

The Logic of Justifications in Bošković's Induction¹

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Abstract

Bošković's induction is described as a reasoning procedure that combines abductive, generalizing and deductive forms of inference. According to Bošković, the application of inductive reasoning is not restricted to natural science. Bošković's critique of the use of the principle of sufficient reason is discussed, and constructive rules of Bošković's inductive logic are proposed from the standpoint of contemporary justification logic. To that end, justification logic could be extended with Bošković's typology of reasons. Hunter's result (1965) is conceived as a confirmation that the inference from the constructive *defectus rationis* is a necessary component of Bošković's induction.

Key words: R. Bošković, induction, abduction, principle of sufficient reason, constructivism.

Explicit justifications play an important role in the scientific methodology of Ruđer Josip Bošković, especially in his inductive method, which comprises, beside inductive generalization, also other ways of reasoning. However, since in inductive methodology there are many steps in reasoning that are not deductively valid, the question of the legitimacy of non-deductive steps is of crucial significance. Bošković pays special attention to this question, and analyzes what kind of reasons justify which steps in inductive reasoning and how different ways of justification agree and compose a unified methodological whole.

1. *Bošković's glimpse of the history of methodology*

At the beginning of his *Philosophia recentior* (I, 50—51), Benedict Stay gives a brief and simplified version of the history of scientific methodology. From Bošković's approving annotation in the same place in Stay's book it is clear that the main criterion for such a history is the role of the

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empirical (and experimental) justification of knowledge. In addition, empirical knowledge has to be combined with reflection and comparison. Bošković characterizes Stay's three phases of the history of methodology in the following way:

1. in old philosophers (*veteres*), there is no empirical justification of knowledge (... *sine ulla phaenomenorum observatione, sine ullis experimentis ...*),
2. in more recent philosophers (*recentiores*, Cartesians), we find only insufficient empirical justification and hence arbitrariness (*Observationes quidem instituerant plerasque, sed non eas, quae satis essent ad cognoscendas generales leges ...*),
3. finally, only Newton (as well as Bošković himself) proposed and used methodology based on full empirical justification (... *naturae indolem, ac leges observando [deprehendere], ac inter se meditando [conferre]*).

Although this is a very simplified historical view,² it shows where the emphasis is: it is on the justification of knowledge by empirical evidence. Also, this is clear evidence of importance which Bošković attributed to inductive methodology. Hence in *De cont. l.*, n. 134 (cf. *Th. phil. nat.*, n. 40) Bošković, more carefully, also says: “With the help of induction, the ancient philosophers, too, always attributed extension, figurability, mobility, and impenetrability to all bodies ...”, although it was for Bošković, obviously, not a systematic inductive methodology but merely casual (and often wrong) induction.

2. Sources and description of the inductive procedure

In several places in his work, Bošković gives an account or illustrative examples of his inductive methodology. For example, in *De lumine* (1748, nn. 27–30) we find his methodology explained with an apt example; in *De continuitatis lege* (1754, nn. 134–135) Bošković gives an almost algorithmic description of the procedure of inductive reasoning; in annotations to B. Stay, *Philosophiae recentioris ...*, l. 1 (1755, pp. 50–63), Bošković gives illustrative as well as conceptual clarifications of the inductive method, and in a supplement to the same work by Stay, one section is exclusively reserved for a concise recapitulation of the inductive method (*supplementum ad librum primum*, §11 *De principio inductionis*, pp. 357–358); in *Theoria philosophiae naturalis* (1763, nn. 40–41) Bošković repeats the account of induction from *De continuitatis lege*.

² For a different and more complex historical picture, see, for example, in V. Bajsić [1987].

The inductive method is for Bošković a procedure of establishing natural laws, as well as laws in other disciplines (e.g., laws of social life). In his description of the inductive procedure Bošković distinguishes three main stages, each of them characterized by an appropriate kind of justification. We start from a hypothesis which we consider a candidate for a (natural) law.

