The underlying logic is mandatory also in discussing the philosophy of quantum physics

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Abstract

It is supposed that any scientific theory (here we consider physical theories only) has an underlying logic, even if it is not made explicit. The role of the underlying logic of a theory T is mainly to guide the proofs and the accepted consequences of the theory's principles, usually described by its axioms. In this sense, the theorems of the underlying logic are also theorems of the theory.

In most cases, if pressed, the scientist will say that the underlying logic of most physical theories is classical logic or some fragment of a set theory suitable for accommodating the theory's mathematical and logical concepts. We argue that no physical theory and no philosophical discussion on the basis of the theory should dismiss its underlying logic, so the arguments advanced by some philosophers of physics in that certain entities (the considered case deals with quantum entities) can be only weakly discerned or be just 'relationals' and that they cannot be absolutely identified by a monadic property, are fallacious if one remains within a 'standard' logico-mathematical framework, grounded on classical logic. We also discuss the claim that some properties (as those that came from logic) would be 'illegitimate' for discerning these entities is not justifiable. Thus, this paper may be interesting for logicians, physicists, and philosophers.

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1 Introduction

With some exceptions, mostly in the fields of mathematics and logic, most scientific theories are first put in an informal manner, that is, they don't emerge axiomatically or as formal theories. Thus, for instance, we may think of Galileo's theory of falling bodies, Darwin's theory of natural selection or Fourier's theory of heat transfer. They are the product of several things, but mainly of scientists' abilities and skills. In the process of the formation of such informal theories, 'anything goes', as Feyerabend

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would say.[1] But once the theory acquires a certain maturity, usually it passes by the scrutiny of rigour and foundational issues. This point of view of course does not exclude that a theory in its 'final' axiomatic form can arise already put this way, that is, axiomatically.

But when one is interested in the meta-study of the theory, a theory can be seen as an ordered pair $\mathscr{T} = \langle F, M \rangle$, where F is a 'mathematical formalism' (a formal system) and M is a class of set-theoretical structures, the *models* of the theory [10, p.60].

Of course, this is an idealisation. The underlying logic is encoded in the 'F' part of the schema.[2] For a discussion, at least in principle we can suppose that any theory can be written in the above way. The discussion whether the 'F' part should be written in first or in higher-order languages will be not touched here, despite its importance.[3]

This entails that the F theorems are theorems of $\mathscr{T}$, and no one should dispute this. So, if F is first-order logic encompassing an amount of set theory (say, from the ZFC system), which apparently is enough for standard physical theories, then for any theory the following are examples of theorems, being α, β formulas, and x and y individual variables (ranging over the domain of discourse):

1. $\alpha \to (\neg\alpha \to \beta)$ — the Explosion Rule
2. $\forall x \forall y((\alpha(x) \leftrightarrow \alpha(y)) \to x = y)$ — the Identity of Indiscernibles
3. (With the Axiom of Choice) Every set is equipotent to an ordinal.
4. In a standard set theory such as the ZFA (or ZFC) system, given any object a it can be discerned from *any other* object by the predicate we call 'the identity of a', namely, $I_a(x) := x \in \{a\}$.

The Explosion Rule says that if one derives (or assumes) two contradictory formulas or sentences α and $\neg\alpha$, then *anything* β, written in the language of the theory, can be derived. Since this is a theorem of first-order logic, it is also a theorem of any theory grounded in classical logic. Even in the informal version of the theory, it is acknowledged that the presence of two contradictory sentences is bad for the theory, so we can accept that the Explosion Rule is valid even in this case.[4]

The Identity of Indiscernibles, a version of Leibniz's Principle of the Identity of Indiscernibles (PII) [22] says that if anything (in F, something expressed by a formula) holding for x holds also for y, then x and y cannot denote distinct objects: they are (numerically) *identical*. By the way, this is the way identity is interpreted in standard logic: *sameness*.

