Machines, Logic and Wittgenstein⋆

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Abstract Wittgenstein's "machines-as-symbols" are considered with respect to their historical sources and their symbolic and logical nature. Among these sources and precursors, along with Leonardo's drawings of machines, there are illustrated "machine books" (*theatra machinarum*), a kind of book published in the period from the 16th to the 18th centuries which consist of pictures and descriptions of a variety of mechanical devices. Most probably, these books were one of Wittgenstein's inspirations for his view of machines as components of language games (not just for his earlier philosophy of depicting symbols in TLP). The picture of *homo volans* in Vrančić's (Verantius) machine book (1615/16) possessed by Wittgenstein is taken as an example. In particular, *homo volans* is shown to contain patterns of logical laws and rules and to be abstractly interpretable as a logical symbol. A machine (or its picture), taken as a symbol, is shown (a) to be a precondition of a meaningful "overview" of a mechanical work (including logical formalisms) that exceeds the limits of decidability; (b) to possess causal features if causality is understood teleologically and in a deeper sense of a "binding" life.

Keywords machine, picture, logical pattern, use of language, forms of life, working of a machine, causality, Ludwig Wittgenstein, Faust Vrančić

1 Introduction

In many places in Wittgenstein's work, examples of machines are used to illuminate the use and meaning of symbols and to describe the nature of logic.

For instance, a well known example is the passage of *Philosophical Investigations* (PI, Wittgenstein, 2009, §§193–194) (cf. RFM, Wittgenstein, 1998, I-122–125 pp. 84–87, and UW, Wittgenstein, 1976c, 405–406) on machines (or their pictures) as symbols. The question naturally arises whether the so-called "machine books" (*theatra machinorum*) possessed by Wittgenstein might have in any way influenced his reflections: in particular, whether conceptual presuppositions of this corpus can be traced in Wittgenstein's work. In the scholarly literature on Wittgenstein, machine books are mentioned or referred to several times. Spadoni and Harley (1985) and Hide (2004) mention some machine books as belonging to Wittgenstein's, and later, to Russell's library. Flowers III (1999) notices that Wittgenstein took Vrančić's (Verantius) *Machinae novae* with him from Germany to England, where he was to begin his study of engineering.[1] Machine books constitute the corpus of illustrated machine books (*theatra machinarum*), consisting of drawings, etchings, or engravings of machines (broadly conceived, including, for example, bridges, buildings, and fountains) with short or extensive descriptions.[2] Among the books Wittgenstein, at that time without funds, sold to Bertrand Russell in 1919 below their real prices,[3] we find, for example, the machine books by Faust Vrančić (Faustus Verantius) *Machinae novae*, 1615/16, *Theatrum machinarum novum* by Georg Andreas Böckler, 1661, works by Jacob Leupold, for example, *Theatri machinarium hydraulicarum tomus I, oder: Schau-Platz der Wasser-Künste Erster Theil*, 1724, and *Theatrum machinarium, oder: Schau-Platz der Heb-Zeuge*, 1725, as well as *Architectura civilis*, ca. 1668, by Johann Wilhelm. Wittgenstein also owned an edition of the manuscripts by Leonardo da Vinci possesed by the *Institut de France* (1881–1891). In addition, there were books

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[1] Flowers III (1999, II, 113) incorrectly gives Vrančić's nationality as Italian, although Vrančić was a Croat, born in Šibenik, in that period under the rule of the Venetian Republic and today a part of Croatia. When Vrančić declares that he is writing in his own language (the language spoken in his homeland), he is writing in Croatian (*Xivvot nikoliko izabraniih divviicz*, see Vrančić/Verantius, 1995, signed in his archaic orthography as 'Favst Vrancsich'). He signed his Latin works as 'Faustus Verantius' and 'Faustus Verancius', sometimes with the addition 'Sicenus' (he also used pseudonyms, such as 'Yustus Verax Sicenus'). Among other duties, Vrančić served as a secretary of the Holy Roman Emperor Rudolph II in Prague and was the titular bishop of Csanád in Transylvania.

[2] This is a special genre of books that were published in the time span of ca 150 years from the sixteenth to the eighteenth centuries. Together with the illustrated treatises, such as, for example, Georg Agricola's *De re metallica* (2nd ed. 1561), these books make a link between the early Renaissance and the 19th century engineering and kinematics. In *theatra machinarum*, machines are usually presented as wholes, in parallel or central, sometimes combined, projections (see Thüringen 2015), and often with open walls, floor or roof for the interior to be visible, or in a section view. Occasionally, some of the parts of a machine are shown separately (sometimes in "expanded view", e.g., Besson 1582, fig. 18) or with added floor plan (e.g. Vrančić/Verantius 1993, fig. 12, 13). Separate, detailed presentations of the particular mechanisms of a machine are characteristic for Leupold, which was pointed out by Franz Reuleaux (1875, p.11–12) (cf. also Moon, 2007, pp. 71, 145, 388–389). The influence of the machine book tradition can be traced deep into the 20th century (e.g., Ferguson, 1992, p. 120, cf. Ferguson, 1977).

[3] See Spadoni and Harley (1985, especially pp. 32, 43), Hide (2004), search `http://digitalcollections.mcmaster.ca/digitalrussell` (cf. `http://digitalcollections.mcmaster.ca/russell-lib/media/machinae-novae-fausti-verantii-siceni`).

on the first hot air balloon flights (for example, *La Description des Expériences Aérostatiques de MM. de Montgolfier*, 1783, by Faujas de Saint-Fond, and its German translation from 1784 and 1785).

Until now, the role and place of these books have not been comprehensively considered with respect to Wittgenstein's philosophical views, especially with respect to his reflections on the symbolic aspect of a machine. Spadoni and Harley explain Wittgenstein's interest in the machine books by his study of engineering.[4] Hide (2004, p. 69) emphasizes the importance of these books for Wittgenstein's "mechanistic conception of language" in TLP and the influence of the method of "explicatory picture texts" of the "machine books" on Wittgenstein's "aphoristic" discourse (Hide, 2004, p. 70, 75–76). A possible influence of machine books (including *Machinae novae* by Vrančić) on Wittgenstein is mentioned in (Kovač, 2020) and (Kovač, 2019, footnote 5).

It is reasonable to accept that these books left a trace on Wittgenstein's mind even after he sold them to Russell. It will be argued that machine books, in an essential sense, particularly conform with some of Wittgenstein's later philosophical views, and might have been one of the incentives for his reflections on machines-as-symbols, along with, for example, the motivations that came from his considerations on Turing's theory of computation, Reuleaux's kinematics, and Plato's views on knowledge and language.

