Supplement to “Metalinguistic Gradability”

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This document contains further technical discussion of the semantic framework developed in “Metalinguistic Gradability” (Rudolph and Kocurek, 2024). There, we developed a novel semantics for constructions like the following:

(1) a. Ann is more a linguist than a philosopher.
    b. Ann is (exactly) as much a linguist as a philosopher.
    c. Ann is very much/sorta a linguist.

We use the term **metalinguistic gradability** (or **metagradability** for short) to classify these constructions. In many ways, these constructions parallel their “ordinary” gradable counterparts, as in (2).

(2) a. Ann is taller than Ben.
    b. Ann is (just) as tall as Ben.
    c. Ann is very/sorta tall.

But in some respects, the morphosyntactic and semantic properties of these two constructions diverge. Our task in Rudolph and Kocurek 2024 was to develop a general theory of metalinguistic gradability that would encompass not just metalinguistic comparatives, such as in (1a), which have been studied extensively elsewhere (Huddleston and Pullum, 2002, Giannakidou and Yoon, 2011, Giannakidou and Stavrou, 2009, Morzycki, 2011, Wellwood, 2014, 2019, Rudolph and Kocurek, 2020), but also metalinguistic equatives, degree modifiers, and conditionals.

The basic idea we develop in Rudolph and Kocurek 2024 is to analyze metalinguistic gradability in terms of the relative strength of a speaker’s commitment to an interpretation. Roughly, (1a) expresses a stronger commitment towards interpretations which count Ann exclusively in the extension of ‘is a linguist’ than towards those which count Ann exclusively in the extension of ‘is a philosopher’. This leads to an approach we call **semantic expressivism**, which holds that when speakers make assertions, they express not only their factual commitments, i.e., their beliefs about the world, but also their semantic commitments, i.e., their plans for how to interpret linguistic expressions (cf. Barker 2013, Kocurek et al. 2020, Mena 2023, Kocurek 2023).

For ease of exposition, we start in section A by quickly reviewing the basic semantics for metagradable constructions developed in Rudolph and Kocurek 2024. Section B discusses amendments that are needed to the basic semantics to ensure the transitivity of the metalinguistic equative. Section C introduces a degree-theoretic formulation of this generalized semantics. Section D generalizes the semantics further to allow nontotal semantic orderings. Section E extends the semantics with quantifiers, which allows us to formalize the different metalinguistic "superlative" readings.

A Review of the Basic Semantics

We start by analyzing metalinguistic comparatives (MCs). To do this, we introduce an operator $\succ$, where $A \succ B$ can be read "It is more the case that A than that B". We also introduce an operator $\approx$ for metalinguistic equatives (MEs), where $A \approx B$ stands for "It is exactly as much the case that A as it is that B".

Here, then, is our starting language:

$$A ::= Pa_1 \ldots a_n \mid \neg A \mid (A \wedge A) \mid (A \succ A) \mid (A \approx B)$$

We'll define the other boolean connectives $\vee$, $\supset$, and $\equiv$ in the standard way. Throughout, I'll use lowercase $p, q, r, \ldots$ for propositional variables, which can be understood as 0-place predicates. We define $\top := (p \vee \neg p)$ and $\bot := \neg\top$. Finally, we further define a "weak" metalinguistic comparative $\succcurlyeq$, where $A \succcurlyeq B$ stands for "It is at least as much the case that A as it is that B", as follows:

$$(A \succcurlyeq B) := (A \succ B) \vee (A \approx B).$$

In the basic semantics, we could define $A \succcurlyeq B$ as $\neg(B \succ A)$ and $A \approx B$ as $\neg(A \succ B) \wedge \neg(B \succ A)$. But this definition depends on the assumption that our ordering of interpretations is total, which we consider dropping in section D.

The truth of a formula is evaluated relative to three parameters: a world, an interpretation, and a semantic ordering. Given a nonempty set of worlds W and nonempty set of objects D, an **interpretation** is a function i where:

(i) $i(a)\colon W \to D$ for each name a

(ii) $i(P^n)\colon W \to \wp(D^n)$ for each n-place predicate P^n

A **semantic ordering** is a total preorder $\leqslant$ on a set I of interpretations, i.e., for all $i, j, k \in I$:

- *reflexivity:* $i \leqslant i$
- *transitivity:* if $i \leqslant j \leqslant k$, then $i \leqslant k$
- *totality:* either $i \leqslant j$ or $j \leqslant i$.

A model in our framework, then, is a tuple $\langle W, D, I, \leqslant\rangle$, where W is a set of worlds, D is a set of objects, I is a set of interpretations over W and D, and $\leqslant$ is a semantic ordering over I. Throughout, the background model will be left implicit.

The truth conditions for atomics and booleans are standard. Here are the truth conditions for our specifically metalinguistic operators ($\succ$, $\approx$, $\succcurlyeq$):

$$\llbracket A \succ B \rrbracket^{\leqslant,i,w} = 1 \quad \text{iff} \quad \exists i' \leqslant i\colon \text{(i) } \llbracket A \wedge \neg B \rrbracket^{\leqslant,i',w} = 1$$
$$\text{(ii) } \forall i'' \leqslant i\colon \llbracket B \wedge \neg A \rrbracket^{\leqslant,i'',w} = 1 \Rightarrow i'' < i'.$$

$$\llbracket A \succcurlyeq B \rrbracket^{\leqslant,i,w} = 1 \quad \text{iff} \quad \forall i' \leqslant i\colon \text{if } \llbracket B \wedge \neg A \rrbracket^{\leqslant,i',w} = 1,$$
$$\text{then } \exists i'' \leqslant i\colon \llbracket A \wedge \neg B \rrbracket^{\leqslant,i'',w} = 1 \;\&\; i' \leqslant i''.$$

$$\llbracket A \approx B \rrbracket^{\leqslant,i,w} = 1 \quad \text{iff} \quad \forall i' \leqslant i\colon \text{(i) if } \llbracket A \wedge \neg B \rrbracket^{\leqslant,i',w} = 1,$$
$$\text{then } \exists i'' \leqslant i\colon \llbracket B \wedge \neg A \rrbracket^{\leqslant,i'',w} = 1 \;\&\; i' \leqslant i''$$
$$\text{(ii) if } \llbracket B \wedge \neg A \rrbracket^{\leqslant,i',w} = 1,$$
$$\text{then } \exists i'' \leqslant i\colon \llbracket A \wedge \neg B \rrbracket^{\leqslant,i'',w} = 1 \;\&\; i' \leqslant i''.$$

In many of the proofs, it is cumbersome to keep mentioning $\leqslant$ and w, which never change as our language does not include operators that shift them. For readability, then, we'll often drop mention of $\leqslant$ and w. Thus, we may write $\llbracket A \rrbracket^i$ in place of $\llbracket A \rrbracket^{\leqslant,i,w}$ where $\leqslant$ and w are implicit.

