + a D_x^3 u(x,t) = 0, \quad t > 0, 0 < \alpha \le 1, \tag{2}$$

where α is the parameter describing the order of fractional derivatives, $u(x,t)$ is the function of x and t, and a, λ are constants. The fractional derivatives are described in the Caputo sense. NIM converges rapidly to the exact solution compared to FRDTM, and only at the 2nd iteration does the proposed method yield very encouraging results. The accuracy of the proposed method can further be increased by taking higher-order approximations.

2. Definitions

In this section, we have stated some definitions which are relevant to our work.

Definition 1. A function $g(y), y > 0$, is said to be in space $C_\eta, \eta \in \mathbb{R}$, if there exists a real number $p > \eta$, such that $g(y) = y^p g_1(y)$, where $g_1(y) \in C(0,\infty)$. The function $g(y)$, $y > 0$, is said to be in space $C_\eta{}^\lambda$ if only if $g^\lambda \in C_\eta$, $\lambda \in N$.

Definition 2. The R-L fractional integral operator of order $\alpha \geq 0$ of a function $g \in C_\eta, \eta \geq -1$, is as follows:

$$\begin{aligned} J_a^\alpha g(y) &= \frac{1}{\Gamma(\alpha)} \int_a^y (y-\eta)^{\alpha-1} g(\eta) d\eta, \quad \alpha > 0, y > a, \\ J_a^0 g(y) &= g(y). \end{aligned} \tag{3}$$

Because of certain disadvantages of R-L fractional derivative operator, Caputo proposed modified fractional differential operator ${}_cD^\alpha$ as follow.

Definition 3. Caputo fractional derivative of $g(y)$ takes the following form.

$${}_cD_a^\alpha g(y) = \frac{1}{\Gamma(\lambda-\alpha)} \int_a^y (y-\eta)^{\lambda-\alpha-1} g^\lambda(\eta) d\eta, \tag{4}$$

where

$$\begin{aligned} \lambda - 1 < \alpha \leq \lambda, \\ \lambda \in N, \\ y > a, \\ g \in C_{-1}^\lambda. \end{aligned} \tag{5}$$

Definition 4. If $\lambda - 1 < \alpha \leq \lambda$, $\lambda \in N$, and $g \in C_\eta^\lambda, \eta \geq -1$, then ${}_{\mathrm{RL}}D_a^\alpha J_a^\alpha g(y) = g(y)$ and $J_a^\alpha {}_cD_a^\alpha g(y) = g(y) - \sum_{k=0}^{\lambda-1} g^{(k)}(a)(y-a)^k/k!, y > a$.

The properties of the operator $J_a{}^\alpha$ are shown as follows:

(i) $J_a^\alpha g(y)$ exists for almost every $y \in [a,b]$.

(ii) $J_a^\alpha J_a^\beta g(y) = J_a^{\alpha+\beta} g(y)$.

(iii) $J_a^\alpha J_a^\beta g(y) = J_a^\beta J_a^\alpha g(y)$.

(iv) $J_a^\alpha (y-a)^\gamma = (\Gamma(\gamma+1)/\Gamma(\alpha+\gamma+1))(y-a)^{\alpha+\gamma}$.

In the equations above, $g \in C_\eta^\lambda$, $\alpha, \beta > 0$, $\eta \geq -1$, and $\gamma \geq -1$.

3. New Iterative Method

The basic mathematical theory of NIM is described as follows.

Let us consider the following nonlinear equation:

$$v(y) = f(y) + \xi(v(y)) + \aleph(v(y)), \tag{6}$$

where $f(y)$, $y = (y_1, y, y_3, \ldots, y_n)$, is the known function and ξ and N are the linear and nonlinear functions of $v(y)$, respectively. According to the basic idea of NIM, the solution of the above equation has the series form.

$$v(y) = \sum_{k=0}^{\infty} v_k(y). \tag{7}$$

The linear operator ξ can be decomposed as

$$\sum_{k=0}^{\infty} \xi(v_k) = \xi\left(\sum_{k=0}^{\infty} v_k\right). \tag{8}$$

The decomposition of the nonlinear operator $\aleph$ is as follows:

$$\aleph\left(\sum_{k=0}^{\infty} v_k\right) = \aleph(v_0) + \sum_{k=1}^{\infty}\left\{\aleph\left(\sum_{i=0}^{k} v_i\right) - \aleph\left(\sum_{i=0}^{k-1} v_i\right)\right\}. \tag{9}$$

