The Topological Realization

Abstract

In this paper, I argue that the newly developed network approach in neuroscience and biology provides a basis for formulating a unique type of realization, which I call topological realization. Some of its features and its relation to one of the dominant paradigms of realization and explanation in sciences, i.e. the mechanistic one, are already being discussed in the literature. But the detailed features of topological realization, its explanatory power and its relation to another prominent view of realization, namely the semantic one, have not yet been discussed. I argue that topological realization is distinct from mechanistic and semantic ones because the realization base in this framework is not based on local realisers, regardless of the scale (because the local vs global distinction can be applied at any scale) but on global realizers. In mechanistic approach, the realization base is always at the local level, in both ontic (Craver 2007, 2013) and representational accounts (Bechtel and Richardson 2010). The explanatory power of realization relation in mechanistic approach comes directly from the realization relation-either by showing how a model is mapped onto a mechanism, or by describing some ontic relations that are explanatory in themselves. Similarly, the semantic approach requires that concepts at different scales logically satisfy microphysical descriptions, which are at the local level. In topological framework the realization base can be found at different scales, but whatever the scale the realization base is global, within that scale, and not local. Furthermore, topological realization enables us to answer the "why" questions, which according to Polger (2010) make it explanatory. The explanatoriness of topological realization stems from understanding mathematical consequences of different topologies, not from the mere fact that a system realizes them.

1. Introduction

The last two decades in neuroscience and biology we have witnessed a very rapid development and overwhelming spread of a network analysis. Network analysis is used to explain behaviours and properties of a variety of systems. Those systems are called real networks (Newman 2010). Such real networks are for example brains, the World Wide Web, the Internet, transportation systems, social groups and many others. In network analysis elements of a system are represented as nodes or vertices of a network and their interactions or connections are represented as edges or links. A network is defined simply as a set of nodes (vertices) linked by connections (edges) (Barabasi 2002; Bullmore and Sporns 2009; Newman 2010; Sporns 2012; Fortunato 2010).

A classical example of topological explanation is the Watts and Strogatz (1998) small-world graph model. This model was built in such a way that starting from a ring lattice it has *n* vertices and *k* edges. The structural properties of such a graph are quantified by using its characteristic path length, which measures a typical separation between two nodes in the graph, which is expressed as L(p), and the clustering coefficient C(p), which measures the cliquishness of a typical neighbourhood of nodes. The small-world networks are characterized by low L(p) values, due to a few long-range links, together with a high C (p). Such 'short-paths' connect nodes that would otherwise be much farther apart and in effect would shorten the path lengths between the whole neighbourhoods, and neighbourhoods of neighbourhoods. For example, an explanation of why infectious disease will spread more rapidly through a population which instantiates a small-world topology, refers to these structural features: pathogens can reach a greater number of nodes more rapidly if the L(p) is low and the C(p) is high. More specifically, the small-world topology tells us what portion of the population will be infected or, in the case when the pathogen is so infectious that it affects the whole population regardless of its structure, the small-world topology tells us that the time in which it is done is a function of L(p) and C(p), i.e. the lower the L(p) and the higher the C(p), the faster it would spread. Or, as Watts and Strogatz put it: "…(small-world topological) model illuminates the dynamics as an explicit function of structure…" (Watts and Strogatz 1998, 442). This pattern of explanation will work for many types of real systems and for a variety of different *explananda*. For example, it will explain computational or metabolic economy: in small-world networks the energy required to send a signal across network is lower because small-world topology enables the signal to be transmitted much more efficiently because it connects distant groups of nodes as if they were in the same group. It is also used to explain synchronicity, stability, robustness, resilience and many others. The small-world topology is just one such graph-theoretical example.

In topological explanation, the explanatory relation (the relation between the explanans and explanandum) stands between a physical fact or a property and a topological property. In the Watts and Strogatz (1998) example, we have seen that the explanation of the physical fact is a function of the system topology, i.e. in this example, small-world topology shortens the path lengths between the whole neighbourhoods, and neighbourhoods of neighbourhoods and in that way the infectious disease can spread much more rapidly.

The topological approach should be sharply distinguished from the neural nets approach from the '80 that is most famously represented in the Connectionism (McClelland 1988). Neural nets are used to study behaviours of systems, in economics, decision theory, AI, cognitive science and philosophy of mind. This approach is by and large mechanistic and functionalist in nature because it postulates networks as elements of a mechanism, each network is a unit, and they may be arranged and connected in different ways that produces different outcomes. The neural nets approach doesn't study the features of connectedness within the networks, but rather their causal and functional arrangements that can give different outcomes. Whereas in topological approach a description of network properties and certain consequences of topology is what is doing the explanatory work.

When it comes to the research on brain, its functions and capacities a systematic discussion of the features of topological explanations in philosophy of neuroscience is lacking so far. Furthermore, even though the discussions on the distinctness of topological explanations in relation to mechanisms is developing rapidly, its relation to one of the very popular approaches in the debates on metaphysics of realization and explanations, namely the semantic one (Endiccott 2005), still has not been discussed in the literature.

My aim in this paper is to further discuss explanatory features of topological approach in terms of realization relation and its key distinctions, especially in cognitive neuroscience, and the relation between topological and semantic realizations as another pervasive explanatory strategy in philosophy. This will allow for a much more comprehensive and fine grained understanding of the topological approach. In order to do so, in the next section I discuss in detail how this approach is used in cognitive neuroscience. I will first introduce some basic topological notions in neuroscience and then present a case of the brain controllability.

