# The Origins of Telicity 

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## 1. Introduction

The distinction between telic and atelic predicates has been described in terms of the algebraic properties of their meaning since the early days of model-theoretic semantics. This perspective was inspired by Aristotle's discussion of types of actions that do or do not take time to be completed ${ }^{1}$ which was taken up and turned into a linguistic discussion of action-denoting predicates by Vendler (1957). The algebraic notion that seemed to be most conducive to express the Aristotelian distinction appeared to be the mereological notion of a part, applied to the time at which these predicates hold: atelic predicates, like push a cart, have the subinterval property, that is, whenever they are true at a time interval, then they are true at any part of that interval; this does not hold for telic predicates, like eat an apple, cf. Bennett \& Partee (1972), Taylor (1977), and Dowty (1979) ${ }^{2}$. Bach (1986) integrated these insights into a semantics based on events.
Work by Garey (1957), Verkuyl (1972) and Platzack (1979) showed that the aspectual properties of a predicate result from two sources, the nature of the verbal head and the nature of a nominal argument. For example, whereas eat two apples is telic, eat apples is atelic. The early accounts of this phenomenon used syntactic features to describe this phenomenon; for example, Verkuyl analyzed eat two apples by saying that eat has a feature [+ ADD To] that allows for the feature [+ SPECIFIED QUANTITY] to percolate from the object NP an apple to the VP eat two apples. These feature percolation rules were understood as semantically motivated; for example, Verkuyl says that for verbs like eat, "the quantities of [the object] X involved are expressible in terms of linearly ordered sets of temporal entities" (p. 96). Several researchers, starting with Dowty (1979) and Hoepelman \& Rohrer (1980), attempted to make this property of verbs like eat formally explicit. More recent approaches made use of the notion of a part not only for events, but also for objects, and assume that verbs like eat relate object parts and event parts to each other (cf. Hinrichs (1985), Krifka (1989a), Krifka (1989b), Krifka (1992), Dowty (1979), Verkuyl (1993), Jackendoff (1996)).

This paper attempts to generalize the approach that was developed for cases like eat an apple to other cases, in particular, predicates that express movement in space, as in walk from the university to the capitol, and predicates that express changes of properties, like bake the lobster. While such cases have been considered in earlier work, the treatments proposed were either not explicit enough, or worked with structures that were richer than required. In particular, they assumed discrete representations of paths and times - paths are seen as a collection of points in space, time intervals are seen as sets of time points (cf. e.g. Verkuyl (1993), Pustejovski (1991), Krifka (1995c)). This leads to the "filmstrip" model of change, a perspective that may seem plausible after the advent of movie cameras, but arguably is not the way how movement and change is conceptualized (cf. Jackendoff 1996). We do not see a moving object as appearing in a succession of distinct locations; rather, we see it as moving continuously along a path.

[^0]This article has three main parts. In § 2 I develop several algebraic structures for objects, paths, directed paths, times and events. I will also introduce the important notion of an extensive measure function. In § 3 I deal with semantic properties of thematic relations that allow us to predict, for example, that eat two apples is telic, whereas eat apples is atelic. These conditions are defined with the part relation. They are similar in spirit, but different in detail, to what I have proposed in earlier work. In § 4 I consider semantic properties that can be defined with the adjacency relation. They turn out to be suitable for the modeling of a wider range of cases, in particular, movements and quality changes.

Model-theoretic semantics in the tradition of Montague, Lewis and Cresswell has often been seen as opposed to cognitive approaches as developed by Lakoff, Langacker, Wierzbicka, Jackendoff, Bierwisch, and others. It was believed that model-theoretic semantics is forced to a 'realistic' view, in which natural-language expressions are interpreted by real entities, like objects and possible worlds, whereas cognitive semantics is concerned with cognitive models of reality. I don't see that model-theoretic semantics has to be realistic in this sense. We can make use of the techniques developed in the model-theoretic tradition and assume that expressions are interpreted by elements of conceptual structures that in turn are related to 'real' entities by some extra-linguistic matching. This is how I would like to understand the algebraic structures discussed below: As attempts to capture certain properties of the way how we see the world, not as attempts to describe the world how it is.

## 2. Conceptual Structures

In this section I will develop the algebraic structures necessary for the various types of incremental relationships that we observe in a wide variety of events, such as eating an apple, painting a door, or walking a mile. Our main task will be to spell out the relation between two dimensions involved in such events that have variously been called the "ADD-TO" property (Verkuyl 1972, 1993), "measuring out" (Tenny (1987; Tenny (1994)), "graduality" (Krifka 1987, 1992), "incremental theme" (Dowty 1991), or "structure-preserving binding relations" (Jackendoff 1996).

### 2.1. Sums and Parts

The basic structure we are working with are part structures, which have been used to model the semantics of mass nouns and plurals (cf. e.g. Link (1983)). There is a wide variety of mereological structures (cf. Simons (1987)). For our purpose we see a part structure P as follows:

$$
\begin{equation*}
\mathrm{P}=\left\langle\mathrm{U}_{\mathrm{P}}, \oplus_{\mathrm{P}}, \leq_{\mathrm{P}}, \leq_{\mathrm{P}}, \otimes_{\mathrm{P}}\right\rangle \text { is a part structure iff } \tag{1}
\end{equation*}
$$

a. $\quad U_{P}$ is a set of entities;
b. $\oplus_{P}$, the sum operation, is a function from $\mathrm{U}_{\mathrm{P}} \times \mathrm{U}_{\mathrm{P}}$ to $\mathrm{U}_{\mathrm{P}}$ that is idempotent, commutative, and associative, that is:

$$
\forall \mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{U}_{\mathrm{P}}\left[\mathrm{x} \oplus_{\mathrm{P}} \mathrm{x}=\mathrm{x} \wedge \mathrm{x} \oplus_{\mathrm{P}} \mathrm{y}=\mathrm{y} \oplus_{\mathrm{P}} \mathrm{x} \wedge \mathrm{x} \oplus_{\mathrm{P}}\left(\mathrm{y} \oplus_{\mathrm{P}} \mathrm{z}\right)=\left(\mathrm{x} \oplus_{\mathrm{p}} \mathrm{y}\right) \oplus_{\mathrm{p}} \mathrm{z}\right]
$$

c. $\leq_{\mathrm{P}}$, the part relation, defined as: $\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{P}}\left[\mathrm{x} \leq_{\mathrm{P}} \mathrm{y} \leftrightarrow \mathrm{x} \oplus_{\mathrm{P}} \mathrm{y}=\mathrm{y}\right]$
d. $<_{\mathrm{p}}$, the proper part relation, defined as: $\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{P}}\left[\mathrm{x}<_{\mathrm{P}} \mathrm{y} \leftrightarrow \mathrm{x} \leq_{\mathrm{p}} \mathrm{y} \wedge \mathrm{x} \neq \mathrm{y}\right]$
e. $\otimes$, the overlap relation, defined as: $\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{P}}\left[\mathrm{x} \otimes_{\mathrm{P}} \mathrm{y} \leftrightarrow \exists \mathrm{z} \in \mathrm{U}_{\mathrm{P}}\left[\mathrm{z} \leq_{\mathrm{P}} \mathrm{x} \wedge \mathrm{z} \leq_{\mathrm{P}}\right]\right]$
f. Remainder principle: $\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{P}}\left[\mathrm{x}<_{\mathrm{p}} \mathrm{y} \rightarrow \exists!\mathrm{z}\left[\neg\left[\mathrm{z} \otimes_{\mathrm{P}} \mathrm{x}\right] \wedge \mathrm{x} \oplus_{\mathrm{P}} \mathrm{z}=\mathrm{y}\right]\right]$

These are all reasonable requirements for part structures. The requirements for the sum operation, (1.b), makes part relations a join semi-lattice. The part relation, as defined from the sum operation, can be shown to be reflexive, transitive, and antisymmetric, that is, a partial order relation. Not any
partial order will do; orders like the one depicted by the Hasse diagram (a) and (b) are excluded because not every two elements have a unique sum. The notion of a remainder, or relative complement, says that whenever an element $x$ is a proper part of another one, $y$, then there is exactly one third element z that does not overlap with x , such that x and z taken together make up y . The remainder principle excludes structures with bottom elements, like (c), but also structures like (d) and (e). An acceptable structure is shown in (f). This structure has a top element; the principles assumed guarantee that finite part structures have a top element (as the sum operation can be applied any number of times), but infinite part structures need not have a top element.
a.


b.

c.

d.

e.

f.


We can define two types of predicates, cumulative and quantized predicates, with respect to a part structure P .

$$
\begin{align*}
& \forall \mathrm{X} \subseteq \mathrm{U}_{\mathrm{P}}\left[\mathrm{CUM}_{\mathrm{P}}(\mathrm{X}) \leftrightarrow \exists \mathrm{x}, \mathrm{y}[\mathrm{X}(\mathrm{x}) \wedge \mathrm{X}(\mathrm{y}) \wedge \neg \mathrm{x}=\mathrm{y}] \wedge \forall \mathrm{x}, \mathrm{y}\left[\mathrm{X}(\mathrm{x}) \wedge \mathrm{X}(\mathrm{y}) \rightarrow \mathrm{X}\left(\mathrm{x} \oplus_{\mathrm{P}} \mathrm{y}\right)\right]\right]  \tag{3}\\
& \forall \mathrm{X} \subseteq \mathrm{U}_{\mathrm{P}}\left[\mathrm{QUA} \mathrm{P}_{\mathrm{P}}(\mathrm{X}) \leftrightarrow \forall \mathrm{x}, \mathrm{y}\left[\mathrm{X}(\mathrm{x}) \wedge \mathrm{X}(\mathrm{y}) \rightarrow \neg \mathrm{y}<_{\mathrm{p}} \mathrm{x}\right]\right]
\end{align*}
$$

Examples of a cumulative predicates are water or apples: If x and y fall under apples, then the sum of x and y falls under apples as well. Examples of quantized predicates are three liters of water or three apples: If x falls under three apples, then it cannot have a proper part y that also falls under three apples. Cumulativity of a predicate X implies that X applies to at least two distinct elements. This makes sure that no predicate can be both cumulative and quantized.

Nothing in the definition of part structures either excludes the existence of atoms, that is, of smallest elements with respect to the part relation. However, it will be useful to have the notion of an atom with respect to a property, and the notion of an atomic property, that is, a property that applies to entities made up of minimal entities with this property:

$$
\begin{align*}
& \forall \mathrm{X} \subseteq \mathrm{U}_{\mathrm{P}} \forall \mathrm{x} \in \mathrm{U}_{\mathrm{P}}\left[\mathrm{ATOM}_{\mathrm{p}}(\mathrm{x}, \mathrm{X}) \leftrightarrow \mathrm{X}(\mathrm{x}) \wedge \neg \exists \mathrm{y} \in \mathrm{U}_{\mathrm{P}}\left[\mathrm{y}<_{\mathrm{P}} \mathrm{x} \wedge \mathrm{P}(\mathrm{y})\right]\right]  \tag{5}\\
& \forall \mathrm{X} \subseteq \mathrm{U}_{\mathrm{P}}\left[\operatorname{ATM}(\mathrm{X}) \leftrightarrow \forall \mathrm{x} \in \mathrm{U}_{\mathrm{p}}\left[\mathrm{X}(\mathrm{x}) \rightarrow \exists \mathrm{y} \in \mathrm{U}_{\mathrm{P}}\left[\mathrm{y} \leq_{\mathrm{P}} \mathrm{x} \wedge \operatorname{ATOM}_{\mathrm{P}}(\mathrm{y}, \mathrm{X})\right]\right]\right] \tag{6}
\end{align*}
$$

That is, an element x is an X -atom iff it has the property X and does not contain any proper part with the property X . And a property X is atomic iff every element with this property has an X -atom as a part. For example, the atoms of the predicate three or more apples are sum individuals that consist of three apples. And this predicate is atomic in the sense that every element it applies to contains atomic elements.

### 2.2. Extensive Measure Functions

Simple predicates in natural language typically are cumulative. Mass nouns like water, arguably the simplest types of nominal predicates, are cumulative: The sum of two quantities to which we can apply water is water again. This means that simple predicates come only with a qualitative criterion of application. quantitative criteria of application lead to non-cumulative, quantized predicates. They
are expressed by extensive measure functions, like liter, kilogram, or hour (cf. e.g. Krantz \& e.a. (1971)).

Measure functions in general are functions that relate an empirical relation, like 'be cooler than', for physical bodies, to a numerical relation, like 'be smaller than', for numbers. For example, the measure function for degree Celsius, ${ }^{\circ} \mathrm{C}$, has the property that, if x is cooler than y , then ${ }^{\circ} \mathrm{C}(\mathrm{y})<{ }^{\circ} \mathrm{C}(\mathrm{x})$. Extensive measure functions are also based an operation of concatenation, which is related to arithmetical addition. For example, if we talk about rods, and if we use $x^{\wedge} y$ for the concatenation of two rods $\mathrm{x}, \mathrm{y}$, then we have that $\mathrm{cm}\left(\mathrm{x}^{\wedge} \mathrm{y}\right)=\mathrm{cm}(\mathrm{x})+\mathrm{cm}(\mathrm{y})$. This property is called additivity ( cf . (7.b)). Another property of extensive measure functions is comensurability, also called the 'Archimedian property' (cf. (7.c)). It ensures that the measure of the whole is commensurate with the measure of the parts. I render this as a requirement that, when the whole x yields a measure greater than 0 , then a part y yields a measure greater than 0 as well. The condition that y is a part of x is expressed by saying that y concatenated with some z makes up x .
(7) $m$ is an extensive measure function for a set $U$ with respect to concatenation " $\wedge$ " iff:
a) $m$ is a function from $U$ to the set of positive real numbers.
b) $\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}\left[\mathrm{m}\left(\mathrm{x}^{\wedge} \mathrm{y}\right)=\mathrm{m}(\mathrm{x})+\mathrm{m}(\mathrm{y})\right]$ (additivity)
c) $\left.\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}\left[\mathrm{m}(\mathrm{x})>0 \wedge \exists \mathrm{z} \in \mathrm{U}\left[\mathrm{x}=\mathrm{y}^{\wedge} \mathrm{z}\right] \rightarrow \mathrm{m}(\mathrm{y})>0\right]\right]$ (comensurability)

The notion of an extensive measure function can be applied to part structures, with the sum operation as concatenation. However, notice that while concatenation operations are commutative (that is, $\left.x^{\wedge} y=y^{\wedge} x\right)$ and associative (that is, $\left.x^{\wedge}\left(y^{\wedge} z\right)=\left(x^{\wedge} y\right)^{\wedge} z\right)$, they fail to be idempotent $\left(x^{\wedge} x \neq x\right)$. Hence the sum operation cannot be taken as a concatenation at face value. But we can restrict concatenation to non-overlapping entities. In particular, we can define the notion of an extensive measure function m for a part structure P as follows:

If $\mathrm{P}=\left\langle\mathrm{U}_{\mathrm{P}}, \oplus_{\mathrm{P}}, \leq_{\mathrm{P}},\left\langle_{\mathrm{P}}, \otimes_{\mathrm{P}}\right\rangle\right.$ is a part structure,
and $m$ is an extensive measure function for (subsets of) $U_{P}$ with concatenation ${ }^{\wedge}$,
then m is an extensive measure function for P iff the following holds:
For all $x, y \in U_{P}, x^{\wedge} y$ is defined only if $\neg x \otimes_{P} y$, and if defined, $x^{\wedge} y=x \oplus_{P} y$.
This notion may at first seem overly restrictive, as it apparently does not permit the computation of the kg-value of the sum of $a_{1} \oplus a_{2}$ and $a_{2} \oplus a_{3}$, as they share one part, $a_{2}$. Not so; we have $\left(a_{1} \oplus a_{2}\right)$ $\oplus\left(a_{2} \oplus a_{3}\right)=a_{1} \oplus\left(\left(a_{2} \oplus a_{2}\right) \oplus a_{3}\right)$, due to commutativity and associativity, which is $\left.a_{1} \oplus\left(a_{2} \oplus a_{3}\right)\right)$, due to idempotence. As these individuals do not overlap, this is $\mathrm{a}_{1} \wedge\left(\mathrm{a}_{2}{ }^{\wedge} \mathrm{a}_{3}\right)$. As kg is an extensive measure function, we have $\operatorname{kg}\left(\mathrm{a}_{1} \wedge\left(\mathrm{a}_{2} \wedge \mathrm{a}_{3}\right)\right)=\operatorname{kg}\left(\mathrm{a}_{1}\right)+\left(\operatorname{kg}\left(\mathrm{a}_{2}\right)+\operatorname{kg}\left(\mathrm{a}_{3}\right)\right)$, as expected. ${ }^{3}$ We can define a part relation for extensive measure functions as follows. Notice that whenever $m$ is an extensive measure function for a part structure P , then $\mathrm{x}<_{\mathrm{m}} \mathrm{y}$ implies $\mathrm{x}<_{\mathrm{P}} \mathrm{y}$.
(9) If $m$ is an extensive measure function with concatenation $\wedge$, then $<_{\mathrm{m}}$, the part relation for m , is defined as follows:
For all $\mathrm{x}, \mathrm{y}$ in the domain of $\mathrm{m}, \mathrm{x}<_{\mathrm{m}} \mathrm{y}$ iff there is a z such that $\mathrm{y}=\mathrm{x}^{\wedge} \mathrm{z}$.
Extensive measure functions can be used to define quantized predicates. Take two liters of water; the nominal predicate water is cumulative, and the predicate two liters of water is quantized. This follows if the measure function denoted by liter is an extensive measure function under the following analysis:

[^1]Proof: If (10) were not quantized, then it would apply to an $x$ and a $y$ such that $y<x$. As it applies to $x$ and $y$, we have $\operatorname{LITER}(x)=2$ and $\operatorname{LITER}(y)=2$. By $y<x$ and the remainder property (1.f) we have a z such that $\neg[\mathrm{z} \otimes \mathrm{y}]$ and $\mathrm{y} \oplus \mathrm{z}=\mathrm{x}$. This means that $\operatorname{LITER}(\mathrm{y} \oplus \mathrm{z})=\operatorname{LITER}(\mathrm{y})+\operatorname{LITER}(\mathrm{z})$, by $\operatorname{additivity},=2+\operatorname{LITER}(\mathrm{z})$. As z is a part of x , and $\operatorname{LITER}(\mathrm{x})>0$, we have, by comensurability, that $\operatorname{LITER}(\mathrm{z})>0$, which means that $\operatorname{LITER}(\mathrm{y} \oplus \mathrm{z})>2$. But we have $\mathrm{x}=\mathrm{y} \oplus \mathrm{z}$, contradicting the assumption $\operatorname{LITER}(x)=2$. Hence the predicate in (10) is indeed quantized. This proof shows that the remainder principle is critical for part structures.
The notion of an extensive measure function seems shows up in certain grammaticality distinctions. We can form nominal measure constructions in general with extensive measure functions, but not with non-extensive ones (cf. Krifka (1992)). Examples like (11.b) show that containers can be used as measure functions; they are certainly extensive in this use. But examples like (c), (d) show that even established measure functions cannot be used in this construction types if the measure function is not extensive.
a. two kilograms of apples
c. *sixty degree Celsius of water
b. two bags of money
d. *eighteen carats of gold

Another interesting property of measure phrase is that they can be applied only to predicates that are not quantized yet :
a. hundred grams of wool
b. five hundred meters of wool
c. *hundred grams of five hundred meters of wool

It seems that the function of measure phrase like two kilograms is to "cut out" entities of a certain size from the extension of a predicate like apples in which we find a continuum of entities of various sizes. This condition can be described as follows: two kilograms of apples applies to individuals x that fall under apples and that have a weight of 2 kg provided that every proper part of x with respect to the concatenation function for kg (which is simply $<_{\mathrm{P}}$ in the present case) falls under apples, and that there are such proper parts. This requirement is a presupposition, which I will mark by $\partial$.

$$
\begin{equation*}
\text { two kilograms: } \quad \lambda \mathrm{P} \lambda \mathrm{x}\left[\mathrm{P}(\mathrm{x}) \wedge \mathrm{KG}(\mathrm{x})=2 \wedge \partial \exists \mathrm{y} \in \mathrm{U}_{\mathrm{P}}\left[\mathrm{y}<_{\mathrm{KG}} \mathrm{x} \wedge \forall \mathrm{z} \in \mathrm{U}_{\mathrm{P}}\left[\mathrm{z} \leq_{\mathrm{KG}} \mathrm{x} \rightarrow \mathrm{P}(\mathrm{z})\right]\right]\right] \tag{13}
\end{equation*}
$$

Notice that the quantification is over parts of $U_{p}$. This is important, as we may work with a whole range of universes for different sorts. For example, we can assume one universe for individuals and another one for masses, and assume a materializing function from one to the other (cf. Link (1983)). In this case, the parts of an x that fall under apples are individual apples, not apple-parts.