Hence, the general equation of (6) takes the following form:

$$\begin{aligned} v(y) = \sum_{k=0}^{\infty} v_k(y) &= f + \xi\left(\sum_{k=0}^{\propto} v_k\right) + \aleph(v_0) \\ &+ \sum_{k=1}^{\infty}\left\{\aleph\left(\sum_{i=0}^{k} v_i\right) - \aleph\left(\sum_{i=0}^{k-1} v_i\right)\right\}, \end{aligned} \tag{10}$$

From this, we have

$$\begin{aligned} v_0 &= f, \\ v_1 &= \xi(v_0) + \aleph(v_0), \\ v_2 &= \xi(v_1) + \aleph(v_0 + v_1) - \aleph(v_0), \\ v_{m+1} &= \xi(v_m) + \aleph(v_0 + v_1 + \cdots + v_m) \\ &\quad - \aleph(v_0 + v_1 + \cdots + v_{m-1}), \\ &\qquad m = 1, 2, 3 \ldots \end{aligned} \tag{11}$$

The k-term series solution of the general equation (6) takes the following form:

$$v = v_0 + v_1 + \cdots + v_{k-1}. \tag{12}$$

4. Applications

Example 1 (damped Burger equation).
Consider the damped Burger equation

$$D_t^{\alpha}u(x,t)+u(x,t)D_xu(x,t)-D_x^2u(x,t)+\lambda u(x,t)=0,\quad t>0,\ 0<\alpha\le 1, \tag{13}$$

together with IC

$$u(x,0)=\lambda x, \tag{14}$$

where λ is a constant. The exact solution of (13) is of the following form:

$$u(x,t)=\frac{\lambda x}{2e^{\lambda t}-1}. \tag{15}$$

Using the operator J^{α} on both sides of (13) using the initial condition and Definition 4 yields

$$u(x,t)=\lambda x+J^{\alpha}\left(-u(x,t)D_xu(x,t)+D_x^2u(x,t)-\lambda u(x,t)\right), \tag{16}$$

where $\xi(u)=J^{\alpha}(D_x^2u(x,t)-\lambda u(x,t))$ and $\aleph(u)=J^{\alpha}(-u(x,t)D_xu(x,t))$.
According to (11), we have

$$\begin{aligned}
u_0(x,t)&=\lambda x,\\
u_1(x,t)&=-\frac{2\lambda^2t^{\alpha}x}{\Gamma(\alpha+1)},\\
u_2(x,t)&=\frac{2\lambda^3t^{2\alpha}x\left(3-\left(2\lambda t^{\alpha}(\Gamma(2\alpha+1))^2/(\Gamma(\alpha+1))^2\Gamma(3\alpha+1)\right)\right)}{\Gamma(2\alpha+1)}.
\end{aligned} \tag{17}$$

The three-term approximate solution of the above equation is

$$u(x,t)=\lambda x-\frac{2\lambda^2t^{\alpha}x}{\Gamma(\alpha+1)}+\frac{2\lambda^3t^{2\alpha}x\left(3-\left(2\lambda t^{\alpha}(\Gamma(2\alpha+1))^2/(\Gamma(\alpha+1))^2\Gamma(3\alpha+1)\right)\right)}{\Gamma(2\alpha+1)}. \tag{18}$$

Example 2 (Sharma-Tasso-Olver equation).
One can consider

$$D_t^{\alpha}u(x,t)+aD_xu^3(x,t)+\frac{3}{2}aD_x^2u^2(x,t)+aD_x^3u(x,t)=0,\quad t>0,\ 0<\alpha\le 1, \tag{19}$$

together with IC

$$u(x,0)=\sqrt{\frac{1}{a}}\tanh\left(\sqrt{\frac{1}{a}}x\right), \tag{20}$$

where a is a constant. The exact solution of (19) for $\alpha=1$ is of the following form:

$$u(x,t)=\sqrt{\frac{1}{a}}\tanh\left(\sqrt{\frac{1}{a}}(x-t)\right). \tag{21}$$

Using the operator J^{α} on both sides of (19) using the initial condition and Definition 4 yields