2. The topological approach in cognitive neuroscience

2.1. Basics of topological approach in cognitive neuroscience

Although philosophical discussions about this approach have already started (Huneman 2010, 2015; Craver 2016; Bechtel and Levy 2013; Silberstein and Chemero 2013; Woodward 2013) many of its aspects still require more philosophical foundation. The first contribution to this debate is Huneman's (2010) paper on topological explanations in biology in which he introduced this approach as distinct from mechanisms. His discussion draws on research in evolutionary biology and ecology. He has shown not only that topological explanations are ubiquitous in the sciences but also that they are different from mechanisms in a number of ways, most notably in that they don't use heuristics such as decomposition and localization, and they don't describe a mechanism in the *explanans*. They explain by identifying a topological property and its consequences. Furthermore, he argues that sometimes even the *explanandum* of topological explanation is different from mechanistic ones. It is typically some trait or the outcome of a system that is explained, but not an activity, which is a typical *explanandum* in the mechanistic framework. Much of these claims are true not only in ecology and evolutionary biology but in cognitive neuroscience as well. However, topological explanations can be used to understand even dynamics in some cases. For example, in cognitive neuroscience, they use network control theory to understand how structural features of the brain networks determine features of its cognitive dynamics (Gu et al 2015). On this approach, various brain states are represented as nodes, "a state is defined as the magnitude of neurophysiological activity across brain regions at a single time point." (Gu et al 2015, p. 2)[1]. The idea is to find out the topological constraints on the brain network dynamics (e.g. cognitive control) and thereby answer whether the brain is topologically controllable and to what extent. This example will show that in some cases even the *explanandum* of topological explanation can be the same as in mechanistic one, i.e. the system dynamics. However, before a detailed discussion of this example, some preliminary clarifications and introductory notes are required so that the example and the argument would be easier to follow.

It is very important to note that although networks are by definition structures that are static and have no moving parts or dynamics, they sometimes can still explain temporal dynamics or dynamics in general as a function of topological structure. In those cases, the system's topology allows us to understand the system dynamics as a function of its structure. The system's dynamics here appears as a consequence of the system's topology through which some activity drives the system towards diverse states, and the brain controllability case exemplifies this idea.

[1] The nodes in this case are the states, not what is in a state.

Topological properties can be realized at different scales even in the same system. For example, the brain can be represented as a network of brain regions (macro-scale), the network of “voxels” (meso-scale), which are three-dimensional units of the data sets obtained through various neuroimaging techniques. Voxel is commonly used to describe the resolution of a CT or MRI scan of the brain. The smaller the voxel, the higher the resolution scan. Or as in the Connectome project, the network of all neurons and their connections (micro-scale) (Sporns 2012; Seung 2012). The key here is to understand that the realisation is not based on local realisers, which may be different at different scales. The realization base is at the global level of network topology. So it’s not really the case that the micro-structure determines the realized properties, it’s rather the global topology of the system. For example, at different scales the brain networks represent different elements and their connections, i.e. at the micro-scale they might represent molecules and their interactions, or neurons (cellular scale) and the anatomical connections among them, whereas at the macro-scale a network may represent brain regions and their inter-connections. However, a property of stability or robustness, or even a function (e.g. infectiousness in Watts and Strogatz’s example) are realized by the global network topology at a given scale, and not by the local elements that are found at a given scale.

This in effect means that if we understand that certain functions are results of particular network topology we can also infer required anatomical features from that topology, we can infer from it even the number of neurons or the number of edges (connections) (Alexander-Bloch et al 2012; Bressler 1995; Honey et al 2010; Hutchison et al 2013; Ponten et al 2010; Sporns, Honey and Kötter 2007). But as opposed to mechanistic and semantic approaches the topological properties that are the realization base are not defined at the local level, as we have seen in the example of small-world topology and the spread of infectious disease in the Watts and Strogatz (1998) model: the topology that realizes the faster spread stands at the global level.

The best way to understand these claims is to first give a general introduction of topological approach in neuroscience and then to focus in detail on one example that illustrates these claims.

In network neuroscience the brain is represented as a system of interacting networks at different scales (Bassett and Bullmore 2009; Bassett and Siebenhühner 2013; Sporns 2010, 2012; Seung, 2012). At the micro-scale individual neurons are connected to one another through synapses and they form networks in which information flows as electrical impulses which are called *action potentials* (Bassett and Muldoon 2016, p. 1). In such networks the individual neurons are represented as nodes and their connections as edges. At a larger scale, the synchronous activity of groups of neurons produces oscillatory signals and in that way form brain regions that are connected by bundles of axons (extended parts of neural cells through which a neuron establishes connections with other neurons) which are called *white matter tracts*. On this view, the brain is organized into both structural and functional networks (Bullmore and Basset, 2006). Structural networks are constructed from actual physical connections between individual neurons or brain regions, whereas functional networks are “constructed from functional connections that quantify statistical similarities in activity” (Bassett and Muldoon 2016, p. 2). Of course, structural and functional networks are deeply intertwined (Sporns 2013) and so can be changed by modulating cognitive states, disease or injury (Basset and Bullmore, 2009). In representing the brain as networks, the choice of nodes and edges depends on the type of data, e.g. if one uses EEG data then nodes may represent surface sensors, or for fMRI data nodes can represent single voxels or aggregates of voxels that in effect represent anatomical or functional areas in the brain (Achard et al. 2006; Power et al. 2011). Choosing the edges also relies on the type of data and thus the edges can be rough edges, or depending on the strength of a connection can be weighted, i.e. weight represents the number of tracts connecting a node. Brain networks, especially the functional networks, can also be directed or undirected, i.e. in directed networks the connection from A to B is stronger whereas a connection from B to A is weaker. In undirected networks the connection between A to B and *vice versa* the strength of the connection is equal.

