Special Relativity: A Reexamination of the Second Postulate and of Space Contraction and Time Dilation

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Abstract

Pure entities consist of mass or energy without the presence of the other: the inertial rest mass is all mass and no kinetic energy (no velocity); the photon is all kinetic energy and no (rest) mass. Pure entities may be compared at the ontological level (for form, progression in a dimension, extension in a dimension and storage). From this analysis it is shown that Einstein's second postulate of special relativity (constant speed of light) is actually derivative from a more fundamental attribute of all pure entities. Part two of this essay focuses on the space contraction and time dilation of moving physical objects. Arguments against attributing these changes to space and time itself (Minkowski) are offered. Instead, the roles of kinetic energy and of de Broglie wave effects are presented as a better explanation.

Special relativity has been with us for over a century and its predictions have been confirmed by countless experiments. Nevertheless, its conceptual and ontological foundations are not completely satisfactory. This essay will offer a critical review of two main aspects of special relativity: 1) the desirability/validity of erecting a postulate for a single phenomenon, namely the constant speed of light, and 2) the challenge of explaining why different inertial observers measure space and time differently (space contraction, time dilation). But one cannot analyze special relativity without considering the photon and the nature of radiant energy. Hence this essay will also look at how radiation went from quantity to entity.

Part I - The Constant Speed of Light

1.0 Introduction

The Scientific Revolution of the seventeenth century was mostly about mechanics and the laws of motion of material entities. Hence when scientists began to explain energy processes they naturally invoked material entities as causal agents. Thus the energy of combustion was regarded as due to the release of a material substance (philogiston). Heat itself was another assumed material substance (the caloric) that self-repelled so it would migrate from a hot object to a cold object. And, of course, in the nineteenth century radiation was regarded as a wave disturbance of the all-pervasive material aether.

In the last half of the nineteenth century theories of energy became dependent upon experiments (e.g., Joules) and became more "scientific." Great progress was made in thermodynamics and in the kinetic theory of gasses. By 1900 all of the preceding energy theories that involved material entities were dead or on death's door (the aether). Energy had become simply a quantitative measure in contrast with mass which was an entity with properties and a quantitative

measure. **For physics in 1900, mass was an entity and energy was a quantity**. And then things began to change somewhat.

In December of 1900 Max Planck announced that radiation energy was quantized. That took some years to be accepted but other theoretical or experimental confirmations followed in 1905 (Einstein's paper on the photoelectric effect), 1917 (Einstein's assertion that radiation quanta have momentum) and 1923 (Arthur Holly Compton's experiment of X-rays losing energy when bouncing off electrons). By the mid-1920s the quantized nature of radiation could not be denied and a quantum of light was given the name "photon." Quantized radiation retained its identity while traversing space just as a quantized matter retained its identity while traversing time. This suggested that the photon is an entity and that it constitutes an exception to the rule that energy is merely a quantity and never an entity. It is useful to compare the photon with the material entity, but before we can do that we must clarify our terminology.

1.1 Nomenclature

The concept of quantity is straightforward. It may involve the counting of discrete units such as atoms or molecules, or it may involve measurement relative to some standard yielding a definite amount of kilograms, volts, meters or seconds. Quantitative measures may be factual but they are not physically real. Something is physically real if it has a dimensional presence (space or time) and can be said to exist or occur. For physics we may assume that only mass and energy have a dimensional presence and either exist or occur.

"Field" became a protean word in twentieth century physics. It is used here in the primitive sense of something that exists and extends in space. Thus a material particle is a field (with a density measure) and so is the potential energy that surrounds a charged particle or an ion. A field is the ontological opposite of a wave: the former exists, the latter occurs. Also, the word "kinetic" means or implies patent, open or obvious; hence for physics it means "unstored." On the other hand, the word "potential" equates to latent or hidden and hence means "stored" (e.g., potential energy is stored energy). The precise term for matter (rest mass) is "kinetic mass" since it is mass that is unstored, unhidden. In what follows the term "inertial" is shorthand for an object or system that is force-free and regarded as space-stationary for a local observer.