The third fact entails that to any set we can associate an ordinal, and since ordinals are well-ordered sets, the elements of the set need to be distinct from each other. Cardinals are particular ordinals, more precisely, they are those ordinals that

[1]My reading of this slogan does not go to some common interpretations which suppose that the slogan indicates that any theory (or logic) is as good as any other. In my opinion, the slogan is better read if indicates that the scientist is free for investigating. See [23].

[2]By 'logic' we mean here 'great logic' (*Logica Magna*) [1, p.202], encompassing higher-order logic (at least second-order) or at least an amount of set theory, such as ZFC or ZFA. When just first-order logic is assumed, this is mentioned explicitly.

[3]For a defence of the second-order approach, see [5], [24].

[4]In fact, the presence of contradictory sentences can be accepted without trivialisation only if the underlying logic is modified to some paraconsistent logic; see [7].

are not equipotent to any ordinal less than themselves. For instance, the set O of odd natural numbers is equipotent to the set $\omega = \{0, 1, 2, \ldots\}$ of the natural numbers (just take the bijection $f(x) = 2x + 1$ to show that). But it is also equipotent to the ordinal $\omega + 1 = \{0, 1, 2, \ldots, \omega\}$; define $f : \omega + 1 \to O$ by $f(\omega) = 0$ and $f(x) = 2x + 1$ for any $x < \omega$. But since $\omega < \omega + 1$ in the ordinal ordering (membership)[5] and the set of odd natural numbers is not equipotent to any ordinal less than ω, then ω is its cardinal ($\omega + 1$ is not a cardinal), which in the language of cardinals we call '$\aleph_0$'. This item has importance in the foundations of quantum physics; if we agree with most physicists that particles of the same kind are indiscernible but that a collection of them may have a cardinal (expressing how many systems there are), then how can we attribute a cardinal to such a collection without committing it to an ordinal, which would discern its elements? This point is not touched here despite its importance; see [15, 16].

The fourth case is also relevant for the philosophy of quantum mechanics. Really, it is well known that quantum theory has brought a lot of questionings concerning several philosophical and foundational aspects, and this was also with logic. It is my opinion that the *logic of quantum theories*, and not only the field of quantum logics [9] needs to be further considered. For instance, the fourth condition above says that an object a of the domain of our theory grounded in a classical setting is always *distinguishable* from any 'other' object by a *monadic property*, its *identity*, expressed by the monadic predicate I_a; in other words, an object is 'absolutely indistinguishable' just from itself. We shall be back to this point below. Thus, if the identity of the elements of the domain is counted among the available properties, then the identity of indiscernibles is a theorem of our theory and every object can be discerned from any other by a monadic property.

Summing up, we need to acknowledge that all logical and mathematical theorems of the theory's underlying logic are theorems of the theory also. This fact poses a challenge to some philosophers who do not pay due attention to the underlying logical facts in the considered physical theories.

From the few cases exemplified above, it is clear that the role played by the underlying logic of physical theories, with a special emphasis on quantum theory, is extremely relevant, even if such a logic is taken only informally. Here we focus on the discussion of the very nature of the basic quantum entities and mainly in the third and fourth above points. The paper is organised as follows. In the next section, we show that if one does not pay attention to the underlying logic, then some mistakes may appear; the case of *relationals* in quantum theory is discussed, and in particular the case of *weakly discernibility.* In the sequence, we turn to another usually assumed fact which is also problematic under the consideration of the underlying logic, namely, the possibility of discerning kinds of properties.

