What if we have only one universe and closed timelike curves exist?

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Abstract

David Deutsch provided us one possible solution to the grandfather paradox, Deutsch's closed timelike curves, or simply Deutsch CTC. Deutsch states that this gives us a tool to test many-worlds (Everettian) hypothesis since Deutsch CTC requires Everettian understanding. This paper explores the possibility of co-existence of Deutsch CTC with contextual/epistemic understanding of quantum mechanics. Then this paper presents the irrelevance hypothesis and the hypothetical application to quantum complexity theory.

1 Introduction: Deutsch's closed timelike curves

Closed timelike curves (CTC) are known to be possible in some solutions of the Einstein field equation in general relativity. These solutions generally are considered implausible for different reasons. This paper will not discuss them and rather assume that CTC is possible. This should not be conflated with the author's opinion on time travel.

It is known that naive interpretation of CTC leads to the Grandfather Paradox via time travel. The Grandfather Paradox allows a person to kill the person's grandfather in the past - thereby causing contradiction in the very existence of that person. When evaluated classically assuming CTC exists, the paradox is unavoidable unless one assumes Novikov self-consistency principle [3]. One alternative option that invokes quantum mechanics is Deutsch CTC by David Deutsch [2]. Following the explanation of Aaronson 2008 [1], one may simply understand the resolution as:

$$\Phi(\rho) = \rho$$

where Φ is some quantum operation and ρ is density matrix. That is, CTC requires that any quantum operation results in a fixed point. This means that density matrix is "fixed" by a quantum operation used in CTC. Every quantum operation has a fixed point, and thus this resolution works for every quantum operation. Thus, in case of the Grandfather Paradox the grandfather may be dead or the grandfather may be alive. However, suppose the following: in the

past the grandfather was alive by quantum measurement. Now one returns to the past from the future and has some possibility of killing the grandfather. Suppose by probabilistic whims that the person succeeds in killing the grandfather. Then, regardless of density matrix consistency, the person changes the history - again making the very existence of that person questionable.
Thus, Deutsch argues that this forms the test of Many-worlds interpretation of quantum mechanics, since unless the person comes to be in a different universe, Deutsch CTC causes problems. Thus, the following classical logic holds if Deutsch's arguments are to be believed:

$$\textbf{CTC} \rightarrow \textbf{Deutsch CTC} \rightarrow \textbf{Many-worlds}$$

where $\rightarrow$ refers to implication. Thus any disproof of many-worlds would mean that closed timelike curves do not exist.
I will present the alternative interpretation building from Deutsch CTC that can be consistent with the Neo-Copenhagen interpretation based on contextual and epistemic nature of quantum mechanics.

2 Contextual/epistemic understanding of Deutsch's closed timelike curves

Recall that in some interpretations of quantum mechanics, wavefunction and density matrix are epistemic. They only reflect our probabilistic knowledge rather than they themselves being real (ontological). Also quantum measurement in some interpretations is assumed to be contextual - that is, once measurement is done, density matrix is updated to reflect the measurement - or so-called "wavefunction collapse". Thus, it is possible to say that at two different time points probabilistic knowledge differs. That is, looking from the future, the fixed point consistency for ρ has to be maintained. However, from the past does not have to respect that consistency. Thus this interpretation eliminates the troubling fixation of ρ in the past by a future quantum operation in Deutsch CTC.
Now let us assume that only one universe, consistent with epistemic and contextual nature of quantum mechanics, exists. Then the Grandfather Paradox appears again even in Deutsch CTC. Now comes the "God plays dice only once" (or "dice only once") principle, to borrow words from Albert Einstein:

> God does play dice for our world (quantum mechanics), but God only plays once.

Thus, every past measurement remains unchanged even under CTC with this principle. Again, epistemic understanding of quantum mechanics is crucial for this principle. Suppose that the past measurement is not known to a future agent. Then this future agent only has probabilistic understanding of the past. By knowledge of quantum operations and CTC, the agent imposes the fixed

point consistency to obtain ρ. Depending on contexts, the agent may or may not get the measurement result back, but this does not change the fact that the agent did not know the past measurement and thus before the measurement result is known cannot deterministically figure out the past measurement.
Now suppose the agent knows the past measurement result. Then by epistemic nature of ρ there is no need to use fixed-point consistent ρ to get the past measurement. The past measurement is already the best knowledge.
Thus the dice-only-once principle can be re-named as "CTC complementarity" in a similar spirit to several complementarity in physics such as black hole complementarity. Thus existence of CTC does not really matter for the past - the past works as if CTC is not there. This inspires the "CTC invariance" hypothesis:

> Even if CTC exists, its existence can be ignored.

CTC invariance hypothesis is not mathematical, and thus equivalence to CTC complementarity cannot be proven. I would rather say that CTC invariance is a natural extension of CTC complementarity (with non-multiverse, contextual and epistemic understanding).

3 Hypothetical application of CTC invariance hypothesis to quantum complexity theory

As Aaronson 2008 [1] showed, given existence of Deutsch CTC, $P_{CTC} = PSPACE$. CTC complementarity still does not eliminate the possibility of a CTC computer. By definition of complexity class P and P_{CTC}, probabilistic nature of quantum mechanics does not affect the final output of a P_{CTC} decision problem - the difference only is that CTC is used. But by CTC invariance hypothesis, the world should work as if CTC does not exist.
One hypothetical resolution/application is that $P_{CTC} = PSPACE$ is in fact not just true for CTC, but also for non-CTC, that is $P = PSPACE$. This eliminates implausible difference between CTC and non-CTC given that the past measurement is fixed by CTC complementarity. So far, computer scientists believe this outcome to be unlikely - and this may count as evidence against existence of CTC.

4 Conclusion

In general, the cleanest argument indeed is that closed timelike curves simply do not exist for their paradoxical nature and physical non-existence. However, CTC has not been ruled out completely and thus the paper explores an alternative interpretation of Deutsch CTC but without Many-worlds attached. Then I explore a plausible hypothesis leading from this interpretation and applies it to quantum complexity theory that results in hypothetical $P = PSPACE$. Now the hypothesis of course is non-rigorous and it is unclear whether the hypothesis really is equivalent to CTC complementarity. But I will leave the question behind for now.

References

[1] Aaronson, S. and Watrous, J. (2008). Closed timelike curves make quantum and classical computing equivalent. Proceedings of the Royal Society A 465, pp. 631-647

[2] Deutsch, D. (1991). Quantum mechanics near closed timelike lines. Physical Review D Vol. 44 No. 10

[3] Friedman, J., Morris, M. S., Novikov, I. D. and Echeverria, F., Klinkhammer, G., Thorne, K. S., Yurtsever, U. (1990). Cauchy problem in spacetimes with closed timelike curves. Physical Review D Vol. 42 No. 6