Entity, but no Identity

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Abstract

Inspired in Quine's well known slogans “To be is to be the value of a variable” and "No entity without identity", we provide a way of enabling that non-individual entities (as characterized below) can also be values of variables of an adequate "regimented" language, once we consider a possible meaning of the *background theory* Quine reports to ground his view. In doing that, we show that there may exist also entities *without* identity, and emphasize the importance of paying attention to the metalanguage of scientific theories, for they may be also fundamental in determining the theory's ontological commitment.

Key Words: ontological commitment, quantum objects, quantum ontology, non-individuals, quasi-set theory.

Introduction

Inspired in Quine's well known slogan "No entity without identity" (Quine 1969, p. 23; see also Chateaubriand 2003), we provide a way of assuming that also non-individual entities (as characterized below) may be values of variables, once we consider an adequate metamathematical theory which here plays de role of Quine's *background theory*. In other words, we consider the background theory as the metatheory we use to speak about the considered (object) theory. Thus, by a convenient change in the metalanguage, we shall be able to sustain that "To be is to be the value of a variable", and that this holds not only for *individuals*, but for *non-individual* entities as well, here understood as entities to which the concept of identity described by classical logic does not apply. This case study suggests that not only the 'object languages' of our scientific theories are relevant for their ontological commitments, but also the metalanguages we use to formulate them should deserve a careful attention as well. At a first glance, the results achieved here can be said to be *motivated* by Quine's ideas, but we are not aiming at to provide an exegesis of his views, for instance, in comparing our use of the metalanguage and Quine's background theory.

Indiscernibility and structures

Quine's criterion of ontological commitment is sufficiently well known and widespread in the literature to be recalled once more here. We shall just make some remarks in order to fix the terminology and the main ideas we would like to emphasize. Specifying about a theory's ontological commitment, Quine remarks that

> "Ontology is indeed doubly relative. Specifying the universe of a theory makes sense only relative to some background theory, and only relative to some choice of a manual of translation of the one theory into the other. (…) We cannot know what something is without knowing how it is marked off from other things. Identity is thus a piece with ontology. Accordingly, it is involved in some relativity, as may be readily illustrated. Imagine a fragment of economic theory. Suppose its universe comprises persons, but its predicates are incapable of distinguishing between persons whose incomes are equal. The interpersonal relation of equality of income enjoys, within the theory, the substitutivity property of the identity relation itself; the two relations are indistinguishable. It is only relative to a background theory, in which more can be said of personal identity than equality of income, that we are able even to appreciate the above account of the fragment of economic theory,

hinging as the account does on a contrast between persons and incomes." (Quine 1969, pp. 54-5)

Then, although people may have the same incomes and even if they cannot be distinguished one each other by the predicates of the (considered) economic theory, they *are* individuals, for (by hypothesis) they can be distinguished from one another in the richer 'background theory'. Here, we shall understand the background theory as the metalanguage in which we speak about our object language and describe its semantic concepts.[1] Thus, the sample advanced by Quine looks quite similarly to what happens when we speak about indiscernible objects within a certain mathematical structure $A = \langle D, (R_i)_{i\in I}\rangle$, characterizing them as elements of the domain that are invariant by the automorphisms of the structure. For instance, a certain set of people can be taken as D and some set of relations on D taken as the relevant relations (operations and distinguished elements can be reduced to relations in the standard way) to fit Quine's sample, so that people with the same income cannot be distinguished by any relation. But, since standard mathematical structures can in principle be built in a set theory (suppose Zermelo-Fraenkel,ZF for simplicity), we can take ZF as the background language, which seems to suffice for almost all physical theories.[2] But the 'whole' ZF, seen as a structure (for instance, the cumulative hierarchy $V = \langle V,\in \rangle$ can be seen as a structure), is rigid, that is, its only automorphism is the identity function. Furthermore, since we can prove in ZF that every structure can be extended to a rigid one, it results that although we can speak of two objects x and y that they are indiscernible relatively to a certain structure A, if they are not the same object (that is, if $x\neq y$), then they *can* be distinguished *from the outside* of A. In Quine's sample, this means that we will ever find outside the economic theory a certain relation which distinguishes among distinct people. The general rule is as follows: indiscernibility can be achieved in standard mathematics only within (that is, if we remain confined to) a certain mathematical structure, but in the *whole mathematics* (read, in the whole ZF), every entity is an individual, in the sense of obeying the classical (Leibnizian) theory of identity (we shall be back to this point soon).

