Is the individuality interpretation of quantum theory wrong ?

U. Klein[*]

University of Linz

Institute for Theoretical Physics

A-4040 Linz, Austria

July 27, 2012

Abstract

We analyze the question whether or not quantum theory should be used to describe single particles. Our final result is that a rational basis for such an 'individuality interpretation' does not exist. A critical examination of three principles, supporting the individuality interpretation, leads to the result that no one of these principles seems to be realized in nature. The well-known controversy characterized by the names of Einstein (EPR), Bohr and Bell is analyzed. EPR proved 'predictive incompleteness' of quantum theory, which implies that no individuality interpretation exists. Contrary to the common opinion, Bell's proof of 'metaphysical completeness' does not invalidate EPR's proof because two crucially different meanings of 'completeness' are involved. The failure to distinguish between these two meanings is closely related to a fundamentally deterministic world view, which dominated the thinking of the 19th century and determines our thinking even today.

1 Introduction

The question formulated in the title of this essay requires first clarification of a semantic point. An interpretation cannot be wrong in the same sense as an experimentally verifiable theoretical prediction. The term 'wrong' is used here in the sense of 'extremely misleading'. In this sense a wrong interpretation leads to paradoxical contradictions or to internal inconsistencies such as unsolvable theoretical problems. I will first formulate some basic assumptions and explain what 'individuality interpretation' means before I will try to answer the question.

I start by formulating the most important assumptions underlying this work. A physical theory is a set of equations together with a number of rules how to compare theoretical and experimental results. Predictions, i.e. real numbers obtained by solving the basic equations using input data referring to earlier times, represent the core of a physical theory. The interpretation of the mathematical terms is also part of a physical theory. It gives 'meaning' to the mathematical variables, but does not affect the predictive core of the theory. Several different interpretations of one mathematical formalism are possible.

---

[*]ulf.klein@jku.at

They lead to slightly different physical theories but cannot make a physical theory right or wrong, since they do not affect the predictive core of the theory. We may say that a theory is given by a set of predictions (which constitute the invariant core) and an interpretation.

What, exactly, does 'individuality interpretation' mean ? A physical theory which allows prediction of single (individual) events, in particular predictions about single particles, must necessarily be interpreted in this sense. For classical mechanics this individuality interpretation is obviously correct. Such a theory could also be called a deterministic theory, because it claims to predict the behavior of individual particles 'with certainty'. There is no room in such a theory for uncertainty, all predictions of this individualistic theory have a probability equal to one . Testing the predictions of this theory requires a single experiment.

On the other hand there are physical theories, expressed in a completely 'deterministic' mathematical form, whose output cannot be verified in single experiments because the scattering of data is too large to be neglected. Clearly, the output of a physical theory must be testable. If it cannot be tested in individual experiments, essentially the only possibility left is a statistical test. In this case the output of the theory is given by statistical quantities like probabilities or expectation values. These numbers can be compared with experimental results and verified or falsified just as the output of the deterministic theories discussed above. But in order to do this an infinite number of individual systems, all prepared in the same way, have to be studied experimentally. This is the way statistical measurements have to be performed in principle (in practice simpler possibilities exist), no other testable meaning (namely "frequentist probability') can be ascribed to the term probability in a physical context. A statistical theory can obviously not be used o make predictions about individual events, because such predictions cannot be verified in individual experiments.

From the present discussion one would expect - considering the above, rather weak assumptions as evident - that *no* physical theory whose output is given by statistical quantities can be interpreted in an individualistic sense. In particular, one would expect that this holds true for quantum theory (QT), whose output is of a probabilistic nature. On the other hand, the dominating interpretation of QT tells us that quantum mechanics *is* a theory about individual particles. We have been using phrases like 'the Schrödinger equation of a single electron' or 'the quantum mechanical description of a single particle' an infinite number of times. This kind of talking determines our thinking. The idea that QT describes individual events and particles presents the basis for much, if not most, of current research on foundations of physics.