In Rudolph and Kocurek 2024, we defined two notions of consequence: truth-preservation ($\vDash$) and acceptance-preservation ($\Vdash$). The former is defined in the standard way: $A_1, \ldots, A_n \vDash B$ iff $\llbracket B \rrbracket^{\leqslant,i,w} = 1$ whenever $\llbracket A_1 \rrbracket^{\leqslant,i,w} = \cdots = \llbracket A_n \rrbracket^{\leqslant,i,w} = 1$. The latter can be defined in terms of the former using the following abbreviation:

$$\boxminus A \coloneqq (A \succ \neg A).$$

Here, $\boxminus A$ stands for the claim that A is accepted, i.e., it is true at all the top-ranked interpretations.

Fact 1. $\llbracket \boxminus A \rrbracket^{\leqslant,i,w} = 1$ iff $\forall i' \equiv i\colon \llbracket A \rrbracket^{\leqslant,i',w} = 1$.

Using this, we can define $A_1, \ldots, A_n \Vdash B$ as short for $\boxminus A_1, \ldots, \boxminus A_n \vDash \boxminus B$ (i.e., whenever the premises are accepted, the conclusion is, too).

Since $\vDash$ captures truth preservation across *all* ranked interpretations while $\Vdash$ captures truth preservation across all the *top-ranked* interpretations, we immediately have the following:

Fact 2. If $A_1, \ldots, A_n \vDash B$, then $A_1, \ldots, A_n \Vdash B$.

As we note in Rudolph and Kocurek 2024, (section 4.4), the converse isn't true. In particular, $A, \neg B \Vdash A \succ B$ (in line with Observation 3 from section 2.5) even though $A, \neg B \nvDash A \succ B$. By the same token, we observed that $\Vdash$ is nonclassical as it invalidates proof by cases: as desired, $\nVdash \boxminus A \vee \boxminus \neg A$ even though $A \Vdash \boxminus A$ and $\neg A \Vdash \boxminus \neg A$ (and $\Vdash A \vee \neg A$).

We now record the following entailment facts, including the other observations from section 2.5.

Fact 3.

(a) $A \succ B, A \wedge B \not\models \bot$ and $A \succ B, \neg A \wedge \neg B \not\models \bot$ (Observation 1)

(b) $A \succ B, B \models A$ (Observation 2)

(c) $A \approx B \models \neg(A \succ B) \wedge \neg(B \succ A)$ (Observation 4)

(d) $A \approx \neg A \not\models \bot$ (Observation 5)

(e) $A \succ B \models \neg(B \succ A)$ ($\succ$ is strict)

(f) $\models \neg(A \succ A)$ ($\succ$ is irreflexive)

(g) $A \succ B, B \succ C \models A \succ C$ ($\succ$ is transitive)

(h) $A \succ B \models \neg B \succ \neg A$ ($\succ$ obeys contraposition)

(i) $A \succ (B \vee C) \models (A \succ B) \wedge (A \succ C)$ ($\succ$ obeys right strengthening)
(in fact: if $B \models C$, then $A \succ C \models A \succ B$)

(j) $(A \wedge B) \succ C \models (A \succ C) \wedge (B \succ C)$ ($\succ$ obeys left weakening)
(in fact: if $A \models B$, then $A \succ C \models B \succ C$)

(k) $\models A \approx A$ ($\approx$ is reflexive)

(l) $A \approx B \models B \approx A$ ($\approx$ is symmetric)

(m) $A \approx B \models \neg A \approx \neg B$ ($\approx$ obeys contraposition)

(n) $\models (A \succ B) \vee (B \succ A) \vee (A \approx B)$ (totality; see section D)

Reminder: In the proofs below, we allow ourselves to drop mention of $\leqslant$ and w in the indices for readability, writing $[\![A]\!]^i$ in place of $[\![A]\!]^{\leqslant,i,w}$.

Proof. Most of these are straightforward to verify. The most difficult one is (g), i.e., the transitivity of $\succ$. We'll assume (A) and (B) to establish (C) below:

(A) $\exists i_a \leqslant i\colon [\![A \wedge \neg B]\!]^{i_a} = 1 \,\&\, \forall j \leqslant i\colon [\![B \wedge \neg A]\!]^j = 1 \Rightarrow j < i_a$. $(A \succ B)$

(B) $\exists i_b \leqslant i\colon [\![B \wedge \neg C]\!]^{i_b} = 1 \,\&\, \forall j \leqslant i\colon [\![C \wedge \neg B]\!]^j = 1 \Rightarrow j < i_b$. $(B \succ C)$

(C) $\exists i_c \leqslant i\colon [\![A \wedge \neg C]\!]^{i_c} = 1 \,\&\, \forall j \leqslant i\colon [\![C \wedge \neg A]\!]^j = 1 \Rightarrow j < i_c$. $(A \succ C)$

In particular, we'll show that we can take i_c to be either i_a or i_b, depending on which (if any) is higher.

Suppose first that $i_a \leqslant i_b$. By (A), $[\![A]\!]^{i_b} = 1$. (So $[\![A \wedge \neg C]\!]^{i_b} = 1$.) Suppose for reductio that $i_b \leqslant j \leqslant i$ where $[\![\neg A \wedge C]\!]^j = 1$. If $[\![B]\!]^j = 1$, then we violate (A), since $i_a \leqslant i_b \leqslant j$ and $[\![B \wedge \neg A]\!]^j = 1$. If $[\![B]\!]^j = 0$, then we violate (B), since $i_b \leqslant j$ and $[\![C \wedge \neg B]\!]^j = 1$. Contradiction. Hence, if $j \leqslant i$ and $[\![\neg A \wedge C]\!]^j = 1$, then $j < i_b$.

Now suppose $i_b \leqslant i_a$. By (B), $[\![C]\!]^{i_a} = 0$. (So $[\![A \wedge \neg C]\!]^{i_a} = 1$.) Suppose for reductio that $i_a \leqslant j \leqslant i$ where $[\![\neg A \wedge C]\!]^j = 1$. If $[\![B]\!]^j = 1$, then we violate (A). If $[\![B]\!]^j = 0$, then we violate (B). Contradiction. Hence, if $j \leqslant i$ and $[\![\neg A \wedge C]\!]^j = 1$, then $j < i_b$. $\square$

B The Revised Semantics: Transitivity for MEs

Though the basic semantics presented can explain quite a few entailment patterns governing MCs and MEs, there is one entailment governing MEs that it fails to predict: the transitivity of MEs. Intuitively, such inferences seem quite plausible:

(3) a. Ann is as much a linguist as she is a philosopher.
  b. Ann is as much a philosopher as she is a psychologist.
  c. Thus, Ann is as much a linguist as she is a psychologist.