2.2. Brain controllability: the topological constraints on the cognitive dynamics

Topology plays a central role in understanding cognitive dynamics of the brain. It also embodies the Watts and Strogatz motto that the dynamics is the function of the structure, i.e. it helps us understand how the topology enables that some activity drives the system to diverse states. The topological approach in network neuroscience is able to catch the dynamics by using network control theory (Gu et al 2015). In network control theory various systems (that have components that are interconnected) are represented as graphs which consist of vertex and edge sets. The network control theory is used to answer questions such as "how to control such an interconnected and complex system"? To this effect the notion of cognitive control is analogous to a mathematical notion of control used in engineering and mathematics (Blondel et al. 2012; Leith et al. 2000; Pasqualetti et al. 2014). The control in this context means to perturb the system in order to reach a desired state. To be clear, there are two notions of controllability here. One is the notion of cognitive control, which should be understood as the system function or its dynamics, that can be embodied in changes in regional activity that is a result of neurofeedback in real-time fMRI, or elicited by external stimuli, or changes in regional activity provoked by non-invasive brain stimulation. The other sense of control here that we want to understand is a mathematical notion of network controllability, that in various ways constrains the types of control that can be exerted on the distributed brain networks, and to that effect it is used to understand brain's cognitive control. It is however, crucial to distinguish these two senses of control in this context, and to understand that one is called the network controllability, and it's a highly mathematical notion, and the other is called the brain controllability and it's about various functions, dynamics or the brain activities. Gu and colleagues say it explicitly in the following passage:

"Importantly, this notion of control is based on a very detailed mathematical construct and is therefore necessarily quite distinct from the cognitive neuroscientist's common notion of 'cognitive control' and the distributed sets of brain regions implicated in its performance. To minimize obfuscation, we henceforth refer to these two notions as 'network control' and 'cognitive control', respectively." (Gu et al 2015, p. 8)

To answer such questions we must know what is a network connectivity of the interconnected components and also how the components act, which taken together provide predictors about the system function (Gu et al, 2016: 8). This allows to answer questions about system's dynamics and to have predictions about its functions. The difference between network control theory and the more traditional graph theory is that network control theory provides predictors about network dynamics, whereas traditional graph theory offers descriptive statistics of network structure.

In this view, some brain activity has a trajectory, which represents dynamics as a path in the state space of the brain, "…where a state is defined as the magnitude of neurophysiological activity across brain regions at a single time point." (Gu et al, p. 2).

In this way the topology, enables some cognitive functions by connecting nodes to some difficult to reach nodes and in that way enables the brain to move into difficult to reach states, for example moving the brain from a resting state into the state of doing a mathematical calculation. The same principle applies in inhibiting inappropriate behavioural responses or linking multiple sources of information to solve problems. Gu and colleagues use the mathematical notion of control in which the topological structure predisposes certain elements of the structure to specific control actions (Gu et al 2015, 2).

Based on network control theory, we can "quantitatively examine how the network structure constrains the types of control that nodes can exert" (Gu et al, 2016: 8). In such a graph the edges have weights (some numerical value that represents the strength of connections) out of which we define a weighted adjacency matrix. In such a graph a real value is associated with each node (given that nodes represent brain states that are defined as magnitudes of neurophysiological activity) and the nodes are collected into a vector of network states. Thereafter we define a map to describe an evolution of a network state over time, thus describing the network dynamics. Based on the network structure and its dynamics we can understand ways in which network structure constrains the types of control that nodes might have.

Like in Watts and Strogatz (1998), the idea here is to understand "…how structural features of a brain network determine temporal features of cognitive dynamics" (Gu et al 2015, 2). More specifically, Gu and colleagues addressed two questions: 1) is the brain topologically controllable, i.e. is it possible to manipulate the properties of its topology through which some activity drives the system to diverse states in the brain; and 2) if it is, which nodes in the brain network topology are most influential in driving the changes in trajectories of brain states merely by their position in network topology (Gu et al 2015, 2).

They used data from the diffusion spectrum imaging (DSI) to build structural brain networks (Gu et al 2015, 3) and to answer these questions they used measures of average controllability, modal controllability and boundary controllability (Gu et al 2015, 9).

The measure of average controllability quantifies a node's impact in moving the system in many easily reachable states. The average controllability measure shows that the easy to reach states are facilitated by the “rich club” topology[2], in which the hubs (nodes that maintain much higher number of edges statistically compared to normal nodes) facilitate controllability of easy to reach states. This is expected because the rich club topology is characteristic of densely connected areas, which then facilitate the movement of the brain to many easily reachable states, e.g. from one resting state to another resting state. The “rich club” topology is mathematically characterized by the presence of hubs that are densely interconnected among each other. Thus it is not surprizing that areas that are involved in the easy to reach states have hubs that are well connected among each other. The high connectivity between the hubs is actually why the brain states are easy to reach. The brain areas with highest measure of average controllability are: precuneus, posterior cingulate, posterior frontal, paracentral, precentral and subcortical structures. These regions seem to be hubs, i.e. they have high network degrees (which is actually defined as average weight of edges that go out of a region in question).