$$u(x,t)=\sqrt{\frac{1}{a}}\tanh\left(\sqrt{\frac{1}{a}}x\right)+J^{\alpha}\left(-aD_xu^3(x,t)-\frac{3}{2}aD_x{}^2u^2(x,t)-aD_x^3u(x,t)\right), \tag{22}$$

where $\xi(u)=J^{\alpha}(-aD_x^3u(x,t))$ and $\aleph(u)=J^{\alpha}(-aD_xu^3(x,t)-3/2aD_x^2u^2(x,t))$.
According to (11), we have

$$\begin{aligned}
u_0(x,t)&=\sqrt{\frac{1}{a}}\tanh\left(\sqrt{\frac{1}{a}}x\right),\\
u_1(x,t)&=-\frac{t^{\alpha}\sec h^2\left(x/\sqrt{a}\right)}{a\Gamma(1+\alpha)},\\
u_2(x,t)&=\frac{t^{2\alpha}\sec h^2\left(x/\sqrt{a}\right)}{a^{5/2}}\left\{\begin{array}{l}-\dfrac{3\sqrt{a}t^{\alpha}\left(-3+2\cosh\left(2x/\sqrt{a}\right)\right)\Gamma(1+2\alpha)\sec h^4\left(x/\sqrt{a}\right)}{(\Gamma(1+\alpha))^2\Gamma(1+3\alpha)}\\ -\dfrac{2a\tanh\left(x/\sqrt{a}\right)}{\Gamma(1+2\alpha)}-\dfrac{6t^{2\alpha}\Gamma(1+3\alpha)\sec h^4\left(x/\sqrt{a}\right)\tanh\left(x/\sqrt{a}\right)}{(\Gamma(1+\alpha))^3\Gamma(1+4\alpha)}\end{array}\right\}.
\end{aligned} \tag{23}$$

The three-term approximate solution of the above equation is as follows:

$$u(x,t)=\sqrt{\frac{1}{a}}\tanh\left(\sqrt{\frac{1}{a}}x\right)-\frac{t^{\alpha}\sec h^{2}\left(x/\sqrt{a}\right)}{a\Gamma(1+\alpha)}+\frac{t^{2\alpha}\sec h^{2}\left(x/\sqrt{a}\right)}{a^{5/2}}\cdot\left\{\begin{array}{l}-\dfrac{3\sqrt{a}t^{\alpha}\left(-3+2\cosh\left(2x/\sqrt{a}\right)\right)\Gamma(1+2\alpha)\sec h^{4}\left(x/\sqrt{a}\right)}{(\Gamma(1+\alpha))^{2}\Gamma(1+3\alpha)}\\ -\dfrac{2a\tanh\left(x/\sqrt{a}\right)}{\Gamma(1+2\alpha)}-\dfrac{6t^{2\alpha}\Gamma(1+3\alpha)\sec h^{4}\left(x/\sqrt{a}\right)\tanh\left(x/\sqrt{a}\right)}{(\Gamma(1+\alpha))^{3}\Gamma(1+4\alpha)}\end{array}\right\}. \tag{24}$$

Table 1: Comparison of numerical results of NIM and FRDTM at $\alpha=1$ and $\lambda=1$.

x	t	5th-order FRDTM [18]	2nd-order NIM	Exact solution	Absolute error
−5	0.002	−4.980060	−4.98006	−4.98006	1.19501×10^{-7}
	0.004	−4.960239	−4.96024	−4.96024	9.52046×10^{-7}
	0.006	−4.940535	−4.94054	−4.94054	3.19985×10^{-7}
	0.008	−4.920949	−4.92096	−4.92095	7.55346×10^{-6}
−3	0.002	−2.988036	−2.98804	−2.98804	7.17009×10^{-8}
	0.004	−2.976143	−2.97614	−2.97614	5.71228×10^{-7}
	0.006	−2.964321	−2.96432	−2.96432	1.91991×10^{-6}
	0.008	−2.952569	−2.95257	−2.95257	4.53208×10^{-6}
3	0.002	2.988036	2.98804	2.98804	7.17009×10^{-8}
	0.004	2.976143	2.97614	2.97614	5.71228×10^{-7}
	0.006	2.964321	2.96432	2.96432	1.91991×10^{-6}
	0.008	2.952569	2.95257	2.95257	4.53208×10^{-6}
5	0.002	4.980060	4.98006	4.98006	1.19501×10^{-7}
	0.004	4.960239	4.96024	4.96024	9.52046×10^{-7}
	0.006	4.940535	4.94054	4.94054	3.19985×10^{-6}
	0.008	4.920949	4.92096	4.92095	7.55346×10^{-6}