2 Carelessness with the underlying logic

Using 'standard' mathematical frameworks in the underlying logic, say a system of set theory such as ZFC or ZFA, which we can assume is grounded on first-order logic [13], we need to acknowledge that we should not be carelessness with some results entailed by such systems. One of the main ones which interest us here is that *everything becomes an individual.* By an 'individual' we understand (according to our supposed informal acknowledgement of the notion) a *something* that (i) is an

[5]We are assuming von Neumann's ordinals, so $\lambda < \nu$ means $\lambda \in \nu$; see [13, p.19].

one of a kind, say a person, a chair, a pen; (ii) it presents *identity* in the sense that it can be recognised *as such* in a certain situation and (iii) can be re-identified *as such* individual in other contexts or at different times. This last condition is essential; Julius Caesar *was the same* person in Rome or passing the Rubicon. I other words, diachronic identity applies to individuals.[6]

Our 'classical' logic, standard mathematics and classical physics were elaborated with such kind of entities in our mind. A physical object, a triangle (in analytic geometry) or whatever thing we describe using such theories, are individuals in this sense.

But quantum physics (both orthodox quantum mechanics, henceforth 'QM' and quantum field theories) challenge this notion, since quantum entities can be *absolutely indiscernible*, and indiscernibility, or indistinguishability, is a core concept in these disciplines; as Anthony Zee says, "[i]indistinguishability has astounding consequences in the quantum domain" [27, p.300]. Really, the issue of *indistinguishability*, or *indiscernibility* (we use both words interchangeably) is a fundamental notion in this discipline and we relegate it on a pair with nonlocality, contextuality and entanglement (see [11]). Indistinguishable things cannot be discerned, and this is (of course) the meaning of the word. But we can distinguish among several types of indistinguishabilities, and here we consider only two: *relative* indistinguishability, when two or more things share *some* properties or relations,[7] and *absolute* indistinguishability, when they share *all* their properties and relations, the meaning of 'all' being postponed by now. The other way around is also useful to be mentioned: things are *relatively discerned* when there is at least one relation which distinguishes them, and they are *relationals* when can be discerned by relations only, but not by (monadic) properties [20]. Furthermore, things are *absolutely discernible* when they can be distinguished by a monadic property.

The sample case is in quantum physics, when we try to understand quantum objects.[8] One of the most propagated claims is that which asserts that quantum objects cannot be distinguished *absolutely*, but only by relations; to some philosophers, in certain situations, such as the two electrons in a neutral Helium atom, we should acknowledge that they are only *weakly discernible*, that is, share an irreflexive but symmetric relation such as 'to have spin opposite to' but no monadic property can discern them [19]. We ought to say that these assumptions ignore the underlying logic and that yes, things represented in the above mentioned 'classical' settings *can* be discerned absolutely, but this will depend on the setting and of our understanding of the term 'property'. I shall consider in a moment the argument that these discerning properties given by the underlying logic would not be considered as 'legitimate'.

The problem, as I see it, is that in general philosophers don't make reference to the mathematical framework they are presupposing, and we should agree that they are presupposing some. Fred Muller, for instance, claims that "the ultimate constituents of physical reality are relationals; that is, entities that are discernible by relations but not by properties. I defend this position." [20] But in his paper

[6]Philosophers use to distinguish between *synchronic* and *diachronic* identities. The first makes respect to an individual a at time t being the same (identical) to individual b at the same time t, while the second states that individual a at time t is identical with individual b at time t'.

[7]'Properties' are relations of arity one. They can be dealt with in the language of the theory either by means of primitive predicate symbols or as formulas with just one free variable.

[8]We use the term 'object' as a neutral term to denote anything that can be conceptualised, in particular the 'objects' typical of quantum physics, such as electrons, protons, atoms and even molecules.

with Saunders [19], where they say that fermions such as the electrons of a neutral Helium atom are just weakly discernible but not absolutely discernible, they explicitly mention that they are working within the ZFC system. Good, but not enough because in being so, they should not ignore the consequences of assuming ZFC (ZFA or any 'standard' mathematical framework you chose which encompasses the standard theory of identity), namely, that such a theory entails that any object it describes can be discerned 'absolutely' from any other.