2 The conceptual basis of the individuality interpretation of quantum theory

According to the assumptions above, an individuality interpretation of a probabilistic theory does not make sense. This means that the boundaries of this simple framework have to be left far behind in order to establish QT as a theory about single particles. There are individuality interpretations which require more than a single universe or the participation of the observer's brain [3]. We shall not discuss such interpretations here but restrict ourselves to the standard, Copenhagen interpretation (CI). In order to overcome the fundamental conflict between deterministic and probabilistic predictions the CI denies

the reality of unobserved properties [21]. The properties 'come into being' by the act of measurement in a way which is unknown and presents an unsolvable 'measurement problem'. The CI 'solves' the fundamental conflict in a sophistic sense because it is not the task of a physical theory to make predictions about non-existing things. But it does not answer the question how things come back to reality. The CI's claim for an individuality interpretation may also be expressed by the statement that QT is a 'complete' theory as regards the description of individual particles; a more detailed analysis of the term 'complete' will be given in section 4.

The CI shows several strange features, which have as a common origin the switching forth and back between reality and un-reality of properties as observation begins and ends. This problem becomes more stringent if two conjugate properties (non-commuting observables) have to be measured at the same time. A number of principles or concepts have been introduced, by the founders of the CI, in order to support the individuality interpretation and to explain its strange features. There seem to be essentially three such principles. Let us begin with

- Heisenberg's uncertainty principle, the well-known inequality expressing the impossibility to measure position and momentum of a single particle simultaneously with arbitrary high precision.

This principle will be referred to as individual uncertainty principle (IUP). The IUP presents the most important cornerstone of the CI because it supports, if true, the idea that certain (conjugate) properties of a single microscopic system cannot be simultaneously real. The second concept supporting the CI is

- the particle-wave duality: Depending on the experimental situation individual microscopic systems may behave either like particles or like waves.

This idea can be considered as a complement to the IUP. In fact, according to the IUP particles have either sharp values of position or of momentum, depending on the experimental conditions. The former case corresponds to the particle picture, the latter to the wave picture. The degree of reality of these two pictures is determined by the measurement arrangement. The third idea supporting the CI is expressed as an assertion about

- the classical limit of QT: Classical mechanics may be regarded as the limiting case of quantum mechanics when $\hbar$ tends to zero [7]

The relevance of this last point for the individuality interpretation is obvious. If QT really describes individual particles (for nonzero $\hbar$), then it should not change its character - as a theory describing individual particles - if the limit $\hbar \to 0$ is performed. This limit must agree with a classical individualistic theory, namely classical mechanics. Otherwise the CI as an individuality interpretation must be called into doubt.

An important point to note is that no one of these three principles is part of the quantum theoretical formalism. This means each one needs justification from experiment or theory. A second important point is that these principles have been set up in the first half of the last century and that enormous technological progress has been made since then. A re-examination, taking today's results into account, seems useful.

3 Critical discussion of the conceptual basis of the individuality interpretation

The philosophical idea that unobserved things cannot be ascribed reality goes back to Aristotle and has Descartes and Berkeley as prominent advocates. It is in disagreement with the common sense philosophy of physicists but this does, of course, not mean that it can be rejected right from the start. Starting our analysis with the

- Uncertainty principle,

we should mention at the very beginning that Kennard's inequality [14], which is commonly written in exactly the same form as the IUP, has a very different meaning. It is a statistical relation which has nothing to do with the simultaneous measurement of position and momentum but expresses the relationship of the statistical fluctuations of independently measured quantities. Its mathematical derivation from the framework of QT is still sometimes erroneously considered as a confirmation of the IUP. The second point to note is that Heisenberg's famous Gedanken experiment connects the error in position to the disturbance in momentum, and not to the error in momentum (the momentum is assumed to be accurately known at the time of position measurement); see e.g. Margenau and Park [20]. Therefore it seems that the universal acceptance of the IUP is based on two historically-grown misunderstandings, namely the failures to distinguish individual from statistical measurements and measurements from preparations.