### 2.3. Adjacency and Paths

In addition to the part relation we will make use of a relation of two entities being externally connected, or adjacent, for which I will write " $\mathrm{x} \infty \mathrm{y}$ ". ${ }^{4}$
$\mathrm{A}=\left\langle\mathrm{U}_{\mathrm{A}}, \oplus_{\mathrm{A}}, \leq_{\mathrm{A}},<_{\mathrm{A}}, \otimes_{\mathrm{A}}, \infty_{\mathrm{A}}, \mathrm{C}_{\mathrm{A}}\right\rangle$ is an adjacency structure iff
a. $\left\langle\mathrm{U}_{\mathrm{A}}, \oplus_{\mathrm{A}}, \leq_{\mathrm{A}},\left\langle_{\mathrm{A}}, \otimes_{\mathrm{A}}\right\rangle\right.$ is a part structure,
b. $\infty_{\mathrm{A}}$, adjacency, is a two-place relation in $\mathrm{U}_{\mathrm{A}}$ such that

[^2]i) $\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{A}}\left[\mathrm{x} \infty_{\mathrm{A}} \mathrm{y} \rightarrow \neg \mathrm{x} \otimes_{\mathrm{A}} \mathrm{y}\right]$
ii) $\forall \mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{U}_{\mathrm{A}}\left[\mathrm{x} \infty_{\mathrm{A}} \mathrm{y} \wedge \mathrm{y} \leq_{\mathrm{A}} \mathrm{z} \rightarrow \mathrm{x} \infty_{\mathrm{A}} \mathrm{z} \vee \mathrm{x} \otimes_{\mathrm{A}} \mathrm{z}\right]$
c. $\mathrm{C}_{\mathrm{A}} \subseteq \mathrm{U}_{\mathrm{A}}$, the set of convex elements, is the maximal set such that
$$
\forall \mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{C}_{\mathrm{A}}\left[\mathrm{y}, \mathrm{z} \leq_{\mathrm{A}} \mathrm{x} \wedge \neg \mathrm{y} \otimes_{\mathrm{A}} \mathrm{z} \wedge \neg \mathrm{y} \infty_{\mathrm{A}} \mathrm{z} \rightarrow \exists \mathrm{u} \in \mathrm{C}_{\mathrm{A}}\left[\mathrm{u} \leq_{\mathrm{A}} \mathrm{x} \wedge \mathrm{u} \infty_{\mathrm{A}} \mathrm{y} \wedge \mathrm{u} \infty_{\mathrm{A}} \mathrm{z}\right]\right]
$$

The condition for adjacency says that adjacent elements do not overlap, and that, if an element x is adjacent to an element y that is a part of an element z , either x is also adjacent to z , or x overlaps z . The condition for convex elements says that all convex parts that do not overlap or are adjacent are connected by a convex element.
With adjacency we can characterize an important notion, namely, paths. We will make use of paths to describe movements in various dimensions. The notion of path that I propose here is applicable to non-branching, non-circular and non-crossing paths only. I will construct path structures as a special case of adjacency stuctures: Paths are elements that are convex and linear, a notion that can be enforced by adjacency.

Path structures: $\mathrm{H}=\left\langle\mathrm{U}_{\mathrm{H}}, \oplus_{\mathrm{H}}, \leq_{\mathrm{H}},\left\langle_{\mathrm{H}}, \otimes_{\mathrm{H}}, \infty_{\mathrm{H}}, \mathrm{C}_{\mathrm{H}}, \mathrm{P}_{\mathrm{H}}\right\rangle\right.$, such that
a. $\left\langle\mathrm{U}_{\mathrm{H}}, \oplus_{\mathrm{H}}, \leq_{\mathrm{H}},\left\langle_{\mathrm{H}}, \otimes_{\mathrm{H}}, \infty_{\mathrm{H}}, \mathrm{C}_{\mathrm{H}}\right\rangle\right.$ is an adjacency structure,
b. $\mathrm{P}_{\mathrm{H}} \subseteq \mathrm{C}_{\mathrm{H}}$ is the maximal set such that

$$
\forall \mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{P}_{\mathrm{H}}\left[\mathrm{y}, \mathrm{z} \leq_{\mathrm{H}} \mathrm{x} \wedge \neg \mathrm{y} \otimes_{\mathrm{H}} \mathrm{z} \wedge \neg \mathrm{y} \infty_{\mathrm{A}} \mathrm{z} \rightarrow \exists!\mathrm{u} \in \mathrm{P}_{\mathrm{H}}\left[\mathrm{u} \leq_{\mathrm{H}} \mathrm{x} \wedge \mathrm{y} \infty_{\mathrm{H}} \mathrm{u} \infty_{\mathrm{H}} \mathrm{z}\right]\right]
$$

d. $\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{H}}\left[\neg \mathrm{x} \otimes_{\mathrm{H}} \mathrm{y} \wedge \neg \mathrm{x} \infty_{\mathrm{H}} \mathrm{y} \rightarrow \exists \mathrm{z} \in \mathrm{P}_{\mathrm{H}}\left[\mathrm{x} \infty_{\mathrm{A}} \mathrm{z} \infty_{\mathrm{A}} \mathrm{y}\right]\right]$

Condition (b) says that two disjoint, non-adjacent parts of a path are always connected by exactly one subpath. This excludes circular and branching paths. The set of paths should be the maximal set that satisfies this condition. Thus, even though the set of paths is not closed under sum formation - if x and y are paths, then $\mathrm{x} \oplus \mathrm{y}$ is not necessarily a path - it is as "closed" as possible. For example, if $x$ and $y$ are paths that do not overlap but are externally adjacent at the end, then $\mathrm{x} \oplus \mathrm{y}$ will form a path as well. Condition (c) says that each two disjoint, non-adjacent elements are connected by a path. This ensures general adjacency, or, that there is a path between any two locations.

It may be helpful to have a model that illustrates some paths for the following discussion:


This illustrates a number of path formations. For example, $a \oplus b \oplus c$ is a path, but a $\oplus c \oplus d$ is not, as it contains two parts, e.g. a and c , that are not connected by a subpath. Also, $\mathrm{a} \oplus \mathrm{b} \oplus \mathrm{c} \oplus \mathrm{h}$ is not a path; it violates uniqueness of connecting subpaths, as both $b$ and $b \oplus h$ are parts that connect between a and c . Also, $\mathrm{c} \oplus \mathrm{d} \oplus \mathrm{e} \oplus \mathrm{f} \oplus \mathrm{i} \oplus \mathrm{h}$ is not a path. This is because, for example, there are two parts that connect non-overlapping c and f , namely $\mathrm{d} \oplus \mathrm{e}$ and $\mathrm{h} \oplus \mathrm{i}$.
Another useful notion for directional structures is that two paths are tangential at an endpoint. The paths $\mathrm{a} \oplus \mathrm{b}$ and $\mathrm{c} \oplus \mathrm{d}$ are externally tangential, and the paths $\mathrm{a} \oplus \mathrm{b} \oplus \mathrm{c}$ and $\mathrm{b} \oplus \mathrm{c}$ are internally
tangential, but, for example $\mathrm{b} \oplus \mathrm{c}$ and h are not tangential, even though they are connected, and neither are $\mathrm{a} \oplus \mathrm{b} \oplus \mathrm{c}$ and b .

> a. $\forall \mathrm{x}, \mathrm{y} \in \mathrm{P}_{\mathrm{H}}\left[\operatorname{ETANG}_{\mathrm{H}}(\mathrm{x}, \mathrm{y}) \leftrightarrow \mathrm{x} \oplus_{\mathrm{H}} \mathrm{y} \in \mathrm{P}_{\mathrm{H}} \wedge \mathrm{x} \infty_{\mathrm{H}} \mathrm{y}\right]$
> b. $\forall \mathrm{x}, \mathrm{y} \in \mathrm{P}_{\mathrm{H}}\left[\operatorname{ITANG}_{\mathrm{H}}(\mathrm{x}, \mathrm{y}) \leftrightarrow \exists \mathrm{z} \in \mathrm{P}_{\mathrm{H}}\left[\neg \mathrm{x} \otimes_{\mathrm{H}} \mathrm{z} \wedge \mathrm{y}=\mathrm{x} \oplus_{\mathrm{H}} \mathrm{z}\right]\right]$
> c. $\mathrm{TANG}_{\mathrm{H}}=\operatorname{ETANG}_{\mathrm{H}} \cup \mathrm{ITANG}_{\mathrm{H}}$

Path structures as defined so far are applicable to a wide variety of concepts, e.g. for paths in space, but also for paths that describe qualitative changes of properties, like change of temperature. Such qualitative structures are typically seen as changes in a one-dimensional domain. We can identify one-dimensional path structures as those for which it holds that any two paths are part of a path:

$$
\begin{equation*}
\text { A path structure } \mathrm{H} \text { is one-dimensional iff } \forall \mathrm{x}, \mathrm{y} \in \mathrm{P}_{\mathrm{H}} \exists \mathrm{z} \in \mathrm{P}_{\mathrm{H}}\left[\mathrm{x} \leq_{\mathrm{H}} \mathrm{z} \wedge \mathrm{y} \leq_{\mathrm{H}} \mathrm{z}\right] \tag{18}
\end{equation*}
$$

### 2.4. Directed Paths

Some path structures exhibit an additional property, insofar as their paths are directed. I will call such directed paths directed paths. The text of a book is a path, insofar as a contiguity relation is defined for it (for example, chapter 1 is contiguous to chapter 2, and chapter 2 is contiguous to chapter 3), but in addition we have a precedence relation: chapter 1 precedes chapter 2, and chapter 2 precedes chapter 3. I will denote this relation by «. It is not necessary that directions are defined for all paths, hence I will assume a subset D of P for directed paths, or directed paths.

$$
\begin{equation*}
\mathrm{D}=\left\langle\mathrm{U}_{\mathrm{D}}, \oplus_{\mathrm{D}}, \leq_{\mathrm{D}},\left\langle_{\mathrm{D}}, \otimes_{\mathrm{D}}, \infty_{\mathrm{D}}, \mathrm{P}_{\mathrm{D}}, \mathrm{C}_{\mathrm{D}},<_{\mathrm{D}}, \mathrm{D}_{\mathrm{D}}\right\rangle\right. \text { is a directed path structure iff } \tag{19}
\end{equation*}
$$

a. $\left\langle\mathrm{U}_{\mathrm{D}}, \oplus_{\mathrm{D}}, \leq_{\mathrm{D}},{\leq_{\mathrm{D}}}, \otimes_{\mathrm{D}}, \infty_{\mathrm{D}}, \mathrm{P}_{\mathrm{D}}\right\rangle$ is a path structure;
b. $\mathrm{D}_{\mathrm{D}} \subseteq \mathrm{P}_{\mathrm{D}}$, the set of directed paths, is the maximal set, and ${ }_{\mathrm{D}}$, precedence, is a two-place relation in $\mathrm{D}_{\mathrm{D}}$ with the properties (20) to (23):

$$
\begin{align*}
& \forall \mathrm{x}, \mathrm{y} \in \mathrm{D}_{\mathrm{D}}\left[[ \neg \mathrm { x } \mu _ { \mathrm { D } } \mathrm { x } ] \wedge \left[\mathrm{x}{\left.\left.\kappa_{\mathrm{D}} \mathrm{y} \rightarrow \neg \mathrm{y} \mu_{\mathrm{D}} \mathrm{x}\right] \wedge\left[\mathrm{x} \mu_{\mathrm{D}} \mathrm{y} \wedge \mathrm{y} \mu_{\mathrm{D}} \mathrm{z} \rightarrow \mathrm{x} \mu_{\mathrm{D}} \mathrm{z}\right]\right]}\right.\right.  \tag{20}\\
& \forall \mathrm{x}, \mathrm{y} \in \mathrm{D}_{\mathrm{D}}\left[\mathrm{x} «_{\mathrm{D}} \mathrm{y} \rightarrow \neg \mathrm{x} \otimes_{\mathrm{D}} \mathrm{y}\right]  \tag{21}\\
& \forall \mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{D}_{\mathrm{D}}\left[\mathrm{x}, \mathrm{y} \leq_{\mathrm{D}} \mathrm{z} \wedge \neg \mathrm{x} \otimes_{\mathrm{D}} \mathrm{y} \rightarrow \mathrm{x} «_{\mathrm{D}} \mathrm{y} \vee \mathrm{y} «_{\mathrm{D}} \mathrm{x}\right]  \tag{22}\\
& \forall \mathrm{x}, \mathrm{y} \in \mathrm{D}_{\mathrm{D}}\left[\mathrm{x} «_{\mathrm{D}} \mathrm{y} \rightarrow \exists \mathrm{z} \in \mathrm{D}_{\mathrm{D}}\left[\mathrm{x}, \mathrm{y} \leq_{\mathrm{D}} \mathrm{z}\right]\right] \tag{23}
\end{align*}
$$

(20) says that precedence is irreflexive, asymmetric and transitive, (21) says that precedence only holds for non-overlapping elements, (22) says that whenever two subpaths of a directed path do not overlap, one must precede the other, and (23) says that only parts of a directed path can stand in the precedence relation to each other.
Many important directed path structures are one-dimensional, for example, time. Such structures are total, in the sense that for each two convex, non-overlapping directed paths $\mathrm{x}, \mathrm{y}$ it holds that either x precedes y , or y precedes x .