The basic semantics does not validate this inference: $A \approx B, B \approx C \not\models A \approx C$. Figure 1 contains a minimal counterexample. To make the counterexample more intuitive, let's interpret A, B, and C as follows:

- A = "Ann is a linguist"
- B = "Ann is a philosopher"
- C = "Ann is a psychologist"

Then, intuitively, the following inference seems valid. Yet in situations like the one depicted in Figure 1, (4a–b) are true according to the basic semantics while (4c) is false (specifically, it predicts Ann is more a linguist than a psychologist).

(4) a. Ann is as much a linguist as she is a philosopher.
  b. Ann is as much a philosopher as she is a psychologist.
  c. Thus, Ann is as much a linguist as she is a psychologist.

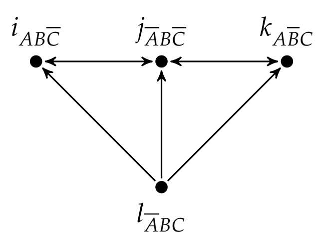

Figure 1: Counterexample to the transitivity of $\approx$ in the basic semantics from section 4.2. Subscripts indicate which of A, B, and C is true at that interpretation. Reflexive and transitive arrows are omitted from the diagram.

Examining this counterexample, it seems like this example *should* satisfy $A \approx C$: while there is a top-ranked interpretation making A true and C false, it is ranked equally highly with a $(A \wedge C)$-interpretation and with a $(\neg A \wedge \neg C)$-interpretation. That should be sufficient to make $A \approx C$ true. As it stands, however, our current semantics doesn't predict this, since the top-ranked $(A \wedge \neg C)$-interpretation is not matched by any equally-ranked $(\neg A \wedge C)$-interpretation.

Interestingly, as we observed in Fact 3(g), the basic semantics *does* validate transitivity for $\succ$. Still, the failure of transitivity for $\approx$ suggests we need to revise the semantics of MEs. And if we weaken the semantics for MEs so that $A \approx C$ comes out true in Figure 1, we must also strengthen the semantics for MCs so that $A \succ C$ and $A \approx C$ are incompatible. This means we must revise the semantics for MCs, even though $\succ$ doesn't suffer from transitivity failures.

Here is our proposed amendment to the semantics of MEs:

$$
\begin{aligned}
[\![A \approx B]\!]^{\leqslant,i,w} = 1 \quad \text{iff} \quad & \text{either:}\\
& \text{(i) } \forall i' \leqslant i:\\
& \quad \text{(a) } [\![A \wedge \neg B]\!]^{\leqslant,i',w} = 1 \Rightarrow\\
& \quad\quad \exists i'' \leqslant i : [\![B \wedge \neg A]\!]^{\leqslant,i'',w} = 1 \;\&\; i' \leqslant i''\\
& \quad \text{(b) } [\![B \wedge \neg A]\!]^{\leqslant,i',w} = 1 \Rightarrow\\
& \quad\quad \exists i'' \leqslant i : [\![A \wedge \neg B]\!]^{\leqslant,i'',w} = 1 \;\&\; i' \leqslant i''\\
& \text{or:}\\
& \text{(ii) } \forall i' \leqslant i : \text{if } [\![A \leftrightarrow \neg B]\!]^{\leqslant,i,w} = 1, \text{ then:}\\
& \quad \text{(a) } \exists i'' \leqslant i : [\![A \wedge B]\!]^{\leqslant,i'',w} = 1 \;\&\; i' \leqslant i'' \text{ and}\\
& \quad \text{(b) } \exists i'' \leqslant i : [\![\neg A \wedge \neg B]\!]^{\leqslant,i'',w} = 1 \;\&\; i' \leqslant i''.
\end{aligned}
$$

In words, $A \approx B$ is true iff either every $(A \wedge \neg B)$-interpretation is matched with a $(B \wedge \neg A)$-interpretation that's ranked at least as high and vice versa (i.e., the semantics in section 4.2), or else every $(A \wedge \neg B)$-interpretation is matched with some $(A \wedge B)$-interpretation and some $(\neg A \wedge \neg B)$-interpretation that's ranked at least as high and similarly for every $(B \wedge \neg A)$-interpretation—exactly the kind of situation depicted in Figure 1.

Using totality as a guide for how to revise the semantics of MCs, we obtain the following:[1]

$$
\begin{aligned}
[\![A \succ B]\!]^{\leqslant,i,w} = 1 \quad \text{iff} \quad & \exists i' \leqslant i:\\
& \text{(i) } [\![A \wedge \neg B]\!]^{\leqslant,i',w} = 1\\
& \text{(ii) } \forall i'' \leqslant i : [\![B \wedge \neg A]\!]^{\leqslant,i'',w} = 1 \Rightarrow i'' < i'\\
& \text{(iii) either:}\\
& \quad \text{(a) } \forall i'' \leqslant i : [\![A \wedge B]\!]^{\leqslant,i'',w} = 1 \Rightarrow i'' < i' \text{ or:}\\
& \quad \text{(b) } \forall i'' \leqslant i : [\![\neg A \wedge \neg B]\!]^{\leqslant,i'',w} = 1 \Rightarrow i'' < i'.
\end{aligned}
$$

In words, $A \succ B$ is true iff there is an $(A \wedge \neg B)$-interpretation that's ranked higher than any $(B \wedge \neg A)$-interpretation (i.e., the semantics in section 4.2) and there aren't both $(A \wedge B)$-interpretations and $(\neg A \wedge \neg B)$-interpretations that are ranked at least as high as it. Applied to Figure 1, condition (iii) is what ensures that $A \succ C$ is false.

[1]This semantics can be generalized to cover failures of totality; see section D.