Much (if not all) of their argument against absolute discernibility is grounded on their hypothesis that the discerning properties should have a 'physical content', although the 'physical content' cannot be achieved from purely syntactical means, that is, by logic, requiring semantics; we shall be back to this point soon. Let us analyse T. Bigaj's discussion given at chapter 4 of his book [2], adapting and simplifying the notation, since he resumes much of what is discussed in [20, 19] and in other papers by these authors.

Let L be a first-order language and $\mathfrak{A}$ an interpretation with domain D. Then

1. Objects $a, b \in D$ are *absolutely discernible* ($Abs(a, b)$) iff there is an open formula $\phi(x)$ in one free variable such that $\mathfrak{A} \models \phi(a)$ but $\mathfrak{A} \not\models \phi(b)$ (ibid., p.73).

2. Objects $a, b \in D$ are *relatively discernible* ($Rel(a, b)$) iff there is an open formula $\phi(x, y)$ in two free variables such that $\mathfrak{A} \models \phi(a, b)$ but $\mathfrak{A} \not\models \phi(b, a)$ (ibid., p.76). Notice that the hypothesis requires that a and b are discernible, and no criterion is given for that.

3. Objects $a, b \in D$ are *weakly discernible* ($Weak(a, b)$) iff there is a formula $\phi(x, y)$ in two free variables such that $\models \phi(a, b)$, but $\mathfrak{A} \not\models \phi(b, a)$, $\mathfrak{A} \not\models \phi(a, a)$ and $\mathfrak{A} \not\models \phi(b, b)$ (ibid., p.76). Again, this definition presupposes that a and b can be discerned and nothing is said about that.

Then Bigaj proves (p.81) that

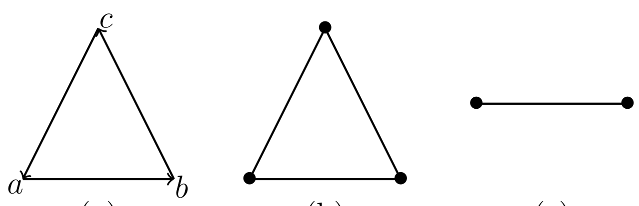
$$Abs(a, b) \to Rel(a, b) \to Weak(a, b), \tag{1}$$

but says that the converse arrows do not hold. To state that, he recurs to graphs. He simply says that "there are cases of objects that are weakly but not relatively discerned (and therefore absolutely discernible), and cases of relatively but not absolutely discernible. Examples of these cases are usually depicted in the form of graphs, where vertices (nodes) represent objects, and connecting arrows represent binary relations."

The problem with this example is that most books on graph theory treat them as sets. By the way, I didn't find a book that didn't (see also [21, 26]). In considering this, the above reasoning seems misleading. Let us consider the approach given in [4]. There, the authors Bondy and Murty (adapting the notation), say that a graph is an ordered triple (hence, a set) of the form $\mathcal{G} = \langle V, E, \psi \rangle$, where V is the *set* of vertices, E is the *set* of edges, and ψ is the incidence function (hence also a set) which associates an unordered pair $\{u, v\}$ (whose elements are not necessarily distinct) to each edge $e \in E$. The pair $\{u, v\}$ is written 'uv' and we say that the edge e *joins* the vertices u and v.

An isomorphism between two graphs $\mathcal{G} = \langle V, E, \psi \rangle$ and $\mathcal{G}' = \langle V', E', \psi' \rangle$ is an ordered pair (θ, ϕ) of functions such that $\theta : V \to V'$ and $\phi : E \to E'$, and $\psi(e) = uv$ iff $\psi'(\phi(e)) = \theta(u)\theta(v)$. It is easy to prove that an isomorphism is an equivalence relation. Then, any representative of an equivalence class of isomorphic graphs is called an *unlabelled* graph (op.cit., p.14).

As we see, even unlabelled graphs are *sets* and the elements of sets in standard set theories are absolutely discerned from one another by their identity properties. Unlabelled graphs, then, 'ignore' the nature of the vertices and edges, keeping with the 'format' of the graph only (see Figure 1). But to ignore the identity of vertices and edges does not correspond to say that these identities do not exist: really, they *do exist* in the underlying mathematics.



