Generality

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Abstract

Hossack's *The Metaphysics of Knowledge* develops a theory of facts, entities in which universals are combined with universals or particulars, as the foundation of his metaphysics. While Hossack argues at length that there must be negative facts, facts in which the universal *negation* is combined with universals or particulars, his conclusion that there are also general facts, facts in which the universal *generality* is combined with universals, is reached rather more swiftly. In this paper I present Hossack with three arguments for his conclusion. They all draw, as does Hossack's theory of facts, on views Russell expressed in various writings. Two arguments are based on Russell's explanation of universals as aspects of resemblance; the third on Russell's observation that general propositions do not follow logically from exclusively particular premises. Comparison with other metaphysics of generality show them to be wanting and Russell's and Hossack's accounts superior.

1 Introduction

In Plato's *Sophist* the Stranger from Elea gives Theaetetus good advice: 'In fact, my friend, it's inept to separate everything from everything else. It's the sign of a completely unmusical and unphilosophical person.' (259d-e) It is advice I can imagine Keith Hossack – my *Doktorvater*, a completely musical and philosophical person – giving his student. It was a privilege to study with a philosopher whose thought exemplifies rigour, intellectual honesty and an encompassing philosophical vision, and I am grateful for our ongoing discussions and friendship. The question 'Would Keith let me get away with this argument?' is still a driving force in getting clear in my philosophical thinking. This paper concerns an aspect at the heart of Hossack's philosophy: realism, or a theory of why it's inept to separate everything from everything else.[1]

[1]My greatest debt in coming to grips with the topic of this paper is to Keith. It has also profited greatly from discussions with Javier Cumpa and a seminar in London with, besides Keith, Julien Dutant, Dorothy Edgington, Guy Longworth, Hamid Taieb and Mark Textor. Thank you also to Jonathan Nassim and Bahram Assadian for comments on the final version. While finishing the paper I was supported by the Alexander von Humboldt Stiftung during a time as a research fellow at the University of Bochum.

Hossack's *The Metaphysics of Knowledge* contains a sustained argument that there are *negative facts*, facts that contain the universal *negation* in principal position. (Hossack, 2007, Sections 2.2–2.6) Negative facts are required to distinguish sense from nonsense, a distinction in turn required in the definition of particulars, universals and the arity of universals. Negation is a logical constant. The universal *negation* is the referent of the symbol for negation in logic. Complex facts are facts that contain referents of logical symbols. Consequently, there are complex facts. But are there any other complex facts besides those containing *negation*? The sections arguing for the existence of negative facts are followed by a much briefer section considering whether there are complex facts containing universals referred to by two other expressions of logic, conjunction and universal quantification. Hossack accepts that besides the negative facts there are also complex facts containing the universal *conjunction* and those containing the universal *generality*. These are the conjunctive and the general facts. (Hossack, 2007, Section 2.7) But Hossack gives no argument comparable to the intricate line of reasoning that concludes that there is a universal *negation*. Hossack arrives at his conclusion after having found the theory of deduction of Wittgenstein's *Tractatus* wanting. Its positive support is rather brief. I will here only consider generality. Hossack bases his acceptance of general facts on 'broader metaphysical considerations', of which 'by far the most important is that the existence of general facts is required if we are to give a realist account of laws of nature: for a law is a general fact'. (Hossack, 2007, 72) While the existence of negative facts is concluded from an argument at the very foundations of Hossack's metaphysics, the existence of general facts is presented as a requirement of more remote developments of his views.

In this article, I will offer Hossack three arguments for his conclusion that are of a foundational character. They draw heavily upon the views of Russell, one of Hossack's philosophical heroes. Two of the arguments appeal to the nature of universals and one to the nature of logical inference.

For most of this paper I will restrict consideration to generality of first-order, the generality that corresponds to quantification in first-order predicate logic, to avoid complications with paradoxes such as Russell's and the Liar. Russell did not accept that it is possible to quantify over absolutely everything, and the issues I am going to address already arise in the case of quantification over particulars. At one point there will be occasion to mention generality of higher-order. Hossack rejects any such distinction of orders and avoids the threat of paradox by other means. An exposition of the details would go too far here, but I see no reason why the arguments given here may not be adapted to his framework.