A directed path structure V is called one-dimensional iff

$$
\begin{equation*}
\forall \mathrm{x}, \mathrm{y} \in \mathrm{D}_{\mathrm{D}}\left[\neg \mathrm{x} \otimes_{\mathrm{D}} \mathrm{y} \rightarrow \mathrm{x} «_{\mathrm{D}} \mathrm{y} \vee \mathrm{y} «_{\mathrm{D}} \mathrm{x}\right] \tag{24}
\end{equation*}
$$

### 2.5. Times and Events

In the following I will assume a one-dimensional directed path structure for time, where the relation « is interpreted as temporal precedence.
(25) A time structure T
is a one-dimensional directed path structure $\left\langle\mathrm{U}_{\mathrm{T}}, \oplus_{\mathrm{T}}, \leq_{\mathrm{T}},\left\langle_{\mathrm{T}}, \otimes_{\mathrm{T}}, \infty_{\mathrm{T}}, \mathrm{P}_{\mathrm{T}}, \mathrm{C}_{\mathrm{T}}, «_{\mathrm{T}}, \mathrm{D}_{\mathrm{T}}\right\rangle\right.$.
We do not require that time structures are atomic, neither that they are non-atomic. By avoiding the issue of atomicity we will be able to reconstruct changes without relying on the film-strip model of time presupposed in much previous work.
An important sort of individuals for natural language semantics are events. Events form a part structure (for example, if Mary sings from 3 p.m. to 5 p.m., then her singing from 3 p.m. to 4 p.m. is a part of that singing event). Events also are subject to a temporal precedence relation (e.g., Mary's singing from 3 p.m. to 4 p.m. precedes her singing from 4 p.m. to 5 p.m.). This precedence relation is related to the precedence relation for times. It is possible to start out with an event structure and reconstruct times from events (cf. Landman (1992)). But my current interest is not in analytical reconstruction of entities, but in their structural relationship, and hence I will simply assume that we start out with times and add events as another type of entities.

$$
\begin{equation*}
\mathrm{E}=\left\langle\mathrm{U}_{\mathrm{E}}, \oplus_{\mathrm{E}}, \leq_{\mathrm{E}},<_{\mathrm{E}}, \otimes_{\mathrm{E}}, \mathrm{~T}_{\mathrm{E}}, \tau_{\mathrm{E}}, \infty_{\mathrm{E}},{\left.{ }_{\mathrm{E}}, \mathrm{C}_{\mathrm{E}}\right\rangle \text { is an event structure iff }}^{2}\right. \tag{26}
\end{equation*}
$$

a. $\left\langle\mathrm{U}_{\mathrm{E}}, \oplus_{\mathrm{E}}, \leq_{\mathrm{E}},\left\langle_{\mathrm{E}}, \otimes_{\mathrm{E}}\right\rangle\right.$ is a part structure,
b. $\mathrm{T}_{\mathrm{E}}$ is a time structure $\left\langle\mathrm{U}_{\mathrm{T}}, \oplus_{\mathrm{T}}, \leq_{\mathrm{T}},<_{\mathrm{T}}, \otimes_{\mathrm{T}}, \infty_{\mathrm{T}}, \mathrm{P}_{\mathrm{T}}, \mathrm{D}_{\mathrm{T}},<_{\mathrm{T}}\right\rangle$,
c. $\tau_{\mathrm{E}}$, the temporal trace function, is a function from $\mathrm{U}_{\mathrm{E}}$ to $\mathrm{U}_{\mathrm{T}}$,
$\infty_{\mathrm{E}}$, temporal adjacency, is a two-place relation in $\mathrm{U}_{\mathrm{E}}$, $<_{\mathrm{E}}$, temporal precedence, is a two-place relation in $\mathrm{U}_{\mathrm{E}}$, $\mathrm{C}_{\mathrm{E}}$, the set of temporally contiguous events, is a subset of $\mathrm{U}_{\mathrm{E}}$, with the properties (27) to (31):

$$
\begin{align*}
& \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\tau_{\mathrm{E}}\left(\mathrm{e} \oplus_{\mathrm{E}} \mathrm{e}^{\prime}\right)=\tau_{\mathrm{E}}(\mathrm{e}) \oplus_{\mathrm{T}} \tau_{\mathrm{E}}\left(\mathrm{e}^{\prime}\right)\right]  \tag{27}\\
& \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\mathrm{e} \infty_{\mathrm{E}} \mathrm{e}^{\prime} \leftrightarrow \tau_{\mathrm{E}}(\mathrm{e}) \infty_{\mathrm{T}} \tau_{\mathrm{E}}\left(\mathrm{e}^{\prime}\right)\right]  \tag{28}\\
& \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\mathrm{e}<_{\mathrm{E}} \mathrm{e}^{\prime} \leftrightarrow \tau_{\mathrm{E}}(\mathrm{e})<_{\mathrm{T}} \tau_{\mathrm{E}}\left(\mathrm{e}^{\prime}\right)\right]  \tag{29}\\
& \forall \mathrm{e} \in \mathrm{C}_{\mathrm{E}}\left[\tau_{\mathrm{E}}(\mathrm{e}) \in \mathrm{P}_{\mathrm{T}}\right] \tag{30}
\end{align*}
$$

$U_{E}$ is the smallest set such that $C_{E} \subseteq U_{E}$, and for every $e, e^{\prime} \in U_{E}, e \oplus_{E} e^{\prime} \in U_{E}$.
The temporal trace function $\tau_{\mathrm{E}}$ maps events to their run time, the time at which an event is going on. (27) says that it is a homomorphism with respect to the sum operations for events and times: The run time of the sum of two events $e, e^{\prime}$ is the sum of the run time of e and the run time of $\mathrm{e}^{\prime}$. (28) and (29) define temporal adjacency for events $\infty_{E}$ and temporal precedence for events $«_{E}$ in relation to the corresponding run times. (30) says that temporally contiguous events are events with a contiguous run time, and (31) says that the set of all events is the closure of the contiguous events under sum formation.
The axioms for event structures entail several interesting properties. For one thing, the fact that temporal precedence $<_{T}$ is an irreflexive, asymmetric and transitive relation makes event precedence $<_{\mathrm{E}}$ irreflexive, asymmetric and transitive as well, (32). But note that two distinct events can go on at the same time. Furthermore, we have that if e is a part of $\mathrm{e}^{\prime}$, then the run time of e is a part of the run
time of $e^{\prime}$. (33). This follows from the definition of $\mathrm{e} \leq_{\mathrm{E}} \mathrm{e}^{\prime}$ as e $\oplus_{\mathrm{E}} \mathrm{e}^{\prime}=\mathrm{e}^{\prime}$ and the homomorphism property (27). Also, we have that two mereologically overlapping events overlap temporally (34). This mereological overlap relation for events should not be confused with purely temporal overlap, which holds for any two events if their run time overlaps. Also, mereologically overlapping events cannot precede each other (35).

$$
\begin{align*}
& \forall \mathrm{e}, \mathrm{e}^{\prime}, \mathrm{e}^{\prime \prime} \in \mathrm{U}_{\mathrm{E}}\left[\neg\left[\mathrm{e}<_{\mathrm{E}} \mathrm{e}\right] \wedge \neg\left[\mathrm{e}<_{\mathrm{E}} \mathrm{e}^{\prime} \wedge \mathrm{e}^{\prime}<_{\mathrm{E}} \mathrm{e}\right] \wedge\left[\mathrm{e}<_{\mathrm{E}} \mathrm{e}^{\prime} \wedge \mathrm{e}^{\prime}<_{\mathrm{E}} \mathrm{e}^{\prime \prime} \rightarrow \mathrm{e}{«_{\mathrm{E}}} \mathrm{e}^{\prime \prime}\right]\right]  \tag{32}\\
& \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\mathrm{e} \leq_{\mathrm{E}} \mathrm{e}^{\prime} \rightarrow \tau_{\mathrm{E}}(\mathrm{e}) \leq_{\mathrm{T}} \tau_{\mathrm{E}}\left(\mathrm{e}^{\prime}\right)\right]  \tag{33}\\
& \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\mathrm{e} \otimes_{\mathrm{E}} \mathrm{e}^{\prime} \rightarrow \tau_{\mathrm{E}}(\mathrm{e}) \otimes_{\mathrm{T}} \tau_{\mathrm{E}}\left(\mathrm{e}^{\prime}\right)\right]  \tag{34}\\
& \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\mathrm{e} \otimes_{\mathrm{E}} \mathrm{e}^{\prime} \rightarrow \neg \mathrm{e}<_{\mathrm{E}} \mathrm{e}^{\prime}\right] \tag{35}
\end{align*}
$$

We will have occasion to refer to the initial and the final parts of an event. An event $\mathrm{e}^{\prime}$ is an initial part of e if it is not preceded by any part of e, and similarly for final parts:

$$
\begin{align*}
& \text { a. } \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\mathrm{INI}_{\mathrm{E}}\left(\mathrm{e}^{\prime}, \mathrm{e}\right) \leftrightarrow \mathrm{e}^{\prime} \leq_{\mathrm{D}} \mathrm{e} \wedge \neg \exists \mathrm{e}^{\prime \prime} \in \mathrm{U}_{\mathrm{E}}\left[\mathrm{e}^{\prime \prime} \leq_{\mathrm{E}} \mathrm{e} \wedge \mathrm{e}^{\prime \prime}<_{\mathrm{E}} \mathrm{e}^{\prime}\right]\right]  \tag{36}\\
& \text { b. } \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\mathrm{FIN}_{\mathrm{E}}\left(\mathrm{e}^{\prime}, \mathrm{e}\right) \leftrightarrow \mathrm{e}^{\prime} \leq_{\mathrm{D}} \mathrm{e} \wedge \neg \exists \mathrm{e}^{\prime \prime} \in \mathrm{U}_{\mathrm{E}}\left[\mathrm{e}^{\prime \prime} \leq_{\mathrm{E}} \mathrm{e} \wedge \mathrm{e}^{\prime}<_{\mathrm{E}} \mathrm{e}^{\prime \prime}\right]\right]
\end{align*}
$$

We now can define the notion of a telic predicate. As I argued in Krifka $(1989,1992)$, it is misleading to think that particular events can be called "telic" or "atelic". For example, one and the same event of running can be described by running (i.e., by an atelic predicate), or by running a mile (i.e., a telic, or delimited, predicate). Hence the distinction between telicity and atelicity should not be one in the nature of the object described, but in the description applied to the object. This is similar to the way how we refer to objects: One and the same entity can fall under the predicates apples and two apples.
How should we define telicity for predicates of events? From the original characterization of telicity in Aristotle's remarks in Metaphysics, Book $\theta 6$ to the more recent ones starting with Garey (1957) and Vendler (1957), the crucial property that distinguishes telic from atelic actions or verbs is that the former require some time till they are completed. They have to reach a "set terminal point", in Vendler's words. But these descriptions also implicitly take account of the starting point. For example, Vendler compares run and run a mile, and says that if a person stops in between, then this person did run, but did not run a mile. Hence the same event, with the same starting point, is compared under two descriptions. This suggests that if a telic predicate applies to an event e, then it does not apply to a part of e that begins or ends at a different time. Avoiding talk about time points, we can characterize telicity as the property of an event predicate X that applies to events e such that all parts of e that fall under X are initial and final parts of e .

$$
\begin{equation*}
\forall \mathrm{X} \subseteq \mathrm{U}_{\mathrm{E}}\left[\mathrm{TEL}_{\mathrm{E}}(\mathrm{X}) \leftrightarrow \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\mathrm{X}(\mathrm{e}) \wedge \mathrm{X}\left(\mathrm{e}^{\prime}\right) \wedge \mathrm{e}^{\prime} \leq_{\mathrm{E}} \mathrm{e} \rightarrow \mathrm{INI}_{\mathrm{E}}\left(\mathrm{e}^{\prime}, \mathrm{e}\right) \wedge \mathrm{FIN}_{\mathrm{E}}\left(\mathrm{e}^{\prime}, \mathrm{e}\right)\right]\right] \tag{37}
\end{equation*}
$$

It is obvious that quantized predicates are telic: If a quantized predicate X applies to some event e , then it does not apply to any proper part of e, hence the only $\mathrm{e}^{\prime}$ such that $\mathrm{X}\left(\mathrm{e}^{\prime}\right)$ and $\mathrm{e}^{\prime} \leq \mathrm{e}$ is e itself, which is both an initial and final part of e. But not every telic predicate is quantized; quantization is the stricter notion. For example, assume that X is a predicate that applies to all events that have a run time from 3 p.m. to 4 p.m.; X is telic, but not quantized. Cumulative predicates, on the other hand, are typically atelic. As soon as a cumulative predicate X applies to at least two events e, é that are not contemporaneous, that is, for which there is a $e^{\prime \prime}$ with $e^{\prime \prime} \leq e$ and $e^{\prime}$ « $e^{\prime \prime}$, then it will be atelic: $X$ applies to both e and e $\oplus \mathrm{e}^{\prime}$, due to cumulativity, and these events end at different times.

### 2.6. How many events?

Event structures should be sufficiently "rich". For one thing, we do not expect that there is any time at which no event is going on. This is a consequence of the reconstruction of times by events in the approaches discussed in Landman (1992), but we can enforce it explicitly:

$$
\begin{equation*}
\forall \mathrm{t} \in \mathrm{U}_{\mathrm{T}} \exists \mathrm{e} \in \mathrm{U}_{\mathrm{E}}\left[\tau_{\mathrm{E}}(\mathrm{e})=\mathrm{t}\right] \tag{38}
\end{equation*}
$$

Another way to guarantee "rich" event structures is to require that the richness of subevents is matched by the richness of the time structure. That is, whenever a time $t^{\prime}$ is a part of the run time tof an event $e$, then there exists an event $e^{\prime}$ that is part of $e$ and has $t^{\prime}$ as its run time. But such a requirement would clearly be too strong. For consider two events of quite different levels of granularity, like the writing of a dissertation and the ticking of a watch. For the ticking of a watch we need events of the magnitude of seconds, but it is not intuitive that events like writing a dissertation have subevents of that magnitude.
But it is reasonable to ask for the following: If an event has a temporal part, then it has a complement to that temporal part, in the following sense:

$$
\begin{align*}
\forall \mathrm{e}, \mathrm{e}^{\prime} \in & \mathrm{U}_{\mathrm{E}} \forall \mathrm{t} \in \mathrm{U}_{\mathrm{T}}\left[\mathrm{e}^{\prime} \leq_{\mathrm{E}} \mathrm{e} \wedge \neg \mathrm{t} \otimes_{\mathrm{T}} \tau\left(\mathrm{e}^{\prime}\right) \wedge \tau(\mathrm{e})=\mathrm{t} \oplus_{\mathrm{T}} \tau\left(\mathrm{e}^{\prime}\right)\right.  \tag{39}\\
& \left.\rightarrow \exists \mathrm{e}^{\prime \prime} \in \mathrm{U}_{\mathrm{E}}\left[\tau\left(\mathrm{e}^{\prime \prime}\right)=\mathrm{t} \wedge \mathrm{e}=\mathrm{e}^{\prime} \oplus_{\mathrm{E}} \mathrm{e}^{\prime \prime}\right]\right]
\end{align*}
$$

That is, if the temporal traces of e and its part $e^{\prime}$ differ in $t$, then e has also a part $e^{\prime \prime}$ with $t$ as its temporal trace. For example, if e is a running from 3 p.m. to 5 p.m., and $\mathrm{e}^{\prime}$ is the part that runs from 3 p.m. to 4 p.m., then there is a part $\mathrm{e}^{\prime \prime}$ that covers the stretch from 4 p.m. to 5 p.m.

Another source for the richness of event structures is the assumption (31), which says that noncontiguous events are sums of contiguous events. This ensures, for example, that an event of running that goes on from 3 p.m. to 4 p.m. and from 5 p.m. to 6 p.m. is composed of (at least) two events of running that go on from 3 p.m. to 4 p.m., and from 5 p.m. to 6 p.m. Furthermore, the fact that the run time function is a function and not just a relation guarantees that one and the same event cannot go on at distinct times; hence the sleeping of Mary yesterday and the sleeping of Mary today will be distinct.
Richness of event structures also follows from how events are put to use in semantic interpretation. Following Davidson (1967), I will propose that verbal predicates of natural language come with an event argument. An intransitive verb like sleep will be analyzed as a two-place relation SLEEP that relates a person (a sleeper) to an event (the sleeping event), and a transitive verb like read will be analyzed as a three-place relation READ that relates a person and a text to an event, the reading event. That is, I assume that all n-place verbal predicates have interpretations $\alpha$ that can be applied to $n$ regular arguments and an event argument:

If $\alpha$ is the translation of an n-place verbal predicate,
then $\alpha$ is an ( $\mathrm{n}+1$ )-place relation, with the last argument restricted to the domain of the event structure, $\mathrm{U}_{\mathrm{E}}$.
Now, one principle that guarantees richness of events is that it should not be the case that one and the same event has different participants. One and the same sleeping event cannot have different sleepers; one and the same reading event cannot have different readers, or be a reading of different texts. We can formulate this principle, which we may call uniqueness of participants (cf. Carlson (1984)) schematically as follows:
(41) For all n-place verbal predicates the following should hold, where $\alpha$ stands for the interpretation of the predicate:

$$
\forall \mathrm{x}_{1}, \ldots \mathrm{x}_{\mathrm{n}}, \mathrm{e}\left[\alpha\left(\mathrm{x}_{1}, \ldots, \mathrm{x}_{\mathrm{n}}, \mathrm{e}\right) \rightarrow \forall \mathrm{x}_{1}{ }^{\prime}, \ldots \mathrm{x}_{\mathrm{n}}{ }^{\prime}\left[\alpha\left(\mathrm{x}_{1}^{\prime}, \ldots, \mathrm{x}_{\mathrm{n}}{ }^{\prime}, \mathrm{e}\right) \rightarrow \mathrm{x}_{1}=\mathrm{x}_{1}{ }^{\prime} \wedge \ldots \wedge \mathrm{x}_{\mathrm{n}}=\mathrm{x}_{\mathrm{n}}{ }^{\prime}\right]\right]
$$

This does not exclude, say, a complex event e of Mary and John sleeping; it just says that e itself cannot also be an event of Mary sleeping (rather, the principle of Mapping to Objects below will tell us that e has a part $e^{\prime}$ that is a sleeping of Mary). However, (41) is problematic for cases like see and touch. For example, with one event of seeing we can both see the Mona Lisa and her eyes, and with one event of touching we can touch a cup and its handle. We should entertain (41) only as a
general principle that holds in case there aren't any more specific rules that may override it. A rule for see is that if an object $x$ was seen from a certain perspective in an event e, then every part of $x$ that was visible from that perspective was seen in e, and a rule for touch says that as soon as a material part x of an object y was touched, then y itself was touched. ${ }^{5}$
Other source that guarantee that events are sufficiently rich come with the semantic requirements of particular verbal predicates. For example, it seems plausible to require that, if e is a reading of a text $x$, and $y$ is a part of $x$, then there should be a part $e^{\prime}$ of e such that $e^{\prime}$ is a reading of $y$. Not every predicate comes with such requirements; for example, see the book does not imply that the book was sent in a piecemeal fashion. We will investigate such differences in the next section.