Modal controllability measures node’s ability to control each evolutionary mode of a dynamical network, and it is used to identify states that are not so easily controllable from a set of control nodes. It helps us find areas in the brain that can drive it to difficult to reach states. In this sense, to control means to be able to reach target states whatever the actual activity is in question. The measures showed that the modal controllability is highest in postcentral, supramarginal, inferior parietal, pars orbitals, medial orbitofrontal and rostral middle frontal cortices (Gu et al, 3). The measure of modal controllability has shown that areas with high modal controllability have a low degree of connectivity i.e. they do not have very many links or edges, which means that they don’t include hubs. This indicates that “difficult-to-reach states require the control of sparsely connected areas” (Gu et al 2015, 3). That is to say moving the brain from a resting state into a state of doing some mathematical calculation or retrieving information from memory requires engagement of sparsely interconnected brain regions that are normally not reachable from densely connected areas.

Boundary controllability is used to measure ability of a set of nodes to decouple trajectories of disjoint brain regions, which means that it identifies brain areas that can steer the system into states where different cognitive systems are either coupled or decoupled. Boundary controllability is computed from a robust partition of a brain network by identifying a set of boundary nodes. The boundary nodes are then assigned controllability values until all nodes have a boundary controllability value.

Average, modal and boundary controllability are used jointly to provide a scalar value for each brain region, and this is further used to answer if the brain is controllable and to what degree. The answer to these questions is actually based on understanding the constraints that the topology exerts on types of control, in our case it is the cognitive control.

In all controllability measures, i.e. in case of average controllability where the control is facilitated by a rich-club topology and in the case of modal controllability which is facilitated by sparsely connected areas, the way in which topology facilitates control is

[2] The rich-club organization is characterized by the fact that network hubs are densely interconnected to one another.

that it tells us what are the mathematical constraints of the topological structure, through which some activity drives the system to diverse states in the brain.

In this example, even though the network is constructed from anatomical structures in the brain, i.e. from brain regions and their connections the explanation of brain's controllability only tracked different topological properties, but not the anatomical structures. Different scales at which we decide to construct a network from the same anatomical structure give us different topologies and the topological explanation only tracks topological dependencies in the model, not the ontic anatomical dependencies.

Having discussed the basics of topological approach, and one of its most prominent features that the realization base is not tied to the local level, in the next section I turn to discussing the features of topological realization. I will first give a definition of the topological realization and then discuss where it stands regarding some of the key distinctions about realization relation in the literature.

3. Topological realization

In this paper I argue that there is a distinctly new type of realization, which I call topological realization, because the realizers in this framework are topological properties at different scales. Topological properties are actually properties of connectedness in various systems that remain invariant under continuous transformations. For example, the stability of an ecological community can sometimes be explained as a consequence of the network of predation relations having a scale-free topology. This means that the interactions among species in a community, or at least the predation relations, instantiate scale-free topology, which enables the ecological community to remain stable regardless of the introduction or extinction of some species in it (Huneman 2010). The notion of stability is tightly connected to the notion of invariance in this case, i.e. mathematically speaking: the more invariance, the more stability.

An example in which the *explanandum* is not so directly tied to some topological notion such as invariance is an explanation of synchronicity of fireflies flashing or neuronal firing (Buchanan 2003; Mirollo and Strogatz 1990; Strogatz 2003). Small-world topology embedded in the network of their communication enables groups as well as distant and sparsely connected nodes (i.e. fireflies or neurons/brain regions, depending on the system) to interact as if they were the first neighbours. In this way the signal (flashing of the fireflies or synaptic connections) can traverse globally across the entire network in a way as it would have traversed locally. This further enables even the most distant individuals in the network to communicate as if they were first neighbours and thus to establish and maintain synchronicity (Buchanan 2003, p. 47; Strogatz 2003, pp. 11-40; Watts and Strogatz 1998).

The topological realization can be formulated as follows:

(TR): The realization relation stands between a topology *T* and a system *S*, such that the system *S* realizes topology *T* when the *elements* of *S* are interconnected in ways that display the pattern of connectivity characteristic of *T*.

Elements in this definition can refer to some spatial objects (such as single neurons, species or brain regions) in spatially embedded networks, but as it is more often the case, it refers to some abstract representation of data, e.g. the voxels or brain states that are defined as magnitudes of neurophysiological activity or even points in a state space.

It's important to understand that as opposed to mechanistic approach, wherein the realization relation itself is explanatory, e.g. the mapping a model onto a mechanism or showing how the mechanism is embedded into a causal structure, in topological explanation realization relation is not explanatory in itself. In the topological explanation the explanatory relation stands between the topology and its mathematical consequences. Think for example of Watts and Strogatz (1998) model where the explanation of the speed and spread of infectious disease is not due to a fact that a population instantiates a small-world topology, but it is due the fact that in small-world topologies the fewer long range connections and high clustering enable distant neighbourhoods of nodes to be connected as if they were the first neighbours. This feature is a mathematical consequence of topology, not a fact about realization.