We note that Wittgenstein also possessed an edition of works by Leonardo da Vinci. Leonardo's drawings of machines (especially flying machines) might have well attracted Wittgenstein's attention, given his interest in engineering and aeronautics. Leonardo's work precedes the emergence of machine books, and seems to presuppose a philosophical viewpoint to which Wittgenstein's position in *Tractatus* is akin. Hamilton (2001, p. 81) likens Wittgenstein's analysis of propositions in TLP into elementary propositions to Leonardo's pictorial analysis of the ratchet (depicted as a whole and as separated into elements). Hagberg (2016) emphasizes Leonardo's capacity not just to depict reality but rather a visual imagination of what is possible, as contained in Wittgenstein's "seeing as" ("aspect-perception", PPF ii; PI pp. 117, 125–126, 137).

Whereas Leonardo's viewpoint is visual-artistic, with painting conceived as a fundamental science, machine books are straightforwardly technically oriented.[5] In addition, Wittgenstein, most probably unknowingly, shares his intertwined interests in machines, logic and language with one of the machine

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[4] "Prior to coming to Cambridge in 1911 in order to study the philosophy of mathematics with Russell, Wittgenstein had been engineering student; this explains his interest in collecting antiquarian material related to that subject."

[5] For Leonardo, see, for example, Capra (2013), bypassing the distinction between "depicting" and "showing": "In order to paint nature's living forms, Leonardo felt he needed a scientific understanding of their intrinsic nature and underlying principles; in order to analyze the results of his observations, he needed his artistic ability to depict them" (p. x). "He preferred to *depict* the forms of nature rather than *describe* their shapes, and he analyzed them in terms of their proportions rather than measured quantities" (p. 8). "He portrays nature's forms—in mountains, rivers, plants, and the human body—in ceaseless movement and transformation" (p. 9).

book authors, Faust Vrančić (Faustus Verantius). Besides *Machinae novae*,
Vrančić is the author of a short logic (which includes a critical examination of
the subject of metaphysics) as well as of a dictionary of five languages. These
works of Vrančić's, taken together, offer a pattern of interconnections between
logic, language and machines that might help in understanding the logical,
linguistic, and philosophical nature of Wittgenstein's machines-as-symbols.

We will first show that *theatra machinarum* clearly anticipates the symbolic
nature of a machine (or of its picture) and might serve as one of the "models"
for Wittgenstein's ostensive way of teaching and learning. We then show that
a machine can be understood as a logical symbol. Thereafter, the problem of
(un)decidability will be considered from the Wittgensteinian viewpoint of a
machine-as-symbol and related to Turing machines. Finally, we address the
question of the causality of machines regarding their symbolic nature and
language.[6]

2 Theatrum, language games, and forms of life

In the passage of *Philosophical Investigations* mentioned earlier (PI §§193–
194, cf. RFM I-122–125 and UW 405–406), Wittgenstein considers a machine
and a picture (*Bild*) of a machine as symbols (*Maschinensymbol*).[7] We show
that Wittgenstein's considerations on machines-as-symbols have essential re-
semblances with the approach of "machine books" (*theatra machinarum*).

Wittgenstein points to a specific "language game" in which "we use a ma-
chine, or the picture of a machine, as a symbol of a particular mode of operation
of the machine" (PI §193).[8] A machine ("its structure", *Bau*, RFM I-122 p. 84)
symbolizes its action. "A particular mode of operation" [*Wirkungsweise*] is,
according to Wittgenstein, a "given way of moving", "the way it [a machine]
moves" (PI §193), "the possible movements of a machine" (PI §194).[9] Moreover,
as Wittgenstein indicates, these movements are "determined" by the meaning
of the machine-as-symbol: a machine (or its picture) is just the beginning
of a series of (pictures of) the movements of the parts of the machine, and
we can "derive" this series from the symbol on the ground of our acquaintance

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[6] For the background and an essential interrelation of the concepts of a machine and
mechanical process (a process that "could be carried out by a machine," Turing, 1965b, p.
160) with Turing machines and logic, let us recall that the general concept of a mechanical
process (procedure) can be defined by means of Turing machines, that the concept of a
formal system S is equivalent with the Turing machine producing the theorems of S, and
that a halting Turing machine is equivalent with a valid inference of first-order logic. His-
torically, see (Turing, 1965a, pp. 118, 135, 138, 145–149) and, for example, (Gödel, 1986,
'Postscriptum' 1964 pp. 369–370, 346).

[7] Compare also the following quotations from LFM (Wittgenstein, 1976a): "For a machin-
ery often stands as a symbol for a certain action" and "The fact is, we use the mechanism
as a symbol for a certain kind of behaviour" (pp. 194, 195).

[8] Translations from Wittgenstein's work are modified.

[9] For comparison, according to an earlier text (PG, Wittgenstein, 1978, 17), the truth
matrix for negation is a "sign of negation," and that, "only by means of the way how it
works – I mean, how it is used in the play."

with the machine.[10] Wittgenstein emphasizes that what is meant is the proper functioning of a machine, its "regular use" ("custom") according to what we have learned and to our "practice of playing" (cf. PI §§197–199 on obeying a rule); it is not about the actual behavior of a machine, where a "distortion of parts" ("bending, breaking off, melting") could happen. Thus, Wittgenstein conceives the meaning of a machine-as-symbol as determined by the language game in which we derive the machine's regular movements if the machine, or its picture, is shown to us (cf. PI §§188, 190).[11]

The use of machines-as-symbols as Wittgenstein describes it is essential in "machine books." A machine book contains a series of pictures or drawings of machines, accompanied by short or extensive descriptions, in order to show the reader how these machines work. On the ground of our foreknowledge and previous practice, with the help of an added description, it is expected that we will understand the working of a displayed machine. We point out three aspects in which Wittgenstein's machines-as-symbols resemble the pictures of machines in the machine books: (1) focus on a regular way of working, (2) importance of the ostensive way of learning, and (3) grounding in "needs" and "forms of life". What is crucial is that all three aspects are related to the symbolic and "theatric" nature of machines or their pictures.

(1) Like Wittgenstein's machine-as-symbol, no machine drawing in a machine book is meant as a mere realistic depiction and as indifferently showing or indicating various physical features of a machine, including its possible deformations and deviations from its regular working. The authors of machine books present the general structure and the regular ways of how a machine should work (cf. *rite suo officio fungatur*, Vrančić/Verantius, 1993, decl. XX). They focus on the "form" and "mode" (*forma, modus*, Verantius) of a machine, its "species" and "genus" (*Art, Gattung*, Böckler), give general instructions about the material of which the machine should be built, and occasionally mention physical wear or possible malfunctioning of devices (e.g. Vrančić/Verantius, 1993, decl. III, XXV).[12] In Wittgenstein's terms, these pictures should "serve" as "paradigms" of machines and their working (RFM I-105 pp. 75–76) and, for example, as in the case of reversal (of numerals), we learn a sequence of "forms", "a formal property of forms" (RFM IV-50 p. 25).[13]

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[10] Wittgenstein describes: "...we give someone such a picture [of a machine] and assume that he will derive the movement of the parts from it" (PI §193 p. 78); "I show you the possibility of a movement, say by means of a picture of the movement" (PI §194 p. 79).