## 3. Telicity by Sums and Parts

### 3.1. Part Structures in Predication

A predication establishes a relation of a specified type between a number of parameters, or semantic arguments. For example, a sentence like John slept establishes that there is a relation of the SLEEPtype that holds of John for some event. Mary ate the apple says that there is a relation of the EATtype between Mary, the apple, and some event. Bill walked from the university to the capitol establishes that there is a relation of the walk-type between Bill, an event, and some path in space that stretches from the university to the capitol. Such semantic arguments may be realized by syntactic arguments, like subjects or objects, but also by adjuncts, like from the university to the capitol. Also, natural-language predicates typically do not come with a fixed number of semantic arguments. For example, sleep may come with a path argument in John slept from Lyon to Paris (think of John being a passenger of a train).
I will adopt a type of semantic representation illustrated in the following example, for Mary ate apples. Here, APPLES is a cumulative predicate, m denotes Mary, R and S are variables for two- and three-place relations. The last step, existential closure, creates a formula from an event predicate. I disregard tense.

$$
\begin{array}{ll}
\text { a. ate: } & \lambda \mathrm{x}, \mathrm{y}, \mathrm{e}[\operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e})]  \tag{42}\\
\text { b. apples } & \lambda \mathrm{S} \lambda \mathrm{x}, \mathrm{e} \exists \mathrm{y}[\operatorname{APPLES}(\mathrm{y}) \wedge \mathrm{S}(\mathrm{x}, \mathrm{y}, \mathrm{e})] \\
\text { c. ate apples: } & \lambda \mathrm{S} \lambda \mathrm{x}, \mathrm{e} \exists \mathrm{y}[\operatorname{APPLES}(\mathrm{y}) \wedge \mathrm{S}(\mathrm{x}, \mathrm{y}, \mathrm{e})](\lambda \mathrm{x}, \mathrm{y}, \mathrm{e}[\operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e})]) \\
& =\lambda \mathrm{x}, \mathrm{e} \exists \mathrm{y}[\operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e})] \\
\text { d. Mary } & \lambda \mathrm{R} \lambda \mathrm{e}[\mathrm{R}(\mathrm{~m}, \mathrm{e})] \\
\text { e. Mary ate apples: } & \lambda \mathrm{R} \lambda \mathrm{e}[\mathrm{R}(\mathrm{~m}, \mathrm{e})](\lambda \mathrm{x}, \mathrm{e} \exists \mathrm{y}[\operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e})]) \\
& =\lambda \mathrm{e} \exists \mathrm{y}[\operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{m}, \mathrm{y}, \mathrm{e})] \\
\text { f. } & \text { Existential closure: } \\
\exists \mathrm{e} \exists \mathrm{y}[\operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{m}, \mathrm{y}, \mathrm{e})]
\end{array}
$$

The semantic arguments of a predication may be elements of part structures, in which case certain mereological relations may hold between them. These are the relations that have been addressed under names like "add-to", "measuring out", "graduality", or "incremental theme". The mere-

[^3]ological notions set up in § 2 will help us to define a variety of such relations between semantic arguments.
In the following I will discuss mereological relations between the event argument and one other argument. For example, for the predicate eat we will investigate the relation between the event argument and the object argument. I will typically use $\theta$, reminiscent of thematic role, to refer to the relation under discussion. The object thematic role for eat is $\{\langle y, e\rangle \mid \operatorname{Eat}(x, y, e)\}$, which gives us (for every interpretation of $x$ ) the set of pairs $\langle y, e\rangle$ such that $y$ was eaten in the event e (by $x$ ). ${ }^{6}$

### 3.2. Cumulativity and Strict Incrementality

We have mentioned one important global property of eventive predicates, namely, uniqueness of participants (41). It follows that any specific thematic relation should have this property as well:
(43) $\theta$ shows uniqueness of participants, $\mathrm{UP}(\theta)$, iff

$$
\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{P}} \forall \mathrm{e} \in \mathrm{U}_{\mathrm{E}}[\theta(\mathrm{x}, \mathrm{e}) \wedge \theta(\mathrm{y}, \mathrm{e}) \rightarrow \mathrm{x}=\mathrm{y}]
$$

One important and very general property of theta roles is that they form a relational homomorphism with respect to the part relations of the semantic arguments:

$$
\begin{align*}
& \theta \text { is cumulative, } \operatorname{CUM}(\theta) \text {, iff: }  \tag{44}\\
& \forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{P}} \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \theta\left(\mathrm{y}, \mathrm{e}^{\prime}\right) \rightarrow \theta\left(\mathrm{x} \oplus_{\mathrm{P}} \mathrm{y}, \mathrm{e} \oplus_{\mathrm{E}} \mathrm{e}^{\prime}\right)\right]
\end{align*}
$$

This relation, which has been called "summativity" in Krifka (1992), is cumulativity generalized to two-place predicates. It does not give us very much. In particular, it cannot distinguish incremental relations between objects and events, as in eat the apples, from non-incremental ones, as in push the cart. It is true that, if, say, $\mathrm{e}_{1}$ is the eating of $\mathrm{a}_{1}$ and $\mathrm{e}_{2}$ is the eating of $\mathrm{a}_{2}$, then $\mathrm{e}_{1} \oplus \mathrm{e}_{2}$ is the eating of $a_{1} \oplus a_{2}$. But the same structural relationship also holds for the pushing of carts. A property that may be more successful in distinguishing between incremental and non-incremental relations is the following one, which we may call mapping to events:

$$
\begin{equation*}
\theta \text { shows mapping to events, } \operatorname{ME}(\theta) \text {, iff } \tag{45}
\end{equation*}
$$

$$
\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{P}} \forall \mathrm{e} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \mathrm{y} \leq_{\mathrm{P}} \mathrm{x} \rightarrow \exists \mathrm{e}^{\prime}\left[\mathrm{e}^{\prime} \leq_{\mathrm{E}} \mathrm{e} \wedge \theta\left(\mathrm{y}, \mathrm{e}^{\prime}\right)\right]\right]
$$

That is, when $\theta$ holds for an object $x$ and an event $e$, and $y$ is a part of $x$, then $y$ stands in the relation $\theta$ to a part $\mathrm{e}^{\prime}$ of e. This seems a plausible requirement for incremental relations. However, as it stands it is quite weak, because it allows, for example, that a proper part $y$ of $x$ is associated with $e$ itself (notice that $\mathrm{e} \leq e$ ). Hence we rather would like to have the following property, which we may call mapping to subevents:
$\theta$ shows mapping to subevents, $\operatorname{MSE}(\theta)$, iff

$$
\begin{equation*}
\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{P}} \forall \mathrm{e} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \mathrm{y}<_{\mathrm{P}} \mathrm{x} \rightarrow \exists \mathrm{e}^{\prime}\left[\mathrm{e}^{\prime}<_{\mathrm{E}} \mathrm{e} \wedge \theta\left(\mathrm{y}, \mathrm{e}^{\prime}\right)\right]\right] \tag{46}
\end{equation*}
$$

That is, whenever $\theta$ holds for an object $x$ and an event $e$, then every proper part $y$ of $x$ stands in the relation $\theta$ to some proper part $\mathrm{e}^{\prime}$ of e . This is certainly true for eat the apples: If apples $\mathrm{a}_{1} \oplus \mathrm{a}_{2}$ are eaten incrementally in an event $e$, then there is a proper part $\mathrm{e}^{\prime}$ of e such that the apple $\mathrm{a}_{1}$ was eaten at $\mathrm{e}^{\prime}$. And it does not hold for push the cart, it does not even make sense to say that a proper part of

[^4]a cart was pushed. But it may hold, if somewhat perversely, for cases like see a picture. Assume that Mary sees a picture $p$ during an event $e$, then we may conclude, first, that she sees $p$ during every part of $e$, and furthermore, that she sees every part of $p$ that is visible from the same perspective. Also, we may assume that she sees every part of $p$ during every part of e. But then (46) holds for this case, though it is clearly non-incremental. A similar argument can be made for cases like touch a cup. Cases like see and touch were identified as peculiar earlier, as they do not satisfy uniqueness of participants, (41). Hence we may require this property (which is a default property for eventive predicates anyway) in order to single out incremental relations.

We may strengthen mapping to subevents by claiming that, in addition, the subevents that correspond to subobjects are unique:

$$
\begin{equation*}
\theta \text { shows uniqueness of events, } \mathrm{UE}(\theta) \text {, iff } \tag{47}
\end{equation*}
$$

$$
\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{P}} \forall \mathrm{e} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \mathrm{y} \leq_{\mathrm{P}} \mathrm{x} \rightarrow \exists!\mathrm{e}^{\prime}\left[\mathrm{e}^{\prime} \leq_{\mathrm{E}} \mathrm{e} \wedge \theta\left(\mathrm{y}, \mathrm{e}^{\prime}\right)\right]\right]
$$

This is certainly the case for eat the apples: If apples $\mathrm{a}_{1} \oplus \mathrm{a}_{2}$ are eaten incrementally in event e , then there is a unique part $e^{\prime}$ of $e$ at which apple $a_{1}$ is eaten. Notice that this property does exclude cases like see a picture or touch a cup described above. If Mary sees the Mona Lisa between 3:00 p.m. and 3:10 p.m., then she sees her eyes during the same event, but there might be distinct subevents during which she sees the eyes as well. We may be tempted to give up uniqueness of participants as a property relevant for incremental relations, and just require uniqueness of events. However, this would allow for cases in which, say, x and all the parts of x are related to one and the same event e , which is certainly not what we want for incremental relations.
Two other natural properties for incremental relations, in some sense the mirror image of mapping to events and mapping to subevents, are the following ones:
$\theta$ shows mapping to objects, $\mathrm{MO}(\theta)$, iff

$$
\begin{equation*}
\forall \mathrm{x} \in \mathrm{U}_{\mathrm{P}} \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \mathrm{e}^{\prime} \leq_{\mathrm{E}} \mathrm{e} \rightarrow \exists \mathrm{y}\left[\mathrm{y} \leq_{\mathrm{P}} \mathrm{x} \wedge \theta\left(\mathrm{y}, \mathrm{e}^{\prime}\right)\right]\right] \tag{48}
\end{equation*}
$$

$\theta$ shows mapping to subobjects, $\mathrm{MSO}(\theta)$, iff

$$
\begin{equation*}
\forall \mathrm{x} \in \mathrm{U}_{\mathrm{P}} \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \mathrm{e}^{\prime}<_{\mathrm{E}} \mathrm{e} \rightarrow \exists \mathrm{y}\left[\mathrm{y}<_{\mathrm{P}} \mathrm{x} \wedge \theta\left(\mathrm{y}, \mathrm{e}^{\prime}\right)\right]\right] \tag{49}
\end{equation*}
$$

Again, (49) is a stronger property; (48) allows for a proper part of e to be mapped to the whole object x . Take eat the apples: If the apples are eaten incrementally at an event e , then it holds that for every proper part é of e there is a proper part of the apples that was eaten at é'. Again, it does not hold for a case like push the cart. It does, however, hold for cases like see the picture; hence we still need uniqueness of participants to rule that out. Uniqueness of participants entails that the object part y in (48) and (49) is unique for $\mathrm{e}^{\prime}$, a property that we should keep in mind:
(50) $\theta$ shows uniqueness of objects, $\mathrm{UO}(\theta)$, iff

$$
\forall \mathrm{x} \in \mathrm{U}_{\mathrm{P}} \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \mathrm{e}^{\prime} \leq_{\mathrm{E}} \mathrm{e} \rightarrow \exists!\mathrm{y}\left[\mathrm{y} \leq_{\mathrm{P}} \mathrm{x} \wedge \theta\left(\mathrm{y}, \mathrm{e}^{\prime}\right)\right]\right]
$$

One important consequence of uniqueness of objects is that it cannot be that for any $\mathrm{x}, \mathrm{e}, \mathrm{e}^{\prime}$ we have $\theta(x, e), \theta\left(x, e^{\prime}\right)$ and $e^{\prime}<_{E} e$ at the same time. In other words, whenever $\theta(x, e)$ holds, then it takes the whole event e to $\theta$ the object $x$, not just a part of e. This can be seen as the reason why eat the apples is quantized, but push the cart is not (cf. § 3.3 below): If e is an event of eating the apples, then it takes the whole e, not just part of it, to qualify for an eating of the apples. But if the pushing of the cart goes on in an event e, then it typically goes on during parts of e as well.
Uniqueness of events and objects (47), (50) and mapping to subevents and subobjects (46), (49) together establish that, if $\theta(\mathrm{x}, \mathrm{e})$ is the case, then $\theta$ is a one-to-one mapping between the parts of x and the parts of e. This is how we understand a process like eat the apples or draw the circle: Every
part of the eating corresponds to exactly one part of the apples, and vice versa; every part of the drawing corresponds to exactly one part of the circle, and vice versa. ${ }^{7}$
Are these then the properties that we should single out as "incremental"? Not quite; we should exclude situations in which both the object x and the event e are atomic. One example is perhaps make a dot; it doesn't take time to make a dot, and a dot doesn't have parts. Hence the notion of incrementality is applicable only if the entities a relation is applied to are extended, that is, have proper parts. We therefore define the notion of incrementality, as follows. For reasons that will become clear later, in § 3.6, I will call it strict incrementality.
(51) $\theta$ is strictly incremental, $\operatorname{SINC}(\theta)$, iff

## i) $\operatorname{MSO}(\theta) \wedge \mathrm{UO}(\theta) \wedge \operatorname{MSE}(\theta) \wedge \mathrm{UE}(\theta)$ (where UO is a consequence of UP$)$

ii) $\exists x, y \in U_{P} \exists e, e^{\prime} \in U_{E}\left[y<x \wedge e^{\prime}<e \wedge \theta(x, e) \wedge \theta\left(y, e^{\prime}\right)\right]$

In the next sections I will show that cumulativity and strict incrementality give us the tools that are sufficient to describe the semantic behavior of predicates like eat two apples and eat apples. However, there is a problem with the combination of these two properties. Take a case like read a novel, which behaves just like eat an apple in the aspectual tests. Assume two distinct events e, e' that are readings of the novel $x$. With cumulativity, we have that $e \oplus e^{\prime}$ is a reading of $x$ as well. But then mapping to subobjects cannot hold, as both $\mathrm{e} \oplus \mathrm{e}^{\prime}$ and e are related to the same object, x . Also, uniqueness of subevents does not hold, as a proper part y of x will have two readings, a part of e and a part of $\mathrm{e}^{\prime}$. The problematic configuration, that we have $\theta(\mathrm{x}, \mathrm{e}), \theta\left(\mathrm{x}, \mathrm{e}^{\prime}\right)$ and $\mathrm{e} \neq \mathrm{e}^{\prime}$, is possible for verbs like read, but not for verbs of consumption or creation like eat and draw. We can eat an apple only once, we can draw a circle (as a concrete physical object) only once. The thematic relation of such verbs has the following property:
$\theta$ shows general uniqueness of events, GUE $(\theta)$, iff
$\forall \mathrm{x} \in \mathrm{U}_{\mathrm{P}} \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \theta\left(\mathrm{x}, \mathrm{e}^{\prime}\right) \rightarrow \mathrm{e}=\mathrm{e}^{\prime}\right]$

Hence strict incrementality and cumulativity are compatible if we restrict our consideration to thematic roles that have the GUE property. I will work with such relations first, and then develop a notion of general incrementality in § 3.6.

### 3.3. Cumulativity and Quantization

Let us see how we can formally reconstruct the mereological difference between predicates like eat apples and eat two apples. I will concentrate on VP meanings, as this is a level were the difference between eat apples and eat two apples shows up, if expressions like in an hour or for an hour are applied to VPs. Tense, being irrelevant for our purpose, will be left unspecified.
a. eat apples:
$\lambda \mathrm{x}, \mathrm{e} \exists \mathrm{y}[\operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e})]$
b. eat two apples: $\lambda \mathrm{x}, \mathrm{e} \exists \mathrm{y}[2 \operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e})]$

We assume that the relation between the object argument and the event argument of eat is strictly incremental and cumulative:

[^5]\[

$$
\begin{equation*}
\operatorname{SINC}(\{\langle\mathrm{y}, \mathrm{e}\rangle \mid \operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e})\}) \text { and } \operatorname{CUM}(\{\langle\mathrm{y}, \mathrm{e}\rangle \mid \operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e})\}) \tag{54}
\end{equation*}
$$

\]

It can be shown that eat apples is cumulative for fixed subject arguments, due to cumulativity of APPLES and cumulativity of the object-event relation of eat. Proof: Assume that x, e fall under eat apples, that is, there is a y such that $[\operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e})]$, and that $\mathrm{x}, \mathrm{e}^{\prime}$ falls under it, that is, there is a $\mathrm{y}^{\prime}$ such that $\left[\operatorname{APPLES}\left(\mathrm{y}^{\prime}\right) \wedge \operatorname{EAT}\left(\mathrm{x}, \mathrm{y}^{\prime}, \mathrm{e}^{\prime}\right)\right]$. Due to cumulativity of apPLES we have $\operatorname{APPLES}\left(\mathrm{y} \oplus \dagger \mathrm{y}^{\prime}\right)$, and due to cumulativity of EAT we have $\operatorname{EAT}\left(\mathrm{x}, \mathrm{y} \oplus \mathrm{y}^{\prime}, \mathrm{e} \oplus \mathrm{e}^{\prime}\right)$. Hence eat apples is cumulative. The predicate eat two apples is not cumulative in the general case, as the nominal predicate two apples is not cumulative. This means that it is not the case, in general, that whenever x , e fall under eat two apples and x , $\mathrm{e}^{\prime}$ fall under it, then x and $\mathrm{e} \oplus \mathrm{e}^{\prime}$ fall under it. We can also show that eat two apples is quantized for fixed subject arguments. Proof: Assume to the contrary that there are entities $x, e, e^{\prime}$ such that $\exists y[2 \operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e})]$ and $\exists \mathrm{y}\left[2 \operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}\left(\mathrm{x}, \mathrm{y}, \mathrm{e}^{\prime}\right)\right]$ and $\mathrm{e}^{\prime}<\mathrm{e}$. Let y be an individual with $2 \operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e})$, and $\mathrm{y}^{\prime}$ be an individual with $2 \operatorname{APPLES}\left(y^{\prime}\right) \wedge \operatorname{EAT}\left(x, y^{\prime}, e^{\prime}\right)$. Due to mapping to subobjects we can derive from $\operatorname{EAT}(x, y, e)$ and $e^{\prime}<e$ that there is a $y^{\prime \prime}$ with $y^{\prime \prime}$ < $y$ such that $\operatorname{EAT}\left(x, y^{\prime \prime}, e^{\prime}\right) ; y^{\prime \prime}$ is unique according to uniqueness of objects, hence we have $y^{\prime}=y^{\prime \prime}$, and hence $y^{\prime}<y$. But this is incompatible with the assumption that 2APPLES, a quantized predicate, applies to $y$ and $y^{\prime}$. We can, of course, also show that a phrase like eat it is quantized, where it refers to a particular entity y. Proof; Assume to the contrary that there are entities $x, e, e^{\prime}$ such that $\operatorname{Eat}(x, y, e)$ and $\operatorname{EAT}\left(x, y, e^{\prime}\right)$, with $e^{\prime}<e$. By uniqueness of subobjects we have that there is a $y^{\prime}, y^{\prime}<y$ such that $\operatorname{EAT}\left(x, y^{\prime}, e^{\prime}\right)$, and as this $y$ is unique, we have $y^{\prime}=y$, contradicting $\mathrm{y}^{\prime}<\mathrm{y}$.