I argue that topological realization provides a unique perspective for understanding the recently developed way of doing science. My argument is based on the distinction between local and global realizers, i.e. that the realization base on this view is at the global level of system's topology, and not at the local level of micro-scale descriptions of functional or causal roles, which is prominent in the metaphysics of semantic realization where the realization relation is explanatory only when the realized properties can satisfy some set of microphysical descriptions, e.g. watery properties are realized by satisfying a microphysical description H_2O (Levine 2001, Chalmers 2010). And it is not a matter of finding and describing a mechanism and then embedding it into the causal structure also at the micro-scale like in the mechanist approach (MDC 2000; Craver 2007, 2013) where the explanation is based on the model-mechanism-mapping explanatory constraint where variables in a model correspond to components, activities or organizational features of the target mechanism and the dependencies between variables "in the model correspond to causal relations among the components of the target mechanism" (Kaplan 2011: 347). In topological realization, the same topology can be realized at different scales depending on the choice of what nodes and edges stand for. For example, at the macro-scale the brain can be represented as a network of interacting brain regions, or at the meso-scale, the network of voxels, and finally at the micro-scale each individual neuron can be represented as a node and each synaptic and dendritic connection as an edge (which is the leading idea in the Connectome project). However, in topological realization the realization base stands at the global level. Even though it is true that the local relations in a system in a seemingly obvious sense determine or fix the topology (because the nodes and edges represent some things in the world), the topological realization is not tied to local levels whatever the scale. It is not the neurons, brain regions, or species in an ecological community, which are particular to any of these diverse systems or scales, that realize topology. Topology is realized at the global level of patterns of connectivity.

After having laid out the fundamentals of topological realization in this section I turn now to discussing its explanatory power in the next section 4. In section 5 and lay out in detail the relation between the topological and semantic realizations, and finally in section 6 I discuss the multiple realizability in the topological framework.

4. Explanatory role of topological realization

4.1. Taxonomies of realization relation

In topological approach an explanation is a function of topology, according to which an explanatory relation stands between some behaviour, function, or dynamics, e.g. robustness, stability, synchronicity, controllability, and topological properties. The realization relation in topological framework is not explanatory in itself, i.e. the fact that a system instantiates certain topology doesn't explain anything. The explanation here is based on the understanding of mathematical consequences of topologies, not on the fact *that* the topology is realized. The fact that various systems have equivalent topologies, and that the explanatory relation between these topological properties and various *explananda* are equivalent across various systems tell us that topologies are multiply realized, and furthermore that their multiple realizability can be understood as equivalence between different classes of topologies, and not as equivalence between local elements in each of these systems. Since the topological explanation explains by describing a topological property and its consequences, explanatory power of topological realization stems from topology that is realized in the system. The explanation doesn't track the objective dependencies that vary across system (e.g. nodes may represent different things even in the same system as we have seen in the case of brain), but it only tracks the topological properties and their mathematical consequences (e.g. the more invariance the more stability, or the higher the small-worldness the more efficient computational economy, or faster the spread of infectious disease). There can be indeed many different types of topological *explananda*. In this case, once we understand what stability or robustness mathematically mean we also understand that they are mathematically dependent on the notion of invariance. But in cases where the *explanandum* is a physical fact, such as for example controllability of the brain (e.g. it's ability to retrieve information from memory), where the mathematical dependence between the topology and the physical fact is not immediately obvious, we also don't need any additional or extra-topological devices, it does suffice to understand that certain topologies constrain types of control that nodes may have in a network.

To better understand what topological realization brings to the table it is important to make several distinctions that would situate topological realization within the various accounts of realisation that have been proposed in the literature.

There are several, somewhat overlapping, taxonomies of the realization relations. For example, Endicott (2005) argues that there are three basic views of realization: a mathematical, wherein realization is thought of as a form of mapping between objects or domains; a logico-semantic in which properties are realized if realized properties can be interpreted to satisfy some predicates or conditions relevant for the realizing properties; and finally the most common view of realization holds that realizing properties determine the realized properties in a metaphysical way. On the other hand Carl Gillett (2010) distinguishes between M, L and A realizations. M realization could roughly be understood as a metaphysical and mechanistic view but it has at least three varieties (subset, flat and dimensioned). Linguistic or L-realization is a version of logico-semantic realization. But most importantly there is the A realization which can be thought of as an abstract mathematical realization. Due to its abstract and mathematical features topological realization seems to belong to the A type relation according to Gillett's taxonomy.

Perhaps, more conducive distinction for understanding the topological realization is the epistemological one introduced by Wilson and Craver (2007) according to which, on the one hand, the realization relation studies logical, and nomological relations between target concepts or on the other hand, it is in the business of finding an empirical substrate of certain capacities, functions or behaviours. The starting distinction here is methodological, and it concerns what we want to know about the realization relation. Wilson and Craver argue that the realization relation serves two masters, one is a metaphysician and the other is a cognitive scientist. In serving the metaphysician the realization is formulated in terms of logical dependency relations, such as supervenience, metaphysical sufficiency and nomic necessity in which normally changes at one level logically determine the changes at another. This view is very nicely encapsulated in the supervenience slogan: "there cannot be an A-difference without a B-difference". Prominent tool in this kind of setting is conceptual analysis which is used to probe various positions regarding these dependency relations.