1. In the first stage we verify the hypothesis empirically.
 - a) The hypotheses should be verified in each observationally or experimentally decidable case. Each such case, as we may add, gives for itself a *strong* justification of the law candidate, but does not say anything about how general the law candidate may be with respect to the things that are unavailable (e.g., too small) for empirical verification.³
 - b) Besides, the hypothesis should be verified in a large number of cases according to the law of probability and the law of large numbers.⁴ Here the hypothesis acquires its *probable* justification – the probability of hypothesis increases with an increase in the number of cases that confirm the hypothesis. Obviously, for Bošković, where there are not enough observations or experiments available, we cannot establish a (natural) law.
2. In the second phase we try to reconcile the hypothesis as a probable law with seemingly contradictory cases. Conceptual work has an important role here. Sometimes we may interpret one and the same appearance in two or more different ways. With the inductive methodological principle, we choose the interpretation that is in accordance with a law candidate. Let us call such a justification of a law candidate *conciliatory* justification.⁵ – We note that here Bošković clearly deals with a problem of modern belief revision theories – how to revise a theory in the presence of contradictory information – and gives an interesting (inductive) solution.
3. Finally, we examine whether there is some positive reason against the generalization of the law candidate beyond the limits of observation. For example, we should exclude a law candidate if it includes properties essentially dependent either on sensibility, on the whole, or on the composition, since unobservables are non-sensible, simple (not wholes) and uncomposed (not aggregates). At this stage, the conceptual work is obviously even more important than in stage 2. The evidence is in this stage *strong*, but *negative*.

³ We may include here also the testing of hypotheses on paradox solving, such as testing the law of continuity on solving the known “Achilles“ paradox by means of convergent geometric series (*De cont. l.*, nn. 41–51). Bošković uses testing on paradoxes also in mathematics (for the Galilean paradox of the bowl and the cone, see *De natura et usu infinitorum...* and I. Martinović [1984, 1985]).

⁴ See detailed discussions in M. Carrier [1985], B. Gower [1993], and D. Škarica [2000], pp. 117-133, 138-152.

⁵ As an example, see in D. Škarica [2004] a discussion on how Bošković explains the collision of bodies, which seemingly contradicts the law of continuity.

3. *Induction beyond natural science*

Although Bošković describes the inductive method primarily as a method of natural science, it is important to note that Bošković's induction represents a methodology common to a wider area of disciplines. According to Bošković, induction is the most appropriate method in all empirical disciplines (*facultates*) – “medicine, anatomy, optics, astronomy and many others” (*De cont. l.*, n. 136). Besides, induction is a method of reasoning in the “practice of human life” (Stay, p. 56, adn.). It seems that Bošković has in mind a kind of application of the inductive method in the area of social cognition. This is also confirmed by Bošković's commentary on Stay's example of the inductive uncovering of laws in some imagined unknown republic just on the ground of the observation of the behaviour of the inhabitants of that republic and without reading the laws themselves (Stay, pp. 52–3, and adn. 1; see also D. Škarica, pp. 135–138). We note that this description closely resembles, for instance, Quine's “radical translation” example – a field linguist exploring an unknown indigenous language just on the ground of the empirical investigation of the linguistic communication without any aid of linguistic manuals for that language (*Word and Object*, ch. 2). Another of Bošković's examples of the inductive method applied outside natural science is the deciphering of an unknown script (see the next section).

4. *Abduction, generalization, and deduction*

Bošković's induction is an integrated procedure consisting of three components.

- a) One component is abduction (or “retroduction” in the terminology of the late Peirce, that is, adopting an explanatory hypothesis⁶), to which we here add analogical induction (which is for Peirce a mixed kind of reasoning, containing abduction, induction as well as deduction⁷). Abduction and analogy make a component by means of which we conjecture a general law (conjectural justification).⁸
- b) The second component is inductive generalization from observed facts with its probable justification of the hypothesis.
- c) Finally, deduction is a part of inductive methodology by means of which we verify or establish consistency; it is a starting point of the conciliatory part of the inductive procedure, and the key part of the refutation procedure on the ground of positive reasons (naturally, it strongly justifies its results).

⁶ See Ch. S. Peirce [1992], pp. 140–141.