Russell's views on generality are intriguing and merit a historical investigation, which will be the subject of another paper. (Kürbis, ms) They constitute one of Russell's famous changes of mind. The view Hossack attributes to Wittgenstein, according to which there are no general facts and general statements are made true by their instances, was also held by Russell in the first edition of *Principia Mathematica*. (Whitehead and Russell, 1910, 47f) In *The Philosophy of Logical Atomism*, Russell had moved to accepting that there are general facts. Whereas in the first edition of *Principia*, there are evidently no general facts, only general judgements, in *The Philosophy of Logical Atomism*, Russell takes it as evident that there are and even goes as far as saying that it cannot be doubted that there are general facts. (Russell, 1919b, Lecture V, 200) Russell

does not, on the other hand, say much about the nature of general facts. This is a difference between Russell and Hossack we should note right from the start. In *The Philosophy of Logical Atomism*, Russell doubts that there are molecular facts, facts of the kind Hossack calls 'complex' (Russell, 1919b, Lecture III, 39, 42), but accepted that there are negative facts. Around the same time, Russell also explicitly rejected the view that negation refers to something, so negative facts are not molecular. The distinction between negative and positive facts is one of form or quality, not of constituents. 'It must not be supposed that the negative fact contains a constituent corresponding to the word "not." It contains no more constituents than a positive fact of the correlative positive form. The difference between the two forms is ultimate and irreducible.' (Russell, 1919a, 4) Russell is less explicit about what distinguishes general facts. Hossack, by contrast, is: they contain the universal *generality*. In the light of the argument to be given in the next section, it is somewhat surprising that Russell did not draw Hossack's conclusion: it is suggested by Russell's view on the nature of universals, and at one point Russell even mentions 'abstract, logical universals' of which we may have knowledge by acquaintance (Russell, 1912, 171). Be that as it may, Russell's notion of general fact is rather different from Hossack's: Hossack's general facts are molecular or complex facts, Russell's apparently are not. It may well be that Russell had in mind that general facts, and possibly others one might be inclined to call molecular or complex, are really further forms of facts. But to argue this point is a matter for another occasion.[2]

2 Resemblance between Universals. I

Russell's characterisation of the nature of universals and how they are distinguished from particulars allows for a swift argument for the existence of general facts and Hossack's conclusion as to their nature. The distinction between universals and particulars is explained in an especially clear and striking fashion in *The Problems of Philosophy*. Russell reflects on the characteristics of words such as 'whiteness' and 'justice':

> The word will be applicable to a number of particular things because they all participate in a common nature or essence. [...] We speak of whatever is given in sensation, or is of the same nature as things given in sensation, as a particular; by opposition to this, a universal will be anything which may be shared by many particulars, and has those characteristics which, as we saw, distinguish justice and whiteness from just acts and white things. (Russell, 1912, 143, 145)

Some things resemble each other, others don't, and this is a matter of fact, not of opinion or perception. As Hossack puts it: 'Things resemble because they literally have something in common, namely a universal. [...] Resemblance is an objective matter: it is not just that some things strike us humans as similar

[2] Kürbis (ms) is a historical investigation into Russell's changes of mind concerning general facts and different options of what their nature might be. I should mention a third option regarding the latter issue. Elkind (2021) argues that Russell had the material at hand for an account on which completely generalised propositions, those of formal logic, are made true by completely general facts, facts that have no constituents at all and which are non-atomic, non-molecular 'logical structures' (Elkind, 2021, 17). On Hossack's account, completely general facts contain only logical universals.

by human standards; some things really are similar, whether they strike us as similar or not.' (Hossack, 2007, 35) A universal can be shared by different things, the things that resemble each other. Russell also points out that at least *resemblance* must be a universal, but that an attempt to do away with all other universals and keep only *resemblance* is doomed to failure. (Russell, 1911, 8ff) Things do not merely resemble each other, but do so in certain respects, and these respects are the universals. A universal is an aspect of resemblance.[3]

Another point that may be added is that objects not only resemble each other in certain respects, but may also fail to do so. In such a case, they do not share these universals. Assuming Leibniz' Law, objects that are different do not resemble each other in certain respects.

Russell focusses on characteristics that may be shared by particulars. Socrates, Plato and Aristotle resemble each other in being men. Bucephalus and Incitatus resemble each other in being horses. Once we have accepted that there are universals, we may note that there are relations between them. General facts are of this kind. Russell gives as an example the fact that all men are mortal (Russell, 1919a, Lecture V, 200f), which consists in a relation between the universals *man* and *mortality*. Another example is the fact that all horses are hoofed, which consists in a relation between the universals *horse* and *hoofed*. We may now note that these pairs of universals resemble each other. The pair *man* and *mortality* has something in common with the pair *horse* and *hoofed*: they resemble each other in that the first is subsumed under the second. Given Russell's explanation of the nature of universals, we should therefore conclude that there is a universal corresponding to what in *Principia Mathematica* is called *formal implication*. (Whitehead and Russell, 1910, 21f) The same relation holds between the two pairs.