On the other hand, in serving the cognitive scientist the realization is framed in terms of how specific functions and capacities are realized by "particular psychological and neurological structures and mechanisms" (Wilson and Craver, 2007, p. 82). On this account, instead of conceptual analysis the major tools are models and mechanisms in their view.

Given the way in which the topological approach is used in neuroscience, it seems that topological realization can well serve the cognitive scientist. But instead of finding and describing a mechanism, in topological approach, describing a connectivity property in a network explains some macro-scale behaviour or property. As we have seen, robustness, stability, and even temporal dynamics, such as brain controllability, can sometimes be explained as consequences of certain brain topologies.

To that effect, the most prominent feature of topological explanation, compared to other types of explanation, is that it is not delimiting of the different realization bases at local levels within or even across different scales that is explanatory, it's the understanding of what topologies at global levels of systems mathematically mean that does the explanatory work. At the global level of system's topology all the local details are highly idealized, i.e. regardless of the actual system, its elements and scales, the topological models represent the elements merely as nodes in a network and their relations as edges, which in effect is intentionally misrepresenting the system. Showing how the topological model latches onto the world or how we build the model based on the information about the local level doesn't seem to add anything to our understanding of a phenomenon. We can't understand it without understanding the topology and its mathematical consequences in these cases.

To put it more explicitly: the concrete real system has parts that are interconnected at different scales (think of the brain at the brain-region scale, the scale of voxels, cellular scale or even molecular scale), but regardless of the scale, the topological properties are realized globally, i.e. regardless of what are the local elements that are interconnected at each scale, the topology is a global mathematical property of a network at each scale.

4.2. Descriptive vs explanatory accounts of realization relation

Further epistemological distinction that can help us better understand topological realization and its explanatory power is between descriptive and explanatory theories of realization relation (Polger 2010).

Polger claims that the descriptive theories of realization simply tell us “when” and “that” the realization relation holds, whereas explanatory theories of realization tell us “why” and “how” it holds (Polger 2010, p. 200). He postulates the distinction between them thusly:

“The key difference between descriptive and explanatory approaches to realization is that explanatory approaches can be discriminate with respect to whether the objects that instantiate the properties are in mereological relations, constitution relations, identity relations, and so forth. Only in some cases is it correct to say that the properties of one thing are realized by the properties of another.” (ibid)

For example, synchronicity plays a very important role in understanding certain behaviours of very diverse systems, e.g. synchronous flashing of fireflies plays a key role in their sexual selection, synchronous firing of neurons enables information to be processed at all in the brain, or synchrony of heart pace cells enables heart to pump blood (Strogatz 2003, pp. 11-40). Different mechanisms facilitate execution of all these functions depending on the target system, e.g. flashing of the fireflies, action potential building in neuron membrane, etc. But regardless of a particular system, their synchronicity is explained as a function of the topology that is realized in all of these diverse systems. Just to be clear, in all these cases we want to understand how the synchronicity is achieved and maintained, not the role of synchronicity in evolutionary functions (sexual selection) or physiological processes (contractions of a heart muscle or information processing in the brain). In all these examples the elements represented as nodes in a network are taken to be coupled oscillators, and just as in the case of the Watts and Strogatz model (Watts and Strogatz 1998) it is the (small-world) topology, mathematically characterized by the fewer long range connections and high clustering that explains the synchronization. In their seminal work, Mirollo and Strogatz (1990) discovered that in the case of fireflies mutual synchronization occurs only when a firefly actually sees the flashing of another firefly and then shifts its flashing rhythm accordingly (Mirollo and Strogatz 1990, p. 1646). They considered the population of fireflies as a network of pulse-coupled oscillators which interact through a variety of pulse coupling, i.e. when an oscillator fires it pulls all the other oscillators up by an amount or pulls them to fire (Ibid.). In order for them to synchronize, apart from these non-linear dynamics constraints, they have to be able to see all the other fireflies and adjust their flashing rhythm accordingly. Adjusting the flashing rhythm according to the flashing of others, even the most distant fireflies in the swarm, can only be achieved if they can see the flashing of the distant fireflies as if they were the first neighbours. Small-world topology that is realized as a pattern of their interaction is what enables or constrains the synchronous adjusting of the flashing rhythm. Here again, just as in the case of infectious disease, the key is to understand that the topological structure enables some activity or dynamics to go through. And here again as well it is the small-world topology which is realized as a pattern of their interaction.

To explain the phenomenon we need to distinguish the realization base from other possible realization bases, i.e. we want to know why something is the case or why something is happening given the realizer story. Understanding the network topology helps a great deal to distinguish the realization base from other possible realization bases, and to precisely delimit topologies that are explanatory of certain physical facts, properties or behaviours from those which are not, effectively answering the "why" question. In our example with synchronization it is the understanding that the firefly interaction realizes a small-world topology that is the key for fully understanding why and how they synchronize. It is because without them being able to see every other firefly as if they were the first neighbours and adjust their rate of firing accordingly, they would have never been able to synchronize. As opposed to Polger's claim that distinguishing the exact types of realization relations is what makes different accounts of realization explanatory, in topological framework it is distinguishing between different realization bases (i.e. understanding what different topologies mathematically imply) that is explanatory.

Admittedly, this applies only to topological realization, and probably cannot be used as a general objection to Polger's distinction.