⁷ See Ch. S. Peirce, “A theory of probable inference”, *Writings of Charles S. Peirce*, Vol. 5, pp. 408–450.

⁸ On abduction in Bošković, see H. Festini [1989] and Z. Čuljak [1998].

The following quotations show more or less clearly that Bošković has in mind the interconnection of the three components of inductive reasoning. The first quotation is related to the inductive deciphering of an unknown script. Both examples confirm the important role of (abductive) conjectures.

De lumine, n. 27 (*Phil. rec.*, I, 60, adn.):

... *conjectando* primum, et plures positiones inter se *conferendo*, ad vocularum quarundam expositionem devenitur; tum illas ipsas positiones jam retinendo pro reliquis, jam *corrigeno*, paulatim post frequentissimos errores, devenitur tandem ad *clavim* aliquem generalem ... [our emphasis].

Phil. rec., I, 54, adn.:

Haec methodus ... leges generales e plurimorum singularium phaenomenorum observationibus *colligendas* et *conjectando* extendendas, ac saepissime *ad examen revocandas* praescribit ... [our emphasis].

5. *Abduction and analogy*

Let us pause for a moment solely on the abductive component, since the theory of abduction was only elaborated after Bošković by Ch. S. Peirce. The abductive component in Bošković was discovered by Heda Festini (see [1989]), where it was put in connection with Peirce's theory of abduction, as well as in a broader perspective of the modern theory of induction and scientific methodology in general.⁹

Peirce describes the form of abductive reasoning in the following way:

A surprising fact C is observed

If A then C

There is reason to suspect that A

(See Ch. S. Peirce, "Pragmatism and abduction", in *Collected Papers*, 5.189).

⁹ See also Z. Čuljak [1998], where abduction ("hypothetical induction" in Harman's sense) is seen in Bošković primarily as a method of inference on the unobservable. Let us mention that H. Festini [1985] has also described and discussed the elements of Peircean abductive methodology in Gjuro Pulić.

We can easily recognize this sort of reasoning, for example, in the conciliatory stage of Bošković's induction. To stay with Bošković's example, let us consider the fact that oil works its way through marble (C). This is at first sight (*prima fronte*, as Bošković says) a surprising fact, inconsistent with the law of impenetrability. But we can find an explanation of this fact which is in accordance with the law: marble possesses (invisible) pores (A). Clearly, we can reasonably conjecture that A is the true reason of the fact C.

Further, a law is itself an abductive conjecture to explain surprising facts. For Peirce, as well as for Bošković, regularity is a surprising fact since irregularity is much more common, or much more probable in nature than regularity (for Peirce, see S. Psillos, p. 133; for Bošković see *Theoria philosophiae naturalis*, n. 543, in connection with one of his arguments for the existence of God). Therefore, to explain regularities is also a special case of explaining surprising facts, and that by conjecturing an appropriate law.

In a more elaborate form abduction is also contained in reasoning by analogy. We present analogical inference by means of the following two patterns, which we can find employed in Bošković's abductive reasoning:

$$\begin{array}{l} \forall x (x \text{ is } M \rightarrow (x \text{ is } P_1 \& \dots \& P_n)) \\ \forall x (x \text{ is } S \rightarrow (x \text{ is } P_1 \& \dots \& P_n)) \\ \hline \forall x (x \text{ is } S \rightarrow x \text{ is } M) \end{array}$$

where conjectured M is an explanation for each S having properties $P_1 \& \dots \& P_n$. We recognize abductive reasoning in the analogical inference above in the following way: (1) a surprising fact is $\forall x (x \text{ is } S \rightarrow (x \text{ is } P_1 \& \dots \& P_n))$, (2) an explanation for that fact is the following: if $\forall x (x \text{ is } S \rightarrow x \text{ is } M)$, then $\forall x (x \text{ is } S \rightarrow (x \text{ is } P_1 \& \dots \& P_n))$, therefore (3) the conclusion is $\forall x (x \text{ is } S \rightarrow x \text{ is } M)$. But in the analogical inference we come to (2) on the ground that each M (and M is more general than S) has all the observed properties of each S (the first premise of the analogical inference above).