"Entity" is also a protean word (e.g., legal entity, public entity). In the realm of physics entities must be composed of either mass or energy. Keeping a material entity in mind as a template, we can set out entity requirements. **Entities must: 1) have a form which implies a dimensional presence; 2) have a quantitative measure; and 3) store their opposite.** For the material entity the form is that of field, the dimensional presence (extension) is space, mass is the quantitative measure and energy is what is stored.

Since the 1920s physicists have treated the photon as an entity: an object of study and experimentation to ascertain its properties. The photon also meets the requirements we have set out for an entity. Photons oscillate and therefore extend in the dimension of time. They have the waveform, their quantitative measure is energy (joules) and they store their opposite, namely (relativistic) mass. A photon may not seem to have much in common with an inertial mass but since they are both entities they are very similar at the ontological level.

1.2 Pure entities

The photon is pure kinetic energy and is devoid of kinetic (rest) mass. The inertial massy particle or object is pure kinetic mass and is devoid of kinetic energy for a local observer. Accordingly, the photon and the inertial mass are **pure entities** since they do not mix mass and energy in their kinetic forms. The fact that one is energy that occurs while the other is mass that exists makes them very different. But the fact that both of them are entities makes for ***formal*** equality providing we recognize that the two entities interchange attributes that are appropriate for existence (kinetic mass) on the one hand and occurrence (kinetic energy) on the other.

Obviously both pure entities are quantized and they extend in a dimension. The inertial mass extends over a space interval and the photon, because of its oscillation, necessarily extends over a time interval. Both entities are also at rest and therefore located in their extension dimension.[1] The inertial mass is at rest in space by its own measure and the photon is at rest in time since time stands still at the speed of light.

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[1] To be stationary in a dimension means to be located there, even if an object is its own reference.

Both entities also progress at the maximum rate in that dimension where they do not extend. The photon extends in time where it is stationary but progresses (advances) at the speed of light in space. The inertial mass extends in space where it is stationary (by its own measure) but progresses in time at the maximum possible rate (were the inertial mass to acquire any velocity in space its clocks would slow down). Both entities follow Aristotle in having a form, but of course the forms differ: a field is suited to existing inertial mass progression in time whereas a wave is suited to occurring photon progression in space.

Storage is the last formal parallel we need to mention here. The inertial mass stores energy: thermal energy stored within the mass plus the energy the mass represents via $E = mc^2$. The photon stores mass, namely, the relativistic mass released upon photon impact/termination. The two pure entities exchange mass-energy in their interactions. The photon gives up stored mass (and its momentum) when terminating upon a material target (absorption). The material object/atom gives up stored energy when releasing a photon (emission).

Because pure entities store their opposite this gives them two identities. They have an unstored, kinetic identity which is mass for the inertial mass and energy for the photon. They also have a stored, potential identity which is potential energy for the inertial mass and potential (relativistic) mass for the photon. To summarize our two pure entities:

Inertial Mass	Photon
Quantized	Quantized
Pure mass (conserved)	Pure energy (conserved)
Stationary in space	Stationary in time
Located in space	Located in time
Extends in space	Extends in time
Max. progression in time	Max. progression in space
Field form	Waveform
Stores energy	Stores mass

We are now ready for a definition of a pure entity.

A pure entity is composed of field form mass or waveform energy that stores its opposite which gives it a kinetic identity and a potential identity.

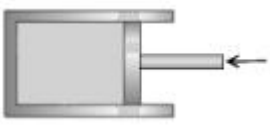
It extends in one dimension wherein it is stationary, located and discrete.

It progresses continuously over paths in the opposite dimension.

The significance of entities and the parallel nature of pure entities have been quite ignored by both physicists and philosophers of science. But it is not our task here to remedy that. Our present concern is narrowly focused upon certain issues of special relativity first of which is the constant speed of light for all observers. This topic is best approached indirectly by looking at how it is possible to stress an entity.

