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EDITORIAL

Aims and Scope of the Special Issue, “Quantum Foundations: Informational Perspective”

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During the past 30 years, the field of quantum information theory has produced a variety of novel information technologies whose full potential is as yet unknown. This quantum information revolution has also renewed the interest in the foundations of quantum theory, to the extent that fundamental concepts are now reconsidered in terms of a new information-theoretical perspective [1–10]. Indeed, the crucial strengthening of the quantum-informational aspect of quantum mechanics has gone beyond merely stimulating traditional foundational studies, prompting a deep and thoroughgoing reconsideration of quantum foundations. In particular, there is now a flourishing research effort that studies quantum foundations from a purely informational perspective. This issue is composed of contributions by leading researchers in quantum foundations, especially from informational and probabilistic perspectives, and it presents their expert viewpoints on a number of foundational problems.

One cluster of related papers in this issue concerns the elusive structures known as *SICs,* or more fully as SIC-POVMs: symmetric, informationally complete, positive-operator-valued measures [11,12]. A SIC is easy to define. Simply take a d-dimensional complex vector space, and find d^2 unit vectors such that the angle between any two vectors, as measured by the magnitude of their inner product, is the same. By the rules of quantum theory, this describes a measurement that can be performed on a d-level quantum system, and each vector corresponds to a possible outcome of that measurement. Moreover, given a system and probability distribution over the outcomes of a SIC measurement upon it, one can compute the probability distribution over the outcomes of any *other* measurement on that system. Indeed, a

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SIC establishes a mapping that takes all the familiar mathematical structures of quantum theory—states, channels, measurements—into purely probabilistic language, and nothing is lost in the translation. Moreover, the image of quantum state space under a SIC mapping is, geometrically speaking, quite a rich structure [13].

It is not yet known whether SICs exist for every choice of dimension d. But by combining the brute power of computer search with some amount of conceptual insight, researchers have found exact solutions and high-precision approximations in all dimensions from $d = 2$ through $d = 151$ inclusive, and in a handful of larger dimensionalities as well [14–17]. The known solutions display exotic mathematical characteristics, enjoying additional symmetries above and beyond the basic definition and relating to subjects that had seemed far removed from quantum measurement theory. The article by Stacey in this issue describes one such connection [18]: The first SICs to be discovered, which appear in dimensions 2, 3 and 8, have hidden links to lattices of integers constructed in the complex numbers, the quaternions and the octonions. Another paper in this issue, by Appleby *et al.,* explores a complementary discovery about SICs [19]. This connection, which applies to SICs discovered more recently than those discussed by Stacey, involves the geometric phases among sets of three vectors in a SIC. A careful study of these phase factors revealed that they have a deep significance for algebraic number theory. Appleby *et al.* reported this finding in an earlier publication [20], but because algebraic number theory is a topic foreign to most physicists, a more accessible treatment was much desired, and we are pleased to include that exposition here.

Bengtsson's contribution to this special issue also pertains to SICs, and specifically to the algebraic number theory thereof [21]. In the simplest possible case, which turns out to be a SIC in dimension $d = 4$, the number theory can be approached with pencil and paper. As Bengtsson demonstrates, the entire SIC can be built up from a single complex number, a unit in a particularly special number field. This paper provides an explicit example, amenable to hand calculation, of the unforeseen connection between advanced algebraic number theory and quantum information.

We note that the analogue of the SIC problem in *real* vector space is also of mathematical interest, and has even attracted some attention in the semi-popular science media [22]. Entertainingly, when we first viewed that article online, the Web site presented three related articles at the foot of it, chosen no doubt because they were also tagged "mathematics" in the magazine's database. These suggestions were about higher-dimensional sphere packing [23], graph isomorphism [24] and combinatorial designs [25], and all three of those areas are closely related to SICs! (See, respectively, [18]; [26]; and [27,28].) As this demonstrates, although the definition of a SIC is concise, investigating them leads us to travel widely through mathematics and physics. The SIC problem looks small, but it is bigger on the inside.

The contribution by DeBrota and Fuchs to this issue considers the problem of representing quantum theory in probabilistic terms, generalizing beyond the SICs themselves [29]. Their work is an extension of theorems proven by Zhu [30], which indicate that by one measure, the SIC representation is an optimal one. Modifying Zhu's measure, and presenting a natural class of measures to which it is seen to belong, DeBrota and Fuchs find new discrete structures in quantum state space with significant information-theoretic properties. While this issue's other articles on SICs focus on their mathematical surprises, DeBrota and Fuchs are driven by quantum physics. In particular, their interest in SICs and related structures arises from *QBism,* an interpretation of quantum mechanics according to which quantum theory is a "user's manual" which an agent can use to better navigate the natural world. "According to QBism", states a key reference, "quantum mechanics is a tool anyone can use to evaluate, on the basis of one's past experience, one's probabilistic expectations for one's subsequent experience" [31]. Developing this philosophical view naturally led to the question of representing quantum theory in wholly probabilistic terms, and thus motivated investigations into SICs. Although an interest in either QBism or SICs does not *compel* an interest in the other, the histories of the subjects are entwined [32–34]. Those who pursue the intersection of these subjects hope that the probabilistic representation of quantum theory enabled by SICs will reveal fundamental physical principles, aspects of the natural world which make quantum physics a good "hero's handbook" for living within it, implicit in quantum theory but obscured by its traditional formalism.