Objectuation, the primitive act of our mind

In virtue of what can an object be said to be an individual? Toraldo di Francia says that *objectuation*, the primitive act of dividing the world in objects, is the first act of ours in forming our knowledge of the world (Toraldo di Francia 1986, p. 23). So, accepting at least partially this view, we can say that we do *individualize* the things first, but this does not entail that they are *individuals* in our conception (*Weltanchaung*) of the world. Jean Piaget, in describing how a child *constructs* the notion of an object (Piaget 1955, chapter 1), says that in the first days or weeks of her life, although a child plays with objects, she has not yet constructed the *notion* of object (we prefer to say: the notion, concept, or idea of an individual). Only later, by circa of eighteen months, she has elaborated, or *constructed*, that notion. Piaget's view is in certain sense Kantian, but he disagrees with Kant in that our categories of understanding are not a priori, but dependent of several factors, determined by our evolution as human beings. An *individual* is identified as such only when the child attributes to the object a notion of *permanence*, being able to recognize it as the very same object in two different opportunities or occurrences of it. In her first days, the child plays with the object, but if it leaves her field of attention, and another one takes its place, she will not realize that *that* previous object is missing. Although the child *individuates* the object, for she plays with it in

[1] I am not claiming that this interpretation of the background theory fits Quine's. As I said before, I am just *inspired* by his ideas, and my argumentation is independent of whatever exegetical analysis. Thus, I am assuming the so called 'model theoretical view', that is, accepting that for every (object) language there exist a metalanguage in which we can talk about the object language itself and express for instance its semantic concepts.

[2] Of course there are various non equivalent set theories, as Quine's NF system (in the Rossser's version) is not equivalent to ZF, so as structures can be considered in the framework of category theory. But our argumentation can be developed by considering ZF.

distinction from other of her toys, even from quite similar ones, it is not yet (to her) an individual. The notion of individual is relative.

The difficulty here is to make precise some terms like permanence. Without pursuing a discussion on this topic (which we intend to do in another work), we shall assume that we have an intuitive account to the idea of permanence, in the sense that a thing being permanent (for a certain period of time) means that it endures in that time, or that it is a *continuant* in that period of time (time is of course another concept that deserves explanation in this context, and here is taken as subjective). We shall also leave aside the reasons we could point out in discussing in virtue of what a certain thing has permanence, for instance by mentioning the two basic approaches to the subject, namely the theories of substratum and the so called bundle theories (something in this direction, taken quantum physics in mind, can be seen in French and Krause 2006). Thus, an individual is something that to a certain child has permanence as *that* thing, and can always be distinguished from any other by some quality. Since, as some authors suggest, these intuitive phenomenological conceptions originate not only classical logic but also classical mathematics and classical mechanics, it seems reasonable (according to us) to postulate that something is an individual if it obeys the rules of the classical theory of identity (CTI).

By CTI we understand either the first order theory of identity or a higher order theory, encompassing set theory. The first order theory, as it is well known, is characterized by the axioms of reflexivity and substitutivity of equality (the symbol of equality, or identity, is taken as a primitive binary predicate symbol), as in Mendelson's book (1997, p. 95). Semantically, we aim at to interpret this predicate in the diagonal of the given interpretation, namely, the set $\Delta = \{\langle x,x\rangle : x \in$ D}, being D the domain of the interpretation. But it would be realized that the above axioms do not "characterize" Δ without ambiguity, that is, no first order language can individualize the elements of the domain up to an equivalence relation (Mendelson op.cit. p.100). Alternatively, we can think of a "classical" second order logic (or a higher order logic), where Leibniz Law (LL) can be taken as the definition of identity, namely, $x=y =_{\mathrm{D}} \forall F(Fx \leftrightarrow Fy)$, where x and y are individual variables and F is a variable for properties of individuals. In set theory, taken as a first order theory, we add to the axioms of reflexivity and substitutivity the axiom of extensionality. Roughly speaking, this is CTI. The objects that obey such a theory are individuals in our sense, that is, they can always be distinguished from one another either for having a certain peculiar property or by the existence of a set to which it belongs to but the others do not.

Characterizing non-individuality

Our use of the expression *non-individual* follows a tradition that came from the seminal work of Max Planck in 1900, when he derived his law of the black body radiation. In deriving that law, Planck assumed that the way of counting in how many ways P energy elements can be distributed in N linear oscillators, arriving at his well known formula (Planck 1901):[3]

$$R = \frac{(\mathrm{N}+\mathrm{P}-1)!}{(\mathrm{N}-1)!\,\mathrm{P}!}$$

Later, Ehrenfest realized that such a hypothesis (namely, the division by P!) conduces to the indiscernibility of the energy elements (the quanta), for the division by P! entails that permutations of indiscernible quanta are not regarded as giving raise to different arrangements. Continuing with our analogy concerning the way a child "constructs" the world around her, the situation involving quanta is something like the child in her first weeks of life, who does not make distinctions between

[3] For historical details not referred to here, see French and Krause 2006, chapter 3.