[11] Machine-as-symbol should be distinguished from merely imagined ("dreamed") machines, without sufficient detail about the building and functioning of the machine. See an example of an "envisioned" (*erdichtet*) flying machines in CV (Wittgenstein, 2006, pp. 60–61).

[12] For instance, Leupold points to the disadvantages of one's being able only to literally stick to the pictures: "ein *Mechanicus*, der sich bey allen Machinen sogar genau an den Vorriß binden will und muß, wird selten grosse Thaten thun, und klüger handeln, wenn er solches andern überlasset; ...und diesem ist genug, wenn er die *Invention*, generalen Verhältnisse und nöthigsten *Observationes* findet" (Leupold, 1724b, Vorrede).

[13] Cf. Wittgenstein: "What we call '*descriptions*' are instruments for particular uses. Think of a machine-drawing, a cross-section, an elevation with measurements, which an engineer has before him. Thinking of a description as a word-picture of the facts has something

As pointed out by Wittgenstein, this "formal" and "paradigmatic" structure and working of a machine is in itself essentially different ("much stricter and harder," RFM I-128 p. 88) from an experiential causal connection between the parts of a machine. Wittgenstein sometimes describes paradigmatic machines as "ideally rigid" (*ideal starr*, RFM I-125 p. 86), "super-rigid" (e.g., LFM p. 199), "absolutely" ("infinitely") hard" (LFM pp. 196, 198) and as a "super-strong connection" (PI §130 p. 88).[14] With the *caveat* that these qualifications must not be understood in the sense of the superlative of a material rigidity, but rather, if at all, as a (perfect) rigidity of a very different kind and origin (RFM I-123,124 p. 86).[15] Wittgenstein conceives this rigidity not as "ethereal" (RFM I-119; material, though "perfectly hard" RFM I-120,121), but as symbolic and thus comes close to the 'machine theater" tradition.

(2) Wittgenstein insists on ostensive learning and teaching (cf. PI §208), prior to mastering any specialized, formal theory. Rather than "explanation", he emphasizes "description" and "insight into". This approach by ostension, "showing", "seeing", "perspicuity", "surveyability", prompts us to connect machine examples from Wittgenstein's work with machine books. Machine books intuitively and vividly demonstrate, on the ground of some foreknowledge and additional instructions, a machine's regular way of operation and its intended possible movements. According to (1), the purpose of such machine pictures is not merely to depict the actual working of a machine, but to "show" what the machine is, the way (rules) it works, and what is its use. By *looking* at a picture in a machine book, with the help of foreknowledge and an accompanying remark, we are expected to "see" and understand how the presented machine works and how it should be used. In addition, we should "see" and "read off" not just the working of particular mechanisms of the parts of the machine but also the working of the machine as a *whole*. By itself, the word *theatrum, Schau-platz*, means the place where something will be shown, played, especially visually and in words.[16] The explicit title of "theater" (*Schauplatz*)

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misleading about it: one tends to think only of such pictures as hang on our walls, which seem simply to depict how a think looks ... (These pictures are, as it were, idle.)" (PI §291).

[14] Compare an example in (Plato, 1967) (207d–208a, near the place on "primary elements" quoted by Wittgenstein in PI §46): we could always write 'Theaetetus' (correct) and always 'Teodorus' (incorrect) instead of (correctly) 'Theodorus.' Thus, it is argued, although 'Theaetetus' is correct, it is not sufficiently grounded so as to be knowledge—not "rigid" enough—because of an inconsistency between the two manners of writing.

[15] Wittgenstein's inspiration for introducing the term "rigidity" is kinematics, and thus its tradition from Leonardo and machine books to Reuleaux. On this tradition, see Moon (2007). Unfortunately, in the context of "machine books", Moon does not mention Vrančić's *Machinae novae*. On the role of Wittgenstein's technical education, especially regarding TLP, see, e.g., (Hamilton, 2001). Recently, for Reuleaux's influence on Wittgenstein's idea of "machines-as-symbols", see (Gandon, 2019).

[16] Leupold speaks in his *Theatrum machinarum generale* (Leupold, 1724a) (not on the list of Wittgenstein's books given to Russell) about what "will be put in front of the eyes", "as it were in a public play stage" ("for looking at"), and this is (a) not only "fundamenta and principles of mechanics", "by means of lines and figures", but also (b) machines and instruments, by means of an "outline" and "fundamental clarifications."
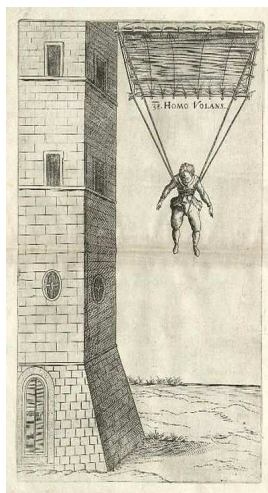
Fig. 1 Homo volans (Vrančić, *Machinae novae*, picture 38)

of many "machine books" points to showing, displaying (as on a stage) and looking.[17]

(3) Wittgenstein focuses on the "particular use" of pictures and descriptions – not depicting facts, but presenting in general the construction and regular working of a machine. This use determines the meaning of the picture as a symbol. It is a learnable custom, a convention (required also for machine books), to use pictures and particular perspectives to present the intended composition of a machine and its working. Also, we can simply declare that a picture presents such and such a machine[18]. Some devices can be easily recognized on the ground of the preexisting use of similar devices as well as on the ground of the "needs of life" which we see can be served and satisfied by the use of the proposed devices. This includes, for example, food production, habitation, irrigation, religious service, travel and transport (over land, see, river, air), printing, time measuring, or ornatus (for *homo volans* in Vrančić/Verantius, 1993: "to jump, without any danger, from a tower or other projecting place"). Each machine book presents a variety of the "needs of life" that the machines serve (including learning and introducing conventions). As a whole, the book

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[17] According to Wittgenstein, theater changes the perspective (e.g., PPF §32)—ordinary, everyday things of life look "uncanny and wonderful" (CV 6e). "And it might well be said of someone who plays longing on the stage, that he experiences or has a picture of longing: not as an *explanation* of his action, but as a description of it" (Z §655 p. 114e, Wittgenstein, 1970).