Another pattern of analogical inference is the following:

$$\begin{array}{l} \forall x (x \text{ is } M \rightarrow (x \text{ is } P_1 \& \dots \& P_n)) \\ \forall x (x \text{ is } S \rightarrow (x \text{ is } P_1 \& \dots \& P_{k < n})) \\ \hline \forall x (x \text{ is } S \rightarrow x \text{ is } P_n) \end{array}$$

where by means of conjectured M we infer that each S has also a property P_n . We see the same sort of abductive reasoning as in the previous example included here.

In the following presentation of Bošković's examples, we also annotate the reasons for which premises and conclusion are accepted. Note that in Peirce's first quoted presentation of abductive inference it is in conclusion explicitly stated that „there is a *reason* to suspect“ C. And still more important, Bošković, as we have seen, explicitly distinguishes among different sorts of justifying reasons in the procedure of inductive reasoning. Let t, u, v be reasons (justifications), and let us denote where a special sort of justification is employed, like ‘gnr’ for justification by inductive generalization, ‘conc’ for conciliatory justification, and ‘law’ for law conjecturing justification.

First, here is Bošković's example of analogous reasoning in solving contradictions by conciliatory justification (*De cont. l.*, n. 134):

$$\begin{array}{l} \text{gnr}(t) : \forall x \text{ x is impenetrable} \\ u : \forall x(x \text{ is not impenetrable} \rightarrow x \text{ is entered by some substances)} \\ v : \forall x(x \text{ possesses pores} \rightarrow x \text{ is entered by some substances)} \\ w : \forall x(x \text{ is marble} \rightarrow x \text{ is entered by some substances)} \\ \hline \text{conc}(t \times u \times v \times w) : \forall x(x \text{ is marble} \rightarrow x \text{ possesses pores)} \end{array}$$

We formalize Bošković's example of conjecturing a law in the following way:

$$\begin{array}{l} t : \forall x(x \text{ is impenetrable} \rightarrow (x \text{ prevents bodies from occupying x's} \\ \text{position } \vee x \text{ gives way to them})) \quad [\text{not vice versa}] \\ u : \forall x(x \text{ is body} \rightarrow (x \text{ prevents bodies from occupying its position} \\ \vee x \text{ gives way to them})) \\ \hline \text{law}(t \times u) : \forall x(x \text{ is body} \rightarrow x \text{ is impenetrable)} \end{array}$$

Let us add two final examples of Bošković's controlled use of analogy at the crucial point of (im)possible generalization from observables to unobservables. In the first example, analogy is employed to achieve the full generalization of conjectured law (*De cont. l.*, n. 134):

$t : \forall x(x \text{ is observable} \rightarrow (x \text{ is mobile} \ \& \ x \text{ is inert} \ \& \ x \text{ is impenetrable}))$

$u : \forall x(x \text{ is unobservable} \rightarrow (x \text{ is mobile} \ \& \ x \text{ is inert}))$

$\text{an}(t \times u) : \forall x(x \text{ is unobservable} \rightarrow x \text{ is impenetrable})$

The conclusion is derived as analogically justified on the ground of “simplicitas quaedam et analogia naturae” (see Stay, 56, adn.)

In another example, analogical justification is impossible for strong (deductive) reasons (*positiva ratio obstat*) (*De cont. l.*, n. 135):

[$t : \forall x(x \text{ is observable} \rightarrow (x \text{ is mobile} \ \& \ x \text{ is inert} \ \& \ x \text{ is impenetrable} \ \& \ x \text{ is coloured}))$

$u : \forall x (x \text{ is unobservable} \rightarrow (x \text{ is mobile} \ \& \ x \text{ is inert} \ \& \ x \text{ is impenetrable})) \ \& \ \exists x x \text{ is unobservable}]$

$\text{ded}(v) : \forall x(x \text{ is coloured} \rightarrow x \text{ is observable})$

$\text{ded}(v) : \neg \forall x(x \text{ is unobservable} \rightarrow x \text{ is coloured})$

Besides, ‘ $\forall x(x \text{ is not observable} \rightarrow x \text{ is not coloured})$ ’ deductively follows from the sole third premise. We cannot transfer the property of being coloured from observables to unobservables on pain of contradiction: the predicate ‘coloured’ (being relative to ‘observable’) contradicts the subject ‘unobservable’. We therefore have to restrict analogous reasoning in such cases.