Considerable efforts have been undertaken to derive Heisenberg-like relations from QT. This requires, however, additional assumptions, forcing individuality concepts into the statistical formalism of QT. The most important of these is the 'projection postulate', the assumption that the state vector jumps after a measurement into an eigenspace of the corresponding operator. The details of this process, which is sometimes referred to as 'non-unitary time evolution', are unknown. This beautiful expression is used to hide the fundamental difference between individual and statistical predictions. The projection postulate was suggested by the ingenious mathematician von Neumann and seems very convincing from the point of view of mathematical simplicity. But it may be a simple error. In this context it must be mentioned that von Neumann's proof of the non-existence of hidden variables contains a simple error. It was quoted many times, as an argument in favor of 'completeness' of QT, during a period of more than thirty years (!), until the 'silly error' in its derivation became widely known [21]. All attempts to derive Heisenberg-like relations from QT use this projection postulate, as well as other assumptions formulated in the abstract language of Hilbert space [13]. Depending on the chosen assumptions some authors derive relations similar to Heisenberg's inequality [6] while others obtain different expressions [22]. Several experimental violations of Heisenberg's inequality have been reported, the most recent one by Erhart et al. [10].

Most relevant for the CI's claim of un-reality of unobserved properties is the IUP, i.e. Heisenberg's inequality interpreted as a relation between measurement errors of conjugate properties. Astonishingly, the practical basis for the IUP seems to be still Heisenberg's famous light-microscope Gedanken experiment - despite the fact that it says nothing about the simultaneous measurement of position and momentum. Thus, let us first ask if other "Gedanken experimente" have been designed which show a violation of the IUP. This is indeed the case. Such idealized measurement arrangements have been proposed by Prugovecki [27], Park and Margenau [24], Ballentine [1], Popper [26] and others. We

may, secondly, ask, as a question of primary importance, if the IUP has ever be confirmed experimentally. Not a single experimental confirmation has been reported [6] since Heisenberg's creation of the IUP in 1927. On the other hand, data showing violations of the IUP have been published. We mention, in particular, the realization of Poppers thought experiment [26] by Kim and Shih [15].

Summing up, we find no experimental or theoretical facts supporting the IUP. This principle does not seem to be an element of science, but rather a historically-grown habit or an object of quasi-religious admiration.

- Particle-wave duality

Recent experiments by Tonomura [31] and others have shown that single particles are always particles and never waves. A video on the Hitachi website [30] shows the development of a double-slit interference pattern as a consequence of an increasing number of electrons arriving at the screen. As pointed out by Silverman in his discussion of the Tonomura experiment [29]: "The manifestations of wave-like behavior are statistical in nature and always emerge from the collective outcome of many electron events" Thus, no mysterious transformation between particles and waves is required. The origin of the miraculous 'particle-wave duality' is poor resolution of early experimental data.

- The classical limit

The idea that classical mechanics must emerge as the classical limit of QT was advocated by Bohr, Dirac and others. But this idea led to a large number of open questions and contradictions. The problem becomes much simpler if one admits the possibility that the classical limit of QT differs from classical mechanics. It has been mentioned before that a straightforward application of the limit $\hbar \to 0$ to Schrödinger's equation leads to a classical probabilistic theory and not to classical mechanics [2, 18, 16] A recent, more complete treatment [17] leads to the same conclusion.

To summarize, closer examination shows that neither the IUP, nor the wave-particle duality, nor the claim that classical mechanics emerges as the classical limit of QT present physically well-defined concepts. No support is provided for the philosophical idea that unobserved properties are not real and for the related idea that an individuality interpretation of QT exists. On top of that, this also implies that a fundamental and very successful methodical principle of physics, namely the principle of reductionism, can not be universally valid. This principle is not compatible with the statistical interpretation of QT. As is well-known, the scientific community decided to keep the philosophical dogma of reductionism along with the individuality interpretation of QT. From a psychological point of view this is understandable, since we expect science to yield predictions with certainty, but the question is how much weight should be given to psychological expectations.