Now that I have situated topological realization within various views of realization relation proposed in the philosophy of science, in the next section I'll turn to the more general framework of realization relation in the philosophy of mind. A comparison between these two very different approaches to realization relation will help us to better understand the account of topological realization developed in this paper. As opposed to the topological realization, in philosophy of mind the realization relation is predominantly understood in metaphysical terms, as a semantic and logical relation between various concepts. My discussion in the next section will not only highlight the key features of topological realization, it will also point out some of the major shortcomings of the semantic realization.

5. Topological versus semantic realization: global base of topology vs the microphysical descriptions

A prominent view in philosophy of mind is that a complete explanation of the mind-brain relation has to be epistemically transparent. The issue of transparency of explanations in the philosophy of mind is framed in such a way that in order for an explanation to be transparent all non-basic truths, such as phenomenal truths, i.e. truths about conscious experience from the first person perspective, must be a priori entailed by the basic physical, or rather microphysical truths (Dowell, 2008, p. 93). However, this is just another way of saying that all the macro-physical truths are logically dependent on the micro-physical ones. To situate it in the already made distinctions, this view of explanation according to Endicott (2005) taxonomy implies a logico-semantic type of realization relation in which properties are realized if they satisfy certain predicates associated with the realizing properties. This approach to realization relation also fits nicely the metaphysical view according to Wilson and Craver (2008) because it is concerned with various dependencies regarding conceptual semantics and furthermore it seems to fit the Polger's (2010) distinction of being a descriptive view of realization, rather than explanatory because it is indiscriminate about the exact relation between realized and realizing properties.

The realization relation on this view could be defined in the following way:

(SR): A property X is semantically realized iff the predicates or descriptions of causal or functional roles associated with X logically satisfy predicates associated with a micro-physical description Y.

An abundantly used example in the literature goes something like this:

1) Water plays the watery role, i.e. the macro-physical predicates or descriptions of causal or functional roles of water are for example odourless, colourless liquid that we find lakes, river and oceans, that quenches thirst, boils at 100 C° freezes at 0 C° and so on.

2) (micro-physical description) H_2O plays the watery role, i.e. it satisfies the macro-physical predicates.

3) Water and H_2O satisfy the same micro-physical descriptions.

4) Water is H_2O.

It is often argued, in philosophy of mind in particular, that this kind of analysis purportedly shows that in the case of consciousness or qualia[3], this pattern of explanation would not work because, as the analogy goes, a concept, say pain is not conceptually analysable in terms of the causal roles of C-fibre firing ("C-fibre firing" is a philosophical abstraction which stands for the actual empirical description of what is going on in the nervous system when someone is in pain). And this is what opens up the so-called explanatory gap (Levine 2001).

It is important for understanding the semantic realization to note that the "watery stuff" is actually a conjunction of descriptions which constitute the (primary) intension of the concepts "water", which ought to be satisfied in order for something to be realized via that set of descriptions.

A secondary intension is a function that reflects how the reference is fixed in a counterfactual world, given that the actual world is fixed. We know it from the Kripkean cases: a secondary intension of "water" is "H_2O", because in the actual world it is discovered that water is H_2O. However, primary intension of "water" is "the watery stuff" (the liquid with such-and -such properties), and there is a possible world, considered as actual, in which the watery stuff is not H_2O (but XYZ instead), and in which water is not H_2O.

Primary intension is a function that is determined by how the reference would have been fixed in a given world considered as actual (Block and Stalnaker 1999, p. 33). One can know the primary intension of a given concept and still be ignorant of the secondary intension, because the secondary intension is related to worlds considered as counterfactual, given that the actual world is fixed. In this sense then, the primary intensions should

[3] This is a technical term that denotes the subjective qualities of conscoius experience, or the properties of what it is like to be in some conscious states.

represent the a priori part of the concept's content that remains unchanged when considered through possible worlds.

The primary intension understood in this way represents the description of the causal roles, and thus can be understood as representing the a priori part of the two-dimensional analysis.

To put it differently, a world verifies S iff its primary intension is true at that world, and a world satisfies S iff its secondary intension is true there considered from the actual world.

The main thing is that the semantic views of realization assume that the realization base concerns only the micro-scale properties at a local level, i.e. the watery properties are realized if they satisfy a microphysical description H_2O. Satisfying here means playing the same causal roles. Topological realization on the other hand assumes that the realization base is not at the micro-scale and local level. It's on the global network level where the realization base is some pattern of connectivity, which is individuated non-causally, for example a hub is characterized by its degree value, i.e. the number of connections it has relative to average number of connections other nodes in the network have, and not by what it does locally. To put it another way, the property of having a hub (s) cannot be local because it is determined relative to average degree values of all other nodes in the network, thus it is a global property.

The explanatory relation in the semantic framework of realization is twofold:

a) The realization relation is a truth function from concepts to possible worlds, and that function is established according to some theory of meaning (causal, referential, two-dimensional and others). Recall the aforementioned example: 1) water plays the watery role, 2) H_2O plays the watery role, 3) that's why the water is H_2O. But as it is pointed out, in different possible words some other substance with different microphysical description can play a watery role, and that all depends on what we postulate to be the way in which concepts get their meanings in different possible worlds.

b) The explanations are *a priori* derivable from the structure of realization relations.