We can simplify the truth conditions for $>$ in this revised semantics. Essentially, $A > B$ is true iff there's an $(A \wedge \neg B)$-interpretation that is either higher than any B-interpretation or higher than any $\neg A$-interpretation.

$$[\![A > B]\!]^{\leqslant,i,w} = 1 \quad \text{iff} \quad \exists i' \leqslant i:$$

$$\text{(i)}\ [\![A \wedge \neg B]\!]^{\leqslant,i',w} = 1$$

$$\text{(ii) either:}$$

$$\text{(a)}\ \forall i'' \leqslant i: [\![B]\!]^{\leqslant,i'',w} = 1 \Rightarrow i'' < i' \text{ or:}$$

$$\text{(b)}\ \forall i'' \leqslant i: [\![\neg A]\!]^{\leqslant,i'',w} = 1 \Rightarrow i'' < i'.$$

While it is a chore, one can verify that all of the entailment patterns Fact 3 still hold in the revised semantics for $>$ and $\approx$. Since the proof of transitivity for $>$ was involved even for the basic semantics, we'll provide the proof that this entailment still holds. Throughout, let $\vDash_r$ stand for truth-preservation in the revised semantics.

Fact 4. $A > B, B > C \vDash_r A > C$.

Proof. Recall in the proof of Fact 3(g), we showed either i_a or i_b could be our witness i_c for conditions (i) and (ii) of $A > C$, depending on which is higher. We'll now show that, whichever it is, it also satisfies condition (iii) for $A > C$ (given i_a satisfies (iii) for $A > B$ and i_b satisfies (iii) for $B > C$). Here, we just do the $i_a \leqslant i_b$ case, since the $i_b \leqslant i_a$ is similar.

Suppose $i_a \leqslant i_b$ (so i_b is our witness i_c). Since i_a satisfies condition (iii) for $A > B$, and since $[\![A \wedge B]\!]^{i_b} = 1$, that means i_a satisfies condition (iiib) in particular, i.e., if $i_a \leqslant i'' \leqslant i$, then $[\![\neg A \wedge \neg B]\!]^{i''} = 0$, i.e., $[\![A \vee B]\!]^{i''} = 1$. But if $[\![A]\!]^{i''} = 0$ where $i_a \leqslant i'' \leqslant i$, that means $[\![B]\!]^{i''} = 1$, which violates (A). Hence, if $i_a \leqslant i'' \leqslant i$, then $[\![A]\!]^{i''} = 1$. So if $i_b \leqslant i'' \leqslant i$, then since $i_a \leqslant i_b$, we have $[\![A]\!]^{i''} = 1$, and so $[\![\neg A \wedge \neg C]\!]^{i''} = 0$, i.e., i_b satisfies (iiib) for $A > C$. □

Fact 5. All of the entailment patterns in Fact 3 still hold for $\vDash_r$.

And of course, the revised semantics now validates transitivity for $\approx$, as we'll now demonstrate:

Fact 6. $A \approx B, B \approx C \vDash_r A \approx C$.

Proof. Suppose (A) $[\![A \approx B]\!]^i = 1$ and (B) $[\![B \approx C]\!]^i = 1$. We want to show;

(C1) If (iia) for $A \approx C$ fails, then condition (i) for $A \approx C$ holds.

(C2) If (iib) for $A \approx C$ fails, then condition (i) for $A \approx C$ holds.

Notice that it suffices to establish (C1) given that $A \approx C \vDash \neg C \approx \neg A$. For condition (iib) for $A \approx C$ is the same as condition (iia) for $\neg C \approx \neg A$. So if condition (iib) for $A \approx C$ fails, then condition (iia) for $\neg C \approx \neg A$ fails, which by

(C1) means condition (i) for $\neg C \approx \neg A$, which is the same as condition (i) for $A \approx C$, holds. Hence, we only need to establish (C1).

Suppose (iia) fails for $A \approx C$. So $\exists i' \leqslant i \colon [\![A \leftrightarrow \neg C]\!]^{i'} = 1$ such that $[\![A \wedge C]\!]^{i''} = 0$ whenever $i' \leqslant i'' \leqslant i$. We'll just establish (ia), since the reasoning for (ib) is symmetric. Let $k \leqslant i$ where $[\![A \wedge \neg C]\!]^{k} = 1$. Suppose for reductio that $\forall k' \leqslant i \colon k \leqslant k' \Rightarrow [\![\neg A \wedge C]\!]^{k'} = 0$. We first establish the following:

Claim 1. Let $j \leqslant i$ be such that $i' \leqslant j$ and $k \leqslant j$. Then $[\![A]\!]^j = [\![B]\!]^j = [\![C]\!]^j = 0$.

Proof. By our choice of i', $[\![A \wedge C]\!]^j = 0$. By our choice of k, $[\![\neg A \wedge C]\!]^j = 0$. Hence, $[\![C]\!]^j = 0$.

Now, suppose for reductio $[\![B]\!]^j = 1$, so that $[\![B \wedge \neg C]\!]^j = 1$. By $B \approx C$, there is a $j' \leqslant i$ where $j' \geqslant j$ and either $[\![C \wedge \neg B]\!]^{j'} = 1$ (if clause (i) for $B \approx C$ holds) or else $[\![C \wedge B]\!]^{j'} = 1$ (if clause (ii) holds). Either way, this contradicts the fact that $[\![C]\!]^{j'} = 0$ for all $j' \geqslant j$, since $j \geqslant i', k$. Hence, $[\![B]\!]^j = 0$.

Finally, suppose for reductio that $[\![A]\!]^j = 1$, so that $[\![A \wedge \neg B]\!]^j = 1$. Similar to above, by $A \approx B$, there is a $j' \leqslant i$ where $j' \geqslant j$ and either $[\![B \wedge \neg A]\!]^{j'} = 1$ or $[\![B \wedge A]\!]^{j'} = 1$, which contradicts the fact that $[\![B]\!]^{j'} = 0$ for all $j' \geqslant j$. Hence, $[\![A]\!]^j = 0$. □

Now, if $i' \leqslant k$, then by Claim 1, $[\![A]\!]^k = 0$, contrary to our choice of k. Hence, $k < i'$, and so by Claim 1, $[\![A \leftrightarrow \neg C]\!]^{i'} = 0$, contrary to our choice of i'. Contradiction. □

Thus, we can revise the basic metagradable semantics to accommodate the transitivity of $\approx$ without lose of predictive power. Cases like those depicted in Figure 1 where transitivity fails for $\approx$ are not common, however. We struggle to think of a realistic yet simple example where one would need to appeal to these revised clauses when evaluating MCs and MEs. For most purposes, then, the basic semantics is a good enough approximation to work with.

In fact, we can define the $\succ$ from the basic semantics using the $\succ$ from the revised semantics and vice versa. Let $\succ_b$ have the truth conditions of $\succ$ in the basic semantics and $\succ_r$ have the truth conditions of $\succ$ in the revised semantics. Then:

Fact 7.

(a) $[\![A \succ_b B]\!]^i = [\![(A \wedge \neg B) \succ_r (B \wedge \neg A)]\!]^i$.