(a) (b) (c)

Figure 1: In (a), a labelled oriented graph and an unlabelled non-oriented graph in (b). This would be the case of relational but not absolutely discernible things. In (c), the case of weakly discernible but not relational discernible things [2, p.81]

You could suggest that unlabelled graphs are just vertices linked by edges without any commitment to sets, as Ladyman and Ross seem to do [17, §3.1], and so they could be used to model weak discernibility, as they suggest. We are not questioning that, but one should not take graphs *from nowhere* and use them in a foundational study. Furthermore, in order to prove that the distinctions do exist, it is enough to find an embedding of the structure of graphs within an Euclidean space.

Perhaps you could find an alternative in seeing graphs as sets by resorting to other mathematical frameworks. Let me mention one at least, *multiset theory*. A multiset is a collection of things but some of them may appear more than once and be counted more than once. Thus, the multiset $A = \{a, a, b, c, c, c\}$ has cardinal six and not three as if it was a set of, say, ZFC [3]. Then you could say that the vertices (and edges) are elements of a multiset and think that this could avoid the problem of their distinction. Yes, perhaps this could be done, but the mentioned philosophers should mention that, I mean, they should explain what they are presupposing as their background theory. In not doing that, we are free to suppose that they proceed as the scientist usually does, namely, to work with standard settings, that is, standard sets.

Consequently, as a result of considering the *standard* underlying logic (set theory), once nothing is said about that, we can surely assert that the 'new category of objects' introduced by these authors, namely, just *relationals* or just *weakly discernible* things cannot exist. Since this is a core point of this paper, let us insist: we should not be carelessness with the mathematical basis of our physical theory. If our quantum theory is grounded on one 'classical' system such as one of the above mentioned ones, 'purely logical' properties such as 'to be identical with' cannot be ignored, since they are part of this basis. This will be emphasised next.

3 On discerning kinds of properties

Another question posed by the mentioned authors concerns the distinction between what one could call 'legitimate' and 'illegitimate' properties to be used to make the distinctions among quantum entities, or then that such properties would have a 'physical content' and so on. Other names could be used instead, and we shall try to be as simple as possible.

Suppose we have a (first-order or not) language L with a stock of unary predicate symbols whose arguments are individual terms (they are enough for our discussion here). Can we say that some of them are 'physical', '1egitimate', 'yellow' or 'blue'? Let us consider just 'yellow and 'blue' in our example in order to be neutral; later we go back to the physical case. This distinction aims at to discuss the proposal of some philosophers that the discerning properties in quantum physics would have 'physical content' and should not be 'purely logical', as the defined identity of the elements of the domain seem to be (the property 'I_a' discussed above).

The answer is that syntactically there is no way to distinguish among classes of predicate symbols. You can use many sorted languages or (equivalently, as we shall do now) suitable second-order predicates. But anyway you need to resort to semantics. Let us interpret L in a structure encompassing a non-empty domain D and so that the unary predicate symbols are associated to subsets of D. Hence we could, in principle, chose some of these subsets to stand for the yellow things and other subset for the blue things. Then, a collection of yellow things would be the extension of a 'yellow predicate' (a predicate holding just for yellow things), and the same would hold in the blue case. But this should be arbitrary if not motivated by some reason, that is, we should know in advance that some elements of D are yellow and that some are blue. In this sense, it is not the obedience to the predicates that make them yellow or blue but the other way around; the predicates come only for helping the syntactic description of already known things: we would be begging the question. Furthermore, being elements of a *set*, even the elements of of D of the same colour are discernible from one another and this is due to the force of the mathematics being used.