Table 2: Comparison of numerical results of the 2nd-order NIM and the 5th-order FRDTM for different values of α.

x	t	FRDTM [18] ($\alpha=0.5$)	FRDTM [18] ($\alpha=0.75$)	NIM ($\alpha=0.5$)	NIM ($\alpha=0.75$)
−5	0.002	−4.211841	−4.876244	−4.55366	−4.89911
	0.004	−3.938004	−4.794473	−4.4015	−4.8326
	0.006	−3.744961	−4.724456	−4.29706	−4.7758
	0.008	−3.592556	−4.661485	−4.21704	−4.72486
−3	0.002	−2.527105	−2.925746	−2.7322	−2.93946
	0.004	−2.362803	−2.876684	−2.6409	−2.89956
	0.006	−2.246977	−2.834674	−2.57824	−2.86548
	0.008	−2.155534	−2.796891	−2.53022	−2.83492
3	0.002	2.527105	2.925746	2.7322	2.93946
	0.004	2.362803	2.876684	2.6409	2.89956
	0.006	2.246977	2.834674	2.57824	2.86548
	0.008	2.155534	2.796891	2.53022	2.83492
5	0.002	4.211841	4.876244	4.55366	4.89911
	0.004	3.938004	4.794473	4.4015	4.8326
	0.006	3.744961	4.724456	4.29706	4.7758
	0.008	3.592556	4.661485	4.21704	4.72486

5. Results and Discussion

We have implemented NIM for finding the approximate solutions of the fractional damped Burger equation and fractional Sharma-Tasso-Olver equation. Tables 1 and 2 show the numerical results of the 2nd-order NIM which are compared with those of the 5th-order fractional reduced differential transform method (FRDTM) solution [18] for the fractional-order damped Burger equation. Tables 3 and 4 show the comparison of the proposed scheme with the FRDTM for the fractional-order Sharma-Tasso-Olver equation. Figure 1 shows the comparison of 2D plot of the approximate and exact solution by NIM for the classical damped Burger equation. Figure 2 shows the comparison of the approximate solution for different values of α with the exact solution at $t=0.01$. In Figures 3 and 4, 3D plots of approximate and exact solutions by NIM for the damped Burger equation are given. In Figure 5, the 2D plots of the approximate and exact solution for the classical Sharma-Tasso-Olver equation are given. Figure 6 shows the comparison of the approximate solution for different values of α with the exact solution at $t=0.1$. The 3D plots of approximate and exact solutions for the Sharma-Tasso-Olver equation are given in Figures 7 and 8. Throughout computations, we take $\lambda=1$ and $a=4$.

By forming the numerical values and graphs, it is clear that NIM is a very powerful tool for the solution of fractional partial differential equations. The accuracy of the NIM can further be increased by taking higher-order approximations.

Table 3: Comparison of numerical results of NIM and FRDTM at $\alpha = 1$ and $a = 4$.

x	t	5th-order FRDTM [18]	2nd-order NIM	Exact solution	Absolute error
−5	0.002	−0.493320	−0.49332	−0.49332	7.13918×10^{-12}
	0.004	−0.493334	−0.493334	−0.493334	5.70817×10^{-11}
	0.006	−0.493347	−0.493347	−0.493347	1.92542×10^{-10}
	0.008	−0.493360	−0.49336	−0.49336	4.56141×10^{-10}
−3	0.002	−0.452664	−0.452664	−0.452664	6.65773×10^{-12}
	0.004	−0.452755	−0.452755	−0.452755	5.333×10^{-11}
	0.006	−0.452844	−0.452844	−0.452844	1.80218×10^{-10}
	0.008	−0.452934	−0.452934	−0.452934	4.27726×10^{-10}
3	0.002	0.452484	0.452484	0.452484	6.64085×10^{-12}
	0.004	0.452393	0.452393	0.452393	5.30587×10^{-11}
	0.006	0.452302	0.452302	0.452302	1.78845×10^{-10}
	0.008	0.452211	0.452211	0.452211	4.23386×10^{-10}
5	0.002	0.493294	0.493294	0.493294	7.14723×10^{-12}
	0.004	0.493281	0.493281	0.493281	5.721×10^{-11}
	0.006	0.493267	0.493267	0.493267	1.93192×10^{-10}
	0.008	0.493254	0.493254	0.493254	4.58194×10^{-10}