1.3 Stressing pure entities

Entities can be stressed (compressed or rarefied) in their extension dimension. They cannot be stressed in their progression dimension. A material entity extends in space and that is where it may be stressed. Suppose we have a space-stationary (inertial) material object we wish to stress, say the air within a cylinder with a piston at one end. Pushing on this piston constitutes work the energy of which is transferred to, and stored by, the trapped air. This air is now warmer, denser and higher in pressure. As a material entity the air's extension in space and its stored energy have been changed



by stressing.

Now consider a single photon emitted from a source and progressing toward an observer. To stress this pure entity the observer must compress it in its extension dimension which for the photon is time. This is to say that photon oscillation spans a certain time interval and compressing this interval raises photon frequency. If the observer increases her speed toward the photon's source, work is done which results in two changes. First, the observer has increased her kinetic energy relative to the photon source. Second, the observer has Doppler stressed the approaching photon raising its frequency. A compressed (higher frequency) photon stores more releasable mass due to $E = mc^2$ where E in this case equals Planck's constant times the photon frequency. The photon's extension in time (wave period) and its stored mass have been changed by stressing.

Compressing pure entities constitutes positive work done. The entity compressed has a greater stored quantity and it quanta become closer in space or in time. The opposite case is a diminution of what is stored and this constitutes negative work. A gas that expands gets colder and has less stored energy per mole. A photon heading toward an observer who is moving away from the light source has lower frequency and stores less mass.

1.4 Conclusions

Stressing pure entities affects them in their extension dimension, space for inertial mass and time for the photon. Stresses applied by an observer have no affect whatsoever on pure entity progression. The photon will always progress in space at a constant rate regardless of observer velocity stress. The inertial mass will always progress in time at a constant rate regardless of observer pressure stress. Since the photon is massless and progresses in only one dimension it cannot be a projectile and Galilean velocity addition does not apply to it.

When Einstein had his splendid insight regarding what we know as special relativity he made the constant velocity of the photon into a postulate. This is understandable, even admirable, in the context of 1905 but it is misguided. There are a number of problems with Einstein's second postulate.

- First of all, it is ad hoc. It addresses but does not solve a specific problem. It becomes a substitute for an explanation.
- Second, it is a postulate at the physics level and postulates should reside at a more foundational level.
- Third, it is derivative from a foundational point of view and hence it is unnecessary.

1.4.1 Replacement

The second postulate of special relativity should be replaced by a more fundamental (ontological) assertion from which the constant velocity of light may be derived.

Work done on any pure entity is done upon the extension dimension and has no effect on entity progression.

Replacing the second postulate of special relativity in this way avoids the dilemma posed by the velocity addition requirement of Galilean relativity. Unfortunately we still have no satisfactory explanation for the space contraction and time dilation of material objects. For that we have to extend our analysis of the nature of entities.

Part II - Space Contraction and Time Dilation

2.0 Background

Theories that address the failure of different inertial observers to agree on space and time dimensions tend to fall into two categories. Kinematic (descriptive) theories point out the inevitable dimensional disagreement of observers forced to measure remote and moving objects via light signals. Formulas for translating points (or intervals) from one inertial coordinate system to another are offered (the Lorentz transforms). Dynamic (explanatory) theories go beyond this and suggest what physical changes in matter might account for rods that shrink and clocks that run slow for a

moving system. Dynamic accounts tend to view physical change as real while kinematic accounts tend to view such change as apparent.

Einstein takes the kinematic approach in his 1905 paper[2]. For Einstein space contraction in his 1905 paper was an apparent effect for the observer rather than a real effect.[3]

H.A. Lorentz regarded the space contraction of material objects as physically real since it was a consequence of the effect the aether had on the electrostatic forces holding atoms together. Hence Lorentz has a causal explanation for dimensional transformation and he also retains a Newtonian world where time is separate from space. For this reason his theory has drawn some recent admiration and even adherents.[4]

Hermann Minkowski took the space and time (Lorentz) transforms that Einstein used and showed they could be represented geometrically where time was merely an additional dimension. Minkowski took his four-dimensional space-time ontologically since "it was not merely a geometrical representation of the world…; it was the world.[5] For Minkowski relative velocity merely reveals different spacetime views of a hidden reality.