### 3.4. Measure adverbials

One of the classic tests for telicity is that measure adverbials like for an hour can be straightforwardly applied to atelic predicates, as in eat apples for an hour, but enforce certain reinterpretations when applied to telic predicates, as in eat an apple for an hour. How can we explain this?
Expressions like for an hour are like nominal measure phrases as two kilograms, insofar as they introduce a quantitative criterion of application. The measure function of the noun hour, H , is used as an extensive measure function for events. Of course, His first and foremost an extensive measure for times. But due to the temporal trace function $\tau$ we can construct a measure function $\mathrm{H}^{\prime}$ for events. The typical way how this is done is as follows 8 : We "standardize" $\mathrm{H}^{\prime}$ for events by requiring that $\mathrm{H}^{\prime}(\mathrm{e})=\mathrm{H}(\tau(\mathrm{e}))$ for all temporally contiguous events. The concatenation operation is the sum operation $\oplus_{\mathrm{E}}$ for events whose temporal traces are contiguous, that is, we have that for events e, $\mathrm{e}^{\prime}$ with $\neg \tau(\mathrm{e}) \infty \tau\left(\mathrm{e}^{\prime}\right)$ that $\mathrm{H}^{\prime}\left(\mathrm{e} \oplus \mathrm{e}^{\prime}\right)=\mathrm{H}^{\prime}(\mathrm{e})+\mathrm{H}^{\prime}\left(\mathrm{e}^{\prime}\right)$. An expression like walk for an hour, $\lambda \mathrm{x}, \mathrm{e}\left[\operatorname{WALK}(\mathrm{x}, \mathrm{e}) \wedge \mathrm{H}^{\prime}(\mathrm{e})=1\right]$, is not quantized, as it might apply to two contemporaneous events and their sum. But it is telic. Proof: Assume it were not telic; for instance, assume that we have e, $e^{\prime}$ with $\mathrm{e}^{\prime} \leq \mathrm{e}$ and $\mathrm{H}^{\prime}(\mathrm{e})=1$ and $\mathrm{H}^{\prime}\left(\mathrm{e}^{\prime}\right)=1$, and an event $\mathrm{e}^{\prime \prime}$ such that $\neg \mathrm{e}^{\prime} \otimes \mathrm{e}^{\prime \prime}$ and $\mathrm{e}^{\prime \prime} \leq \mathrm{e}$. Assume $\mathrm{e}^{\prime \prime}$ is the maximal such event, that is, we have $\mathrm{e}=\mathrm{e}^{\prime} \oplus \mathrm{e}^{\prime \prime}$. By the homomorphism property for $\tau$ we have that $\tau\left(\mathrm{e}^{\prime}\right), \tau\left(\mathrm{e}^{\prime \prime}\right)$ do not overlap and are part of $\tau(\mathrm{e})$, by the definition of $\mathrm{H}^{\prime}$ we have $\mathrm{H}(\tau(\mathrm{e}))=\mathrm{H}\left(\tau\left(\mathrm{e}^{\prime}\right)\right)=1$, by comensurability of H we have that $\mathrm{H}\left(\tau\left(\mathrm{e}^{\prime \prime}\right)\right)>0$, by additivity we have that $\mathrm{H}(\tau(\mathrm{e}))=\mathrm{H}\left(\tau\left(\mathrm{e}^{\prime}\right)\right)+$ $\mathrm{H}\left(\tau\left(\mathrm{e}^{\prime \prime}\right)\right)$, which is a contradiction.

[^6]This shows that the result of applying for an hour to a predicate like walk is telic. That the predicate it is applied to must not be telic follows from a similar principle as discussed in $\S 2.2$ for measure functions like kilogram.

$$
\begin{align*}
& \text { for an hour: }  \tag{55}\\
& \lambda \mathrm{R} \lambda \mathrm{x}, \mathrm{e}\left[\mathrm{R}(\mathrm{x}, \mathrm{e}) \wedge \mathrm{H}^{\prime}(\mathrm{e})=1 \wedge \partial \exists \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\mathrm{e}^{\prime}<_{\mathrm{H}^{\prime}} \mathrm{e} \wedge \forall \mathrm{e}^{\prime \prime} \in \mathrm{U}_{\mathrm{E}}\left[\mathrm{e}^{\prime \prime} \leq_{\mathrm{H}^{\prime}} \mathrm{e}^{\prime} \rightarrow \mathrm{R}\left(\mathrm{x}, \mathrm{e}^{\prime \prime}\right)\right]\right]\right]
\end{align*}
$$

The presupposition requires that the event e has proper parts, and that all parts of $\mathrm{e}^{\prime \prime}$ fall under R . The notion of a part $<_{H^{\prime}}$ is the part relation with respect to the measure function $\mathrm{H}^{\prime}$, that is, we have $\mathrm{e}^{\prime}$ $<\mathrm{e}$ iff $\mathrm{e}^{\prime} \leq \mathrm{e}$, and there is an $\mathrm{e}^{\prime \prime} \leq \mathrm{e}$ such that $\neg \tau\left(\mathrm{e}^{\prime}\right) \otimes \tau\left(\mathrm{e}^{\prime \prime}\right)$. If e is a temporally contiguous event, which follows from the fact that $\mathrm{H}^{\prime}(\mathrm{e})$ can be applied to it, then we have that $\mathrm{e}^{\prime \prime}$ precedes or follows $\mathrm{e}^{\prime}$, or that a part of $\mathrm{e}^{\prime \prime}$ precedes and another part follows $\mathrm{e}^{\prime}$, which means that R must not be telic with respect to its event argument.
This interpretation has an advantage over Moltmann (1991), where measure adverbials express a universal quantification over parts of the runtime of an event. The problem is that times may have a structure that is too fine-grained for certain events. For example, Mary ate apples for an hour does not require that every time $t$ included in the run time of an event of eating apples $e$ is the run-time of some apple-eating event by Mary. ${ }^{9}$
In case the presupposition of for an hour is not satisfied, the predicate can be coerced into an imperfective interpretation or an iterative interpretation. The imperfective version of an event predicate X is a predicate that applies to events $\mathrm{e}^{\prime}$ iff there is an event e such that $\mathrm{P}(\mathrm{e})$, and $\mathrm{e}^{\prime}$ is a part of e. The iterative version of $X$ is a predicate that applies to the sum of events $\mathrm{e}_{1} \oplus \mathrm{e}_{2} \oplus \ldots \oplus \mathrm{e}_{\mathrm{n}}$ for some $\mathrm{n}>1$, where P applies to $\mathrm{e}_{1}, \mathrm{e}_{2}, \ldots \mathrm{e}_{\mathrm{n}}$. In these interpretations the presuppositions for measure adverbials like for an hour are satisfied.

### 3.5. Interval adverbials

Another classic test for telicity is that interval adverbials like in an hour can be applied to telic expressions, as in eat two apples in an hour, but lead to certain peculiar effects when applied to atelic expressions, as in eat apples in an hour. How can we explain this difference?

An adverbial like in an hour expresses that an event happened within an interval of one hour. Like other numerical expressions they trigger scalar implicatures (cf. Gazdar (1979), Levinson (1984)). Measure adverbials do this as well; for example, Mary walked for an hour comes with the implicature that Mary did not walk for more than an hour at this occasion. That this is an implicature and not part of the meaning can shown by the fact that Mary walked for an hour, she even walked for one hour and a half is not a contradiction. Now, interval adverbials are peculiar because the implicature goes into the other direction; cf. Mary ate seven apples in an hour, she even ate them in 45 minutes, which is not a contradiction. Hence a sentence like Mary ate seven apples in an hour implicates that Mary didn't eat the apples in less than an hour.
Scalar implicatures can be described formally as follows (cf. Krifka (1995b)): A sentence S that induces a scalar implicature does not only express a proposition $\Phi$, its normal meaning, but also a set of alternative propositions. The effect of scalar implicature is that it denies all alternatives $\Phi^{\prime}$ that are semantically stronger than $\Phi$, that is, for which $\Phi^{\prime} \Rightarrow \Phi$ and $\Phi^{\prime} \nRightarrow \Phi$. The set of alternatives can be generated compositionally when we assume that interval adverbials introduce a set of alternatives. This set of alternatives then is propagated, for which we may follow the rules of Alternative Semantics (cf. Rooth (1985)). We get the following derivation for our example, where I identify meanings by M and alternatives by A :

[^7]a. in an hour: $\quad \mathrm{M}: \lambda \mathrm{R} \lambda \mathrm{x}, \mathrm{e} \exists \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\left[\mathrm{R}(\mathrm{x}, \mathrm{e}) \wedge \mathrm{H}(\mathrm{t})=1 \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right]$
$A:\left\{\lambda R \lambda x, e\left[R(x, e) \wedge \tau(e) \leq_{T} t\right] \mid t \in P_{T}\right\}$
b. ate two apples: $\mathrm{M}: \lambda \mathrm{x}, \mathrm{e} \exists \mathrm{y}[2 \operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e})]$

A: $\varnothing$
c. ate two apples $\mathrm{M}: \lambda \mathrm{x}, \mathrm{e} \exists \mathrm{y} \exists \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\left[2 \operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e}) \wedge \mathrm{H}(\mathrm{t})=1 \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right]$ in an hour:

A: $\left\{\lambda x, e \exists y\left[2 \operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{e}) \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right] \mid \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\right\}$
d. Mary ate two $\mathrm{M}: \lambda \mathrm{e} \exists \mathrm{y} \exists \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\left[2 \operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{M}, \mathrm{y}, \mathrm{e}) \wedge \mathrm{H}(\mathrm{t})=1 \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right]$ apples in an hour: $\mathrm{A}:\left\{\lambda \mathrm{e} \exists \mathrm{y}\left[2 \operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{M}, \mathrm{y}, \mathrm{e}) \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right] \mid \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\right\}$

$$
\begin{aligned}
& \text { e. Existential } \quad \mathrm{M}: \exists \mathrm{e} \exists \mathrm{y} \exists \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\left[2 \operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{M}, \mathrm{y}, \mathrm{e}) \wedge \mathrm{H}(\mathrm{t})=1 \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right] \\
& \text { closure: } \quad A:\left\{\exists \mathrm{e} \exists \mathrm{y}\left[2 \operatorname{APPLES}(\mathrm{y}) \wedge \operatorname{EAT}(\mathrm{M}, \mathrm{y}, \mathrm{e}) \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right] \mid \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\right\}
\end{aligned}
$$

Applying the rule for scalar implicature at this point, and using the fact that $\leq_{T}$ is transitive, we get indeed the result that there is no event $e$ that is an eating of two apples by Mary that can be placed in an interval shorter than one hour. This analysis falls short with respect to one point, as we should quantify just over events within a reference situation. I will not attempt to make this formally explicit. But notice that we can assume that, if e is an event in the situation under discussion, then any part of e will be under discussion as well.

Now, telic and non-telic predicates lead to interesting differences under this analysis of interval adverbials. For telic predicates $\alpha$ it is guaranteed that, if $\alpha$ applies to e, then $\alpha$ does not apply to a part of e that can be placed within a smaller temporal interval. To prove this, let me first introduce the notion of the minimal convex time that covers an event e , for which I will write $\mathrm{c} \tau(\mathrm{e})$ :

$$
\begin{equation*}
\forall \mathrm{e} \in \mathrm{U}_{\mathrm{E}} \forall \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\left[\mathrm{c} \tau(\mathrm{e})=\mathrm{t} \leftrightarrow \tau(\mathrm{e}) \leq \mathrm{t} \wedge \neg \exists \mathrm{t}^{\prime} \in \mathrm{P}_{\mathrm{T}}\left[\tau(\mathrm{e}) \leq \mathrm{t}^{\prime} \wedge \mathrm{t}^{\prime}<\mathrm{t}\right]\right] \tag{57}
\end{equation*}
$$

We want to show that, if $\alpha$ is telic and there are e, $e^{\prime}$ with $\alpha(e), \alpha\left(e^{\prime}\right)$, and $e^{\prime} \leq e$, then $c \tau(e)=c \tau\left(e^{\prime}\right)$. In any case we have $c \tau\left(\mathrm{e}^{\prime}\right) \leq \mathrm{c} \tau(\mathrm{e})$, due to the homomorphism properties of $\tau$, hence we have to show that $\neg \mathrm{c} \tau\left(\mathrm{e}^{\prime}\right)<\mathrm{c} \tau(\mathrm{e})$. Assume to the contrary that we have $\mathrm{c} \tau\left(\mathrm{e}^{\prime}\right)<\mathrm{c} \tau(\mathrm{e})$. Due to the remainder principle, there is a time t with $\neg \mathrm{t} \otimes \mathrm{c} \tau\left(\mathrm{e}^{\prime}\right)$ and $\mathrm{c} \tau(\mathrm{e})=\mathrm{t} \oplus \mathrm{c} \tau\left(\mathrm{e}^{\prime}\right)$. As $\mathrm{c} \tau(\mathrm{e})$ and $\mathrm{c} \tau\left(\mathrm{e}^{\prime}\right)$ are convex times, we have either $\mathrm{t} « \mathrm{c} \tau\left(\mathrm{e}^{\prime}\right)$, or $\mathrm{c} \tau\left(\mathrm{e}^{\prime}\right) \ll \mathrm{t}$, or that a part of t precedes $\mathrm{c} \tau\left(\mathrm{e}^{\prime}\right)$, and a part of t follows $c \tau\left(\mathrm{e}^{\prime}\right)$. - Assume first that t precedes $\mathrm{c} \tau\left(\mathrm{e}^{\prime}\right)$. This means that there is a time $\mathrm{t}^{\prime}, \mathrm{t}^{\prime} \leq \mathrm{t}$, such that $\mathrm{t}^{\prime} \leq$ $\tau(\mathrm{e})$ and $\mathfrak{t}^{\prime}$ « $\tau\left(\mathrm{e}^{\prime}\right)$. Let $\mathrm{t}^{\prime \prime}$ be the maximal such time. Then $\mathrm{t}^{\prime \prime}$ is the remainder of $\tau(\mathrm{e})$ with respect to $\tau\left(e^{\prime}\right)$. Due to (39), there is an event $e^{\prime \prime}$ with $e^{\prime \prime} \leq e$ and $\neg e^{\prime \prime} \oplus e^{\prime}$ such that $\tau\left(e^{\prime \prime}\right)=t^{\prime \prime}$ and $e=e^{\prime} \oplus e^{\prime \prime}$. As $\mathfrak{t}^{\prime \prime}$ « $\tau\left(e^{\prime}\right)$, we have that $\mathrm{e}^{\prime \prime}$ « $\mathrm{e}^{\prime}$. Hence $\mathrm{e}^{\prime}$ cannot be an initial part of e. But this is contrary to our assumption that $\alpha$ is telic and that $\alpha(e), \alpha\left(e^{\prime}\right)$ and $\mathrm{e}^{\prime} \leq \mathrm{e}$. - Assume now that t follows $\mathrm{c} \tau\left(\mathrm{e}^{\prime}\right)$; we arrive at a contradiction as well, as $\mathrm{e}^{\prime}$ cannot be a final part of e. - Assume now that part of t precedes $c \tau(\mathrm{e})$, and part of t follows $\mathrm{c} \tau\left(\mathrm{e}^{\prime}\right)$. Instead of one maximal complementary time $\mathrm{t}^{\prime \prime}$ we have two such times, one of which precedes $\mathrm{e}^{\prime}$, and one of which follows $\mathrm{e}^{\prime}$. Due to (39) this must correspond to an event $e^{\prime \prime}$ complementary to $e^{\prime}$ with respect to $e$. As $e^{\prime \prime}$ has a non-contiguous run time, and all such events are composed of events with a contiguous run time due to (31), we have to assume that $\mathrm{e}^{\prime \prime}$ consists of a part that precedes $\mathrm{e}^{\prime}$, and a part that follows $\mathrm{e}^{\prime}$, hence $\mathrm{e}^{\prime}$ cannot be an initial or a final part of e, which again violates the assumptions we started out with.
Things are of course different for non-telic predicates. If a non-telic predicate $\alpha$ applies to an event e, it could very well apply to a shorter part $\mathrm{e}^{\prime}$, and then a shorter minimal interval $\mathrm{t}^{\prime}$ may have been chosen. In general, it holds that if $\mathrm{e}^{\prime} \leq \mathrm{e}$ and $\neg \mathrm{INI}\left(\mathrm{e}^{\prime}, \mathrm{e}\right)$ or $\neg \mathrm{FIN}\left(\mathrm{e}^{\prime}, \mathrm{e}\right)$, then $\mathrm{c} \tau\left(\mathrm{e}^{\prime}\right)<\mathrm{c} \tau(\mathrm{e})$. Proof: Assume that this is not the case. As $\mathrm{e}^{\prime} \leq \mathrm{e}$ implies $\mathrm{c} \tau\left(\mathrm{e}^{\prime}\right) \leq \mathrm{c} \tau(\mathrm{e})$, this means that $\mathrm{c} \tau\left(\mathrm{e}^{\prime}\right)=\mathrm{c} \tau(\mathrm{e})$. Assume now $\neg \operatorname{INI}\left(e^{\prime}, e\right)$, that is, there is an $e^{\prime \prime}$ with $e^{\prime \prime} \leq e$ and $e^{\prime \prime}$ " $e^{\prime}$. Due to the homomorphism
property of $\tau$ we have $\tau(\mathrm{e})=\tau\left(\mathrm{e}^{\prime}\right) \oplus \tau\left(\mathrm{e}^{\prime \prime}\right)$, and due to the definition of precedence for events, $\tau\left(\mathrm{e}^{\prime \prime}\right)$ « $\tau\left(\mathrm{e}^{\prime}\right)$, contrary to our assumption that $\mathrm{c} \tau(\mathrm{e})=c \tau\left(\mathrm{e}^{\prime}\right)$.
But it might be that a non-telic predicate P is temporally atomic. For such predicates it holds that, whenever they apply to e , then there is a part $\mathrm{e}^{\prime}$ of e that does not contain any parts $\mathrm{e}^{\prime \prime}$ with $\tau\left(\mathrm{e}^{\prime \prime}\right)$ < $\tau\left(\mathrm{e}^{\prime}\right)$ and $\mathrm{P}\left(\mathrm{e}^{\prime \prime}\right)$. Scalar implicature will single out the shortest temporal atom of e , which will lead to the most informative proposition among the alternatives. That it is this weaker property of temporal atomicity, and not telicity, that is required, becomes evident with examples like the following one, which sounds quite fine:
(58) Mary is an incredibly fast eater. Yesterday she ate peanuts in 0.43 seconds!