Table 4: Comparison of numerical results of the 2nd-order NIM and the 5th-order FRDTM for different values of α.

x	t	FRDTM [18] ($\alpha = 0.5$)	FRDTM [18] ($\alpha = 0.75$)	NIM ($\alpha = 0.5$)	NIM ($\alpha = 0.75$)
−5	0.002	−0.493876	−0.49339	−0.49363	−0.493375
	0.004	−0.494098	−0.493447	−0.493755	−0.493421
	0.006	−0.494262	−0.493496	−0.493849	−0.493461
	0.008	−0.494397	−0.49354	−0.493926	−0.493497
−3	0.002	−0.456455	−0.453141	−0.454774	−0.453036
	0.004	−0.457972	−0.453523	−0.455639	−0.453348
	0.006	−0.459102	−0.453856	−0.456286	−0.45362
	0.008	−0.460032	−0.45416	−0.456819	−0.453867
3	0.002	0.448366	0.452001	0.450211	0.452106
	0.004	0.446521	0.451607	0.449182	0.451784
	0.006	0.445064	0.451259	0.448372	0.4515
	0.008	0.443806	0.450937	0.447674	0.451237
5	0.002	0.492686	0.493223	0.492959	0.493238
	0.004	0.492412	0.493165	0.492806	0.493191
	0.006	0.492194	0.493113	0.492687	0.493149
	0.008	0.492007	0.493066	0.492583	0.49311

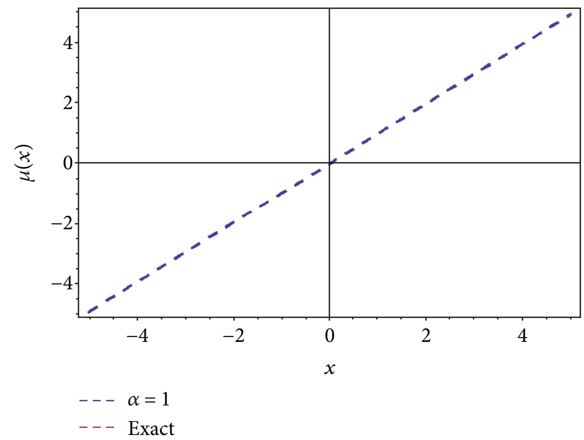


Figure 1: Numerical solution of the classical damped Burger equation with the exact solution at $t = 0.01$.

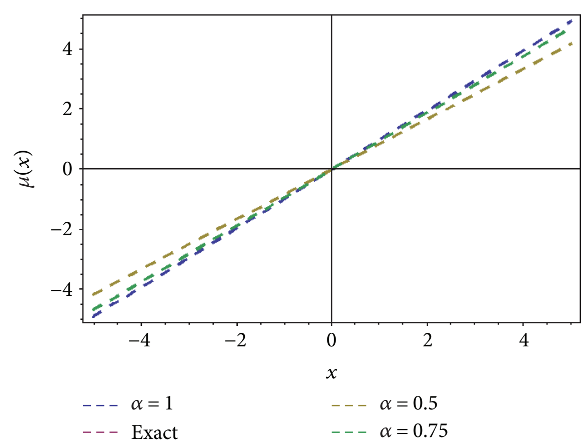


Figure 2: Numerical solution of the fractional damped Burger equation with the exact solution for different values of α at $t = 0.01$.

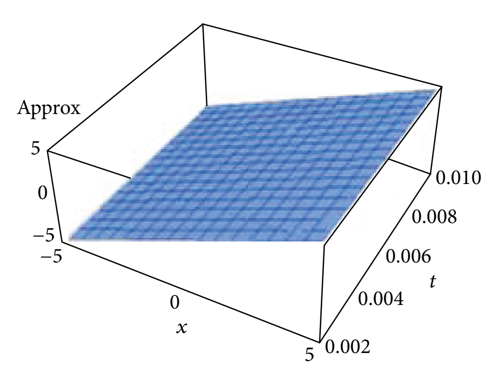


Figure 3: 3D plot of $u(x, t)$ for (DB) equation at $\alpha = 1$.