(b) $[\![A \succ_r B]\!]^i = [\![((A \wedge \neg B) \succ_b \neg A) \vee ((A \wedge \neg B) \succ_b B)]\!]^i$.

In a similar vein, the revised truth conditions for $\approx$ can be defined in terms of the basic truth conditions for $\approx$ and vice versa by using the equivalence between $(A \approx B)$ and $\neg(A \succ B) \wedge \neg(B \succ A)$. The two semantic frameworks only differ, then, in which operator they take to be primitive, $\succ_b$ or $\succ_r$.

C Degree-theoretic formulation

In Rudolph and Kocurek 2024, we mentioned a degree-theoretic formulation of the expressivist semantics (assuming we adopt the revisions in section B to ensure $\approx$ obeys transitivity). More precisely, given an index $\langle \leqslant, i, w \rangle$, we can define a linear order $\langle Deg, \sqsubset \rangle$ and a mapping, deg, of sentences to elements of Deg so that $[\![A \succ B]\!]^{\leqslant,i,w} = 1$ iff $\deg(A) \sqsupset \deg(B)$ and $[\![A \approx B]\!]^{\leqslant,i,w} = 1$ iff $\deg(A) = \deg(B)$.

The key is to take the "metalinguistic degree" of a sentence A (relative to $\langle \leqslant, i, w \rangle$) to be a *set of sets of interpretations*, where each set of interpretations is "ranked as high" as the others. Let $I_i = \{j \mid j \leqslant i\}$. Define the following relation on sets of interpretations $X, Y \subseteq I_i$:

$$
\begin{array}{lll}
X \sim_i Y & \text{iff} & \text{either:} \\
& & \text{(i)}\ \ \forall i' \leqslant i: \\
& & \quad \text{(a)}\ \ i' \in X - Y \Rightarrow \exists i'' \in Y - X\colon i' \leqslant i'' \\
& & \quad \text{(b)}\ \ i' \in Y - X \Rightarrow \exists i'' \in X - Y\colon i' \leqslant i'' \\
& & \text{or:} \\
& & \text{(ii)}\ \ \forall i' \in (X \cup Y) - (X \cap Y), \text{then:} \\
& & \quad \text{(a)}\ \ \exists i'' \in X \cap Y\colon i' \leqslant i'' \text{ and} \\
& & \quad \text{(b)}\ \ \exists i'' \in \overline{X} \cap \overline{Y}\colon i' \leqslant i''.
\end{array}
$$

This definition mirrors the truth conditions for $A \approx B$ from section B. Technically, we should write $\sim_{\leqslant,i,w}$, but again, I'm leaving $\leqslant$ and w implicit throughout to reduce on cumbersome notation. To simplify further, I will also drop mention of i, writing $X \sim Y$ and leaving the choice of i implicit.

It is easy to verify that $\sim$ is reflexive and symmetric, and the proof that $\sim$ is transitive is effectively the same as the proof of Fact 6. Hence:

Fact 8. $\sim$ is an equivalence relation.

Given this, we may define the metalinguistic degree (relative to $\langle \leqslant, i, w \rangle$) of a set of interpretations X as the $\sim$-equivalence class of X, and the metalinguistic degree of a sentence as the degree of the set of interpretations where the sentence is true:

$$
\begin{aligned}
\deg(X) &= [X]_\sim = \{Y \subseteq I_i \mid X \sim Y\} \\
[\![A]\!]_i &= \{j \leqslant i \mid [\![A]\!]^i = 1\} \\
\deg(A) &= \deg([\![A]\!]).
\end{aligned}
$$

Using these definitions, it is easy to establish the following:

Fact 9. $[\![A \approx B]\!]^i = 1$ iff $\deg(A) = \deg(B)$ (i.e., $[\![A]\!]_i \sim [\![B]\!]_i$).

Now to define the ordering on degrees. This definition mirrors the truth conditions for $>$ in section B.

$$
\begin{array}{lll}
X \sqsupset Y & \text{iff} & \exists i' \leqslant i: \\
& & \text{(i)}\ \ i' \in X - Y \\
& & \text{(ii)}\ \ \forall i'' \in Y - X : i'' < i' \\
& & \text{(iii)}\ \ \text{either:} \\
& & \qquad \text{(a)}\ \ \forall i'' \in X \cap Y : i'' < i' \text{ or:} \\
& & \qquad \text{(b)}\ \ \forall i'' \in \overline{X} \cap \overline{Y} : i'' < i'.
\end{array}
$$

Fact 10. $[\![A > B]\!]^i = 1$ iff $[\![A]\!]_i \sqsupset [\![B]\!]_i$.

We can lift this order to an order on degrees as follows:

$$\deg(X) \sqsupset \deg(Y) \quad \Leftrightarrow \quad X \sqsupset Y.$$

We need to show that (1) this definition of $\sqsupset$ over degrees is well-defined in that it does not depend on the choice of X and Y, and (2) $\sqsupset$ is a linear order.

To show that this is well-defined, we need to prove that if $X' \in \deg_i(X)$ and $Y' \in \deg_i(Y)$, then $X' \sqsupset Y'$ iff $X \sqsupset Y$. It suffices to prove the following:

Fact 11. Where $X, Y, Z \subseteq I_i$:

(a) if $X \sqsupset Y$ and $Y \sim Z$, then $X \sqsupset Z$.

(b) if $X \sqsupset Y$ and $Y \sim Z$, then $X \sqsupset Z$.

Proof. We just prove (a), since the proof of (b) is similar. Suppose $X \sqsupset Y$ and $Y \sim Z$. Let $i' \leqslant i$ be our witness to $X \sqsupset Y$, i.e.:

(i) $i' \in X - Y$

(ii) $\forall i'' \in Y - X : i'' < i'$

(iii) either:

  (iiia) $\forall i'' \in X \cap Y i'' < i'$, or:

  (iiib) $\forall i'' \in \overline{X} \cap \overline{Y} : i'' < i'$.

We split the proof into two cases, depending on whether $i' \in Z$.

Case 1: $i' \in Z$**.** Thus, $i' \in Z - Y$.

Claim 2. Condition (iiib) above holds.

Proof. Since $Y \approx Z$, there is some $i'' \geqslant i'$ where either $i'' \in Y - Z$ or $i'' \in Y \cap Z$. Either way, $i'' \in Y$, and so $i'' \in X$, since i' is ranked higher than any interpretation in $X - Y$. Hence, there is an $i'' \in X \cap Y$ and $i'' \geqslant i'$, meaning (iiia) above fails. That means (iiib) holds. $\square$

So $j < i'$ whenever $j \in Y - X$ (by (ii) above) or $j \in \overline{X} \cap \overline{Y}$ (by Claim 2). That means $j < i'$ for any $j \notin X$. Our witness for $X \sqsupset Z$ will then depend on which condition for $Y \sim Z$ holds.