Notice that even though eat peanuts is not quantized, it can be understood as temporally atomic. One chewing move may be a part of an event of eating peanuts, but not yet an event of eating peanuts.

### 3.6. Telicity through Quantization

Let us come back to the mereological conditions that hold between the arguments of predicates. The properties of uniqueness of events (47) and mapping to subobjects (49) were part of the definition of strict incrementality (51), but these properties do not apply to cases like read the article. It often happens that, while reading an article, we back up and re-read a passage. This violates uniqueness of subevents, as the passage is read at two different subevents. And it violates mapping to subobjects: Assume that the article p has 3 paragraphs $\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{p}_{3}$, and that Mary read $\mathrm{p}_{1}$ at $\mathrm{e}_{1}$, that she read $\mathrm{p}_{2}$ at $e_{2}$, that she read $p_{2}$ is read again at $e_{2}^{\prime}$, and that she read $p_{3}$ at $e_{3}$. Then both $e_{1} \oplus e_{2} \oplus e_{2}^{\prime} \oplus e_{3}$ and $e_{1}$ $\oplus e_{2} \oplus e_{3}$, where the latter event is a proper part of the former, are related to the same object, $p$, and due to uniqueness of participants there is no proper part of $p$ that is related to $e_{1} \oplus e_{2} \oplus e_{3}$. This example also shows that read the article is not necessarily a quantized predicate, as it may apply to both $\mathrm{e}_{1} \oplus \mathrm{e}_{2} \oplus \mathrm{e}_{2}{ }^{\prime} \oplus \mathrm{e}_{3}$ and $\mathrm{e}_{1} \oplus \mathrm{e}_{2} \oplus \mathrm{e}_{3}$. Nevertheless, predicates like read the article should be prime examples for incremental relations as well, as they can be combined with interval adverbials. Obviously, verbs of consumption and creation, like eat and draw, are special as we have uniqueness of events for such verbs (52).
So, why are interval adverbials applicable to predicates like read the article? We have assumed in § 3.5 that such adverbials come with the presupposition that the predicate they apply to is telic. We can show that read the article is indeed telic. Consider our example. There are two proper subevents of this event to which the predicate read the article applies, namely, $\mathrm{e}_{1} \oplus \mathrm{e}_{2} \oplus \mathrm{e}_{3}$ and $\mathrm{e}_{1} \oplus$ $\mathrm{e}_{2}{ }^{\prime} \oplus \mathrm{e}_{3}$. Notice that the minimal convex time that covers these two events is the same, $c \tau\left(e_{1} \oplus e_{2} \oplus e_{3}\right)=c \tau\left(e_{1} \oplus e_{2}{ }^{\prime} \oplus e_{3}\right)$, as $e_{2}$ and $e_{2}^{\prime}$ occur temporally in between $e_{1}$ and $e_{2}$. Hence telic predicates can very well be applicable to events like $\mathrm{e}_{1} \oplus \mathrm{e}_{2} \oplus \mathrm{e}_{2}{ }^{\prime} \oplus \mathrm{e}_{3}$ and $\mathrm{e}_{1} \oplus \mathrm{e}_{2} \oplus \mathrm{e}_{3}$ at the same time, and such predicates may satisfy the requirement for interval adverbials. This suggests the following definition for incremental relations in general, that is, relations that do allows for backups, like read. First, we assume that there is a corresponding relation $\theta^{\prime}$ that is strictly incremental (i.e., does not allow for backups). From $\theta^{\prime}$ we can define $\theta$ as the closure of $\theta^{\prime}$ under sum formation of the arguments.
(59) $\theta$ is incremental, $\operatorname{INC}(\theta)$, iff
a. there is a strictly incremental relation $\theta^{\prime}$ such that $\operatorname{SINC}\left(\theta^{\prime}\right)$, and
b. $\theta$ is the smallest relation that contains $\theta^{\prime}$ and is closed under sum formation:

$$
\theta^{\prime} \subseteq \theta \text { and } \forall \mathrm{x}, \mathrm{y} \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \theta\left(\mathrm{y}, \mathrm{e}^{\prime}\right) \rightarrow \theta\left(\mathrm{x} \oplus_{\mathrm{P}} \mathrm{y}, \mathrm{e} \oplus_{\mathrm{E}} \mathrm{e}^{\prime}\right)\right]
$$

Being closed under sum formation is nothing but cumulativity (44), and hence a natural condition. Cumulativity holds for strictly incremental relations like EAT as well, but the property of uniqueness of events rules out the existence of elements $x, e$ and $e^{\prime}$ such that $\theta(x, e), \theta\left(x, e^{\prime}\right)$ and $e \neq e^{\prime}$.
Now, incrementality does not necessarily guarantee telicity. That is, if $\theta$ is incremental, then a predicate like $\lambda \mathrm{e} \theta(\mathrm{x}, \mathrm{e})$ is not necessarily telic. Consider the following example. Assume as before that $p_{1}, p_{2}$ and $p_{3}$ are the three paragraphs of an article, and assume readings like $e_{1} \oplus e_{2} \oplus e_{3} \oplus e_{3}{ }^{\prime}$ or $e_{1}$ $\oplus e_{1}^{\prime} \oplus e_{2} \oplus e_{3}$, that is, we have repetitions at the beginning and the end. These cases do not violate the conditions for incrementality, but they violate telicity, as, for example, read the article would apply to both $\mathrm{e}_{1} \oplus \mathrm{e}_{2} \oplus \mathrm{e}_{3} \oplus \mathrm{e}_{3}{ }^{\prime}$ and a non-final part $\mathrm{e}_{1} \oplus \mathrm{e}_{2} \oplus \mathrm{e}_{3}$. But this type of violation is benign when it comes to the applicability of measure adverbials like in an hour. The pragmatic conditions ask us to identify an event that is situated within the shortest interval, and this clearly is $\mathrm{e}_{1} \oplus \mathrm{e}_{2}$ $\oplus e_{3}$ in this case, as $e_{3}$ precedes $e_{3}{ }^{\prime}$ temporally. Should we change the definition of incrementality to exclude initial or final repetitions, then? I think this is not necessary. While incrementality of a thematic relation $\theta$ does not guarantee telicity of the resulting event predicate $\lambda e \theta(x, e)$, it at least guarantees that the resulting event predicates will always contain a temporally atomic part, like $\mathrm{e}_{1} \oplus \mathrm{e}_{2} \oplus$ $e_{3}$, and thus the requirement for interval adverbials is met.

### 3.7. Extensions

There are various problems that a theory like the one developed here has to face that are not fatal but call for certain extensions. Let me start with the case build a house, a telic predicate. The problem is that the thematic relation lacks mapping to objects in the strict sense: Not every part of building a house is mapped to a part of the house (think of the erection of a scaffold that is removed later). I can think of two possible answers to this problem. First, we can say that we are just interested in conceptual structures, not in reality. Events that do not contribute to parts of the house like erecting the scaffold may not play any role in conceptual structures, and can be disregarded. Or we refine the notion of mapping to objects in the following way: If $\theta(x, e)$, and $\mathrm{e}^{\prime} \leq \mathrm{e}$, then it is either the case that $e^{\prime}$ can be related to a part $x^{\prime}$ of $x$ such that $\theta\left(x^{\prime}, e^{\prime}\right)$, or $e^{\prime}$ is a necessary preparatory event for an event $\mathrm{e}^{\prime \prime}, \mathrm{e}^{\prime \prime} \leq \mathrm{e}$, which in turn is related to a part $\mathrm{x}^{\prime}$ of x such that $\theta\left(\mathrm{x}^{\prime}, \mathrm{e}^{\prime \prime}\right)$. Given an adequate definition of the notion 'necessary preparatory event', it is possible to show that a predicate like build $a$ house is telic.
Another problem appears with predicates like peel an orange. Notice that not every part of the orange is affected by this process, only every surface part. This violates mapping to events. Also, a predicate like read a book does violate mapping to events, as for example the spine, though part of the book, cannot be subjected to a reading event. We can deal with that by assuming that certain verbs select for certain aspects of their objects, and only the parts with respect to the selected aspect count for mapping to events. The verb peel selects for the hull of an object, and the word read selects for the linguistic information an object contains.
A third problem pointed out in White (1994) and White \& Zucchi (1996) can be illustrated with write a sequence of numbers. The nominal predicate a sequence of numbers is not quantized, as it may be that proper parts of a sequence are sequences as well. Nevertheless, write a sequence of numbers is telic; it can be combined with in ten seconds. I would like to contest the notion that we use predicates like a sequence of numbers in a way that makes them non-quantized. We use them typically as referring to a sequence of numbers not flanked by any additional numbers. If someone is given the task to write down a sequence of five consecutive prime numbers, and proposes the solution " $5,9,11,13,17,19,23,25,29$ ", he probably failed, even though this sequence contains a sequence of five consecutive prime numbers. When we observe that a sequence of numbers can properly contain a sequence of numbers, then we apply a slightly more general notion of 'sequence of numbers'. And when we fix this interpretation, then the most plausible way to interpret write a
sequence of numbers in ten seconds is as applying to the writing of a minimal sequence, that is, a sequence of two numbers. ${ }^{10}$
Another interesting case are object NPs that do not seem to refer to entities at all, but rather express a measure of events. An example is Four thousand ships passed through the lock (in one year); notice that it may be that there were just a few ships passing to and fro. Krifka (1991) discusses how an extensive measure function for events can be derived from the nominal measure function inherent in ship. It can be shown that a predicate like four thousand ships pass through the look is quantized and hence telic under this reading.

### 3.8. Issues of scope

An important problem with the account developed so far, pointed out by, White (1994) with reference to an observation by Mittwoch (1982) and White \& Zucchi (1996), is that certain NPs, while not quantized, nevertheless result in telic predicates:
a. Mary wrote something in 10 minutes.
b. Mary ate more than three apples in an hour.
c. Mary ate a quantity of porridge in an hour.

Taken as predicates, something, more than three apples and a quantity of porridge are not quantized, and even cumulative. For example, if we can apply a quantity of porridge to two entities, then we can apply this predicate to their sum as well. Nevertheless, eat a quantity of porridge fits the tests for telicity.
What is going on can be described, following Mittwoch's contrastive analysis of John ate something vs. John ate, by assuming that quantificational indefinites must have wide scope over the verb phrase and its modifiers, possibly by quantifier movement on LF. In contrast, bare indefinites have narrow scope. This difference can be explained with the distinction between strong case and weak case in de Hoop (1992). We then get the following analysis for (60.b).
a. Mary ate more than three apples in an hour.
b. LF: [more than three apples $]_{1}$ [Mary $\left[\left[\right.\right.$ ate $\left.\mathrm{t}_{1}\right]$ in an hour $\left.]\right]$
c. ate $\mathrm{t}_{1}$
$\lambda_{\mathrm{x}, \mathrm{e}\left[\operatorname{EAT}\left(\mathrm{x}, \mathrm{x}_{1}, \mathrm{e}\right)\right]}$
d. in an hour:
$\mathrm{M}: \lambda R \lambda x, \mathrm{e} \exists \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\left[\mathrm{R}(\mathrm{x}, \mathrm{e}) \wedge \mathrm{H}(\mathrm{t})=1 \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right]$
A: $\left\{\lambda R \lambda x, e\left[R(x, e) \wedge \tau(e) \leq_{T} t\right] \mid t \in P_{T}\right\}$
e. [ate $\left.\mathrm{t}_{1}\right]$ in an hour:
$\mathrm{M}: \lambda \mathrm{x}, \mathrm{e} \exists \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\left[\operatorname{EAT}\left(\mathrm{x}, \mathrm{x}_{1}, \mathrm{e}\right) \wedge \mathrm{H}(\mathrm{t})=1 \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right]$
A: $\left\{\lambda \mathrm{x}, \mathrm{e}\left[\operatorname{EAT}\left(\mathrm{x}, \mathrm{x}_{1}, \mathrm{e}\right) \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right] \mid \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\right\}$
f. Mary $\left[\left[\right.\right.$ ate $\left.\mathrm{t}_{1}\right]$ in an hour $]$ : $\quad \mathrm{M}: \lambda \mathrm{e} \exists \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\left[\operatorname{EAT}\left(\mathrm{M}, \mathrm{x}_{1}, \mathrm{e}\right) \wedge \mathrm{H}(\mathrm{t})=1 \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right]$

A: $\left\{\lambda e\left[\operatorname{EAT}\left(\mathrm{~m}, \mathrm{x}_{1}, \mathrm{e}\right) \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right] \mid \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\right\}$
g. Existential closure: $\quad \mathrm{M}: \exists \mathrm{e} \exists \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\left[\operatorname{EAT}\left(\mathrm{M}, \mathrm{x}_{1}, \mathrm{e}\right) \wedge \mathrm{H}(\mathrm{t})=1 \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right]$

A: $\left\{\exists \mathrm{e}\left[\operatorname{EAT}\left(\mathrm{M}, \mathrm{x}_{1}, \mathrm{e}\right) \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right] \mid \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\right\}$
h. $[\text { more than three apples }]_{\mathrm{i}}: \quad \lambda \mathrm{p} \exists \mathrm{x}_{1}\left[\operatorname{APPLES}\left(\mathrm{x}_{1}\right) \wedge \#\left(\mathrm{x}_{1}\right)>3 \wedge \mathrm{p}\right]$

[^8]i. [more than three apples] $]_{\mathrm{i}}$
$$
\left[\text { Mary }\left[\left[\text { ate } \mathrm{t}_{1}\right] \text { in an hour }\right]\right]
$$
\[

$$
\begin{aligned}
& \exists \mathrm{x}_{1}\left[\operatorname{APPLES}\left(\mathrm{x}_{1}\right) \wedge \#\left(\mathrm{x}_{1}\right)>3 \wedge\right. \\
& \left.\quad \exists \mathrm{t} \in \mathrm{P}_{\mathrm{T}}\left[\operatorname{EAT}\left(\mathrm{M}, \mathrm{x}_{1}, \mathrm{e}\right) \wedge \mathrm{H}(\mathrm{t})=1 \wedge \tau(\mathrm{e}) \leq_{\mathrm{T}} \mathrm{t}\right]\right]
\end{aligned}
$$
\]

(61.b) is the LF that we assume. Traces are interpreted as variables (cf. c) that are bound by coindexed antecedents (cf. h, i). The pragmatic conditions for the applicability of the interval adverbial are checked after existential closure (g). On this level the variable $\mathrm{x}_{1}$ acts like a name, and the pragmatic conditions for in an hour are met. A paraphrase that renders the essence of this analysis well is: "Mary ate some object x in an hour, and x were more than three apples".

This is essentially a scope solution, which White \& Zucchi (1996) criticize because it does not explain why the adverbial cannot outscope the NP. Another question that the particular proposal in (61) raises is why the pragmatic conditions are checked after level (g) and not after level (i), where they would not be satisfied. A plausible answer to both questions is that the sentence forces a widescope reading because only this meets the conditions for interval adverbials. The question then arises why we do not find that measure adverbials can be forced into a wide-scope reading, thus satisfying the conditions for sentences like Mary ate a quantity of porridge for an hour. But notice that the information of such sentences under this reading could have been expressed by using a simple bare noun, as in Mary ate porridge for an hour. There must be some reason why this shorter form is avoided; the only plausible reason is that the speaker wants to give a quantity of porridge wide scope, which is impossible for the bare noun porridge. ${ }^{11}$

Are bare indefinite NPs always interpreted in situ? This is not the case. If destressed they can be interpreted in the restrictor of an overt or a generic quantifier, in which case interval adverbials can be applied:

Mary (usually) eats apples in 5 minutes.
'In general, if $x$ are apples and Mary eats $x$ in situation $s$, then she eats $x$ in 5 minutes in s'
However we arrive at the generic reading (cf. Krifka (1995a), Rooth (1995) for proposals), the object argument is bound with scope wider than the interval adverbial, hence its pragmatic requirements are locally satisfied.