4 EPR, Bohr, Bell, and two meanings of 'completeness'

Present research on foundations of QT is strongly influenced by a paper published in 1935 by Einstein, Podolsky, and Rosen (EPR) [9]. There is an enormous secondary literature, see e.g. Fine [11], Ballentine [1], Redhead [28], and the author's website [16]. In this work, EPR claim that the quantum-mechanical description of reality is incomplete. The

CI is attacked 'from inside' because the basic assumptions used in this paper do not reflect the positions of the authors but are part of the CI. The most significant example is EPR's assumption that "The state of the particle is completely characterized by a wave function ψ", a statement in sharp opposition to Einstein's well-documented opinion that ψ describes an ensemble. EPR's conclusion was, of course, attacked by CI's advocates who considered QT as a complete theory. However, a discussion of the specific EPR problem was generally avoided and EPR's claim of incompleteness of QT was attacked on different routes - circumventing the specific problem. Bohr, in his reply, took a very philosophical, elusive route, which was not really convincing for many people.

According to the prevailing opinion this question was decided in favor of Bohr by the work of John Bell [5, 4], about thirty years later. Bell circumvented the specific EPR problem by relating it to the problem of hidden variables. A (local) hidden variable theory is compatible with all predictions of QT providing, however, at the same time, a more detailed (deterministic) description of reality. Physical intuition tells us that such a thing cannot exist but Bell proved that it cannot exist - at least within the framework of his postulates; all no-go proofs are of course only valid within a certain 'universe of discourse' (repeated remarks on this important limitation will be omitted from now on for brevity). He formulated general conditions for local hidden variable theories and derived therefrom an inequality which *differs* from the corresponding prediction of QT. Thus, he showed that hidden variable theories cannot exist if QT is correct. This shows that QT is a 'complete' theory (with the meaning of 'complete' given in context). This reasoning seems correct but the question is what can be concluded from it. The simplest conclusion is that EPR's proof of incompleteness of QT cannot be true because Bell showed that QT is complete. I claim that this simple reasoning is not justified because a subtle semantic trap, concerning the meaning(s) of the word 'complete', has been overlooked.

The word complete has two different meanings. If used to characterize the predictive power of a physical theory it means: "All facts that can be observed can be predicted (with certainty)". This kind of completeness could be called 'predictive completeness', or 'p-completeness' for brevity. In order to find out if a physical theory is p-complete one needs solutions (predictions) of this theory and experiments testing these predictions. This first kind of completeness may equivalently be characterized by saying that an 'individuality interpretation' for this (p-complete) theory exists. The standard example for a p-complete theory is classical mechanics. Classical massless field theories are of a similar nature but do not directly describe individual particles.

The second meaning of the word complete can be described as follows: "No better theory, in the sense of producing more 'definite' (deterministic) predictions, exists". This is a very strong assertion. It entails not only the concrete physical theory under discussion but also an infinite number of other theories (all unknown), which are all not allowed to exist according to the assertion. Such an assertion can of course only be verified within a certain 'universe of discourse', which may possibly be generalized in later steps. But it can be approached nevertheless. Let us call this second kind of completeness 'metaphysical completeness', or 'm-completeness'. As an example, we mention classical probabilistic theories where the uncertainty is only in the initial conditions while the movement in phase space is deterministic [19]. These theories are m-incomplete, with classical mechanics playing the role of the 'better theory'.

How are these two kinds of completeness related to each other ? A p-complete theory is also m-complete. It would not make sense to search for a better theory than classical mechanics (in its range of validity) because classical mechanics makes already predictions

with probability equal to one. This means, the implication

$$\text{p-completeness} \;\Rightarrow\; \text{m-completeness}, \tag{1}$$

holds true. This means that m-incompleteness implies p-incompleteness and that p-incompleteness is a necessary condition for m-incompleteness. On the other hand p-incompleteness is not a sufficient condition for m-incompleteness. A p-incomplete theory may be either m-complete or m-incomplete.