We note that Peirce clearly separated abduction (i.e., “retroduction” or „adopting of an explanatory hypothesis“) and an argument from analogy, that he finds deciphering to be a good example of the use of hypothesis (in distinction to analogy), and that he legitimates historically his distinction (although indirectly) also by reference to Bošković among others (see Bošković’s example in *De*

lumine quoted above).¹⁰ By the way, let us add that Peirce often emphasizes his strong adherence to nonextensional “Boscovichian points”.¹¹

6. Critique of the principle of sufficient reason

Reasons (*rationes*) are, for Bošković, causes or premises. As is known, there is a principle which refers to them – Leibniz's principle of sufficient reason:

$$p \rightarrow \Box p$$

where \Box (which is usually read as “it is necessary that”) means “there is a reason for”. Bošković's critique of the principle shows not only (a) his rejection of absolute necessitarianism, but also (b) his epistemological *constructivist* standpoint.

As to the first point, Bošković argues that the principle of sufficient reason does not hold because it has as a consequence the denial of human and divine free will. Besides, Bošković argues on the ground of the relativity of each created perfection (goodness) so that the best choice cannot be God's reason in creation (*De cont. l.*, nn. 126–127). On the other hand, the principle of sufficient reason holds in physics (*ratio physica in creatis habebitur semper*, *De. cont. l.*, n. 127). Hence, the principle can hold only if we count human and divine will as reasons (*stat pro ratione voluntas*).¹²

Especially interesting is the second of the above-mentioned points of Bošković's criticism of the principle of sufficient reason. Bošković argues that the principle of sufficient reason is of no use in concrete determination and demonstration (*De maris aestu*, n. 87, *De cont. l.*, n. 125). It only serves as a justification of reasoning in general: to infer consequences, premises should be explored, and to

¹⁰ Peirce says: “Several persons versed in logic have objected that I have here quite misapplied the term hypothesis, and that what I so designate is an argument from analogy. It is a sufficient reply to say that the example of the cipher has been given as an apt illustration of hypothesis by Descartes (Rule 10 *Oeuvres choisies*: Paris, 1865, page 334), by Leibniz (*Nouv. Ess.*, lib. 4, ch. 12, § 13, Ed. Erdmann, p. 383 *b*), and (as I learn from D. Stewart: *Works*, vol. 3, pp. 305 *et seq.*) by Gravesande, Boscovich, Hartley, and G. L. LeSage. The term has been used in the following senses: ... 7. most commonly in modern times, for the conclusion of an argument from consequence and consequent to antecedent. This is my use of the term. ...”. Ch. S. Peirce, “Some consequences of four incapacities” 1868, in: *Writings of Charles S. Peirce*, vol. 2, pp. 218-219.

¹¹ “In short, we are logically bound to adopt the Boscovichian idea that an atom is simply a distribution of component potential energy throughout space (this distribution being absolutely rigid), combined with inertia” p. 167 (“Man's glassy essence” 1892, in: *Writings of Charles S. Peirce*, vol. 8). Compare the draft of the same place: “The only logical view, therefore, is that the centre of a molecule is a Boscovichian point, the whole molecule, - that is, its forces, - extending out to infinity. This is a perfectly clear and diagrammatic idea” (p. 404). On “Boscovichian point” cf. also p. 285 (“Periodic Law” 1892, *Writings*, vol. 8.) and several places in [1992]. Hence, Andrew Reynolds concludes: “On the question of atoms, Peirce was, throughout most of his career, fondest of the Boscovichian conception of atoms as immaterial point centers of force (6.82, 6.242, 7.483),” A. Reynolds [2002], p. 79.