All of these interpretations have both their adherents and their defects. Einstein's 1905 version does not explain any of the material transformations.[6] Lorentz requires us to accept an aether we can neither measure nor detect. And Minkowski denies the reality of our familiar world which other branches of science (quantum mechanics, astronomy, geology, biology, etc.) take for granted. Is there any hope for an explanation of dimensional transformations between inertial observers that conforms to our familiar world where mass and energy interact in three dimensions of space plus a separate one in time?

2.1 High velocity and high energy

Dimensional warpage (space contraction and time dilation) and relativistic mass increase are all consequences of high velocity. These changes are objective and are not a simple consequence of signaling problems. For example, unstable particles at high velocities have a retarded disintegration time interval (half life). What is it about high velocity, and the high kinetic energy it creates, that results in these objective changes?

We know that small particles such as electrons with high energy and high velocity exhibit wave behavior. This was first suggested by Louis de Broglie and later confirmed in the laboratory. So what is the difference between such particles and a meter stick when both are traveling at nine-tenths of the speed of light? They both are space contracted by the factor τ and both are time dilated by the inverse factor $1/\tau$ where $\tau = \sqrt{1 - v^2/c^2}$ **.** Of course the meter stick is vastly more massive than the electron, but velocity affects the increase of relativistic mass for both by the same factor: $m_{rel} = m_{rest}/\tau$. Since any rest mass, however small, will generate infinite relativistic mass at the velocity of light no rest mass can achieve that velocity.

Just as the effective mass (rest mass plus relativistic mass) of any material object increases without limit the closer you get to the velocity of light, so does the momentum of an object. And momentum is inversely proportional to the wavelength λ of an object: $\lambda = h/mv$. As a result of the latter, a meter stick and an electron at the same velocity relative to an observer have, for that observer, vastly different wavelengths. The electron's wavelength is large enough to generate diffraction effects in the laboratory, but the meter stick's wavelength is so infinitesimal that diffraction effects are precluded. But diffraction effects are irrelevant when it comes to the fundamental nature of the waveform; waves are waves whatever their wavelength or frequency. At the same high velocity the meter stick and the electron

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[2] Albert Einstein, "On the Electrodynamics of Moving Bodies," translation by George Barker Jeffery and Wilfrid Perrett in *The Principle of Relativity* (London: Methuen and Company, Ltd. 1923). Also available on the web, accessed March, 2014, https://www.fourmilab.ch/etexts/einstein/specrel/www/

[3] Jaykov Foukzon, Israel Institute of Technology, "Generalized Principle of Limiting 4-Dimensional Symmetry. Solution of the 'Two-Spaceship Paradox'," http://arxiv.org/ftp/arxiv/papers/0805/0805.2820.pdf, 4-5, accessed March 2014.

[4] See Harvey R. Brown, *Physical Relativity: Space-time structure from a dynamical perspective* (Oxford: Clarendon Press. 2005). Also J. S. Bell, "How to teach special relativity," in *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press, 1987), 67-80.

[5] William Lane Craig, "The metaphysics of special relativity: three views," in *Einstein, Relativity and Absolute Simultaneity*, ed. William Lane Craig, Q. Smith (Routledge 2004), 13.

[6] Craig, op.cit., 23.

both possess the de Broglie waveform. So how does the waveform relate to the effects we see in special relativity, namely space contraction and time dilation?

2.2 The projectile as hybrid entity

We have seen (Part I) that pure entities extend in one dimension and progress in the opposite dimension. Kinetic mass by itself as a field extends in space and races along in time. Kinetic energy by itself as a wave extends in time and races along in space. Any material object (technically speaking, a field form kinetic mass) at high velocity acquires the de Broglie waveform. Such projectiles therefore have a hybrid nature: part field due to rest mass and part wave due to de Broglie effects. Fast moving projectiles are hybrid because they have material mass field form acquiring de Broglie waveform. When you combine the two forms, wave and field, then extension and progression in both space and time get merged and altered.