Suppose condition (i) for $Y \sim Z$ holds. That means there is an $i'' \geqslant i'$ where $i'' \in Y - Z$. By Claim 2, $i'' \in X$. Since $i'' \geqslant i'$, that means $i'' > j$ for any $j \notin X$. Hence, i'' automatically satisfies conditions (ii) and (iib) for $X \sqsupset Z$: it is ranked higher than any interpretation in $Z - X$ and it is ranked higher than any interpretation in $\overline{Z} \cap \overline{X}$. So i'' witnesses $X \sqsupset Z$.

Suppose now condition (ii) for $Y \sim Z$ holds. So there are some $j, k \geqslant i'$ where $j \in Y \cap Z$, and $k \in \overline{Y} \cap \overline{Z}$. Now, since i' witnesses $X \sqsupset Y$, $j \in X$. Hence, condition (iiia) above fails, and so condition (iiib) above holds. Since $k \geqslant i'$ and $k \notin Y$, that means $k \in X$. Thus, $k \in X - Z$. Moreover, since $k \geqslant i'$, that means $k' < k$ for any $k' \notin X$. Hence, k automatically satisfies conditions (ii) and (iiib) for $X \sqsupset Z$. So k witnesses $X \sqsupset Z$.

Case 2: $i' \notin Z$**.** Thus, $i' \in X - Z$. As in Case 1, if i' satisfies (iiib) above, then i' is ranked above any interpretation not in X, and so automatically satisfies conditions (ii) and (iiib) for $X \sqsupset Z$. So suppose i' does not satisfy (iiib) above. Thus, it satisfies (iiia), i.e., for all $i'' \in X \cap Y$, $i'' < i'$. Since $i'' < i'$ for all $i'' \in Y - X$, that means that for all $i'' \in Y$, $i'' < i'$. We'll now show that i' witnesses $X \sqsupset Z$.

First, we show (ii) for $X \sqsupset Z$. Suppose for reductio that $i'' \in Z - X$ where $i' \leqslant i''$. By the above, $i'' \notin Y$, i.e., $i'' \in Z - Y$. By $Y \sim Z$, there is some $i''' \geqslant i''$ where either $i''' \in Y - Z$ or $i''' \in Y \cap Z$. Either way, $i''' \in Y$, which contradicts $i'' \geqslant i'$ being ranked above all interpretations in Y. Hence, if $i'' \in Z - X$, then $i'' < i'$.

Next, we show (iiia) for $X \sqsupset Z$ holds. Suppose for reductio that $i'' \in X \cap Z$ where $i' \leqslant i''$. Again, $i'' \notin Y$, i.e., $i'' \in Z - Y$. But again, since $Y \sim Z$, this implies that $i''' \in Y$ for some $i''' \geqslant i''$, which contradicts $i'' \geqslant i'$ being ranked above all interpretations in Y. Hence, if $i'' \in X \cap Z$, then $i'' < i'$.

So regardless of whether $i' \in Z$, we have our witness for $X \sqsupset Z$. □

Hence, $\sqsupset$ is well-defined. The fact that $\sqsupset$ is a linear order on degrees follows by the reasoning that shows that $>$ is irreflexive, transitive, and total.

Fact 12. On degrees, $\sqsupset$ is a linear order, i.e., for all $X, Y, Z \subseteq I_i$:

(a) $\deg(X) \not\sqsupset \deg(X)$.

(b) if $\deg(X) \sqsupset \deg(Y)$ and $\deg(Y) \sqsupset \deg(Z)$, then $\deg(X) \sqsupset \deg(Z)$.

(c) if $\deg(X) \neq \deg(Y)$, then either $\deg(X) \sqsupset \deg(Y)$ or $\deg(Y) \sqsupset \deg(X)$.

We also note that this linear order has top and bottom elements: $\deg(\top) = \deg(I_i) = \{I_i\}$ is the top element while $\deg(\bot) = \deg(\varnothing) = \{\varnothing\}$ is the bottom element. (These are, in fact, singletons: one can prove that if $\varnothing \neq X \neq I_i$, then $I_i \sqsupset X \sqsupset \varnothing$.)

Fact 13.

(a) $\deg(\top) = \{I_i\}$ is the maximum element with respect to $\sqsupset$.

(b) $\deg(\bot) = \{\varnothing\}$ is the minimum element with respect to $\sqsupset$.

D Dropping totality

Throughout, we've assumed that semantic orderings are total, i.e., for any i and j, either $i \leqslant j$ or $j \leqslant i$. This means $\vDash (A \succ B) \vee (B \succ A) \vee (A \approx B)$. Here, we generalize that semantics for MCs to avoid the totality assumption. We start by stating the generalization as it applies to the *basic* semantics (the one in section A, before adjustments to ensure $\approx$ is transitive).

$$\begin{aligned}
[\![A \succ B]\!]^{\leqslant,i,w} = 1 \quad \text{iff} \quad & \text{(i*) } \exists i' \leqslant i \colon [\![A \wedge \neg B]\!]^{\leqslant,i',w} = 1 \\
& \text{(ii*) } \forall j \leqslant i \colon [\![B \wedge \neg A]\!]^{\leqslant,j,w} = 1 \Rightarrow \exists j' \leqslant i \colon \\
& \qquad \text{(a) } j < j' \\
& \qquad \text{(b) } [\![A \wedge \neg B]\!]^{\leqslant,j',w} = 1 \\
& \qquad \text{(c) } \forall j'' \leqslant i \colon [\![B \wedge \neg A]\!]^{\leqslant,j'',w} = 1 \Rightarrow j' \nleqslant j''
\end{aligned}$$

If the semantic ordering is non-total, then the ordering effectively "branches" on incomparable interpretations (the branches may converge again later). The above truth conditions state that $A \succ B$ is true iff (i*) there is a ranked $(A \wedge \neg B)$-interpretation, and (ii*) on each branch with a $(B \wedge \neg A)$-interpretation, there's some $(A \wedge \neg B)$-interpretation on that branch that has no $(B \wedge \neg A)$-interpretation ranked higher than it. If the semantic ordering is total, then there's only one "branch" of the ordering. So these truth conditions reduce to those in section 4.2 given totality.

All the entailment patterns highlighted in section 2.5 hold in this generalized semantics, with the exception of (n), which is an object-language statement of totality. Below, we just prove the transitivity of $\succ$, which is the most difficult to establish. Let's use $\vDash_{gb}$ for the "generalized basic semantics", i.e., the generalization of the basic semantics to allow nontotal orderings.