¹² See also I.-P. Sztrillich [1987].

infer effects, causes should be made evident (*De cont. l.*, n.128).¹³ The determinative and demonstrative use of the principle, which Bošković refutes, would consist of inference from the lack of reason (*ex defectu rationis*):

$$\neg \Box p \rightarrow \neg p \text{ (contrapositive of } p \rightarrow \Box p)$$

$$\neg \Box p$$

$$\neg p$$

However, as we can see from Bošković's argument, $\neg \Box p$ is not decidable by any inference procedure and from no knowledge base available to humans. Bošković holds that human knowledge is limited: we do not know all the causes that there can be – all necessary connections of causes with their effects and all determinations of free will (*Suppl. IV*, n. 34). Bošković also argues theologically: if the free will of the Founder can be a reason for the existence of something, all reasons cannot be known to us at all; His determination can be known only in a deficient and non-demonstrative way by induction (*De cont. l.*, n. 128). Bošković finds at least one reason why this is so in the tenuity of our cognition, which is conditioned by sensibility (*tenuitas nostrorum cognitionum*, *De cont. l.*, n. 129, *tenui nostrorum sensuum ope*, n. 131).

Bošković especially argues that each natural theory is incomplete with respect to natural truth, that is, there will always be some truth that is not demonstrable in a given natural theory. Hence, in a given natural theory, we cannot make any determinate conclusion from the lack of reason.

7. Bošković's constructivism

As we have mentioned, Bošković denies the availability of “all reasons” (which should include all possible proofs) to humans. Instead, he requires a defined *procedure* of surveying reasons to inductively confirm a hypothesis (and exclude the antithesis). On the other side, “there is a reason against” or “not all reasons are for” are reduced, for Bošković, to the requirement for an *example* of a concrete “positive reason” that could refute a hypothesis. Those Bošković’s requirements uncover his specific, inductive, constructivism.¹⁴ Hence, to elaborate Bošković's example, Archimedes could not ground his proof of the spherical form of the Earth on the lack of reason for the disturbance of the equilibrium. Archimedes could have said only that a, b, c ... (possibly infinitely of them) are not

¹³ See I.-P. Sztrillich [1987].

¹⁴ For the concept of constructivism, see, for example, Gödel [1995].

reasons for the disturbance of the equilibrium – but this would still leave the possibility that there might be some reason unknown to him for the disturbance of equilibrium. Instead, Archimedes should have given positive evidence for the spherical form of the Earth that would inductively make the hypothesis probable. For any such evidence it should be effectively decidable whether it confirms the hypothesis or not. Hence, instead of inferences from the lack of reasons and from the law of sufficient reason in general, Bošković proposes the procedure of reasoning with individual decidable reasons (justifications) for a hypothesis, and claims the truth of the hypothesis only with probabilistic strength.

Formally expressed, Bošković replaces reasoning that includes operator \square ('there is a reason for ...') with reasoning on the ground of concrete reasons (justifications). This is precisely the main feature of contemporary justification logic (conceived already by Gödel), where the format $t: p$ is used for "t is a justification for p".¹⁵ What is specific for Bošković is a typology of reasons, differing in strength and in the context of the reasoning procedure where they can be used. It is therefore appropriate to conceive Bošković's inductive logic as a kind of justification logic.

8. *Sketch of inductive rules*

We, first, sketch the typology of reasons for Bošković's induction:

ded(t) - deductive evidence

dcd(t) - confirmation in each case that is decidable by observation
or experiment

larg(t) - confirmation in a large number of cases

gnr(t) - sufficient reason for generalization

conc(t) - conciliatory reason

law(t) – conjectural reason in order to establish a law

an(t) - analogical reason

The following special propositional rules corresponding to the reasoning within Bošković's induction can be proposed (with a general operation \times for a combination of reasons):

¹⁵ See Artemov [2001]. The idea of justification logic stems from Kurt Gödel as a solution for constructive formalization of the concept of provability (Zilsel lecture, from 1938), but, unfortunately, it remained unpublished until 1995. In this moment, justification logic was already independently rediscovered by S. Artemov. For the semantics of justification logic see, e.g. Fitting [2005].