Russell's discussion of $2+2=4$ in *The Problems of Philosophy* mentions further characteristics that may be shared by universals:

> In the special case of "two and two are four," even when we interpret it as meaning "any collection formed of two twos is a collection of four," it is plain that we can *understand* the proposition, *i.e.* we can see what it is that it asserts, as soon as we know what is meant by "collection" and "two" and "four." It is quite unnecessary to know all the couples in the world: if it were necessary, obviously we could never understand the proposition, since the couples are infinitely numerous and therefore cannot all be known to us. Thus although our general statement *implies* statements about particular couples, *as soon as we know that there are such particular couples*, yet it does not itself assert or imply that there are such particular couples, and thus fails to make any statement whatever about actual particular couples. The statement made is about "couple," the universal, and not about this or that couple.

[3]This may raise the question whether there can be universals under which only one thing falls, such as *being identical to Socrates*. Then there is only a resemblance in the rather strained sense that Socrates resembles himself in this respect. Some aspects of resemblance may be explicable or definable in terms of others, so not all of them need to be assumed as unanalysed primitives. In the ultimate analysis, not every aspect of resemblance needs to correspond to a universal. For this reason, the examples of universals to be given in this paper, always put in *italics*, are merely heuristic.

> Thus the statement "two and two are four" deals exclusively with universals, and therefore may be known by anybody who is acquainted with the universals concerned and can perceive the relation between them which the statement asserts. (Russell, 1912, 163f)

Couple is a universal, but it is not a characteristic of particulars like Socrates and Plato. It might be a characteristic of pluralities or collections, but if we try to attribute it to any of the entities Russell explicitly accepted at this point, and he was of course sceptical whether classes 'really' exist, then there is little choice but to attribute it to universals: two things are a couple if they share a universal shared only by them. *Having exactly two instances* is a universal of universals, a characteristic shared for instance by the universals *planet closer to the sun than Earth* and *planet between Venus and Jupiter*.

Thus having accepted that there are aspects of resemblance, *i.e.* universals, we can further observe that there are relations between them and that there are aspects of resemblance between the universals themselves. The universals *mortality*, *rationality* and *animality* resemble each other in the respect of being had by every human. Having exactly two instances is an aspect of resemblance between the universals *planet closer to the sun than Earth* and *planet between Venus and Jupiter*. Generality is also something in respect of which universals can resemble each other: *being universally instantiated* is a property shared by every universal had by every particular, such as *being a material object that is extended*, *being mortal if human*, *being hoofed if a horse*. We should thus side with Hossack and conclude that there is a universal *generality*.

We may call such resemblances higher-order resemblances, but resemblances they are nonetheless. Given Russell's explanation of what a universal is, there is no reason not to accept that these higher-order relations or properties are universals. The generality of the examples given is first-order, but a statement of the resemblances between them involves expressions of higher-order logic. *Having exactly two instances* means that there are x, y such that Fx, Fy and x and y are different and anything that is F is either x or y. That two universals F, G resemble each other in this respect is expressed in Fregean fashion by the sentence stating that there is a one-to-one correspondence between the Fs and the Gs, *i.e.* by the corresponding instance of Hume's Principle, which requires second-order quantification.

According to Hossack and Russell, universals combine with particulars and other universals to form facts. (Hossack, 2007, 45ff) The present line of argument concludes that there is a universal *generality*. This universal is a constituent of facts. It combines with such properties as *being mortal if human*, *being hoofed if a horse* and *being a material object that is extended* to form facts corresponding to 'All humans are mortal', 'All horses are hoofed', 'Every material object is extended'. These are what Hossack calls 'general facts'.

Another characteristic that is often attributed to universals is that, contrary to particulars, they can be in many places at the same time. If it makes sense to say this, it also makes sense to say that a universal occurs only once. Adopting terminology from logical syntax, if the universal *generality* is the principal universal in a fact, then the fact is a general fact; if *negation* is the principal universal, it is a negative fact. A fact that corresponds to a double negation would then count as a negative fact, which may not satisfy everyone, but as this

is not the topic of the present article, I shall not attempt to sort out this issue.