[18] See, for example, in (Vrančić/Verantius, 1993): "Hic tibi lector tria horologia exhibeo, unum quod ligne, alterum quod aqua, tertium quod sole operatur" (Decl. VI) and "Hoc est illud horologium nostrum solare, quod in priore tabela, duobus circulis in plano delineauimus: hic autem erectum, et simul compositum tibi exhibemus" (Decl. VII). Cf. Leupold's general introduction to such a (theatric) use of machine pictures in footnote 16 above

can be said to present a "carpet of life" (*Lebensteppich*) and a "bond of life" (*Band des Lebens*, translated also as "ribbon of life" or "weave of life").[19]

In accord with Wittgenstein, the connection between a machine-as-symbol and its meaning is a kind of language game (*Sprachspiel*), which is also connoted in *theatra machinarum* (*Spiel* is German for both 'play' and 'game'). *Theatrum* has a symbolic (or even metaphoric) nature: it is not just accurately depicting pictures but rather a play, whose meaning is something far more general than just what we literally see before our eyes, and is symbolically presented by a scene on the stage.[20] Moreover, our needs and forms of life, into which language games are embedded, run in front of us in each of the *theatra machinarum*.

In Vrančić's *Machinae novae*, as seen from the viewpoint of the whole of his work (which includes logic and linguistic publications), a symbolic nature of machines particularly stands out. With some natural generalization, a picture of a machine, with accompanying *declaratio* (in five languages), can be symbolically understood as an *oratio docens*, a 'teaching discourse' (which makes the subject of logic), comprising definition, division, and argumentation (Vrančić/Verantius, 1616, pp. 1, 7–8). A picture with an accompanying *declaratio* involves 1) a definition (*declaratio*, *descriptio* are some of the synonyms), showing what kind of machine it is, how it works, for what purpose, and so on (these are parts of a definition according to Vrančić); 2) a division (in species or in parts), since, for example, several species of mills or bridges are presented, and each picture clearly shows the essential parts of the machine or even a machine decomposed into its parts; 3) an argument, by showing (in picture and words) the mechanism due to which the machine works as intended (and better than other versions of the machine). In sum, Vrančić's machine book can be seen as a list ("dictionary") of machines, each of them ostensively and declaratively defined, divided and demonstrated.[21]

Finally, there are explicit logical and philosophical components of mechanics which Wittgenstein could have encountered in Leupold's work. In Leupold's (liberal) translation from Cunrad Dasypodius' (Dassipodius) *Heron Mechanicus*, the division of mechanics into logical and "chirurgical" ('manual') is introduced, and "logical mechanics" (*mechanica logica*) is defined through the philosophical and mathematical approaches, which comprise intelligence (*ingenium*) and proofs (*demonstrationes*) (Leupold, 1725, pp. 2, 5, 12) (cf. Dasypodius, 1580, introductory text).

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[19] The *variety* of Turing machines, to which Juliet Floyd refers to as the inspiration for Wittgenstein's late philosophy (Floyd, 2016), was preceded, in Wittgenstein's development, by the machine books in his possession, which he was familiar with already in his early scholarly years.

[20] According to Wittgenstein, theater is one of the language games. Cf. *Theater spielen* ("acting in a play", PI §23) and *Schauplatz unseres Sprachspiels* ("stage for our language game", PI §179).

[21] Again, Vrančić's five-column dictionary (of five languages) can be conceived as a machine: if a Latin word and the name of the language are entered, the translation into this language is returned.

3 *Homo volans* and logical patterns

In several places, Wittgenstein considers the likening of propositions, proofs, and language to mechanisms, machines or machine parts,[22] but at the same time warns of the misleading character of such a comparison (LFM p. 190). Conversely, it is not hard to see that each picture of a machine, if looked at abstractly, displays, in a non-formalised way, not only a particular proposition or a particular proof, but also some logic or part of logic (logical axioms, tautologies, rules of inference, and definitions; cf. RFM pp. 434–435 VII-72). Correspondingly, Wittgenstein also likened a mechanism to a proof: "…a picture may very well convince us that a particular part of a mechanism will move in such-and-such way when the mechanism is set in motion. The effect of such a picture (or series of pictures) is like that of a proof" (RFM IV-21 p. 235).[23]

We show the proof-like character of a machine by the example of *homo volans* from Vrančić's machine book (Figure 1), to which Wittgenstein might have paid attention due to his early interest in the study of aeronautics.[24] What we see in this picture can be described in the following way: a human hangs on a parachute, which hangs *on* the air, that is, according to Vrančić's appended clarification, the air, in the form of the upward wind, artificially produced by the downward movement of the parachute, holds the parachute in the air and prevents its free fall. This configuration depicted, we also see that a human hangs *on* the air (by means of a parachute). We anticipate that by thus hanging in the air, a human will safely land on the ground. Obviously, we can describe what we see by an inference: assume that x, which is a human, hangs on a parachute (as we see in the picture); assume that the parachute hangs on the air (we see a blown parachute wing that holds the parachute construction in the air); thus, we also see (as if "of itself", without any addition to the picture) that x, which is a human, is hanging on the air—this is an evident conclusion, shown immediately by the pictured antecedent assumptions. We can also easily imagine a picture with a negative assumption: if a parachute does not hang on the air (for example, it is quickly falling to the ground, say, because it is too small), and a human hangs on (is attached to) the parachute, then the human will not hang on the air.

Since we are considering machines from a possible Wittgensteinian point of view, we apply modern symbolism of first-order logic to express some logical

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[22] RFM pp. 315, 433, 437; LFM p. 282; PI §559; RPP1 40 (Wittgenstein, 1980); proof as a "house", RFM III-41; earlier: PG 33, 135, TLP 6.1262 (Wittgenstein, 1976b).

[23] See also page 199: "…if I say there is no such thing as the super-rigidity of logic, the real point is to explain where this idea of super-rigidity comes from—to show that the idea of *super-rigidity* does *not* come from the same source which the idea of *rigidity* comes from." The last one "comes from comparing things like butter and elastic things like iron and steel" and the first one "comes from the interference of two pictures—like the idea of superinexorability of law."

[24] Drawings of various aerospace devices by Leonardo da Vinci can be found in his manuscripts from the *Institut de France*, published in the edition possessed by Wittgenstein, for example, in the manuscript B, da Vinci, 1883, ff. 74v, 80r, 83v, 89r. The famous sketch by da Vinci of a parachute is contained in the *Codex Atlanticus*, f. 1058v, https://www.codex-atlanticus.it/#/Detail?detail=1058.

forms as examples of implicit logic of a machine. Of course, in a more traditional fashion, the syllogistic laws can also be used for the same purpose, which would be closer to the logic of Vrančić/Verantius (1616). If we focus on x's (human's) "hanging", we can describe the *homo volans* picture in the following way: (i) "For x that is a human, if x hangs on a parachute, then x hangs on the air". We immediately also see (ii) "If humans hang on a parachute, then humans hang on the air". We also see that we cannot have (i) without (ii), that is, if we abstract from the concrete terms indicated by the picture, the classical propositional axiom scheme $(Ax \to (Bx \to Cx)) \to ((Ax \to Bx) \to (Ax \to Cx))$ is patent.