Let us now reconsider the EPR-Bohr-Bell question using this refined vocabulary. What Bell proved is obviously m-completeness of QT. In EPR's paper both kinds of completeness occur. In the last paragraph EPR express their believe that QT is m-incomplete:

"We believe however that such a [more complete] theory is possible"

The communication of EPR's 'believe' (which had been known for a long time) is of course not the central message of EPR. The central message is given by the logical deductions reported in the body of the paper, i.e. in the whole paper except the last paragraph. So, which kind of completeness is referred to in the body of EPR's paper ? The subject of the paper is the problem of predictions of the values of certain observables, thus what EPR mean by completeness is obviously p-completeness. An assertion of m-incompleteness, i.e. a statement that a better theory then QT must exist, can nowhere be found in the relevant part (the body) of EPR's paper.

The proof of p-incompleteness of QT was of course a necessary prerequisite for EPR's 'believe' in a deterministic replacement of QT. If an analysis had led to the conclusion that QT is p-complete, this had also implied that QT is m-complete. On the other hand, EPR were aware of the fact that p-incompleteness is only a necessary and not a sufficient condition for m-incompleteness. Thus, they were aware of the fact that their proof of p-incompleteness of QT did not imply m-incompleteness of QT. They express this in the first sentence of the last paragraph of their paper in a very clear way:

"While we have thus shown that the wave function does not provide a complete description of physical reality, we left open the question of whether or not such a description exists".

It is a real mystery why this clear statement, separating cleanly the two different kinds of completeness (made even more explicit in the present essay only by means of different names) from each other, has been overlooked by the scientific community.

It follows that Bell's proof of m-completeness cannot be used, even if we accept the assumptions underlying his proof [12], as an argument against EPR's proof of p-incompleteness of QT. Bell was in error, when claiming that his results contradict EPR ! If we accept both Bell's and EPR's findings we arrive at the final conclusion that QT is p-incomplete and m-complete. This conclusion is compatible with a recent derivation of non-relativistic quantum theory from statistical postulates [19, 23] and agrees roughly with the common sense assessment of QT as a correct (complete) statistical (incomplete) theory. It implies that an individuality interpretation of QT is not justified.

The present conclusion can only be avoided if one takes a deterministic point of view of the world, namely that everything that can be observed must in principle be predictable. This then means that p-incompleteness implies m-incompleteness (or the existence of a hidden variable theory). But note that this implication, which eliminates our final conclusion, is not a logical requirement but the consequence of a new (in fact, very old)

philosophical dogma. From the point of view of physics such a deterministic dogma is not required. Interestingly, this deterministic point of view was *shared* by Einstein and Bohr (see [16] for a more detailed explanation), despite their otherwise very different opinions. It was denied by several other authors, in particular by Popper [25]. Unfortunately, today's discussions are still centered around the two alternatives represented by Einstein and Bohr.

5   More recent developments

Recent years have shown a tendency to reinterpret the body of EPR's paper as a proof of m-incompleteness. The sharp contrast between the 'believe' in the last paragraph and the science in the body of the paper had to be hidden somehow. The paper was so to say 'rewritten' as a hidden variable theory, 'elements of reality' became hidden variables. However, the real EPR paper is not a hidden variable theory but the construction of an internal contradiction within the CI. It is certainly true that EPR's conclusion implies the *possibility* of a more complete description. However they have not *shown* that such a more complete description exists. The only way to do this is to construct a 'better' (hidden variable) theory for QT. Nothing like that can be found in EPR's paper.