This line of argument to the conclusion that there is a universal *generality* is not explicitly taken by either Russell or Hossack. Given Russell's discussion of couples, this is somewhat surprising. It may be due to a reluctance to accept that logical constants refer. But the line of argument follows their shared outlook on the nature of universals closely. As will be discussed later, some such line has in fact been taken by some metaphysicians, although I will argue that further aspects of their views mean that they are less satisfactory than those of Russell and Hossack.

3 Resemblance between Universals. II

According to Russell, existence is a property, albeit not a property of individuals: 'Existence is essentially a property of a propositional function. It means that the propositional function is true in at least one instance. [...] Existence is a predicate of a propositional function, or derivatively of a class.' (Russell, 1919c, 195f) This is also fundamentally Frege's view: existence is a concept of concepts. (Frege, 1884, §53) Not wishing to go into either Russell's views on propositional functions or Frege's on concepts, we can, instead, build on material from the last section to form an alternative view on what existence is.

Socrates, Plato and Aristotle are human. Bucephalus and Incitatus are horses. The universals *human* and *horse* have something in common: they are instantiated. There is something that has that property or something with that property exists. Russell explicitly accepted that there are existential facts. (Russell, 1919c, Lecture V, 200) But he shied away from concluding that there is a universal *existence*. Given his explanation of the nature of universals, there may be little reasons for this reluctance. Instantiated universals resemble each other in this respect. Thus there is a universal that is this resemblance. We may call it 'existence'. It is a higher-order universal, shared by some universals. Existential facts are those in which the universal *existence* is combined with other universals, such as *human* or *horse*.

Some properties are distinct from *human* and *horse* in this respect: *unicorn* and *winged horse* are not instantiated. But this means that everything fails to be a unicorn and is other than a winged horse. Russell accepted that there are negative facts without committing to the view that the symbol for negation refers to a constituent of facts. Arguably, however, objects that lack a property resemble each other in the respect of not instantiating this property. Such facts have something in common. This provides further support for Hossack's conclusion that there is the universal *negation*. Either way, existential and negative facts together yield universal facts. Conversely, if there are negative facts and general facts, there are existential facts.

Some philosophers may now feel that surely the account proves too much. There are too many universals, if all that is required for the existence of a universal is some aspect of resemblance. Some philosophers may, for instance, prefer to say that existential propositions are made true by their instances. If Socrates is human, then there are humans. The first fact suffices to ensure the truth of the sentence expressing the latter, and there is no need for existential facts in addition.

Two responses can be made here. Positions according to which properties

are 'abundant' have been argued for.[4] Such a view, however, does not cohere well with Russell's and Hossack's characterisation of the nature of universals. A theory according to which universals are aspects of resemblance will probably not accept that they are 'abundant': not everything resembles everything else in the substantial fashion countenanced by the theory. Such a theory will rather take to heart Socrates' remarks in the *Phaedrus*. There are, says Socrates,

> two kinds of things the nature of which it would be quite wonderful to grasp by means of a systematic art. [...] The first comes in seeing together the things that are scattered about everywhere and collecting them into one kind[. ... The other] is to be able to cut up each kind according to its species along its natural joints, and to try not to splinter any part, as a bad butcher might do. (265d-e)

The first aspect is to collect things together according to their resemblances, the second to look more deeply into what explains the resemblances according to the natures of things. Not every resemblance needs to point to the existence of an unanalysable universal. It suffices that any resemblance can be explained in terms of or reduced to some universals. This line of thought would not exclude the possibility that nature allows for some abundance in her offerings.

Russell happily accepted that there are universal, existential and negative facts, even though one of the former two would appear to be redundant in the presence of the other and negative facts.[5] It would be natural to call an existential fact also a general fact, as does Russell, albeit one of a different kind than a universal fact. Hossack, by contrast, accepts only that there is the universal *generality*, not also the universal *existence*, and only calls facts containing the former in principal position 'general facts'. Existential facts are analysed in terms of generality and negation.[6]

4 The Logic of Generality

While the arguments of the previous two sections were not put forward by Russell, there is an explicit argument for the existence of general facts in *The Philosophy of Logical Atomism*. Here Russell turns away from the treatment of generality of the first edition of *Principia Mathematica*, the same that Hossack finds wanting. General judgments are said to collect together elementary judgments and they are made true by their instances:

> The judgment ["all men are mortal"] does not correspond to one complex, but to many, namely "Socrates is mortal," "Plato is mortal,"

[4] See for instance (Hale, 2013, Chapter 1). See also my review of Hale's book (Kürbis, 2015a).