Fact 14. $A \succ B, B \succ C \vDash_{gb} A \succ C$.

Proof. Suppose $[\![A \succ B]\!]^i = [\![B \succ C]\!]^i = 1$. We first establish clause (i*) for $A \succ C$. For reductio, suppose there is no $i' \leqslant i$ where $[\![A \wedge \neg C]\!]^{i'} = 1$. By (i*) for $A \succ B$, however, there is an $i_a \leqslant i$ where $[\![A \wedge \neg B]\!]^{i_a} = 1$. Hence, by our reductio supposition, $[\![C]\!]^{i_a} = 1$ since $[\![A]\!]^{i_a} = 1$. But now, since $[\![C \wedge \neg B]\!]^{i_a} = 1$, it follows by clause (ii*) for $B \succ C$ that there's a $j_b > i_a$ such that $[\![B \wedge \neg C]\!]^{j_b} = 1$ and $j_b \nleqslant k$ for any $k \leqslant i$ where $[\![C \wedge \neg B]\!]^k = 1$. By our reductio supposition,

$[\![A]\!]^{j_b} = 0$ since $[\![C]\!]^{j_b} = 0$. But now, since $[\![B \wedge \neg A]\!]^{j_b} = 1$, it follows by clause (ii*) for $A > B$ that there's a $k_a > j_b$ such that $[\![A \wedge \neg B]\!]^{k_a} = 1$ and $k_b \nleqslant j$ for any $j \leqslant i$ where $[\![B \wedge \neg A]\!]^{j} = 1$. Again, by our reductio assumption, $[\![C]\!]^{k_a} = 1$. But the fact that $[\![C \wedge \neg B]\!]^{k_a} = 1$ and $k_b > j_b$ contradicts the fact established earlier that $j_b \nleqslant k$ for any $k \leqslant i$ where $[\![C \wedge \neg B]\!]^{k} = 1$.

Now we establish clause (ii*) for $A > C$. Suppose $k_c \leqslant i$ is such that $[\![C \wedge \neg A]\!]^{k} = 1$. We want to find a $k' \leqslant i$ such that $k_c < k'$, $[\![A \wedge \neg C]\!]^{k'} = 1$, and $k' \nleqslant k''$ for any $k'' \leqslant i$ where $[\![C \wedge \neg A]\!]^{k''} = 1$. We split the proof into two cases depending on the value of $[\![B]\!]^{k_c}$.

Case 1: $[\![B]\!]^{k_c} = 1$. By (ii*) for $A > B$, since $[\![B \wedge \neg A]\!]^{k_c} = 1$, there must be an $i_a \leqslant i$ where $i_a > k_c$, $[\![A \wedge \neg B]\!]^{i_a} = 1$, and for all $i' \leqslant i$, if $[\![B \wedge \neg A]\!]^{i'} = 1$, then $i_a \nleqslant i'$. If i_a satisfies (ii*b) and (ii*c) for $A > C$, then we're done. So suppose otherwise.

Claim 3. There is some $j_b \leqslant i$ such that $j_b > i_a$, $[\![B \wedge \neg C]\!]^{j_b} = 1$, and for all $j' \leqslant i$, if $[\![C \wedge \neg B]\!]^{j'} = 1$, then $j_b \nleqslant j'$.

Proof. If $[\![C]\!]^{i_a} = 1$, then clause (ii*) for $B > C$ immediately establishes this, since $[\![C \wedge \neg B]\!]^{i_a} = 1$. If $[\![C]\!]^{i_a} = 0$, then i_a satisfies (ii*b). Since we're assuming it doesn't also satisfy (ii*c), that means there is an $i' \leqslant i$ where $[\![C \wedge \neg A]\!]^{i'} = 1$ and $i_a \leqslant i'$. But since $i_a \nleqslant i''$ for all $i'' \leqslant i$ where $[\![B \wedge \neg A]\!]^{i''} = 1$, that means $[\![B]\!]^{i'} = 0$. Hence, by clause (ii*) for $B > C$, since $[\![C \wedge \neg B]\!]^{i'} = 1$, there is a $j_b > i' \geqslant i_a$ with the desired properties. □

We'll show that j_b is our desired witness k', i.e., it satisfies (ii*b) and (ii*c). First, if $[\![A]\!]^{j_b} = 0$, then j_b contradicts the fact that $i_a \nleqslant j$ for any $j \leqslant i$ where $[\![B \wedge \neg A]\!]^{j} = 1$. Hence, $[\![A]\!]^{j_b} = 1$, and thus $[\![A \wedge \neg C]\!]^{j_b} = 1$, i.e., j_b satisfies (ii*b).

Next, suppose for reductio that $j \leqslant i$ where $[\![C \wedge \neg A]\!]^{j} = 1$ and $j_b \leqslant j$. Since $j > i_a$, that means $[\![B]\!]^{j} = 0$. But the fact that $[\![C \wedge \neg B]\!]^{j} = 1$ and $j_b \leqslant j$ contradicts Claim 3. Hence, j_b satisfies (ii*c).

Case 2: $[\![B]\!]^{k_c} = 0$. By (ii*) for $B > C$, since $[\![C \wedge \neg B]\!]^{k_c} = 1$, there must be a $j_b \leqslant i$ where $j_b > k_c$, $[\![B \wedge \neg C]\!]^{j_b} = 1$, and for all $j' \leqslant i$, if $[\![C \wedge \neg B]\!]^{j'} = 1$, then $j_b \nleqslant j'$. If j_b satisfies (ii*b) and (ii*c) for $A > C$, then we're done. So suppose otherwise.

Claim 4. There is some $i_a \leqslant i$ such that $i_a > j_b$, $[\![A \wedge \neg B]\!]^{i_a} = 1$, and for all $i' \leqslant i$, if $[\![B \wedge \neg A]\!]^{i'} = 1$, then $i_a \nleqslant i'$.