Understood in this way the semantic realization doesn't really tell us much about the phenomenon we want to explain it only tells us about how truth functions of concepts behave in different modal semantics. But the major problem with this view is that apparently every other kind of realization and explanation can be translated into this pattern. In such translation one would only have to substitute "the description of causal or functional roles" with the "description of topological roles" and the explanation would yield the same result, and once again prove the existence of the explanatory gaps, as we have seen in the above case with water and *qualia*.

For example, the argument pattern in this scenario might go something like this:

5) Topology X plays the macro-physical role R, i.e. the topological predicates play a certain role in describing macro-physical behaviour or properties of a system.

6) Micro-physical description Y of a system plays the same topological role R, i.e. it satisfies the macro-physical topological predicates of X.

7) X and Y satisfy the same topological descriptions.

8) X is Y.

But this pattern of explanation is concerned only with different scales of realization, and it is insensitive to understanding of what certain topologies mathematically imply. That's why it is not really explanatory, it doesn't enable us to distinguish different realization bases within the system in question or even within the same domain of phenomena, because the distinction between different realization bases in the semantic framework is made based on theories of meaning (e.g. descriptive, causal, two-dimensional). In a way, it provides an explanation assuming the distinction between different realization bases within the system or a domain of phenomena is already established.

In other words, even if we translated topological explanation into the semantic realization, the semantic realization is indiscriminate towards specific topological *explanantia*, i.e. any description of "topological roles" would suffice for the semantic realization to work. Different topologies on this approach wouldn't yield different conceptual semantics, only different modal semantics give different conceptual semantics. Given that it is indiscriminate regarding the exact relations between the realized and realizing properties, the semantic realization then seems rather descriptive than explanatory, because it cannot really answer the "why" questions in Polger (2010) sense nor in my sense of being able to distinguish different realization bases.

This becomes clearer if we recall the original definition of topological realization:

(TR): The realization relation stands between a topology *T* and a system *S*, such that the system *S* realizes topology *T* when the *elements* of *S* are interconnected in ways that display the pattern of connectivity characteristic of *T*.

Here, the realization relation is not explanatory in itself, the fact that a system realizes some pattern of connectivity means nothing unless we understand what that pattern mathematically means and what are its mathematical consequences, e.g. small-world topology is defined through fewer L(p) connections and high cliquishness, which in effect enables that distant neighbourhoods of nodes be connected as if they were in the same neighbourhood. Understanding that different patterns of connectivity constitute different topologies is in effect distinguishing different realization bases, which is actually explanatory. Whereas in semantic approach the realization relation is a logical function that stands between a concept and a micro-physical description of causal or functional roles, whatever those roles may be. In semantic framework it is presupposed that distinction between the actual realization bases (e.g. between different topologies) is already made. The only realization bases it can distinguish is between bases in different theories of meaning, which although explanatory in regard to different concepts, it is not really explanatory in regard to the phenomenon in question, because any distinction about the realization bases will stem from the theory of meaning one adopts.

Having laid out the basic network notions in the introduction, discussed its use in neuroscience (section 2), and discussed the definition and general features of topological realization (section 3), its explanatory power in section 4 and its relation to semantic realization in this section, in the last section I discuss how to understand multiple realizability in topological realization, which is one of the key issues of any account of realization relation.

6. Multiple realizability in topological realization

The key to understanding multiple realizability in topological realization is in understanding that it is a highly idealized mathematical realization relation. Even though many systems may realize the same topologies, it would be misguided to think about topological multiple realizability as an empirical fact. It would be misguided to think that there is something intrinsically topological in the world, just because the variety of real systems (the logistic networks, friendships, the brain, ecological communities, monetary systems) all can realize the same topology, for example the small-world topology, and some of their behaviours, properties or dynamics can be explained as a consequence of the realized topology. The topological realization and its multiple realizability are more about how we represent and explain the world by using highly idealized network models. The emphasis in multiple realizability of topological properties should rather be on the explanatory power of the realization relation than on finding out why so many systems realize the same topologies. The latter could even be a wrong question in this context, or perhaps impossible to answer. The key lies in understanding topological realization and its mathematical consequences. Topological properties are multiply realized, but what is interesting about it is that the explanation tracks mathematical dependencies of topologies rather than actual ontic details of various systems.

To illustrate this point, consider two topological systems S1 and S2 that instantiate small-world topology. In this sense the S1 and S2 are topologically equivalent, despite the fact that causally they are completely different, i.e. despite the fact that in S1 and S2 the nodes and edges may stand for different things, their topologies are equivalent, i.e. isomorphic and so S1 and S2 have the same topology B1, e.g. a small-world topology. For example, S1 and S2 can be very different in their causal details, e.g. S1 may represent a financial market, and S2 may topologically represent relations of predation in an ecological community or the connections in the brain. But an explanation of any system or set of elements will be a function of topological properties, that are the same, in various systems. It is the topology B1 and its consequences that are equivalent across diverse systems that is explanatory, and not the various ways in which the same topology (B1) is instantiated in those systems.

In summary, what makes topological realization and explanation based upon it unique is the global level of the realization base, or the fact that the realizers are not local, but global, which contrary to semantic and mechanistic realization doesn't bound the realization base to the local level of any scales; and the explanatory power of topological realization doesn't stem from its ability to distinguish the exact type of realization relation between the realized and realizing properties, but from the fact that the physical fact or property is a mathematical consequence or a function of the realized topology.

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