two situations originated by the permutation of two *distinct* but similar objects of her stock of toys (it should be realized that at least in principle we assume that *we*, having "constructed" the notion of object, are able to distinguish two of them, say by some characteristic mark or scratch). The difference between our sample and the quantum case (and of course there are many) is that the child will evolve to elaborate the notion of object, and this cannot be said about quanta. Perhaps we will never *construct* them as individuals, once we assume them to be non-individuals.[4]

Heisenberg, Weyl, and Schrödinger are among those who explicitly have spoken about "non-individuals" (see French and Krause op.cit.). Schrödinger provided some additional insights, suggesting that even the concept of identity has no meaning when applied to elementary particles. In order to approach non-individuals, we follow Schrödinger's intuitions and refuse the theory of identity as applied to them, although in our case we cannot restrict ourselves in assuming a "particle view", for the *quanta* we are considering may be whatever entity a quantum theory makes (implicit) reference to. Thus, there are at least two main aspects of non-individuality to be explained. Firstly, we need a metaphysical account to non-individuals, that is, to develop a *metaphysics of non-individuality*. Secondly, we need a formal description of them. We may assume here the first point informally, due to our characterization of individuals. Really, as put long time ago by Wittgenstein and Ramsey, the traditional concept of identity, that we can assumed to be summed up by Leibniz Law (the second order formula shown above), is not a logical truth, so there is no apparent contradiction in assuming that it can be rejected. The objects which violate LL in the sense of sharing their properties without turning to be the very same object, that is, those objects which violate the Principle of the Identity of Indiscernibles (if there are some), are (formally) our non-individuals. In particular, we may assume, inspired in Schrödinger's ideas, that the relation of equality cannot be applied to them, in the sense that expressions of the form $x=y$ are not formulas of the considered language. Thus, in particular the property "being identical with a", for a certain term a, which we can write as $P_a(x) =_{\mathrm{D}} x=a$, cannot be considered among the properties of the object a. These ideas lie in the core of the *theory of quasi-sets*, we have developed to cope with collections of non-individuals and which we use to answer the second question posed above. We shall not present this theory here for limitations of space (but see French and Krause op.cit., chapter 7).

Of course the metaphysics of non-individuality needs to be further developed, but we shall not do it here. Instead, we shall assume that non-individuals, that is, objects which do not obey the classical theory of identity, could exist. Thus, in assuming that, we shall show how we can provide the grounds for saying that non-individuals *exist* in the sense that they can be values of variables. In doing that, non-individuals can be assumed in the ontology of suitable theories if we assume that the background theory is quasi-set theory. Thus, non-individuals, taken as indistinguishable in the object theory, cannot be distinguished even in the background theory, for they lack the concept of identity. Let us be more explicit on this point.

Non-individuals do exist

In order to show that non-individuals can "exist" in the sense of being values of variables, we may consider quasi-set theory $\boldsymbol{Q}$ as our metamathematical framework (the "background theory"). Since this theory encompasses standard Zermelo-Fraenkel set theory as a sub-theory –really, there is a "copy" of it in $\boldsymbol{Q}$, all standard mathematics can be constructed within $\boldsymbol{Q}$. Thus, we have at our disposal all standard set theoretical machinery for considering the relevant structures of our theories. But since $\boldsymbol{Q}$ is compatible with the existence of non-individuals, we have more machinery to deal with, namely, all mathematical constructions (structures) that can be achieved with non-individuals. Thus, we may suppose a certain mathematical structure (which can stand for our object

[4] There is of course the possibility of considering quanta as individuals, but this assumption requires restrictions either on the states the objects may be in or in the observable to be allowed to them. We will not pursue this case here, since we are exploring the *non-individuals alternative* –but see French and Krause 2006, chapter 6.

theory) that involves non-individuals in its domain (in the terminology of ***Q***, they are called *m*-atoms, and constitute one of the two kinds of *Urelemente* of the theory, the other ones, the *M*-atoms, have the standard properties of the atoms of the theory ZFU, Zermelo-Fraenkel with *Urelemente*). Since the classical identity theory (CTI) does not hold to them, but since they may share properties, they can be completely indiscernible not only within the object theory, but in the background theory as well. Furthermore, since the background theory is ***Q***, adequate indiscernible objects which are values of the variables of the object language cannot be discerned even in the background theory. Thus, in considering a suitable background theory, we can say, contrary to Quine, that there can be entities without identity.

This result has a corollary which may have interesting philosophical consequences: in ***Q***, there may be structures which cannot be extended to rigid structures. This kind of structures enable us to deal with indiscernible objects, as non-rigid structures do in standard mathematics (ZF), but contrarily to what happens in ZF, these structures cannot be extended to rigid ones, that is, the indiscernible objects of the structure cannot be taken as individuals in any way. A structure of this kind, we guess, would be of interest to quantum physics, for it would map more accurately the idea of non-individual quanta. But to pursue this possibility is something to be developed as a research program, and we leave this topic to future works.

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