Proof. If $[\![A]\!]^{j_b} = 0$, then clause (ii*) for $A > B$ immediately establishes this, since $[\![B \wedge \neg A]\!]^{j_b} = 1$. If $[\![A]\!]^{j_b} = 1$, then j_b satisfies (ii*b). Since we're assuming it doesn't also satisfy (ii*c), that means there is an $j' \leqslant i$ where $[\![C \wedge \neg A]\!]^{j'} = 1$ and $j_b \leqslant j'$. But since $j_b \nleqslant j''$ for all $j'' \leqslant i$ where $[\![C \wedge \neg B]\!]^{i''} = 1$, that means $[\![B]\!]^{j'} = 1$. Hence, by clause (ii*) for $A > B$, since $[\![B \wedge \neg A]\!]^{j'} = 1$, there is a $i_a > j' \geqslant j_b$ with the desired properties. □

We'll show that i_a is our desired witness k', i.e., it satisfies (ii*b) and (ii*c). First, if $[\![C]\!]^{i_a} = 1$, then i_a contradicts the fact that $j_b \nleqslant i'$ for any $i' \leqslant i$ where $[\![C \wedge \neg B]\!]^{i'} = 1$. Hence, $[\![C]\!]^{i_a} = 0$, and thus $[\![A \wedge \neg C]\!]^{i_a} = 1$, i.e., i_a satisfies (ii*b).

Next, suppose for reductio that $i' \leqslant i$ where $[\![C \wedge \neg A]\!]^{i'} = 1$ and $i_a \leqslant i'$. Since $i' > j_b$, that means $[\![B]\!]^{i'} = 1$. But the fact that $[\![B \wedge \neg A]\!]^{i'} = 1$ and $i_a \leqslant i'$ contradicts Claim 4. Hence, i_a satisfies (ii*c).

Thus, either way, we have found our witness k'. □

Retaining the original basic semantics for $\approx$, we no longer validate the totality principle: $\nvDash_{gb} (A \succ B) \vee (B \succ A) \vee (A \approx B)$. Figure 2 contains a minimal example where $A \succ B$, $B \succ A$, and $A \approx B$ are all false in this generalized basic semantics. Note that this counterexample crucially relies on the incomparability of j and k. If we made $j \equiv k$, then $A \approx B$ would be true. If we made $j > k$, then $A \succ B$ would be true. And if we made $j < k$, then $B \succ A$ would be true.

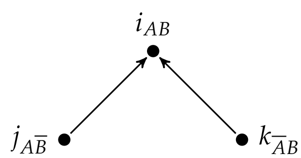

Figure 2: Counterexample to totality in the nontotal semantics for $\succ$.

It is possible to similarly generalize the revised semantics from section B to accommodate nontotal preorders, though we will not provide the gory details here. The key idea is to require the truth conditions for $\succ$ and $\approx$ given in section B to hold within every "branch" rooted at i. This approach will ensure that all the desired entailment patterns hold within every branch, which (apart from totality) suffices to ensure they hold generally.

E Quantifiers and Metalinguistic "Superlatives"

We close by briefly presenting a semantics for first-order and second-order quantifiers that can be used in the analysis of "metalinguistic superlatives" as quantified comparatives (section 6.2).

Let $I_{\leqslant}$ be the set of interpretations ranked by $\leqslant$, i.e., $I_{\leqslant}$ is the field of $\leqslant$. We assume context supplies a set D_1 (the first-order domain) of functions α from ranked interpretations $i \in I_{\leqslant}$ to functions $\alpha(i)\colon W \to D$, as well as a set D_2 (the second-order domain) of functions π^n from ranked interpretations $i \in I_{\leqslant}$ to functions $\pi^n(i)\colon W \to \wp(D^n)$. Given these sets, a **variable assignment** is a function g mapping each first-order variable x to a member of D_1 and each second-order variable X^n to a member of D_2 of the right arity. We write g^x_α for the result of reassigning x to α, and similarly for g^X_π.

A **first-order term** is either a name or first-order variable. A **second-order term** is either a predicate or a second-order variable. Where t is a first-order term, we write $[\![t]\!]^{\leqslant,i,w,g}$ for $i(t)(w)$ if t is a name and $g(t)(i)(w)$ if t is a variable. Likewise for second-order terms T.

Truth is relativized to variable assignments. Thus, the atomic clause looks like this:

$$[\![T^n t_1 \dots t_n]\!]^{\leqslant,i,w,g} = 1 \quad \text{iff} \quad \langle [\![t_1]\!]^{\leqslant,i,w,g}, \dots, [\![t_n]\!]^{\leqslant,i,w,g} \rangle \in [\![T^n]\!]^{\leqslant,i,w,g}.$$

We assume $=$ for first-order terms amounts to coextensionality:

$$[\![t_1 = t_2]\!]^{\leqslant,i,w,g} = 1 \quad \text{iff} \quad [\![t_1]\!]^{\leqslant,i,w,g} = [\![t_2]\!]^{\leqslant,i,w,g}.$$

For second-order terms, which denote properties relative to interpretations, identity is a stronger condition than coextensionality:

$$[\![T_1 = T_2]\!]^{\leqslant,i,w,g} = 1 \quad \text{iff} \quad \forall w \in W\colon [\![T_1]\!]^{\leqslant,i,w,g} = [\![T_2]\!]^{\leqslant,i,w,g}.$$

Finally, the quantifier clauses are given below:

$$[\![\forall x A]\!]^{\leqslant,i,w,g} = 1 \quad \text{iff} \quad \forall \alpha \in D_1\colon [\![A]\!]^{\leqslant,i,w,g^x_\alpha} = 1$$

$$[\![\forall X^n A]\!]^{\leqslant,i,w,g} = 1 \quad \text{iff} \quad \forall \pi^n \in D_2\colon [\![A]\!]^{\leqslant,i,w,g^{X^n}_{\pi^n}} = 1.$$

In Rudolph and Kocurek 2024, section 6.2, we observed we can use quantified comparatives to construct two kinds of metalinguistic "superlatives". One is what we call a *first-order* superlative, which compares individuals, as in (5):

(5) Ann is more a linguist than anyone else.

First-order superlatives have the following form:

$$\forall x(x \neq a \supset (Fa \succ Fx))$$

The truth condition for this formula is as follows:

$$\forall \alpha \in D_1\colon \alpha(i)(w) \neq i(a)(w) \Rightarrow [\![Fa \succ Fx]\!]^{\leqslant,i,w,g^x_\alpha} = 1$$

In other words, this first-order superlative is true iff for each relevant b distinct from a, there's an interpretation where a is F but not b that's ranked higher than any interpretation where the reverse is the case.

There are also *second-order* superlatives, which compare properties, as in (6):

(6) Ann is more a linguist than anything else.

Second-order superlatives have the following form:

$$\forall X(X \neq F \supset (Fa \succ Xa))$$

The truth condition for this formula is as follows:

$$\forall \pi \in D_2\colon i(F) \neq \pi(i) \Rightarrow [\![Fa \succ Xa]\!]^{\leqslant,i,w,g^X_\pi} = 1.$$

In other words, this second-order superlative is true iff for each relevant property G distinct from F, there's an interpretation where a is F but not G that's ranked higher than any interpretation where the reverse is the case.

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