[5] The reason may have to do with Russell's avoidance of referents for logical symbols: whether these kinds of facts are interdefinable depends on whether, and if so, how, forms of facts can be embedded. Depending on the answers, an account that classifies facts according to their forms may require more primitive forms than an account that classifies them by principal and sub-principal universals requires primitive universals.

[6] In correspondence Hossack suggested a further argument for accepting existential or general facts. The real resemblances are the natural divisions underlined by Plato, and these accord with the laws of nature. The laws of logic are laws of nature, and hence there must be logical universals. Instantiation and generality are essential to the laws of logic, and so they are both plausible candidates for logical universals. Whether to choose only one, and if so, which one, or both as primitive may depend on further considerations.

> "Aristotle is mortal," etc. [...] Our judgment that all men are mortal collects together a number of elementary judgments [namely that Socrates is mortal, that Plato is mortal, that Aristotle is mortal, etc.]. It is not, however, composed of these, since (*e.g.*) the fact that Socrates is mortal is no part of what we assert, as may be seen by considering the fact that our assertion can be understood by a person who has never heard of Socrates. In order to understand the judgment "all men are mortal," it is not necessary to know what men there are. (Whitehead and Russell, 1910, 47)

Russell's argument for the existence of general facts is based on the observation that no universal proposition follows from exclusively particular premises: any such inference is invalid. From elementary propositions about individual men's mortality, 'Socrates is a man that is mortal', 'Plato is a man that is mortal', 'Aristotle is man that is mortal' and so on, one cannot infer 'All men are mortal' unless the premise 'All men are amongst those I have enumerated' is also given, which is a general proposition. 'You never can arrive at a general proposition by inference from particular propositions alone. You will always have to have at least one general proposition in your premise.' (Russell, 1919b, 198f) Certain logical inferences require general propositions for their validity. Thus they cannot be reduced to their instances. General propositions must therefore state general facts, rather than collecting together elementary facts. Accordingly, Russell concludes that there are irreducibly general facts:

> I do not think one can doubt that there are general facts. It is perfectly clear, I think, that when you have enumerated all the atomic facts in the world, it is a further fact about the world that those are all the atomic facts there are about the world, and that is just as much an objective fact about the world as any of them are. It is clear, I think, that you must admit general facts as distinct from and over and above particular facts. The same thing applies to "All men are mortal." When you have taken all the particular men that there are, and found each one of them severally to be mortal, it is definitely a new fact that all men are mortal; how new a fact, appears from what I said a moment ago, that it could not be inferred from the mortality of the several men that there are in the world. (Russell, 1919c, 200)

A mere list of mortal men separates each item from every other, as the Stranger from Elea might say, and falls short of the truth about all men: that they are all mortal.

This may still allow for some leeway in what general facts there are. It may, for instance, suffice to list all atomic truths and add the clause that these are all of them. This is the position taken in the introduction to the second edition of *Principia Mathematica*:

> Given all true atomic propositions, together with the fact that they are all, every other true proposition can theoretically be deduced by logical methods. That is to say, the apparatus of crude fact required in proofs can all be condensed into the true atomic propositions together with the fact that every true atomic proposition is one of the following: (here the list should follow). (Whitehead and Russell, 1997, xv)

It may also suffice to list all atomic propositions and then to add that the particulars named in them are all there are. Formally, this is the more straightforward option, at least in the finite domain, as it allows us to remain firmly first-order. Suppose there are only two objects in the universe, a and b name them, and each of them has the property F. Then the inference from premises (1) Fa and (2) Fb to the conclusion $(x) \,.\, Fx$ is invalid. If we add a further premise to the effect that a and b are the only objects there are, *i.e.* (3) $(x) \,.\, x = a \vee x = b$, we can argue as follows: by universal quantifier elimination, from (3) $x = a \vee x = b$; if the first, then by (1) Fx, if the second, then by (2) Fx, hence by disjunction elimination Fx; Fx was derived from premises not containing x free, hence by universal quantifier introduction, $(x) \,.\, Fx$. This pattern of argument can be extended to any finite domain of objects.[7]

It may be that all cases of generality can be reduced to this one kind: once we have listed all humans and observe that each one of them is mortal, or all the facts about human mortality, and add that these are all of them, we may infer that all humans are mortal. But whether there is only one kind of general fact or several, there are then irreducibly general facts.