Philosophers such as Tomasz Bigaj say that we should not consider among the attributes of a thing properties such as "this very object" [2, p.36], which is equivalent to our I_a above, since the use of the symbol 'a' would report to some specific element. He continues (loc. cit.): "it is standard practice to exclude (. . .) [the] so-called *impure* properties, that is, properties that somehow involve reference to individual objects (and the relation of numerical identity)." Others such as Adam Caulton (and many others he mentions) would not agree with my proposed predicate I_a too; as Caulton says, "indiscernibility by monadic formulae [should not] contain individual constants or equality." [6, p.579].

Why to impose such a restriction? In the former case, I_a would be 'impure' in what sense? Why properties that involve reference to individual objects would be 'impure'? A possible answer would be, in my opinion, the supposed fact that in using, say, the individual constant 'a', we need to suppose that the object a is already known, that is, it is identified from the start. The same could be said with respect to the identity relation, since in saying that something is 'identical with something', these 'somethings' must be given already. Another answer would be the desire of the maintenance of the some weak form of PII (which I insist will be no more the very PII since some properties were ruled out of the domain of the universal quantifier) within a standard mathematical framework. These arguments, if cogent (the mentioned philosophers do not speak about their reasons), could be considered with care: in presupposing that the objects are given from the start, we are not necessarily committed with their identities; in supposing that there are eleven electrons in a Sodium neutral atom, we are not committed with the identity of the electrons, as seems quite evident. Even the reference to *some* particular electron, say that one in the outer shell of the Na atom, does not entail that we are specifying an electron in special, but just some conditions (suitable quantum numbers) that need to be fulfilled in order we are authorised to say that there is an electron there. The reference (to the outer shell electron) is not to *the* electron specifically, but to a condition to be satisfied by some quantum entity, that is, a physical law.

In a logical language, we can bypass Caulton's and Bigaj's questioning by defining I_a differently without making reference to a explicitly; by the way, let us recall that the standard language of ZFC doesn't contain individual constants. Let x and y be individual variables (of the language of ZFC) and put $I_y(x) := x \in \{y\}$. Since the unitary set of any y can be get from the axioms, our definition is ready for the identity of y, whatever y can be. Furthermore, notice that the definition doesn't use neither the identity symbol nor any individual constant, and is a legitimate monadic property since it has just one free variable.

But let us consider a little bit further this question as kinked to quantum physics. Really, this discussion is linked to the validity of the Principle of the Identity of Indiscernibles (PII) in quantum mechanics.[9] In a second-order language, PII reads as follows:

$$\forall x \forall y (\forall X (X(x) \leftrightarrow X(y)) \rightarrow x = y), \tag{2}$$

where x and y are individual variables, X is a predicate variable and '=' is numerical identity. It is easy to see that if for every individual a, I_a is included as an attribute of a, then PII becomes trivial. Then philosophers provide 'versions' of PII by restricting the range of the universal quantifier, for instance assuming only monadic properties or also relational ones. But using our above language, let us restrict the range of the quantifier to yellow properties only, thus getting

$$\forall x \forall y (\forall X (Y(X) \rightarrow (X(x) \leftrightarrow X(y))) \rightarrow x = y). \tag{3}$$

But in this case '=' cannot be numerical identity anymore, but some weaker relation of *indistinguishability relative to yellow properties*, since there could exist some 'other' property, not an yellow one, which could discern between the two objects; then, (3) is not PII. Really, PII (and numerical identity), as usually formulated, requires *all* properties; as Joseph Melia guessed, "if logician's '∀' doesn't mean 'all', what *does* it mean?" [18].

So, there is no way to escape the conclusion that it is arbitrary to relegate some properties as 'impure' either in the discussions of PII or in looking for distinguishing properties. Logic is mandatory.