Specifically, if a hybrid entity (projectile) approaches the speed of light, then the rapid time progression of existing mass gets moderated by an occurring (de Broglie) wave identity whose progression is spatial not temporal. That is, the dynamic time progression of kinetic mass as field becomes dominated by the static time extension of kinetic energy as wave. Time slows down (dilates) for the hybrid entity as seen by the "space-stationary" observer. This is a case of progression being superseded by extension for the time dimension.

The reverse happens for the space dimension of the kinetic mass. As a hybrid entity (a projectile) approaches the speed of light the static space extension (interval) of kinetic mass as field is dominated by the dynamic space progression of kinetic wave energy. The space extension of kinetic mass gets squeezed (contracted) by the domination of kinetic energy space progression. Recall that as you approach pure waveform (the photon) all extension must be in time and none in space. Any projectile that approaches photon velocity for some observer will, photon-like, have its extension more in time than in space for that observer. Space contraction in the direction of motion is the result.

The warpage (space contraction, time dilation) of a moving object is a natural consequence of that object possessing two forms that have opposite extension and progression dimensions. Whatever form dominates a moving object, wave or field, that form will impose its version of extension and progression. If both forms are present equally then the resulting object will be half field and half wave and extension and progression will be affected accordingly. All of this is true regardless of the mass of the speeding object. At velocities close to the speed of light the waveform will dominate whatever object is moving: an electron, a meter stick or a planet. With that in mind let's look at two observers who disagree about muon lifetime.

2.3 Muon case study

Muons are short-lived particles of small mass created by cosmic rays interacting with the earth's upper atmosphere. These projectile particles (hybrid entities) have a half-life of less than 2 microseconds and travel very close to the speed of light. Even with their great speed they require over 100 half-lives to reach the surface of the earth which suggests that none of them do. Yet that is not the case because time has apparently slowed down for them. However, an observer accompanying a muon does not experience any change in the progression of time.

For an observer on the earth the muon is mostly kinetic energy waveform due to its extreme velocity. Pure waves (the photon) that are exclusively kinetic energy have time only as extension, not as progression (time is stationary). Accordingly, the muon as near-wave and mostly energy for the earth observer has a small amount of time as progression (due to its rest mass), but much more of time as (static) extension (due to its wave nature). The earth observer measures a drastic slow down of time for the muon as wave-dominated hybrid entity (projectile). Of course, an observer accompanying the muon is only presented with the muon's kinetic mass and measures "proper" (fast) time for the muon's disintegration.

The observer within the muon's inertial system finds the earth approaching at close to the speed of light. This means the earth is now the hybrid entity that is mostly wave (of very short wavelength). We know that pure waves (the photon) have space only as progression, not as extension. With the earth now much more wave than field for the muon, the earth's extension in space as kinetic mass is now dominated by the earth's progression in space as kinetic (wave) energy. The earth's spatial extension has become spatial progression with an inevitable contraction. The observer aboard the muon finds the muon's lifetime to be normal but decides that the surface of the earth can be reached because the distance the muon must travel after birth is much less than the earth observer claims.

For the earth observer space travelled is local (within its inertial system); for the muon observer time lapsed is local (it has the disintegration "clock"). Each of them finds the non-local dimension "warped" because they are confronted with a near-wave hybrid entity approaching them that objectively either has its space mostly progressing rather than extending (space contraction) or its time mostly extending rather than progressing (time dilation). So what has been transformed, the hybrid entity itself or space and time itself?

2.4 Theories of dimensional change

The assumption of physicists, Einstein included, has been that adding energy to material objects via relative velocity does not change these objects. Somehow energy of motion is a benign and observer-relative quantity that can be added and subtracted without effect. So if objects change (contract in space, slow down in time) and their relative velocity (and energy) is not the cause, then it must be space and time itself that changes the objects. This view of space and time as actors has been quite popular over the years and we may cite a few of its supporters here. According to John Norton (my italics):

> "Special relativity, as *a theory of space and time*, cannot make pronouncements by itself on energy, mass and matter. It can only constrain the ways that they can manifest in space and time…."[7]

Michael Friedman agrees that special relativity is all about space and time; he embraces the "physicalization of [Minkowskian] geometry."[8] Vesselin Petkov argues that it is spacetime that determines how objects appear.[9] True reality for Petkov resides with the four-dimensional worldtubes of Minkowski spacetime; our world of three space dimensions is illusory.