To further develop the above example, assuming that what hangs on the parachute hangs on the air, let us imagine that a human does not hang on the air (and is freely falling to the ground). It is clear that (s)he cannot hang on the parachute either (otherwise, (s)he would hang on the air), that is, a picture of the situation with a human not hanging in the air, but hanging on the parachute, is impossible (cf. the classical theorem scheme $(Bx \to Cx) \to (\neg Cx \to \neg Bx)$).[25] Let us now imagine that a human does not hang on the parachute since the human is not hanging on the air (e.g., the parachute has not opened or the parachute cords are broken). If we now imagine the human's hanging on a parachute and try to imagine that human's not hanging on the air, the latter would be impossible (because in that case (s)he could not be imagined to hang on the parachute). Thus, it remains to imagine the human as hanging in the air (compare the axiom scheme $(\neg Cx \to \neg Bx) \to (Bx \to Cx)$). Next, if we see a human hanging on a parachute, whatever else might also be the case, this does not change the fact of seeing the human hanging on the parachute (cf. the classical axiom scheme $Ax \to (Bx \to Ax)$).

Furthermore, it is clear that 'x' in the *homo volans* picture refers to any human satisfying the conditions of the annotation (right proportions of the human's weight and the canopy area of the parachute). This indicates first-order generalization as implied by the picture. Besides, 'A' need not mean a human, but could refer to any other being that is proportionate to the parachute; instead of the air, we could imagine another medium, for instance water, and instead of a parachute, some analogous device (platform, ship, boat) preventing an object from sinking. Thus, the picture can lead us to the second-order generalization, with universally quantified predicate variables instead of schematic predicate letters.

The *homo volans* example is particularly interesting because the displayed "hanging" (of x on the parachute and on the air, and of the parachute on the air) can be understood quite abstractly (as a logical "parachute"), in a formal logical sense of predication: in particular, in the sense of the subject–predicate relation of traditional logic, as well as in the sense of the antecedent–consequent dependence in the conditional predications in modern logic. As to the first case, Kant, in his reflections on logic (Kant, 1924, AAXVI), presents

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[25] Compare Wittgenstein's impossibility examples of not succeeding in imagining a "lamp different from itself" or trying to "draw an object to himself from a distance by mere willing," PI §132 p. 89.
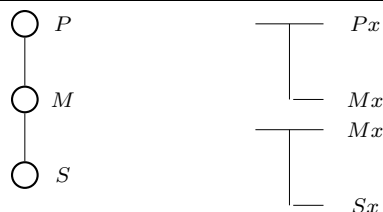
Fig. 2 Logical subordination of terms in Kant (left) and Frege (right)

categorical syllogistic figures by means of "hanging" of a middle term on a predicate term, and of a subject term as "hanging" on the middle term (see the figure on the left below). The terms are here "hooks" (*Haken*), on which other terms might "hang". On the other hand, "hanging" in a logical sense is inherent in Frege's *Begriffsschrift* (Frege, 1988), where the antecedent of a conditional is presented as "hanging" on the consequent (the figure on the right above). With "hanging" taken in the logical sense, *homo volans* (like, analogously, other devices) does not only contain logical patterns (*Muster*) or paradigms (*Vorbild*) of reasoning, but can be viewed in a quite abstract way and become in itself a *logical* symbol for interdependence of P, M and S ('if M is P, then, if S is M, then S is P').[26]

The "intuitiveness" and "surveyability" of logic, as represented by machines-as-symbols, is in accordance with Wittgenstein's view that "surveyabilty" (intuitivity) is essential for a proof ("proof is an intuitive procedure") and that the "proving force" ("cogency") of a proof should be "geometrical" (RFM I-42–43 pp. 173–75). Moreover, a proof can be thought of as a "theater play" (*Theaterstück*) so that "watching" the play "leads" us to something, that is, we can make a prediction of "how it will go" (RFM IV-33 pp. 241–242).

Notwithstanding the correspondences between machines and inferences, Wittgenstein would not say that we literally "prove" something by a machine:

> clearly it would not normally be said of someone turning the wheel that he was proving something. Isn't it the same with someone who makes and changes arrangements of signs as a game; even when what he produces could be seen as a proof?   (RFM V-4 p. 259)

Also, a machine could come into existence and be started by a mere accident (RFM V-2 p. 257). According to Wittgenstein, for a machine to be

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[26] Of course, the logic of (Vrančić/Verantius, 1616) is more akin to Kant's than to Frege's logic. Vrančić shares with Kant the formal concept of logic and the very logical forms with which formal logic should deal: term (concept), proposition (judgments), inference, methodological forms (definition, division, proof), with the distinction that, for Vrančić, methodological forms are the main subject of logic, whereas for Kant it is concept, judgment and inference (see AAXVI, and *Jäsche Logik* in Kant, 1923). It should be noted that Frege chooses conditional, and not, say, conjunction or disjunction, as a primary connective, thus emphasizing logical subordination ("hanging") as a key logical feature, inherent also in the so-called subject–predicate judgments of traditional logic (Frege, 1988, pp. 5–10, 24, Frege, 1998, pp. 20-23).

capable of inferring, it should have a "linguistic [*sprachliche*] function" and be a meaningful symbol.[27] In the case of *homo volans*, there should be a human ("human calculating machine") able, not necessarily to jump by a parachute, but to "read off" and check the picture of a parachute and the symbolized conclusion which states that a human will safely land by means of the depicted parachute.

4 Mechanical decidability and meaningful overview

The "theatric" approach to a machine (as in machine books) and Wittgenstein's symbolic understanding of a machine are not pieces of mechanical knowledge (without reflection) but rather include an overview of the whole working of the machine and an understanding of the meaning and use of the machine.[28] In this perspective, the halting problem need not be reduced to the mechanical 'yes' or 'no' replies of a supposed halting program, which exclude any further reflection. Wittgenstein advocates a viewpoint that, in a way, seems to pass over the halting problem. First of all, Wittgenstein states that by a machine-as-symbol and by means of our familiarity with the machine, all the machine's movements "seem to be already completely determined".[29] This need not mean that there is a mechanical procedure for the "complete determination" of the machine's mode of work, but that it is *our* understanding of the machine symbol that results in a complete knowledge of the work of the machine. That is, it is for a human to whom the machine (its picture) is presented and who "knows the machine" (from learning, practice, use), that the movement of the

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[27] "...it is not logical inference ...for me to make a change from one formation to another (say from one arrangement of chairs to another) if these arrangements have not a linguistic function apart from this transformation" (RFM V-2 p. 257, cf. RFM III-41 pp. 171–172).