## 4. Telicity by Precedence and Adjacency

### 4.1. A Dynamic Theory of Incrementality

The properties of eventive predicates that we have discussed in $\S 3$ were defined solely with respect to the sum operation and the part relation. While sufficient to explain why, for example, eat apples is cumulative and atelic but eat two apples is quantized and telic, they offer a rather static description of what is going on in these events. It is perhaps cognitively more realistic to switch to a dynamic picture in which we incorporate the idea that first one piece of apple is eaten, then another one, and so on, till the two apples are eaten up ${ }^{12}$. Also, when it comes to events like walk a mile, we want to be able to describe events in which first one part of a path is walked, then the next part, and

[^9]so on. To describe such events we will have to make use of structural properties like temporal precedence and external adjacency in addition to the part relation and the sum operation.
The dynamic notion that is necessary for the description of cases like eat two apples I will call expansion; it makes use of the precedence relation for events.
$\theta$ has the property of expansion, $\operatorname{EXP}(\theta)$, iff
\[

$$
\begin{equation*}
\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{P}} \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \theta\left(\mathrm{y}, \mathrm{e}^{\prime}\right) \wedge \mathrm{e}<_{\mathrm{E}} \mathrm{e}^{\prime} \rightarrow \neg \mathrm{x} \otimes_{\mathrm{P}} \mathrm{y}\right] \tag{63}
\end{equation*}
$$

\]

That is, it holds that, whenever x is $\theta$-related to e, and y is $\theta$-related to a following $\mathrm{e}^{\prime}$, then x and y do not overlap. Expansion does not give us strict incrementality as defined in (51). It does not guarantee, if $\theta(x, e)$, that it holds for no $e^{\prime}, e^{\prime}<_{E} e$, that $\theta\left(x, e^{\prime}\right)$. But expansion together with mapping to objects (48) entails that $\theta$ cannot hold between an object and an event and at the same time to a nonfinal part of the event:

$$
\begin{equation*}
\operatorname{EXP}(\theta) \wedge \operatorname{MO}(\theta) \rightarrow \neg \exists \mathrm{x} \in \mathrm{U}_{\mathrm{P}} \exists \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \theta\left(\mathrm{x}, \mathrm{e}^{\prime}\right) \wedge \mathrm{e}^{\prime} \leq_{\mathrm{E}} \mathrm{e} \wedge \neg \operatorname{FIN}\left(\mathrm{e}^{\prime}, \mathrm{e}\right)\right] \tag{64}
\end{equation*}
$$

Proof: Assume to the contrary that we have $x, e, e^{\prime}$ with $\theta(x, e), \theta\left(x, e^{\prime}\right), e^{\prime} \leq e$ and $\neg F I N\left(e^{\prime}, e\right)$, that is, there is an $e^{\prime \prime}, e^{\prime \prime} \leq e$, with $e^{\prime}$ « $e^{\prime \prime}$. By MO and $e^{\prime \prime} \leq e$ we have that there is a $y, y \leq x$, such that $\theta\left(y, e^{\prime \prime}\right)$. By EXP and $e^{\prime}$ « $e^{\prime \prime}$ we have that $\neg x \otimes y$, which contradicts $y \leq x$.
One consequence of this is that a predicate like $\lambda \mathrm{e}[\theta(\mathrm{y}, \mathrm{e})]$ is telic if $\theta$ is expansive and has the mapping to objects property. Also, a predicate like $\lambda \mathrm{e} \exists \mathrm{y}[\alpha(\mathrm{y}) \wedge \theta(\mathrm{y}, \mathrm{e})]$ is telic under these conditions, if $\alpha$ is a quantized predicate. Hence a predicate like eat two apples is telic, as we can assume expansion and mapping to objects for the relation between events and objects with verbs like eat. Consequently, we can explain why eat two apples in an hour is fine, but eat two apples for an hour is in need of a progressive or iterative reinterpretation. - Let me call thematic relations that are expansive and show mapping to objects strictly expansive incremental:

$$
\begin{equation*}
\theta \text { is strictly expansive incremental, } \operatorname{SEINC}(\theta), \text { iff } \operatorname{EXP}(\theta) \text { and } \operatorname{MO}(\theta) . \tag{65}
\end{equation*}
$$

Clearly, this property does not apply to predicates like read a book when we consider readings with backups: Here, two subsequent events may affect overlapping, even identical, text. But we can generalize strictly expansive incremental relations to expansive incremental relations in the same way as we generalized strictly incremental relations to incremental relations in (59).

So it appears to be a matter of taste whether we formulate the properties of thematic relations that are responsible for the telic interpretation of expressions like eat two apples or read a novel in terms of incrementality or in terms of expansiveness. Perhaps expansiveness is the cognitively more natural condition. But there are certain instances that have been discussed in the literature that cannot be handled by expansiveness, because the required structural properties are lacking. Nishida (1994) observes a certain use of the Spanish reflexive clitic se that requires a telic interpretation of the underlying predicate. This explains the following grammaticality contrast:
a. Juan (se) tomó una copa de vino anoche antes de acostarse.
'Juan drank a glass of wine last night before going to bed'
b. Juan (*se) tomó vino anoche antes de acostarse.
'Juan drank wine last night before going to bed'
Interestingly, Nishida observes that se can occur with certain stative predicates as well, as in conocerse la ciudad the Quito 'to be familiar with the (whole) city of Quito' vs. conocer la ciudad the Quito 'to know the city of Quito'. She suggests that se in these cases establishes a mapping between the parts of the city and parts of the knowledge. Now, it is easy to see that bodies of knowledge can be ordered by a part relation (e.g. the knowledge of the old town of Quito is part of the knowledge of Quito). But we certainly do not have a relation of precedence or adjacency defined for bodies of knowledge. Hence it seems that the conceptual model in this case is solely based on the part relation.

### 4.2. Simple Movements

Another form of incrementality that requires an adjacency relation are movements along paths (see Tenny (1995), Jackendoff (1996) for some peculiarities of motion verbs). Movement predicate like WALK relate the parts of a path to the parts of an event. They have at least three arguments, for an object, a path, and an event. The path argument can be made explicit, even though it often remains implicit and gets existentially bound, just like the event argument. In the following, I will assume that paths are elements of a path structure $\mathrm{U}_{\mathrm{H}}$ :
a. Mary hiked the Vernal Falls Path.
b. Mary hiked.

$$
\begin{align*}
& \exists \mathrm{e} \in \mathrm{U}_{\mathrm{E}}[\operatorname{HIKE}(\mathrm{M}, \mathrm{VFP}, \mathrm{e})]  \tag{67}\\
& \exists \mathrm{x} \in \mathrm{U}_{\mathrm{H}} \exists \mathrm{e} \in \mathrm{U}_{\mathrm{E}}[\operatorname{HKKE}(\mathrm{M}, \mathrm{x}, \mathrm{e})]
\end{align*}
$$

The crucial property for movements along a path is that temporal adjacency of movement events is reflected in spatial adjacency of the paths, and vice versa.
$\theta$ has the ADJACENCY PROPERTY, $\operatorname{ADJ}(\theta)$, iff

$$
\begin{align*}
& \forall x, y, z \in P_{H} \forall e, e^{\prime}, \mathrm{e}^{\prime \prime} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \mathrm{e}^{\prime}, \mathrm{e}^{\prime \prime} \leq_{\mathrm{E}} \mathrm{e} \wedge \mathrm{y}, \mathrm{z} \leq_{\mathrm{H}} \mathrm{x} \wedge \theta\left(\mathrm{y}, \mathrm{e}^{\prime}\right) \wedge \theta\left(\mathrm{z}, \mathrm{e}^{\prime \prime}\right) \rightarrow\right.  \tag{68}\\
& \left.\quad\left[\mathrm{e}^{\prime} \infty_{\mathrm{E}} \mathrm{e}^{\prime \prime} \leftrightarrow \mathrm{y} \infty_{\mathrm{H}} \mathrm{z}\right]\right]
\end{align*}
$$

Condition (68) says that whenever two subevents of a movement event are temporally adjacent, then their paths are spatially adjacent, and vice versa. This allows us to define the notion of a strict movement relation, for which we just need in addition Mapping to Objects and a condition that strict movements happen along connected paths:
$\theta$ is a strict movement relation, $\operatorname{SMR}(\theta)$, iff
$\operatorname{ADJ}(\theta) \wedge \operatorname{MO}(\theta) \wedge \forall \mathrm{x} \in \mathrm{U}_{\mathrm{H}} \forall \mathrm{e} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \rightarrow \mathrm{x} \in \mathrm{P}_{\mathrm{H}}\right]$
This excludes certain types of movements. Consider the following movement with respect to the paths in (16), where I specify the corresponding movement events by pairs of a subpath and an event, and assume that $e_{i} \infty e_{j}$ and $e_{i}<e_{j}$ iff $j=i+1$ :
a. $\left\langle\mathrm{a}, \mathrm{e}_{1}\right\rangle,\left\langle\mathrm{b}, \mathrm{e}_{2}\right\rangle,\left\langle\mathrm{c}, \mathrm{e}_{3}\right\rangle,\left\langle\mathrm{d}, \mathrm{e}_{4}\right\rangle,\left\langle\mathrm{e}, \mathrm{e}_{5}\right\rangle,\left\langle\mathrm{f}, \mathrm{e}_{6}\right\rangle$
b. $\left\langle\mathrm{a}, \mathrm{e}_{1}\right\rangle,\left\langle\mathrm{b}, \mathrm{e}_{2}\right\rangle,\left\langle\mathrm{c}, \mathrm{e}_{3}\right\rangle,\left\langle\mathrm{d}, \mathrm{e}_{5}\right\rangle,\left\langle\mathrm{e}, \mathrm{e}_{6}\right\rangle,\left\langle\mathrm{f}, \mathrm{e}_{7}\right\rangle$
c. $\left\langle\mathrm{a}, \mathrm{e}_{1}\right\rangle,\left\langle\mathrm{b}, \mathrm{e}_{2}\right\rangle,\left\langle\mathrm{e}, \mathrm{e}_{3}\right\rangle,\left\langle\mathrm{f}, \mathrm{e}_{4}\right\rangle$
d. $\left\langle\mathrm{a}, \mathrm{e}_{1}\right\rangle,\left\langle\mathrm{b}, \mathrm{e}_{2}\right\rangle,\left\langle\mathrm{c}, \mathrm{e}_{3}\right\rangle,\left\langle\mathrm{c}, \mathrm{e}_{4}\right\rangle,\left\langle\mathrm{b}, \mathrm{e}_{5}\right\rangle$
e. $\left\langle\mathrm{c}, \mathrm{e}_{1}\right\rangle,\left\langle\mathrm{d}, \mathrm{e}_{2}\right\rangle,\left\langle\mathrm{e}, \mathrm{e}_{3}\right\rangle,\left\langle\mathrm{f}, \mathrm{e}_{4}\right\rangle,\left\langle\mathrm{i}, \mathrm{e}_{5}\right\rangle,\left\langle\mathrm{j}, \mathrm{e}_{6}\right\rangle$
f. $\left\langle\mathrm{a}, \mathrm{e}_{1}\right\rangle,\left\langle\mathrm{b}, \mathrm{e}_{2}\right\rangle,\left\langle\mathrm{e}, \mathrm{e}_{5}\right\rangle,\left\langle\mathrm{f}, \mathrm{e}_{6}\right\rangle$
g. $\mathrm{e}_{1},\left\langle\mathrm{a}, \mathrm{e}_{3}\right\rangle,\left\langle\mathrm{b}, \mathrm{e}_{3}\right\rangle,\left\langle\mathrm{c}, \mathrm{e}_{5}\right\rangle,\left\langle\mathrm{d}, \mathrm{e}_{6}\right\rangle$

The pairs in (70.a) can be in a movement relation, as they satisfy the necessary properties. The pairs of (b) and (c) do not satisfy ADJ; (b) has two adjacent subpaths (c, d) that do not correspond to adjacent events, and (c) has two adjacent events $\left(\mathrm{e}_{2}, \mathrm{e}_{3}\right)$ that do not correspond to adjacent paths. The pairs in (d), a movement and a return, do not satisfy ADJ, as the two adjacent events $\mathrm{e}_{3}, \mathrm{e}_{4}$ do not correspond to two adjacent paths - they both are related to the same path, c. The pairs in (e), a movement along a circular path, do not satisfy ADJ, as j and c are adjacent, but $\mathrm{e}_{6}$ and $\mathrm{e}_{1}$ aren't. The pairs in (f) do satisfy ADJ, but their paths are not connected, thus violating the last condition of
(69). Cases like (g) with detached initial or final events that are not related to any path are excluded by MO.
We can show that strict movement relations lead to telic predicates - if $\theta$ is a strict movement relation, then $\lambda_{\mathrm{e}} \theta(\mathrm{x}, \mathrm{e})$ is telic. Proof: Assume it were not telic, that is, there are events e, $\mathrm{e}^{\prime}$ and a path x with $\theta(x, e), \theta\left(x, e^{\prime}\right)$, and there is an $e^{\prime \prime}, e^{\prime \prime} \leq e$ and $e^{\prime \prime}$ « $e^{\prime}$ or $e^{\prime}$ « $e^{\prime \prime}$. Assume $e^{\prime \prime}$ « $e^{\prime}$. By MO, there is a path $y, y \leq x$, such that $\theta\left(y, e^{\prime \prime}\right)$. But by $\theta\left(x, e^{\prime}\right)$ and $e^{\prime \prime}$ « $e^{\prime}$, we have that $y$ and $x$ cannot overlap; otherwise this would lead to a situation like in (70.d). This contradicts $\mathrm{y} \leq \mathrm{x}$. For the other assumption, $\mathrm{e}^{\prime}$ « $\mathrm{e}^{\prime \prime}$, we can derive a similar contradiction.

### 4.3. General Movements

General movements include movements with stops (70.b), circular movements like the walking of prisoners in a prison court (70.e), or movements with backups (70.d). Let me call them stop-n-go movements, Alcatraz movements, and Echternach movements. ${ }^{13}$ These funny movements are still continuous, that is, telekinesis (70.c) is disallowed.
Not any two movements form a general movement; to qualify, the second movement must begin where the first movement has ended. We can express this by using the precedence relation for events, the notion of final and initial parts for events, as defined in (36), and the tangentiality relation for paths, as defined in (17.c).
(71) $\theta$ is a movement relation, $\operatorname{MR}(\theta)$, iff it is the smallest relation that satisfies the following conditions:
a. There is a strict movement relation $\theta^{\prime}$, and $\theta^{\prime} \subseteq \theta$;
b. $\forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{H}} \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \theta\left(\mathrm{y}, \mathrm{e}^{\prime}\right) \wedge \mathrm{e} «_{\mathrm{E}} \mathrm{e}^{\prime} \wedge\right.$
$\forall \mathrm{e}^{\prime \prime}, \mathrm{e}^{\prime \prime \prime} \in \mathrm{U}_{\mathrm{E}} \forall \mathrm{x}^{\prime}, \mathrm{y}^{\prime} \in \mathrm{U}_{\mathrm{H}}\left[\operatorname{FIN}_{\mathrm{E}}\left(\mathrm{e}^{\prime \prime}, \mathrm{e}\right) \wedge \operatorname{INI}_{\mathrm{E}}\left(\mathrm{e}^{\prime \prime \prime}, \mathrm{e}^{\prime}\right) \wedge \theta\left(\mathrm{e}^{\prime \prime}, \mathrm{x}^{\prime}\right) \wedge \theta\left(\mathrm{e}^{\prime \prime \prime}, \mathrm{y}^{\prime}\right) \rightarrow\right.$
$\left.\mathrm{TANG}_{\mathrm{H}}\left(\mathrm{x}^{\prime}, \mathrm{y}^{\prime}\right)\right]$
$\left.\rightarrow \theta\left(\mathrm{x} \oplus_{\mathrm{H}} \mathrm{y}, \mathrm{e} \oplus_{\mathrm{E}} \mathrm{e}^{\prime}\right)\right]$
That is, a movement relation is based on a strict movement relation, and is closed under sum formation under certain restrictions. Namely, if e is a movement event along $x$, and $e^{\prime}$ is a movement event along y , and e precedes $\mathrm{e}^{\prime}$, and the paths of the final parts of e are tangential to the paths of the initial parts of $e^{\prime}$, then $e \oplus e^{\prime}$ is a movement event with the path $x \oplus y$. This allows for stop-n-go movements, Alcatraz movements, and Echternach movements, but excludes telekinesis.
General movement relations $\theta$ do not lead to telic predicates, that is, $\lambda \mathrm{e} \theta(\mathrm{x}, \mathrm{e})$ need not be telic. Consider a movement that goes along a circle $x$ twice, where $e_{1}$ is the event of the first movement around the circle and $e_{2}$ is the event of the second movement, then we have both $\theta\left(x, e_{1}\right)$ and $\theta\left(x, e_{1}\right.$ $\oplus e_{2}$ ), and $e_{1}$ is not a final part of $e_{1} \oplus e_{2}$. However, by (71) all general movements are composed of strict movements. By an argument similar to the one at the end of $\S 3.6$ we can derive that the criteria for the application of interval adverbials like in an hour are met. For example, if the circle $x$ was completed within a shorter time in $\mathrm{e}_{1}$ than in $\mathrm{e}_{2}$, then these criteria will pick out the smallest time interval that contains the run time of $\mathrm{e}_{1}$.
The definition of a general movement relation in (71) is compatible with the definition of a strict movement relation. If e is a strict movement along a path $x$, and if $e$ is sliced in two parts $\mathrm{e}^{\prime}$ and $\mathrm{e}^{\prime \prime}$

[^10]such that $e^{\prime}$ " $e^{\prime \prime}$ and $e=e^{\prime} \oplus e^{\prime \prime}$, then $e^{\prime}$ and $e^{\prime \prime}$ are two strict movements along paths $x^{\prime}$ and $x^{\prime \prime}$ that satisfy the condition (71.b). Hence $e^{\prime}$ and $e^{\prime \prime}$ can be joined again to a movement relation that holds between $e^{\prime} \oplus e^{\prime \prime}$ and $x^{\prime} \oplus x^{\prime \prime}$. Namely, we can show that all final parts of $e^{\prime}$ and all initial parts of $e^{\prime \prime}$ have (externally) tangential paths, if we assume Uniqueness of Participants as a general condition. I show here that the paths of $x^{\prime}$ and $x^{\prime \prime}$ themselves are externally tangential: By MO and UP, we have that $x^{\prime}, x^{\prime \prime} \leq x$. By e $e^{\prime} e^{\prime \prime}$ and $e^{\prime}, e^{\prime \prime} \leq e$ and $\operatorname{SMR}(\theta)$, we have that $\neg x^{\prime} \otimes x^{\prime \prime}$. We also have that $x^{\prime}$ $\infty \mathrm{x}^{\prime \prime}$, as otherwise there is a $\mathrm{x}^{\prime \prime \prime}, \mathrm{x}^{\prime \prime \prime} \leq \mathrm{x}$, such that $\mathrm{x}^{\prime} \infty \mathrm{x}^{\prime \prime \prime} \infty \mathrm{x} \mathrm{x}^{\prime \prime}$ (as x is a path), and there could be no event $e^{\prime \prime \prime}$ such that $\theta\left(x^{\prime \prime \prime}, e^{\prime \prime \prime}\right)$ and $e^{\prime \prime \prime} \leq e$, as $e^{\prime \prime \prime}$ would have to be disjoint from both $e^{\prime}$ and $e^{\prime \prime}$.