Of course what we said relative of yellow properties holds also for 'physical' properties, whatever they are. The mentioned philosophers claim that a physical property must be endowed with a 'physical' significance, so that they can be measured, but this is unjustified out of interpretations, that is, in a purely syntactical setting. To say that a certain property has 'physical significance' makes no part of the theory's underlying logic except if we use a second-order language as described above in (3). Really, you should be able to chose a bunch of 'measurable properties', then represent them in the language and say that only these ones are able to

[9] An important distinction is in order here. We should distinguish between the 'logical' PII, whose formulation is being presented here, and other forms of the principle such as those formulated for instance by Leibniz. We are discussing the logical version, and this 'very (logical) PII' requires the use of *all* properties and relations as we shall comment now.

provide any distinction among the involved objects. But, as in the case of indiscernibility according to yellow properties, this would be just indiscernibility relative to 'measurable' or 'physical' properties'.

Furthermore, how could we justify that we cannot 'measure' whether an element of D does belong to $\{a\}$ or not? What is 'to measure'? Does it depend on the existence of a well equipped laboratory full of PhDs? If we accept a wide definition of 'measurable' saying that a property P is measurable if it can be available in some context and under certain circumstances, say given by the theory's axioms, then we have all the reasons to say that the identity of a is a measurable property.

4 A final example

The carelessness with the underlying logic is not uncommon even among mathematicians. Let me say that even Hermann Weyl made the same mistake in his discussion about combinatorics of aggregates of individuals at the Appendix B of his celebrated book [25]. In short, he was looking for what is important for quantum mechanics regarding such combinatorics, realizing that it is not the individuation of the entities that import, but their 'kinds' and quantities. In order to express that, he considered a set S with n elements endowed with an equivalence relation $\sim$. The equivalence classes $C_1, \ldots, C_k$ of the quotient set $S/\sim$ were taken as the *states* the elements of S may be in, and the important thing for quantum physics is the cardinalities of these classes, so that the ordered "decomposition" $n_1 + \ldots + n_k$ sums n. So, Weyl supposes, one is able to say that there are n_i elements in class C_i without commitment with individualisation of these entities.

Here goes Weyl's mistake. He cannot asserts that the elements of the equivalence classes cannot be discerned one another. As elements of a *set*, the considered entities are distinct elements, and the supposition that we can forget this and pay attention exclusively to the equivalence classes and their cardinals does not eliminate such a fact. Weyl's elements are individuals. For a discussion on this, see [14] and [12, §3.7].

5 Conclusions

To provide an end, let us consider again our language L, of first or second order. Can we say that there are available discerning formulas for any two objects we consider? Such formulas would be excluded from *symmetric* languages, that is, those that obey the following definition [2, p.84]: a language L is symmetric with respect to an intended interpretation $\mathfrak{A}$ iff for every open formula $\phi(x_1, \ldots, x_n)$ in L and any permutation $\sigma : D \to D$, $\mathfrak{A} \models \phi(x_1, \ldots, x_n)$ iff $\mathfrak{A} \models \phi(\sigma(x_1), \ldots, \sigma(x_n))$."

We should acknowledge that the language of ZFC *is not* symmetric since the whole universe of sets, taken as the structure $\mathfrak{A}$, has no nontrivial automorphisms [13, p.66], and so there are open formulas that are not invariant by permutations, for instance, $x \in y$. But you can reply by suggesting that we should use ZFA instead. Due to the existence of permutation models, someone could argue that if we remain within a permutation model (or of another non-rigid structure), then there would be discerning formulas if they are restricted to the domain of the model. This is correct, but does not solve the problem; it is a theorem *about* ZFC (ZFA and the like) that *every structure can be extended to a rigid one* [8], and so, even if some elements look like indiscernible things in the structure, they are seen to be discernible in the extended one.

Summing up, within 'standard' frameworks such as ZF, ZFA, etc., any object can always be discerned from any other at least by properties like its identity, which is a monadic property and there seem to be no indisputable way to say that some properties are yellow, physical, pure or illegitimate. In particular, within a standard framework encompassing classical logic and set theory, we cannot say that there are only relationals or only weakly discernible things. Standard logic forbids that.

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