6. Conclusion

We have successfully applied NIM to time fractional (DB) and (STO) equations. Results reveal that NIM converges to the desired solution in lesser iteration compared to FRDTM. We can conclude that NIM computationally handles many physical and engineering problems in a simple and straightforward way. The accuracy of this method is also better than that of many methods which are computationally difficult to use.

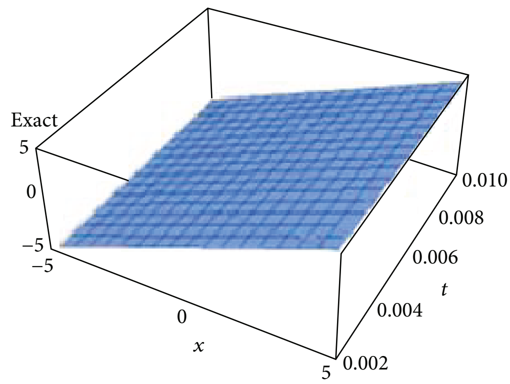


Figure 4: 3D plot of the exact solution for the DB equation.

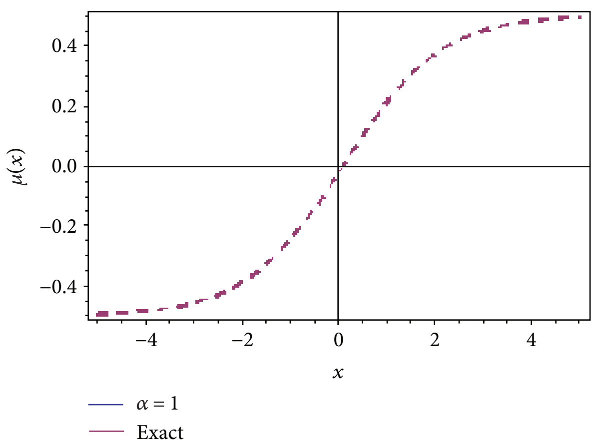


Figure 5: Numerical solution of the classical Sharma-Tasso-Olver equation with the exact solution at $t = 0.1$.

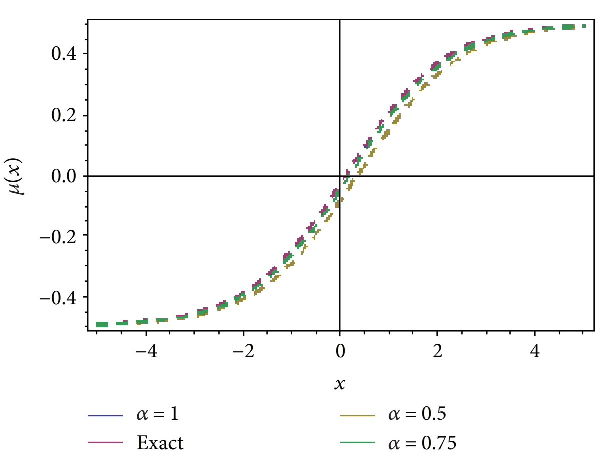


Figure 6: Numerical solution of the fractional Sharma-Tasso-Olver equation with the exact solution for different values of α at $t = 0.1$.

Data Availability

All the data and the metadata regarding the finding of the manuscript have been given in the research manuscript.

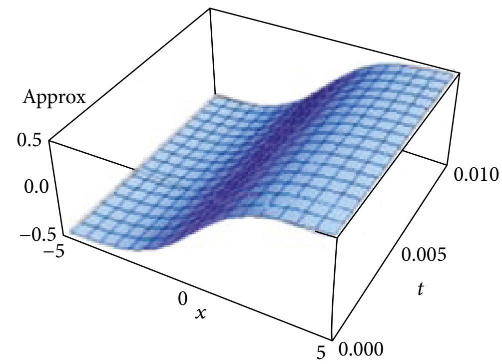


Figure 7: 3D plot of $u(x,t)$ for the STO equation at $\alpha = 1$.