On the other hand, if the truth of elementary propositions is defined in terms of the facts to which they correspond (Whitehead and Russell, 1910, 43f), and facts are explained as containing universals (Russell, 1918, 513), a less austere route is more natural. Although it is not necessary to know who Socrates was in order to understand that all humans are mortal, it is necessary to know the universals *human* and *mortality*, as pointed out in *The Problems of Philosophy*. (Russell, 1912, 166) There it is also pointed out that certain truths consist purely in the relations between universals. Those that give rise to *a priori* knowledge are amongst them. (Russell, 1912, 162) Our evidence for the truth of 'All humans are mortal' comes from our observation of individual instances of human mortality, but its meaning is one that deals only with universals. Russell was never in doubt that there are facts concerning the relations between universals. In particular, there are those about which universals subsume which others. Facts concerning only the relations between universals contain no particulars and are therefore general. Thus it is plausible that there are rather more general facts than just those concerning enumerations of the facts or objects that there are.

Russell observes that if there are general facts concerning the subsumption of one universal under another, it may be hard to resist the conclusion that there are hypothetical facts that link particular instantiations of those universals, such as the fact that Socrates is mortal if a man. (Russell, 1919c, 201)[8] If there were, he would probably have to admit that there is a universal *hypotheticality* or a further form of facts.

Russell resisted the conclusion that the subsumption of one universal under another is also a universal. This may again have something to do with worries concerning logical constants. Russell does not say anything more about the nature of general facts. The first argument given here, however, settles the issue.

[7] This reconstruction of Russell's argument follows Hochberg (Hochberg, 1969, 336f), about whom more later.

[8] See also (Russell, 1919b, Lecture III, 42).

5 Comparison with other Metaphysicians' Views on Generality

Let's compare Russell's and Hossack's views with those of two other contemporary defenders of general facts, Armstrong and Hochberg.

According to Armstrong, if there is a number of facts and these are all of them, then they stand in the '*alling* or *totalling* relation'. (Armstrong, 1997, 199) Armstrong's argument for the existence of such a relation is reminiscent of that of Section 2 above: 'It can hardly be denied that such a relation exists [...]. It looks to be exactly the same relation in each case, to be a true universal if you accept universals.' (Armstrong, 2004, 73) Totality facts present limits to what there is:

> If it is true that a certain conjunction of states of affairs is all the states of affairs, then this is only true because there are no more of them. If there are more, then the proposition is not true. That there are no more of them must then be brought into the truthmaker. But to say that there are no more of them is to say that they are *all* the states of affairs. This, then, must be the fact or state of affairs that the great conjunction *is* all the states of affairs. (Armstrong, 1997, 197)

Totality facts are higher-order facts. More precisely, the totalling relation holds between a mereological aggregate and a property that *all* things that compose the aggregate have: 'A certain mereological object totals a certain property. The mereological object is the whole composed of the existents in question.' (Armstrong, 2004, 72) Such a fact has the form Tot(aggregate, the corresponding property) (Armstrong, 2004, 73). For instance, if all humans are mortal, then the totalling relation holds between the mereological sum of all humans and the property *mortality*.

Armstrong's totalling facts are rather different from the general facts of Russell and Hossack. Armstrong's totality relation is a higher-order relation relating lower-order facts or a relation between a mereological sum and a property. Russell's and Hossack's general facts are not of this kind. Armstrong's totality facts do not have the universal *generality* as constituent. Armstrong's totality facts are relational facts: the totalling relation holds between a property and an object. The universal *generality* requires only a property to form a fact. Armstrong seems to think that these cases boil down to the same thing:

> The centrally important case, though, is Russell's own case: the general fact that all the facts (states of affairs) of lower order are all such facts. The analysis in terms of the Tot relation is the same. The object is the mereological whole (or manifold) of all the lower-order states of affairs. The property involved in the relation is the very abstract one of *being a state of affairs*. (Armstrong, 2004, 74)

But this is not so. Armstrong's totalling relation is a many-place higher-order relation. The universal *generality* is a one-place higher-order relation. The universal *generality* is a universal that is instantiated by first-order universals.[9] These differences in form shouldn't be run together.

[9] This may have to assume that there are compound universals. But if some universals are related to others, this is hardly an issue.