[28] Kripke's understanding of a Turing-machine-as-symbol as a program (Kripke, 1982, pp. 35–36 footnote 24) is questionable if taken as Wittgenstein's interpretation. First, a written program (flowchart, quadruples) is not the same as a visible Turing machine (with its head and tape), although a program might be presented by the pictures of the changes on the tape (or tapes) or the flowchart could contain verbal descriptions ("print", "move", etc.). Second, a program is "superrigid" (that is, it has a conventional meaning), but only as an exact description of the working of a Turing machine, not as a machine itself (or its picture) as a symbol: steps in the working of a Turing machine follow automatically from the program— nothing is left to the viewer to conclude outside automatical reasoning (conclusions from the view of the machine) and thus the complete meaning of the program (e.g., (non-)halting, whether there is the "institution of the end") might be missing. Of course, we should bear in mind that Kripke does not primarily intend to interprete Wittgenstein's text, but wants to present his own reflections on the occasion of Wittgenstein's thoughts on the "following a rule" (Kripke, 1982, pp. vii–ix).

[29] "We use a machine or its picture as a symbol of a particular mode of operation [*Wirkungsweise*]", PI §193. Cf.: "predetermined" movement of the machine-as-symbol; "definiteness" of "the future movements of the machine", where the "future use" of a machine is not taken to be "causally determined" but "logically determined" (PI §220 85) and "is in some sense already present" (PI §193 p. 78, §195 p. 79). See also RFM IV-33: "when I see the picture of a mechanism in motion: that can tell me how a part actually will move."

machine "seems" to be "completely determined".[30] Wittgenstein conveys such a knowledge by the phrase "It is as if we could grasp the whole use of the word in a flash" (RFM I-122 p. 85, I-130 p. 88). And we "grasp" the use of the word because we "understand" the word (i.e. the machine-as-symbol), its meaning, which consists, according to Wittgenstein, in the rules of the use of the word (RFM I-130 p. 89).[31]

Accordingly, from a Wittgensteinian viewpoint, the agent's understanding ("overview", *Überblick* PI §125) of a machine cannot be assumed to function like a hypothetic halting program H (for Turing machines), whose replies can be refuted by the very machine to which H is referring and which H is a part of (diagonal procedure) (see the description of the diagonal process in Turing, 1965a, p. 132). The agent's knowledge should be acquired in a non-mechanical, meaningful procedure based on the ostensive, descriptive, learning of the machine's behavior and use. What we learn in this way is to "derive", from a machine or its picture, a "series of pictures" representing the steps in the working of the machine (PI §193, p. 78) and all the possible movements of the parts of a machine. Even the infinity of the future movements of the machine could be in some way already present in the agent's knowledge: it is not so that "it [a machine] goes on without ever stopping" but rather that the procedure "lacks the institution of the end" (RFM II-45 p. 138).[32] This "lack of the goal", like all steps in the working of a machine, can also be "derived from the starting position" (Z 693 p. 120e). Obviously, such a "deriving" is not a merely meaningless mechanical behavior, but includes "insight", "seeing at" what is shown, and a holistic overview of the intended behavior and use of the machine.[33] Since the agent knows the machine and is able to derive the whole of its work (finite of infinite) from the machine-as-symbol, the machine cannot refute the agent. The agent's knowledge ("grasp") of a machine refers to the whole of a machine—it is not an H "program", which could be refuted by the machine that contains H as its part and continues to work in its own way after H finishes.

A supposed halting program leads to a contradiction, that is, it cannot work; it "leaves no room for action" and is useless (LFM pp. 185, 209, 223).

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[30] In addition, it is always a human being who follows a mechanical rule (calculates). "Turing's machines. These machines are *humans* who calculate" (RPP1 1096 p.191e). Also: "If calculating looks to us like the action of a machine, it is the *human* being doing the calculation that is the machine" (RFM IV-2 p. 234). Cf. (Shanker, 1987).

[31] RFM I-121–133 and PI §§193–202 on machine-as-symbol could be read as Wittgenstein's response to Turing's results in (Turing, 1965a) as put in the broader context of Wittgenstein's engineering knowledge and his philosophy. This can be supported by the facts that before February 11, 1937, Turing sent his 1936/37 paper to Wittgenstein, and after mid-August 1937, Wittgenstein started to work on the early version of PI beyond §189. See (Floyd, 2016, 8–9) and (Wittgenstein, 2009, p. xix). It should also be noted, as Floyd (2016, pp. 33–34) points out, that Turing, "partly under Wittgenstein's influence", eventually came to emphasize, in his own words, "the inadequacy of 'reason' unsupported by common sense."

[32] See also in Z p. 120e]: "The reasoning that leads to an infinite regress is to be given up not 'because in this way we can never reach the goal,' but because here there is no goal."

[33] Compare OC 410: "Our knowledge forms a big system [*ein großes System*]. And only within this system has a particular bit [*das Einzelne*] the value we give it." See also OC 105.

Instead of searching for a halting program, we can stick to a Turing machine as a symbol, in which way the halting problem receives a solution—but on the symbolical (meaningful) level: in the sense that we understand ("see") what is meant by each Turing machine, for instance, by "detecting" its loops ('visible recursions', Lampert, 2018, p. 18). In contrast, a supposed halting program is always a part of a mechanical procedure and does not necessarily correspond to ("overview") the whole procedure.

On the other hand, from the standpoint of a game, a circular "rule" is "not a rule" and the machine does nothing on such an instruction (leaving us with no reply). Nevertheless, we "see" from this standpoint that a circular "rule" is "meaningless" (*unsinnig*), saying "do the same as you are doing". This is the case, for example, in Wittgenstein's presentation of the second diagonal example of Turing ("the simplest and most direct proof") in terms of a game (Z 694 p. 120e, RPP1 1096–1097, see Turing, 1965a, pp. 132–133). Let $F'(n) =_{def} F_n(n)$ and let F' be F_k. F' should follow the general procedure also at the self-referring k in the diagonal: at an arbitrary i do what $F_i(i)$ commands, and hence, at k do what $F_k(k)$ says, which is self-identity ($F_k(k) = F'(k)$), thus, circular and tautologous, meaningless as an instruction, and hence not a defined, predetermined mechanical step ("not a rule").[34]

To conclude, a symbolic ("theatric") approach to a machine has a wider perspective than just mechanical "following a rule"—it overviews (or intends to overview) the whole of a machine and of its use and considers its work in terms of meaningfulness. An engineer always has to bear in mind the overall purpose and meaning of the device and the functioning of its parts within the whole of the structure.[35] Such reflections can also be found in the machine books possessed by Wittgenstein (see, for example, Vrančić's considerations on the unsuccessful and senseless trials to establish an irrigation system of Tiber with possible meaningful corrections, his thoughts on a harmonious and functional composition of a temple, or his proposal for a stable and more efficacious windmill, in Vrančić/Verantius 1993, decl. I, V, VIII/1).