$$\begin{array}{ccc}
\text{dcd}(t) : p & \text{gnr}(t) : p & \text{IND}(t) : p \\
\text{larg}(u) : p & u : (\neg p \rightarrow r) & \text{ded}(u) : \neg p \\
\hline
\text{gnr}(t \times u) : p & v : (q \rightarrow r) & \hline
& w : r & \text{ded}(t \times u) : \neg p \\
& \hline
& \text{conc}(t \times u \times v \times w) : q &
\end{array}$$

$$\begin{array}{cc}
\text{t} : (p \rightarrow (r_1 \wedge \dots \wedge r_n)) & \text{t} : (p \rightarrow (r_1 \wedge \dots \wedge r_n)) \\
\text{u} : (q \rightarrow (r_1 \wedge \dots \wedge r_n)) & \text{u} : (q \rightarrow (r_1 \wedge \dots \wedge r_{n-1})) \\
\hline
\text{law}(t \times u) : (q \rightarrow p) & \hline
\text{an}(t \times u) : (q \rightarrow r_n)
\end{array}$$

IND(t) : dcd(t), larg(t), gnr(t), conc(t), law(t), or an(t).

1. *Appendix: a constructive defect*

Bošković excluded inferences from lack of reason as non-constructive. Does that mean that Bošković must have excluded each form of general inference with reasons? Clearly not – if reasons are restricted to reasons that are (constructively) available to a reasoning epistemic agent. This is precisely the way we can interpret Hunter's [1965] proposal of an “extension of logical theory” from the standpoint of the previous section. Hunter referred to Bošković's elaboration of conditions for inductive reasoning, especially to Bošković's clause *nisi ratio positiva obstat*, as a necessary condition for a valid inductive conclusion.¹⁶ For illustration, here is one of Hunter's examples of inductive reasoning with quantified reasons:

¹⁶ Hunter says (p. 83):

The first person I know of to state clearly the importance in inductive arguments of the clause *nisi ratio positiva obstat* (“unless there is a positive reason against”) was Roger Joseph Boscovich in his *De Lege Continuitatis* (1754). He also makes use of the legal notion of “presumption”.

There is an earlier, Newton's clause “donec alia occurrerint phenomena, per quae [propositiones] aut accuratiores reddantur aut exceptionibus obnoxiae” (in *Philosophiae naturalis principia mathematica*, p. 55-56, *Regula IV*). Bošković's clause aims at the validity of inductive argument (“principles of investigation”) as such, while Newton's, although having a similar content, aims at preventing the rejection of inductive propositions by hypotheses, and at the accurateness and corrections of inductive propositions.

For one comparison of Newton's and Bošković's principles of the inductive method, see in D. Škarica [2002]. Cf. also V. Bajsić [1987].

I know of at least one case of an X being a Y

I know of no positive reason for thinking that an X might not be a Y

I have some reason for thinking that all Xs are Ys

The second premise, “presumption” (or “Boscovichian premise” as we might also term it), is essential to the validity of the argument.¹⁷ As Hunter states, the acceptance of the two premises and the rejection of the conclusion, “though not self-contradictory, is absurd in virtue of the meaning of what is said”. The distinction made to the reasoning *ex defectu rationis* is evident in Hunter's examples – neither the premises nor the conclusion are assertoric, but they are epistemically modalized with “I have some reason for thinking that ...”, “I know of no positive reason for”, or similar expressions. In accordance with our previous analysis, “I have some reason for thinking that ...” should be read as an abbreviation for a concrete reason of which we are aware”, and “I know of no positive reason for ...” should imply a certain procedure of verification of the reasons available to us. From Hunter’s examples we clearly see that the *presumption* is in fact a constructive epistemic transformation of the premise about *defectus rationis*, and that as such it not only contributes, but even makes inductive inference possible.

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¹⁷ Here is another, rather more complex example given by Hunter:

I have good reason for thinking that if p then q and r and s ...

I have good reason for thinking that q and r and s ...

I have no good reason for thinking that any definite suggestion that I know of, other than the suggestion that p, is a true explanation of the facts that q and r and s ...

I have some reason for thinking that p

Presumption is included in the third premise (“no positive reason for thinking otherwise”). Note also that the argument explicitly includes the abductive component of reasoning to the best explanation.

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