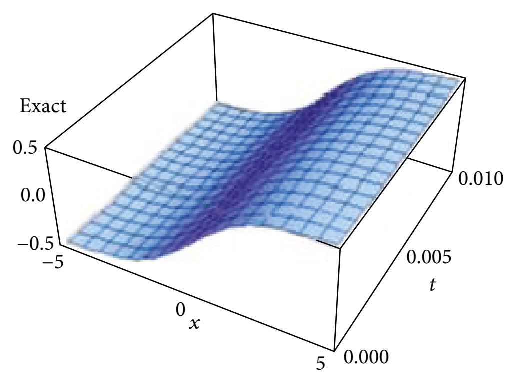


Figure 8: 3D plot of the exact solution of the STO equation.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

Sarhad University of Science and Information Technology, Peshawar, Pakistan, will pay the publication charges of the manuscript.

References

[1] I. Podlubny, *Fractional Differential Equations, Mathematics in Science and Engineering*, Academic Press, San Diego, California, USA, 1999.

[2] X.-J. Yang, J. A. Tenreiro Machado, and H. M. Srivastava, “A new numerical technique for solving the local fractional diffusion equation: two-dimensional extended differential transform approach,” *Applied Mathematics and Computation*, vol. 274, pp. 143–151, 2016.

[3] R. S. Sengar, M. Sharma, and A. Trivedi, “Fractional calculus applied in solving instability phenomenon in fluid dynamics,” *Journal Impact Factor*, vol. 6, no. 5, pp. 34–44, 2015.

[4] X. J. Yang, F. Gao, and H. M. Srivastava, “New rheological models within local fractional derivatives,” *Romanian Reports in Physics*, vol. 69, no. 113, 2017.

[5] Y. Yu, P. Perdikaris, and G. E. Karniadakis, “Fractional modeling of viscoelasticity in 3D cerebral arteries and aneurysms,” *Journal of Computational Physics*, vol. 323, pp. 219–242, 2016.

[6] V. Gill, K. Modi, and Y. Singh, "Analytic solutions of fractional differential equation associated with RLC electrical circuit," *Journal of Statistics and Management Systems*, vol. 21, no. 4, pp. 575–582, 2018.

[7] S. Das, "Analytical solution of a fractional diffusion equation by variational iteration method," *Computers & Mathematics with Applications*, vol. 57, no. 3, pp. 483–487, 2009.

[8] A. Elsaid, "The variational iteration method for solving Riesz fractional partial differential equations," *Computers & Mathematics with Applications*, vol. 60, no. 7, pp. 1940–1947, 2010.

[9] M. Ahmad, A. A. Bhat, and R. Jain, "Space time fractional telegraph equation and its application by using Adomian decomposition method," *Journal of New Theory*, vol. 22, pp. 73–81, 2018.

[10] F. Mabood, W. A. Khan, and A. I. M. Ismail, "MHD flow over exponential radiating stretching sheet using homotopy analysis method," *Journal of King Saud University - Engineering Sciences*, vol. 29, no. 1, pp. 68–74, 2017.

[11] S. Salahshour, T. Allahviranloo, and S. Abbasbandy, "Solving fuzzy fractional differential equations by fuzzy Laplace transforms," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 3, pp. 1372–1381, 2012.

[12] S. Momani, O. Abu Arqub, M.'m. Abu Hammad, and Z. S. Abo-Hammour, "A residual power series technique for solving systems of initial value problems," *Applied Mathematics & Information Sciences*, vol. 10, no. 2, pp. 765–775, 2016.

[13] S. Sarwar and M. M. Rashidi, "Approximate solution of two-term fractional-order diffusion, wave-diffusion and telegraph models arising in mathematical physics using optimal homotopy asymptotic method," *Waves in Random and Complex Media*, vol. 26, no. 3, pp. 365–382, 2016.