Wittgenstein applies the same, symbolic (meaningful) approach to logic: a "habitual" technique of thinking, based on the the distinction of meaningful steps from senseless (unsuccessful) trials (RFM I-131–133 pp. 89–90). In this way, mechanically undecidable questions might be meaningfully solvable. For example, a tautology can be confirmed in case the mechanical procedure "sticks" (circular instruction) or consistency stated in case the mechanical procedure does not terminate (no goal present, "no institution of an end"). Furthermore, logic can still have a usable and meaningful part or a meaningful interpretation despite containing contradictions (e.g., LPM 209–211). On the

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[34] J. Floyd reinterprets the senselessness of a tautologous instruction as a "new", creative moment: we are free to do whatever we please (Floyd, 2012).

[35] Cf. Ferguson (1977, p. 828) "As the designer draws lines on paper, he translates a picture held in his mind into a drawing that will produce a similar picture in another mind and will eventually become a three-dimensional engine in metal. Some decisions, such as wall thickness, pin diameter, and passage area may depend upon scientific calculations, but the non-scientific component of design remains primary. It rests largely on the nonverbal thought and nonverbal resoning of the designer, who thinks with pictures."

other hand, Wittgenstein tries to show that incompleteness theorems lack a meaningful interpretation that would confirm them by a "convincing [*triftig*] reason" independently of any formalism (RFM III-14,15).

5 Objectivity, causality and logic

Although Wittgenstein opened a wider perspective on logical properties and the properties of logic, there are limits to his conception of language games and learning. His "meaning is use" conception sometimes leads him to unacceptable, even horrifying, conclusions, when a language game cannot be established in the expected way: in one such case, he suggests "giving [a child] up as hopeless" and "lunatic" (LFM p. 58).[36] The reason for this lies in the irrationality of his foundations of language games:

> You must bear in mind that the language game is so to say something unpredictable. I mean: it is not based on grounds [*nicht begründet*]. It is not reasonable (or unreasonable) [*nicht vernünftig (oder unvernünftig)*]. It is there—like our life.   (OC 559)

Wittgenstein's foundation of logic and language games in "forms of life" did not result in primitive concepts from which these forms and the forms originating from them would receive a deeper clarification.[37] A means to come to a further clarification of the forms of life could be a deeper concept of causality, traces of which might be to some extent recovered from Wittgenstein's reflections. Despite such traces, we cannot claim that Wittgenstein adopted such a concept of causality. In the rest of this section, we sketch a possible way this problem could be approached, starting from Wittgenstein's concept of machines-as-symbols.

As mentioned, Wittgenstein sometimes rejects the idea of "logical mechanism" (RFM I-119 p. 83, LFM pp. 194, 198–199) and thus, of any mechanical causality "behind logic" and focuses instead on the "machines-as-symbols". Furthermore, given his suggested concept of causality reduced to efficient physical

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[36] Wittgenstein gives an example of an unsuccessful teaching of multiplication:

> Similarly one can show a child how to multiply 24 by 37, and 52 by 96, and then say to it, 'Now multiply 113 by 44 analogously.' ... If he can't justify his action, we should go through it again and again, until we converted him to doing the same as us. ... If we find that he cannot be trained to do it the same as us, then we give him up as hopeless and say he is lunatic.   (LFM p. 58)

Instead, in order to leave open a way for a positive solution, a high grade of teacher's "humbleness" (*humilitas*) seems to be needed in front of such a fact of unsuccessful teaching. As Wittgenstein himself states: "Self-recognition & humility is one" (*Selbsterkenntnis & Demut ist eins*), with the addition: "These are cheap remarks" (Wittgenstein, 2003, pp. 104–105). For the grades of humility, we can return to the author of *Machinae novae* (Vrančić/Verantius, 1616, p. 66).

[37] This in contrast, for example, to Vrančić's theory of definition and predicaments as primitive *loca* (questions, observations, considerations, etc.), from which it should be possible to build a definition (broadly conceived, including description, declaration, narration, or history) of a given concept (Vrančić/Verantius, 1616, 13–15,).

causes, as something "experiential" (RFM I-126, 128 p. 87–88), and having to do merely with "hits" and "impacts" (*Stoss, Anstoss*),[38] he strictly distinguishes between reason and cause, language games and causation. In what follows, we show that Wittgenstein's machines-as-symbols presuppose an idea of causality of a different, non-mechanical and nonexperiential, kind. As a preparation to such a concept of causality, we note that Wittgenstein had a deeper concept of reality (*Wirklichkeit*) that does not reduce to material objects and which consists in the use of language, in language games and forms of life as "facts" and "the given" (PI §654, PPF §345). The use of language is for Wittgenstein the source of objectivity, that is, the applying of 'true' and 'false' is a component of a language game (which does not reduce to mere sentence formation; cf. PI §136). Yet Wittgenstein did not correspondingly extend the concept of causality in order to make explicit the possibility of interconnections ("bond") and interaction ("life") at the level of his deeper concept of reality.

In the sense in which Wittgenstein positively (but with a critical *caveat*) evolves the idea of the kinematic mechanism (or its picture) as "perfectly rigid", "perfectly hard", "completely rigid" or "super-strong" (see RFM pp. 83–84, I-128 p. 88, I-130 p. 89),[39] we could also speak of the "perfectly rigid" causality of machines-as-symbols and of a "logical mechanism". This is in contrast to a material, "experiential" causality of hit and impact (RFM p. 88). To avoid misunderstanding, the right "criterion" of rigidity (RFM I-119 p. 83, LFM 1197) should be applied. The crucial thing is, according to Wittgenstein, to see that the idea of perfect rigidity has quite a different origin than the idea of rigid materials, that is, to distinguish between the "hardness of a rule" and the "hardness of a material" (RFM p. 220, like the "inexorability" of law in contrast to the "inexorability" of a judge, LFM pp. 197, 199, RFM I-118 p. 82). What Wittgenstein aims at is the meaning and use of symbols, for instance, in the "interference of two pictures" (see LFM p.199), not of represented material things. Furthermore, what so to speak "most rigidly" there *is*, is not just "linguistic" items or "reasons" but "real needs" ("needs of life") and the "bond of life", which by means of various "forms" and "patterns" *bind* pieces into "structures" with their "internal properties" ("essence"), into "families" of these structures, and into language games and forms of life.[40] The following expressions of Wittgenstein's that connote causation need not be mere metaphors but might be taken in a "perfectly rigid" sense: for example, "pictures ... force themselves on us" (*uns aufdrängen*, RFM I-14), a "proof compels me" (*zwingt*,

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[38] "Now suppose I were to say that when we speak of cause and effect we always have in mind a comparison with impact; that this is the prototype of cause and effect? ... Imagine a language in which people always said 'impact' instead of 'cause'." (Wittgenstein, 1976c, English, p. 410).

[39] "The connection which is not supposed to be causal, experiential one, but much stricter and harder, so rigid even that the one thing somehow already *is* the other, is always a connection in grammar" (RFM I-128). "... more than experience: seeing a picture" (RFM IV-50).