As a further development, which will be dealt with only briefly, the validity of EPR's final conclusion has been discussed in conjunction with the validity of their basic assumptions. According to the late Einstein, EPR's final conclusion may be rewritten as the statement that not both of the following two assertions (quoted literally from [8]) can be true:

1. The description by means of the Psi-function is *complete.*

2. The real states of spatially separated objects are independent from each other.

Today, the second assertion is frequently split into the assertions of 'locality' and 'reality' (see e.g. Wiseman [32]). Incompleteness with regard to predictions of individual events is a familiar feature. On the other hand a breakdown of 'locality' or 'reality' presents a much stronger and stranger assumption. Thus, the failure of the first assertion seems more natural. Therefrom Einstein's conclusion that QT is 'incomplete'.

If Bell's theorem is (erroneously) used to show that the first assertion is true, then the breakdown of at least one of the fundamental scientific principles of 'locality' or 'reality' is a necessary consequence. Thus, certain strange features associated with single events became a subject of intense research because they could now be described in terms of c.f. a 'breakdown of locality'. Thus the strange ('weird', 'magical') features of QT which always appear, whenever QT is used to describe single particles, became manifest once again, but this time in a supposedly more definite form thanks to Bell's theorem.

If, on the other hand, the two different meanings of the term 'completeness' are clearly distinguished from each other, then Bell's proof of m-completeness cannot be used to eliminate QT's property of p-incompleteness. Then, the above Einstein alternative leads to the same final conclusion as before, namely that an individuality interpretation of QT does not exist. This means that QT is a statistical theory (probably complete in a metaphysical sense, and certainly complete in a metaphysical sense as defined by Bell) which by its very nature cannot be used to describe the behavior of single particles. The 'strangeness' of QT is nothing but the consequence of an unjustified extension of its range of validity. Typically, all the strange things never happen in the laboratory but always in the brain of the interpreter.

6 Conclusion

In the first part of this essay we found that a rational basis for several principles, believed to support the philosophical basis of the CI, does not exist. The mysterious nature of the CI, which contains undefined statements in its definition, can be expected to lead to further obscurities whenever applied to single particles. This is indeed what happens. An attempt by EPR to make the contradictory character of the CI explicit, i.e. to prove the p-incompleteness of QT - or the non-existence of an individuality interpretation for QT - was successful. Bell's proof of m-completeness cannot be used to invalidate EPR's proof of p-incompleteness. Our final conclusion is that QT is p-incomplete and m-complete. The common failure to distinguish the two different meanings of completeness is due to an old 'deterministic dogma' which rules our thinking even today. This dogma invalidates, if true, our final conclusion. However, this deterministic point of view is not a logical neccessity but rather a historical grown intellectual habit. According to the present analysis it is incompatible with the structure of QT and should be abandoned.

References

[1] L. E. Ballentine. The statistical interpretation of quantum mechanics. *Reviews of Modern Physics*, 42:358–381, 1970. 4, 5

[2] L. E. Ballentine. *Quantum Mechanics*. Prentice Hall, Englewood Cliffs, NJ, 1989. 5

[3] J. A. Barrett. *The Quantum Mechanics of Minds and Worlds*. Oxford University Press, Oxford, 1999. 2

[4] John S. Bell. On the problem of hidden variables in quantum mechanics. *Reviews of Modern Physics*, 38:447–452, 1966. 6

[5] John Stuart Bell. On the Einstein-Podolsky-Rosen paradox. *Physics*, 1:195–200, 1964. 6

[6] P. Busch, T. Heinonen, and P. Lahti. Heisenberg's uncertainty principle. *Physics Reports*, 452:155–176, 2007. 4, 5

[7] P. A. M. Dirac. *The principles of quantum mechanics*. Oxford University Press, Oxford, 4. edition, 1958. p. 88. 3

[8] A. Einstein. *Reply to Criticism*, page 665. Harper and Row, New York, 1949. 8

[9] A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.*, 47(10):777–780, May 1935. 5

[10] J. Erhart, S. Sponar, G. Sulyok, B. Badurek, M. Ozawa, and Y. Hasegawa. Experimental demonstration of a universally valid error-disturbance uncertainty relation in spin-measurement. *Nature Physics*, 8:185, 2012. 4