An analysis of universally quantified propositions in terms of the totalling relation falls foul of Russell's argument for the existence of general facts. Armstrong's totalling facts have the form Tot(a, F), where Tot is the higher-order totalling relation, a is a mereological aggregate of particulars and F is a first-order property. From a proposition corresponding to a fact of this form and the proposition that b is one of the aggregate, it does not follow logically that b has the property. From the propositions (1) 'The totality relation relates the mereological sum of all humans and *mortality*' and (2) 'Socrates is human' it does not follow logically that Socrates is mortal. This is because (1) has the form $R(a, F)$, where R is a second-order property and F a first-order property, and hence it is an atomic proposition. Anyone taking Russell's argument for general facts from logical inferences seriously should be equally wary of analysing them in terms of Armstrong's totalling relation.[10]

'Totalling' is a more accurate name than 'alling' for Armstrong's relation, as totalling has little to do with 'all', that is universal quantification. Hossack's general facts are structurally isomorphic to universally quantified propositions, $(x)\,.\,Fx$, where '(x)' refers to the higher-order universal *generality* and 'F' to a first-order property. Armstrong's are not. It is, therefore, preferable to follow Hossack and to analyse general facts as sharing a common universal *generality* rather than in terms of a totalling relation.

Hochberg makes some pertinent observations about how Russell's argument for the existence of general facts fares if there are infinitely many objects. In this case, it is no longer possible to write down a premise stating what every thing is identical to: this would require an infinite disjunction. Someone not worried about the finitary limitations of predicate logic may simply say that although there is no such sentence, there is nonetheless such a proposition and leave it there. We can, however, do a little better than that and use the infinite case to argue even more forcefully that there are general facts.

In the finite case, we were drawing on a complex property that every object in the domain has, namely being either a_1 or being a_2 or being a_3 or ... being an, a property to which we can refer using identity, disjunction and names for each object. We are looking for a similar property that every object has in case the domain is infinite. Let's suppose for simplicity that there are ω objects and that the language has a name for each of them, say a_i, for $i \in \omega$. One option of a property of every object in the domain is 'being identical to something'. This can be expressed using identity and existential quantification. But $\exists x\,.\,x = a_i$ is logically true in classical logic, so adding the infinite collection of premises $\exists x\,.\,x = a_i$, for all i, to an infinite collection of premises Fa_i, for all i, does not get us any further towards a valid inference of $(x)\,.\,Fx$ than not adding them. Using a free logic, where $\exists x\,.\,x = a_i$ is a substantial claim, makes no difference to the issue at hand. We need a premise to the effect that nothing else but the a_i exist,

[10]This appropriates an argument of Bergmann's discussed by Hochberg (Hochberg 1981: 386). Hochberg reports Bergmann's argument to be one 'against the construal of a notion of "essential connection" in terms of a second-order relational property', *i.e.* against the claim that the notion of an essential connection between first-order properties F_1 and G_1 is to be analysed in terms of a second-order relation C_2, such that $C_2(F_1, G_1)$: if there is an essential connection between F_1 and G_1, then any F_1 is G_1, but $C_2(F_1, G_1)$ does not logically entail $(x)\,.\,F_1x \supset G_1x$, as $C_2(F_1, G_1)$ is an atomic proposition. $(x)\,.\,F_1x \supset G_1x$ expresses, of course, a second-order relational property, but the crucial point is that it is not an unanalysed relation, but one formed from F_1 and G_1 by means of logical constants.

and even an infinite list of premises $\exists x \,.\, x = a_i$ does not give us that.[11]

We would require a premise that collects together all the things there are, so that we can conclude that an arbitrary object referred to by a free variable, say x, is one of them, and then something that allows us to infer that x must be identical to one of the things named in the premises Fa_i. Alternatively, assuming $\neg Fx$, where x is a fresh free variable, we must be in a position to derive that x is one of all the things there are, so that we reach a contradiction, as for any thing there is, we have a premise to the effect that it is F. So not only do we need premises stating the Fness of each object and a premises stating that some property is had by all the things there are; we also require a premise connecting the infinitely many premises Fa_i, for all i, with the premise stating a property every a_i has. In other words, we need an additional premise that all the things there are are Fs, and we have gone nowhere in the attempt to avoid general statements of the form $(x) \,.\, Fx$, which correspond to general facts. To conclude with Hochberg, in the infinite case 'every true generalisation would, in fact, require its own fact.' (Hochberg, 1969, 341)[12] The infinite case, so far from being a problem for Russell, is grist for the mill of Russell and Hossack: it shows even more forcefully that there must be general facts. And this is particularly so for Hossack, for whom general facts consist of universals, such as being an F, instantiating the universal *generality*.