[40] "Every sign by itself seems dead. In *use* it lives" (PI §432). "Here ... it just is the mental act of meaning that gives the sentence life" (PI §592). What has to be accepted, the given, is – one might say – *forms of life*" (PPF §345).

RFM I-34) or "impresses a procedure on us" (*prägt ... ein*, RFM I-40), also: "logical compulsion" (*Zwang* RFM I-117,118), and in particular, the "workings of our language" (*Arbeiten unserer Sprache*, PI §109, cf. PI §132; the "functioning" [*Funktionieren*] of words, PI §5)[41]—everything in contrast to the working of a physical machine.

In particular, the "workings of our language" witness a broader concept of causality and as firmly rooted in the western tradition of philosophy, stemming from Aristotle and his fourfold causal structure consisting of form, matter, efficient cause, and ends.[42] "Working" is a causal affair (no matter how abstract it may be, for example, as mental work in contradistinction to manual work, or as conceptual work in distinction to physical work),[43] which bridges Wittgenstein's separation of reasons (motives) and causes, the logical and the "causal" (in the narrow sense, excluding final causes).[44] Such a four-fold causal structure can be described in Wittgensteinian terms in the following way: in a language game, a speaker uses (works according to) a pattern (model, paradigm); (s)he applies a pattern to a given subject-matter (constituent parts, pieces, for instance, sounds, lines, digits, signs, sentences, propositions), that is, arranges parts (pieces) in a pattern with the task of producing (or reproducing) a meaningful symbol (e.g., proof, sequence of signs).[45] It is easy to recognize here four types of causes: formal (pattern, paradigm, model), material (parts, pieces), efficient (working, applying, arranging as activities), and final (a task to be performed).[46] Although Wittgenstein rejects the idea of a mathematical or "logical machinery",[47] this still does not exclude that there is something like the "working" of a machine-as-symbol (or of the symbolic picture of a ma-

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[41] "Framework, out of which our language works [*wirkt*]" (RFM VI-21).

[42] Wittgenstein was not interested in reading Aristotle (Z p. 366) and we do not claim that he intentionally worked with an Aristotelian concept of causality.

[43] See Sutlić (1987) for "work" as the fundamental metaphysical concept stemming from the "old" concept of causality and covering a variety of phenomena: the work of an ancient Greek craftsman and of an industrial manual worker, the functioning of modern society with its services and the "working of" fundamental science. Let us note that, according to Wittgenstein, "thinking" in not an "incorporeal" process (PI p. 116).

[44] In Wittgenstein's view, "how it has come about that we now go by the signpost" excludes "ends" (PI §§198, 220); thus: our problem is not a causal, but a conceptual one (PI §183): proof is a "path" (*Weg*, "picture of the course [*Ablauf*]", RFM VI-2, III-41,69) to be described, not to be causally explained (RFM VII-74).

[45] Compare the following expressions: "mit einem Muster ... arbeiten", PI §56; "the way we work with concepts" (*begriffliches Arbeiten*, RFM VII-45); "language games which work with concepts and others" ("mit Begriffen arbeiten", RFM, VII-71); "working according to the pattern" (LFM pp. 59–60).

[46] Similarly: (i) pieces (*Steine, Stücke*), digits or sounds (ii) are being arranged (*zusammenlegen, zusammensetzen, zusammenfügen*) in (iii) a pattern, model, shape (*Muster, Vorlage, Gestalt, Figur*) (iv) in a particular application with a particular sense, according to a certain task or end (RFM pp. 48, 56, 72, 233, 240, 266, 339); also: there are (i) parts or pieces on which (ii) we "work" (iii) according to a pattern, which is applied (iv) in a certain trained (taught) way, or of which a "new use" is made of (LFM pp. 53, 59–60, 75, 79, 130; "use as", RFM p. 108). Numbers (i)–(iv) are related to material, efficient, formal, and final causes, respectively.

[47] "I am speaking against the idea of a 'logical machinery'. I want to say there is no such thing. The idea of a logical machinery would suppose that there was something behind our

chine), in the sense of the "working of language". Why not consider "working" as a family resemblance describing the use of language?

With respect to logic, on the presupposition of the critical *caveat*, logic can be conceived as an "abstract", "super-rigid" causal structure involving patterns (of propositions, axioms and proofs), pieces (parts) as signs and propositions arranged into wider logical forms (simple and compound propositions, proofs), an efficient "technique of thinking" (RFM I-133 p. 90), aiming at the conclusion as the *end* (RFM III-39 p. 171).[48] Since patterns, paradigms, are present as such once the proof is given, there is essentially no movement in a proof: the proof is simply a "path", "route" (RFM p. 173, 436)—an objective (super-rigid, timeless) causal structure, rooted, like the laws of logic, in our "thinking habits" and the "habit of thinking" that bind the single steps of a proof into one whole.

In general, Wittgenstein's "bond of life" (*Band des Lebens*) can be seen as the all-comprehensive "super-rigid" force that binds together a "carpet of life" (*Lebensteppich*), where logic and the customs of thinking are embedded. Proofs and logical laws are finally grounded in the linguistic and pre-linguistic patterns and "forms of life", which are the real foundation, "the given" ("facts of living", RPP1 630; "what has to be accepted", PPF 345) (see Moyal-Sharrock, 2015). According to Wittgenstein, instead of a "crystalline purity" of logic, logic should, in the last instance, refer to "the fixed point of our real need" (PI §108).

A variety of patterns of life can be vividly displayed as a "theatre" of machines designed in numerous forms and in diverse materials, by various techniques and for various needs of life. At the same time, machines reveal ("show") a variety of aspects of the "'logical structure' of the world".[49] No wonder, if the machine books possessed by Wittgenstein made an impact on Wittgenstein's philosophical views. They also indicate a way to causally and intuitively clarify the procedures of thinking, as it can be implicitly read off from many Wittgenstein's remarks.

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symbols" (LFM p. 194). For Wittgenstein, "It is not something behind, but the proof that proves" (RFM III-42).

[48] For example, Wittgenstein speaks about a "propositional pattern" (*Satzfigur*, RFM pp. 386, 387), pattern, "model" (*Figur, Muster*) or "chain" (*Kette*) as a proof (RFM pp. 48, 50; LFM p. 38). In some places, Wittgenstein is more precise: a proof proves by means of a pattern, but the pattern itself is not the proof (RFM p. 151; cf. RFM III-41 p. 171–173; proof as a "paradigm of a pattern", RFM p. 168, cf. p. 298).

[49] Compare Wittgenstein's example: 'The stove is smoking, so the chimney is out of order again', as an inference that we read off from what we see, in contrast to 'The stove is smoking, and whenever the stove smokes the chimney is out of order; and so …'. In the text that precedes, Wittgenstein remarks: "Logic is a kind of ultra-physics, the description of the 'logical structure' of the world, which we perceive through a kind of ultra-experience (with the understanding e.g.)" (RFM I-8 p. 40).

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