[11] Arthur Fine. *The Shaky Game*. The University of Chicago Press, Chicago and London, 1986. 5

[12] Karl Hess, Hans De Raedt, and Kristel Michielsen. Hidden assumptions in the derivation of the Theorem of Bell. 2011. invited paper presented at FQMT11, available at http://www.arxiv.org/abs/quant-ph/1108583. 7

[13] H. F. Hofmann. Measurement uncertainties in the quantum formalism: quasi-realities of individual systems. In *Proceedings of QIT26*, 2012, available at http://arXiv.org/abs/quant-ph/1205.0073. 4

[14] E. H. Kennard. Zur Quantenmechanik einfacher Bewegungstypen. *Z. Physik*, 44:326–352, 1927. 4

[15] Y. H. Kim and Y. H. Shih. Experimental realization of Popper's experiment: violation of the uncertainty principle ? *Foundations of Physics*, 29:1849–1861, 1999. 5

[16] U. Klein. A website explaining the statistical interpretation of quantum theory. http://statintquant.net. 5, 8

[17] U. Klein. What is the limit $\hbar \to 0$ of quantum theory? *American Journal of Physics*. 'conditionally accepted' for publication, a similar paper is available online at http://arxiv.org/abs/1201.0150. 5

[18] U. Klein. Schrödinger's equation with gauge coupling derived from a continuity equation. *Foundations of physics*, 39:964, 2009. 5

[19] U. Klein. The statistical origins of quantum mechanics. *Physics Research International*, Article ID 808424, 2011. Open access paper available at http://downloads.hindawi.com/journals/phys/2010/808424.pdf. 6, 7

[20] H. Margenau and J. L. Park. The physics and the semantics of quantum measurement. *Foundations of Physics*, 3(1):19–28, 1973. 4

[21] N. D. Mermin. Hidden variables and the two theorems of John Bell. *Reviews of Modern Physics*, 65:803–815, 1993. 3, 4

[22] M. Ozawa. Universally valid reformulation of the Heisenberg uncertainty principle on noise and disturbance in measurements. *Phys. Rev. A*, 67:042105, 2003. 4

[23] M. R. Pahlavani, editor. *Measurements in Quantum Mechanics*, chapter 'A statistical derivation of non-relativistic quantum theory', by U. Klein, pages 141–174. InTech, Rijeka, 2012. online available at http://www.intechopen.com/download/pdf/29091. 7

[24] J. L. Park and H. Margenau. Simultaneous measurability in quantum theory. *International Journal of Theoretical Physics*, 1:211–283, 1968. 4

[25] Karl R. Popper. *The Open Universe - an Argument for Indeterminism*. Rowman and Littlefield, Totowa, 1982. 8

[26] Karl R. Popper. *Quantum theory and the schism in physics*. Rowman and Littlefield, Totowa, 1982. 4, 5

[27] E. Prugovečki. On a theory of measurement of incompatible observables in quantum mechanics. *Can. J. of Physics*, 45:2173–2219, 1967. 4

[28] Michael Redhead. *Incompleteness, Nonlocality, and Realism: A Prolegomenon to the Philosophy of Quantum Mechanics.* Clarendon Press, Oxford, 1990. 5

[29] Mark P. Silverman. *Quantum Superposition.* Springer, New York, 2008. p.10. 5

[30] A. Tonomura. Demonstration of single-electron buildup of an interference pattern, 1989. Video available at http://www.hitachi.com/rd/research/em/movie.html. 5

[31] A. Tonomura, J. Endo, T. Matsuda, and T. Kawasaki. Demonstration of single-electron buildup of an interference pattern. *Am. J. Phys.*, 57(2):117–120, 1989. 5

[32] H. M. Wiseman. From Einstein's theorem to Bell's theorem: a history of quantum nonlocality. *Contemporary Physics*, 47:79–88, 2006. available at http://www.arxiv.org/abs/quant-ph/0509061. 8