To close this section, a few words on whether general facts are positive or negative may be in order. Metaphysicians are in two minds over this question. For Armstrong, they are negative; for Hochberg, they are positive. For Hossack, too, general facts look rather more like positive than negative facts. Negative facts have as a constituent the universal *negation* in principal position. General facts don't. According to Armstrong, we should avoid accepting negative facts in our ontology that are on a par with atomic facts. To do so, Armstrong argues that it is necessary to accept that there are totality facts that set limits, and as such they are negative. 'The *all* state of affairs is itself a "no more" state of affairs.' (Armstrong, 2004, 58, see also 70) It is a negative state of affairs, albeit a higher-order one. One kind of negative fact suffices to explain all first-order negative truths. (Armstrong, 1997, 200) The general facts Hochberg considers are essentially described by lists, and as such they are positive.

Another option is to follow Ayer. Ayer observes that a general distinction between negative and affirmative statements is difficult, if not impossible to draw; at least so far no one has provided a satisfactory one. It looks likely that 'the distinction between affirmative and negative statements [is] a matter of

[11] Besides, in a negative free logic, probably the free logic preferred by metaphysicians of the Russellian type, if F is atomic, $Fa_i \vdash \exists x \,.\, x = a_i$, so once more nothing is added to the argument by adding these premises.

[12] Hochberg's route to his conclusion is, however, flawed. Hochberg fails to distinguish questions concerning the number of things there are and questions concerning which things there are. He writes that 'we can maintain, in view of [Russell's argument], that there is only one general fact for a finite universe: that pertaining to how many things (at most) there are.' (Hochberg 1969: 334) Concerning the infinite case, Hochberg writes that together with the sentence used to state that there were at most a denumberably infinite number of objects, no set of atomic (and negations of atomic) sentences would suffice as premises to derive a generality of the form "$(x)(Fx \supset Gx)$".' (Hochberg, 1969, 341) But this is true in the finite case, too. Even if I add 'There are at most two objects' as a premise to the argument that attempts to conclude $(x) \,.\, Fx$ from Fa and Fb, unless I know that my premises name them both, rather than one of them twice, I'm none the wiser. The premise $(x) \,.\, x = a \vee x = b$ cannot be replaced by the premise that there are at most two things in the domain, $(z)(y)(x) \,.\, z = x \vee z = y$. The latter requires the further premise $a \neq b$.

emphasis.' (Ayer, 1952, 815) Some such distinction can, however, be drawn, even if it may be one with less of a metaphysical impact than one might expect. Positive statements are more specific than negative ones. 'This flower is not yellow' leaves open a range of colours the flower may have which is not left open by 'This flower is blue'. This is why negative statements cannot be reduced to positive ones, although the less specific negative statements are entailed by the more specific positive ones. Ayer concludes that 'logically a negative statement [...] can be verified only through the truth of some more specific statement which entails it; a statement which will itself, by contrast, be counted as affirmative'. (*Ibid*.) A general statement, being one that entails much specific information, should then count as rather more positive than negative.

Russell, on the other hand, rejects that the distinction between negative and positive is applicable to general facts, because 'all' is the negation of 'some not', 'some' the negation of 'all not': 'In regard to general propositions, the distinction of affirmative and negative is arbitrary [...] All general propositions deny the existence of something or other.' (Russell, 1919c, 190f) In light of these disagreements, it may be, as it is so often, wise to side with Russell.[13]

6 Conclusion

Russell concludes his discussion of general facts with an admission and an exhortation:

> I do not profess to know what the right analysis of general facts is. It is an exceedingly difficult question, and one which I should very much like to see studied. (Russell, 1919c, 201)

Hossack has taken up the challenge and proposed a bold thesis about the analysis of general facts, in keeping with his rationalist and realist outlook. Russell and Hossack agree that there are general facts, but Hossack is more forward concerning their nature. Following in their footsteps, I have proposed three arguments for Hossack's account of general facts and the existence of a universal *generality*. They are Russellian in spirit and draw heavily on Russell's writings.

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[13] For further discussion of negative facts and the logic, metaphysics and epistemology of negation, see (Kürbis, 2015b), (Kürbis, 2018), (Kürbis, 2019a), (Kürbis, 2019c, Chapter 4), and (Kürbis, 2019b). The last item is a commentary on (McCabe, 2019).

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