Quantum cellular automata and quantum walks provide a framework for the foundations of quantum field theory. This may be surprising, given the discrete character of cellular automata, in contrast with the emphasis that field theory places upon the continuum. However, starting from very general principles, the equations of motion of free relativistic quantum fields can be derived as the small-wave-vector limit of quantum automata and walks. The paper of Bisio, D'Ariano, and Perinotti [35], "Quantum walks, Weyl equation and the Lorentz group", characterizes the full symmetry group of the Weyl walk, which is shown to be a nonlinear realization of a group that is the semidirect product of the Poincaré group and the group of dilations.

The contribution of Khrennikov [36], "The present situation in quantum theory and its merging with general relativity", studies the foundational issues inherent in the thorny puzzle of unifying quantum theory with general relativity. The basic problems of two basic counterparts of quantum theory, namely quantum mechanics and quantum field theory, are considered in interrelation. It is pointed out that each of these counterparts has its own problems. For quantum mechanics, one can stress its nonrelativistic character and the perplexing "spooky action at a distance". The problem of infinities is considered as the main problem of quantum field theory. The main point of the paper is that it is meaningless to try to unify quantum field theory, so heavily suffering of infinities, with general relativity. An essential part of the paper is devoted to analysis of two basic mathematical constraints of quantum theory, namely, the use of the real number line (and the orthodox complex plane) and Hilbertian state spaces. Khrennikov briefly presents approaches to quantum theory that are non-Archimedean (in particular, p-adic) and non-Hilbertian (based on distribution theory). The main claim of the paper is that, in spite of the Bell theorem, it is still possible to treat quantum phenomena on the basis of a classical-like causal theory. In particular, a random field model generating the quantum formalism is discussed. This emergence viewpoint can serve as the basis for unifying general relativity with a novel quantum theory, a theory that might be strikingly different from presently powerful quantum mechanics and quantum field theory. Furthermore, it may happen that general relativity would also be modified in the course of this unification.

Quantum violation of Bell inequalities is now used in many quantum information applications, and it is important to analyze this phenomenon both technically and philosophically. The contribution of E. R. Loubenets [37], “Bell’s nonlocality in a general nonsignaling case: Quantitatively and conceptually”, analyzes violation of multipartite Bell inequalities by way of a local probability model. This “LqHV” (local quasi-hidden variable) model incorporates the traditional LHV model only as a particular case and correctly reproduces the probabilistic description of every quantum correlation scenario (and more generally, every nonsignaling scenario).

Plotnitsky’s article [38], “On the character of quantum law: complementarity, entanglement, and information”, offers a new perspective on Bohr’s concept of complementarity, a perspective never expressly considered by Bohr himself, although one feels that he must have realized the key implications of this perspective. Complementarity, the article argues, is a reflection of the fact that, as against classical physics or relativity, the behavior of quantum objects of the same type—say, all electrons—is not governed by the same *physical law* in all situations or contexts. This applies specifically in complementary contexts, where complementarity is defined by their mutual exclusivity. On the other hand, the *mathematical formalism* of quantum mechanics offers correct predictions, essentially probabilistic or statistical—sometimes, as in the EPR-type experiments, statistically correlated—in all contexts. The EPR experiment brings out the deeper meaning of complementarity. In connection with this experiment, Bohr accordingly spoke of “an entirely new situation as regards the description of physical phenomena that the notion of *complementarity* aims at characterizing”. This situation, he argued, arises thanks to the irreducible role of measuring instruments in the constitution of quantum phenomena. Here, “quantum phenomena” are considered differently from quantum objects, such as electrons, which in the interpretation that Plotnitsky finds in Bohr are assumed to be beyond representation or possibly even beyond conception. This view is, arguably, the most radical departure from realism currently available, especially if this reality is assumed to be beyond conception (although it is not clear if Bohr was willing to go quite that far). The article then considers, via the EPR experiment, the relationships between complementarity and entanglement, both of which are crucial to our understanding of the EPR experiment. Schrödinger, who introduced the concept of entanglement, saw it as reflecting “not one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought”. Plotnitsky’s article argues that entanglement and complementarity are fundamentally connected via the question of measurement, most especially because any quantum measurement leads to an entanglement between the quantum object under consideration and the measuring apparatus used. This makes any quantum experiment essentially analogous to the EPR experiment. While this type of entanglement has been noted, the relationships between it and complementarity have not been previously considered, including by either Schrödinger (who did not find the idea of complementarity appealing) or Bohr, even though the latter implicitly used this type of entanglement in his analysis of the EPR experiment. Finally, the article addresses the connections between complementarity and entanglement in quantum information theory, and conversely, the bearings of quantum information theory upon these connections. Helped by John A. Wheeler’s pioneering “it-from-bit” vision of quantum reality as emerging from quantum information, the article argues that quantum theory and the character of quantum law are fundamentally defined by the relationships among complementarity, entanglement, and information.

We hope that the reader will enjoy the present issue, which will be useful to experts working in quantum physics and quantum information theory, ranging from experimentalists, to theoreticians, mathematicians (especially those who are interested in the SIC existence problem), and philosophers.

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