Unfortunately these arguments are based upon a false premise, namely that velocity and the kinetic energy it creates cannot affect material objects. One side result of this misconception is that the subject of kinetic energy and relativistic mass due to velocity is absent from discussions on special relativity.

It is very revealing that when special relativity theorists discuss meter sticks moving at very high velocity they ignore relativistic mass and wavelength measures and only discuss space contraction of the meter stick. In contrast, when quantum mechanics theorists discuss electrons moving at very high velocity they discuss relativistic mass increase plus the wave properties of the electron. But except for (irrelevant) wavelength measures, both cases are identical! When you add enough velocity to a material object so that kinetic energy dominates kinetic mass, then waveform dominates field form and you get the following situation: space contracts, time dilates, relativistic mass dominates rest mass and wave effects appear (diffraction being measurable only if the rest mass is very small). By equating wave behavior with diffraction effects, special relativity theorists have overlooked the fact that their large material objects (meter sticks, planets) at extreme relative velocities are in fact subject to wave effects other than diffraction, namely: space as progression not as extension and time as extension not as progression.

2.5 What Is relative?

To assert that space and time in and of themselves are warped by observer velocity is unsupported by the facts of special relativity. Space contraction and time dilation were in need of an explanation after 1905 when opinion turned against the idea of the aether. Making space and time into an agent of change for material objects was an easy leap to make, but it was a mistake.

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[7] John D. Norton. 2004. "Einstein's Special Theory of Relativity and the Problems in the Electrodynamics of Moving Bodies that led him to it." Accessed March 2014, http://www.pitt.edu/~jdnorton/papers/companion.pdf, 34.

[8] Michael Friedman, *Foundation of Space-Time Theories: Relativistic Physics and Philosophy of Science* (Princeton University Press, Princeton, New Jersey, 1983), chap. IV.

[9] Petkov, "The muon experiment...demonstrated that space itself contracts relativistically." From Vesselin Petkov, "Accelerating spaceships paradox and physical meaning of length contraction." 2009 ArXiv document, accessed March 2014, http://arxiv.org/pdf/0903.5128v1.pdf, 4.

The reality is that inertial observers with different velocities relative to a common material object (projectile) are each encountering a different blend of kinetic mass and kinetic energy, of field form and waveform, and therefore a different (hybrid) entity. Since these different observers measure different entities, of course they find different splits between wave and field, between time and space and between rest mass and relativistic mass. Each hybrid entity is unique for a specific observer because that observer alone determines the singular kinetic energy filling out the hybrid entity's identity.

What is fundamentally relative for different observers with different velocities is the reality of the hybrid entity; experiencing space, time and mass measures differently follows from that. It is not the case that the velocity of an inertial system (relative to some observer) changes/warps the space or time metric for that inertial system. Space contraction and time dilation were in need of an explanation after 1905 when opinion turned against the idea of the aether. Making space and time into an agent of change for material objects was an easy leap to make but it was a misstep. Rather it is the material meter sticks and clocks as hybrid entities that undergo the change. This change is objective for the observer yet it is also subjective in the sense that it is only true for that observer.

2.6 Conclusion

Two mistakes have bedeviled relativity discussions for well over a century and they probably still have legs. The first mistake is that light is some kind of projectile progressing in two dimensions (space and time) and its violation of Galilean relativity can be resolved by a postulate. The second mistake is the idea that projectiles are simple so adding kinetic energy (velocity) to kinetic mass does not change the nature or identity of that mass: this wrong idea that two observers in relative motion are measuring the same object which should have the same space and time measurements.

The correction for both of these mistakes is a proper (ontological) understanding of entities in physics. Pure entities of kinetic mass or kinetic energy have a pure form (wave or field) and progress uniformly in only one dimension for all observers; hence the kinetic pure energy entity (the photon) is not subject to Galilean relativity. And when de Broglie waveform and rest mass field form combine as projectile they create a hybrid entity whose mass, space and time measures are unique for each inertial observer.

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