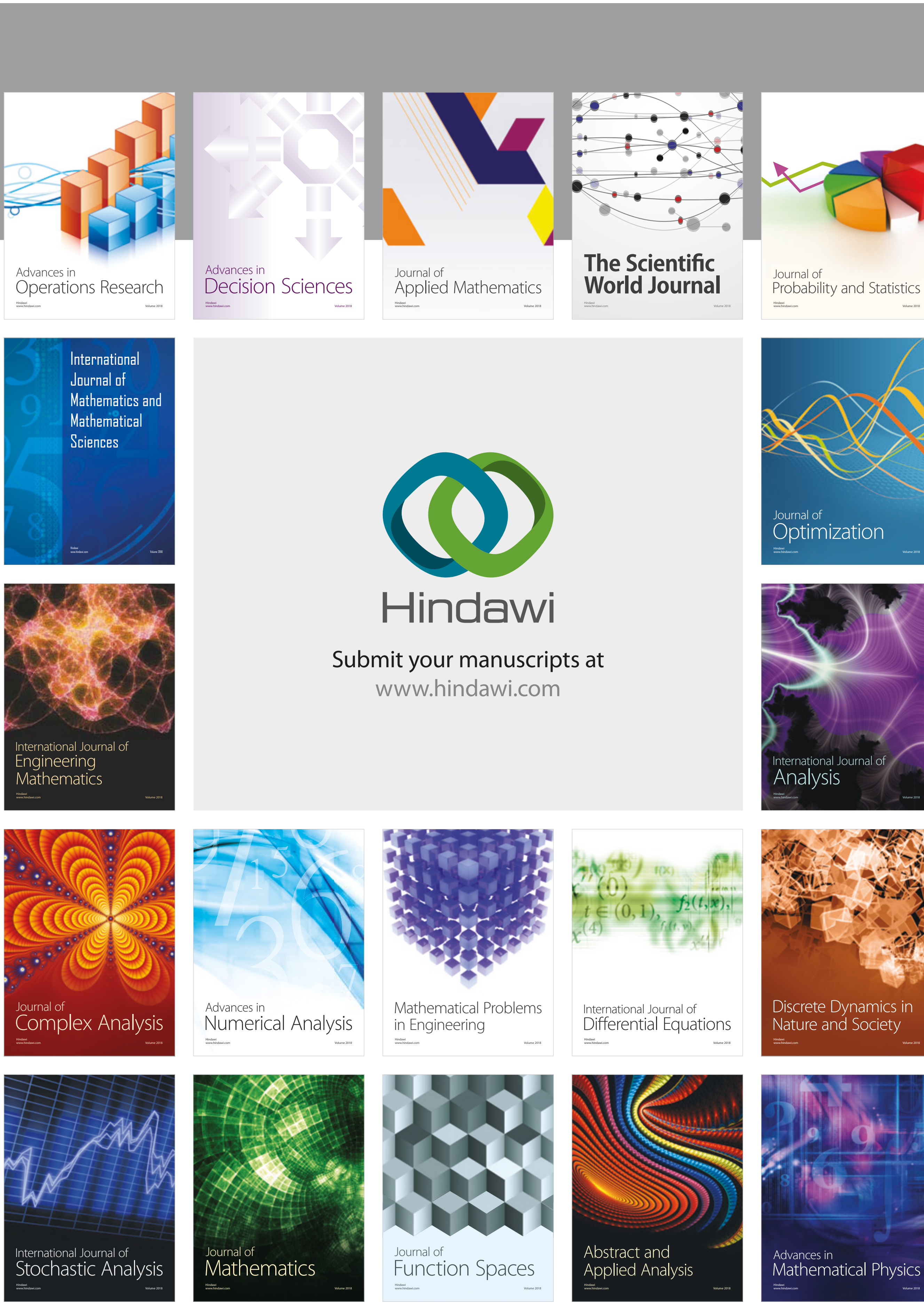
[14] V. Daftardar-Gejji and H. Jafari, "An iterative method for solving nonlinear functional equations," *Journal of Mathematical Analysis and Applications*, vol. 316, no. 2, pp. 753–763, 2006.

[15] A. M. S. Mahdy and N. A. H. Mukhtar, "New iterative method for solving nonlinear partial differential equations," *Journal of Progressive Research in Mathematics*, vol. 11, no. 3, pp. 1701–1711, 2017.

[16] M. S. Al-Luhaibi, "New iterative method for fractional gas dynamics and coupled Burger's equations," *The Scientific World Journal*, vol. 2015, Article ID 153124, 8 pages, 2015.

[17] B. R. Sontakke and A. Shaikh, "Approximate solutions of time fractional Kawahara and modified Kawahara equations by fractional complex transform," *Communications in Numerical Analysis*, vol. 2016, no. 2, pp. 218–229, 2016.

[18] M. S. Rawashdeh, "An efficient approach for time-fractional damped Burger & time-Sharma-Tasso-Olver equation using the FRDTM," *Applied Mathematics & Information Sciences*, vol. 9, no. 3, pp. 1239–1246, 2015.



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