### 4.4. Measurements for Movement

Movement events can be measured by the path that they cover, as in Mary walked two kilometers. That is, KM, a measure function that was originally designed for paths ${ }^{14}$ can be used to define a measure function км $^{\prime}$ for movement events. A natural way ${ }^{15}$ is to standardize Km $^{\prime}$ with respect to the pathlength of strict movements, and generalize it following the definition of general movements:

Assume that $\theta$ is a general movement relation based on the strict movement relation $\theta^{\prime}$, assume that КМ is an extensive measure function for paths, then the corresponding measure function $\mathrm{Km}^{\prime}$ for movement events is defined as follows:

$$
\begin{aligned}
& \text { a. } \forall \mathrm{x} \in \mathrm{U}_{\mathrm{H}} \forall \mathrm{e} \in \mathrm{U}_{\mathrm{E}}\left[\theta^{\prime}(\mathrm{x}, \mathrm{e}) \rightarrow K M^{\prime}(\mathrm{e})=\mathrm{KM}(\mathrm{x})\right] \\
& \text { b. } \forall \mathrm{x}, \mathrm{y} \in \mathrm{U}_{\mathrm{H}} \forall \mathrm{e}, \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \theta\left(\mathrm{y}, \mathrm{e}^{\prime}\right) \wedge \mathrm{e} «_{\mathrm{E}} \mathrm{e}^{\prime} \wedge\right. \\
& \forall \mathrm{e}^{\prime \prime}, \mathrm{e}^{\prime \prime \prime} \in \mathrm{U}_{\mathrm{E}} \forall \mathrm{x}^{\prime}, \mathrm{y}^{\prime} \in \mathrm{U}_{\mathrm{H}}\left[\mathrm{FIN}_{\mathrm{E}}\left(\mathrm{e}^{\prime \prime}, \mathrm{e}\right) \wedge \mathrm{INI}_{\mathrm{E}}\left(\mathrm{e}^{\prime \prime \prime}, \mathrm{e}^{\prime}\right) \wedge \theta\left(\mathrm{e}^{\prime \prime}, \mathrm{x}^{\prime}\right) \wedge \theta\left(\mathrm{e}^{\prime \prime \prime}, \mathrm{y}^{\prime}\right) \rightarrow\right. \\
& \left.\quad \rightarrow \operatorname{TANG}_{\mathrm{H}}\left(\mathrm{x}^{\prime}, \mathrm{y}^{\prime}\right)\right] \\
& \left.\quad \rightarrow\left(\mathrm{e} \oplus \mathrm{e}^{\prime}\right)=\operatorname{KM}^{\prime}(\mathrm{e})+\operatorname{KM}^{\prime}\left(\mathrm{e}^{\prime}\right)\right]
\end{aligned}
$$

This guarantees that, once the km-measure is defined for strict movement events, it can be generalized for movement event that include backtracks and circles. For example, if the named paths in (16) all have a length of 1 km , then the Echternach movement (70.d) is 5 kilometers long.
The measure function $\mathrm{Km}^{\prime}$ that we obtain by (72) is an extensive measure function for movement events. Its concatenation operation is of course restricted to movements that are temporally and spatially adjacent. We can show that a predicate like walk a kilometer is telic under this reconstruction. Proof: Assume a predicate $\lambda \mathrm{e} \exists \mathrm{x}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \mathrm{Km}^{\prime}(\mathrm{e})=\mathrm{n}\right]$, where $\theta$ is a movement relation. This predicate is telic iff for all events e, $e^{\prime}$ such that $\exists \mathrm{x}\left[\theta(\mathrm{x}, \mathrm{e}) \wedge \mathrm{Km}^{\prime}(\mathrm{e})=\mathrm{n}\right]$ and $\exists \mathrm{x}\left[\theta\left(\mathrm{x}, \mathrm{e}^{\prime}\right) \wedge \mathrm{Km}^{\prime}\left(\mathrm{e}^{\prime}\right)=\mathrm{n}\right]$ and $\mathrm{e}^{\prime} \leq \mathrm{e}$ it holds that $\operatorname{INI}\left(\mathrm{e}^{\prime}, \mathrm{e}\right)$ and $\operatorname{FIN}\left(\mathrm{e}^{\prime}, \mathrm{e}\right)$. Assume to the contrary that $\neg \operatorname{INI}\left(\mathrm{e}^{\prime}, \mathrm{e}\right)$, that is, there is an $e^{\prime \prime}, e^{\prime \prime}$ « $e^{\prime}$ and $e^{\prime \prime} \leq e$. Assume that $e^{\prime \prime}$ is the maximal such event, then $e^{\prime \prime}$ is a movement event as well. As $\mathrm{e}^{\prime \prime} \leq \mathrm{e}$ and $K m^{\prime}(\mathrm{e})=\mathrm{n}$, we have that $K m^{\prime}\left(\mathrm{e}^{\prime \prime}\right)>0$. (Sketch of proof: Either e is a strict movement along $x$, then $\mathrm{e}^{\prime \prime}$ is related to some part $\mathrm{x}^{\prime \prime}$ of x by MO, where $\mathrm{Km}\left(\mathrm{x}^{\prime \prime}\right)>0$. Or e is a general movement, then it is composed of strict movements, for each of which we have a $\mathrm{Km}^{\prime}$ measure $>0$, and hence their sum is $>0$ ) We furthermore have that $\mathrm{e}^{\prime \prime} \oplus \mathrm{e}^{\prime}$ is a movement event (as both events are part of the movement event e , and $\mathrm{e}^{\prime \prime}$ is the maximal event that precedes $\mathrm{e}^{\prime}$ ), and as the conditions for (72) are met, we have that $K M^{\prime}\left(\mathrm{e}^{\prime \prime} \oplus \mathrm{e}^{\prime}\right)=K M^{\prime}\left(\mathrm{e}^{\prime \prime}\right)+K M^{\prime}\left(\mathrm{e}^{\prime}\right)>\mathrm{n}$. But this is incompatible with $\mathrm{e}^{\prime}, \mathrm{e}^{\prime \prime} \leq \mathrm{e}$ and $\mathrm{Km}^{\prime \prime}(\mathrm{e})=\mathrm{n}$. (Sketch of proof: Either e is a strict movement; if x is its path,

[^11]then $K m^{\prime}(e)=K M(x)$, then the path $x^{\prime} \oplus x^{\prime \prime}$ of $e^{\prime} \oplus e^{\prime \prime}$ is a part of $x$, hence $K m^{\prime}\left(e^{\prime} \oplus e^{\prime \prime}\right)=K M\left(x^{\prime} \oplus\right.$ $\left.\mathrm{x}^{\prime \prime}\right) \leq K M(\mathrm{x})$. Or e is a general movement, then it is composed of strict movements, and we again have $K^{\prime}\left(\mathrm{e}^{\prime} \oplus \mathrm{e}^{\prime \prime}\right) \leq \mathrm{KM}^{\prime}(\mathrm{e})$.) This contradicts $\neg \operatorname{INI}\left(\mathrm{e}^{\prime}, \mathrm{e}\right)$. (By a similar argument, $\neg \mathrm{FIN}\left(\mathrm{e}^{\prime}, \mathrm{e}\right)$ cannot be the case). Hence, the predicate $\lambda e \exists x\left[\theta(x, e) \wedge \operatorname{Km}^{\prime}(e)=n\right]$ is indeed telic. Consequently, the pragmatic conditions for interval adverbials like in an hour are satisfied.

### 4.5. Source, Goal and Direction

We can specify the source and the goal of a movement, as in walk from the university to the capitol. This is a predicate that applies to movement relations that begin at the university and end at the capitol. That is, all the initial parts of this movement are adjacent to the location of the university, and all the final parts are adjacent to the capitol. Furthermore, no non-initial parts should be adjacent to the university, and no non-final parts should be adjacent to the capitol. We can define the notion of SOURCE and GOAL as follows:

If $\theta$ is a movement relation and it holds for $x$, e that $\theta(x, e)$, then

$$
\text { a. } \begin{align*}
\forall \mathrm{y} \in \mathrm{U}_{\mathrm{H}} & {\left[\operatorname { S O U R C E } ( \mathrm { x } , \mathrm { y } , \mathrm { e } ) \rightarrow \forall \mathrm { e } ^ { \prime } \in \mathrm { U } _ { \mathrm { E } } \forall \mathrm { x } ^ { \prime } \in \mathrm { U } _ { \mathrm { H } } \left[\left[\operatorname{INI}\left(\mathrm{e}^{\prime}, \mathrm{e}\right) \wedge \mathrm{x}^{\prime} \leq_{\mathrm{H}} \mathrm{x} \rightarrow \mathrm{x}^{\prime} \infty_{\mathrm{H}} \mathrm{y}\right] \wedge\right.\right.}  \tag{73}\\
& {\left.\left[\neg \mathrm{INI}\left(\mathrm{e}^{\prime}, \mathrm{e}\right) \wedge \mathrm{x}^{\prime} \leq_{\mathrm{H}} \mathrm{x} \rightarrow \neg \mathrm{x}^{\prime} \infty_{\mathrm{H}} \mathrm{y}\right]\right] }
\end{align*}
$$

b. $\forall \mathrm{y} \in \mathrm{U}_{\mathrm{H}}\left[\operatorname{GOAL}(\mathrm{x}, \mathrm{y}, \mathrm{e}) \rightarrow \forall \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}} \forall \mathrm{x}^{\prime} \in \mathrm{U}_{\mathrm{H}}\left[\left[\operatorname{FIN}\left(\mathrm{e}^{\prime}, \mathrm{e}\right) \wedge \mathrm{x}^{\prime} \leq_{\mathrm{H}} \mathrm{x} \rightarrow \mathrm{x}^{\prime} \infty_{\mathrm{H}} \mathrm{y}\right] \wedge\right.\right.$

$$
\left.\left[\neg \operatorname{FIN}\left(e^{\prime}, e\right) \wedge x^{\prime} \leq_{H} x \rightarrow \neg \mathrm{x}^{\prime} \infty_{\mathrm{H}} \mathrm{y}\right]\right]
$$

We then can give analyses like the following:
Mary walked from the university to the capitol. $\lambda e \exists x[\operatorname{waLk}(\mathrm{~m}, \mathrm{x}, \mathrm{e}) \wedge \operatorname{SOURCE}(\mathrm{x}, \mathrm{u}, \mathrm{e}) \wedge \operatorname{GOAL}(\mathrm{x}, \mathrm{c}, \mathrm{e})]$

Such predicates are telic, which explains why we can apply interval adverbials like in an hour to them. Proof: Assume a predicate like $\lambda \mathrm{e} \exists \mathrm{x}[\theta(\mathrm{x}, \mathrm{e}) \wedge \operatorname{SOURCE}(\mathrm{x}, \mathrm{u}, \mathrm{e}) \wedge \operatorname{GOAL}(\mathrm{x}, \mathrm{v}, \mathrm{e})]$ were not telic, that is, there are e, $\mathrm{e}^{\prime}$ with $\mathrm{e}^{\prime} \leq \mathrm{e}$ and $\exists \mathrm{x}[\theta(\mathrm{x}, \mathrm{e}) \wedge \operatorname{SOURCE}(\mathrm{x}, \mathrm{u}, \mathrm{e}) \wedge \operatorname{GOAL}(\mathrm{x}, \mathrm{v}, \mathrm{e})]$ and $\exists x\left[\theta\left(x, e^{\prime}\right) \wedge \operatorname{SOURCE}\left(\mathrm{x}, \mathrm{u}, \mathrm{e}^{\prime}\right) \wedge \operatorname{GOAL}\left(\mathrm{x}, \mathrm{v}, \mathrm{e}^{\prime}\right)\right]$ such that $\neg \operatorname{INI}\left(\mathrm{e}^{\prime}, \mathrm{e}\right)$ or $\neg \operatorname{FIN}\left(\mathrm{e}^{\prime}, \mathrm{e}\right)$. Take the first disjunct, $\neg \operatorname{INI}\left(e^{\prime}, e\right)$; it implies that no initial part of $\mathrm{e}^{\prime}$ can be an initial part of e. Hence the conditions $\operatorname{SOURCE}(\mathrm{x}, \mathrm{u}, \mathrm{e})$ and $\operatorname{SOURCE}\left(\mathrm{x}, \mathrm{u}, \mathrm{e}^{\prime}\right)$ are contradictory; the first entails that no noninitial part is adjacent to $u$, but the second says that every initial part of $\mathrm{e}^{\prime}$, which is not an initial part of e , is adjacent to u . The second disjunct, $\neg \operatorname{FIN}\left(\mathrm{e}^{\prime}, \mathrm{e}\right)$, leads to a similar problem.
We can also specify the direction of a movement, for example, in walk from the university towards the capitol. Such predicates are not telic; witness * walk from the university towards the capitol in an hour. We can treat such cases by assuming that they refer to initial parts of events that end at the specified goal:
(75) Mary walked from the university towards the capitol.

$$
\lambda \mathrm{e}^{\prime} \mathrm{e}^{\prime}\left[\operatorname{INI}\left(\mathrm{e}, \mathrm{e}^{\prime}\right) \wedge \exists \mathrm{x}\left[\operatorname{WALK}\left(\mathrm{~m}, \mathrm{x}, \mathrm{e}^{\prime}\right) \wedge \operatorname{SOURCE}\left(\mathrm{x}, \mathrm{u}, \mathrm{e}^{\prime}\right)\right] \wedge \operatorname{GOAL}\left(\mathrm{x}, \mathrm{c}, \mathrm{e}^{\prime}\right)\right]
$$

Such predicates are not telic in the technical sense, as they refer to any initial part of an event of walking from the university to the capitol. The representation in (75) is problematic in one point: It assumes the existence of a complete walking to the specified goal (e'). However, this need not exist in the actual world. We can construct instances of the imperfective paradox (Dowty (1979)) with such examples (e.g., Mary walked from the university towards the capitol when she was hit by a truck). Hence a proper treatment of such cases has to be couched in a modal representation. Alternatively, we may just assume that any strict movement along a path from the university to the capitol that starts at the university qualifies. This is certainly the prototypical case, but note that one can walk in funny ways from A to B that include loops, backups, and so on -- think of Kafka's land surveyor K. on his way towards the castle. It is not necessarily the existence of a path from A to B
that makes us classify a movement as one from A towards B, but things like intentions of participants, knowledge of physical laws, and so on.

### 4.6. Movements in other Dimensions

Change of qualities is structurally similar to movement in space. For example, the change of temperature of an object can be seen as a movement in temperature space. When we assume a linear directed path structure, as defined in § 2.4, to model temperature, then we can treat sentences like the following in the same way as we treated Mary walked from the university to the capitol and Mary walked two kilometers.
(76) a. Mary heated the water from $30^{\circ} \mathrm{C}$ to $90^{\circ} \mathrm{C}$ (in an hour).
$\lambda \mathrm{e}\left[\operatorname{HEAT}(\mathrm{m}, \mathrm{w}, \mathrm{e}) \wedge \operatorname{SOURCE}\left(\mathrm{e}, 30^{\circ} \mathrm{C}\right) \wedge \operatorname{GOAL}\left(\mathrm{e}, 90^{\circ} \mathrm{C}\right)\right]$
b. Mary heated the water by 60 centigrades (in an hour).
$\lambda \mathrm{e}\left[\operatorname{HEAT}(\mathrm{M}, \mathrm{w}, \mathrm{e}) \wedge\right.$ CENTIGRADES $\left.\left.^{\prime}(\mathrm{e})=60\right)\right]$
We have to assume that whenever $\operatorname{HEAT}(x, y, e)$ is the case, then there is a $z$, an element of the directed path structure of temperature, such that $\operatorname{HEAT}(x, y, z, e)$. The relation $\theta$ between $z$ and $e$, $\theta=\{\langle\mathrm{z}, \mathrm{e}\rangle \mid \operatorname{HEAT}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{e})\}$, must satisfy the properties of a movement relation. In addition, hEAT identifies a direction for this movement, namely, to higher temperatures; we have for all $\mathrm{z}, \mathrm{z}^{\prime}, \mathrm{z}^{\prime \prime}, \mathrm{e}, \mathrm{e}^{\prime}$, $e^{\prime \prime}$, if $\theta(z, e), \theta\left(z^{\prime}, e^{\prime}\right), \theta\left(z^{\prime \prime}, e^{\prime \prime}\right)$ and $e^{\prime}$ « $e^{\prime \prime}$, then $z^{\prime}$ « $z^{\prime \prime}$. Fixed degrees like $30^{\circ} \mathrm{C}$ denote locations in the directed path space of temperature. And centrigrades' is an extensive measure function for temperature change events that is derived from an extensive measure function CENTRIGRADES for paths in the directed path structure of temperatures, according to the same principles as we derived Kı' from Km in (72). CENTIGRADES itself is a difference measure derived from degree locations; for example, the path between $30^{\circ} \mathrm{C}$ and $90^{\circ} \mathrm{C}$ has a length of 60 centigrades. By this analogy to movements in space we can infer that the verbal predicates in (76.a,b) are telic.
Examples of the type heat the water from $30^{\circ} \mathrm{C}$ to $90^{\circ} \mathrm{C}$ are particularly clear instances of quality changes that can be treated as movements, as the temperature scale is well developed, with established measure functions and names for degrees. But all other instances of quality changes can be treated in the same way, a view that was proposed in Gruber (1965). Take the verb bake, as in bake a lobster. It can be treated as push, as in push a cart, or heat, as in heat the water. Whereas push denotes changes in a spatial path structure and heat denotes a change in the directed path structure of temperature, bake denotes changes in the directed path structure of being cooked by means of baking. Certain degree regions in this directed path structures have names, like raw and done and perhaps over-done. We can also refer to intermediate degrees, like half done or $80 \%$ done. Such proportional measures are typically derived from other, more clearly defined measure functions, e.g. the ones for time or energy spent in the process. For example, a lobster is half done if it is undergoing a change in done-ness from raw to done, and half of the necessary energy or time is already spent.
The goal of such quality changes can be expressed in various ways with verbs like heat and bake. Consider the following examples:
a. Mary baked the lobster till half done (in an hour / *for an hour). $\lambda e[\operatorname{bAKE}(\mathrm{M}, \mathrm{L}, \mathrm{e}) \wedge \operatorname{SOURCE}(\mathrm{e}$, RAW) $\wedge$ GOAL $(\mathrm{e}$, HALF DONE) $]$
b. Mary whipped the cream stiff (in an hour / $*$ for an hour).
$\lambda \mathrm{e}[$ WHIP $(\mathrm{m}, \mathrm{c}, \mathrm{e}) \wedge \operatorname{SOURCE}(\mathrm{e}$, UNWHIPPED $) \wedge \operatorname{GOAL}(\mathrm{e}$, STIFF $)]$
c. Mary baked the lobster (in an hour / for an hour).
i) $\lambda \mathrm{e}[\operatorname{BAKE}(\mathrm{m}, \mathrm{L}, \mathrm{e}) \wedge \operatorname{SOURCE}(\mathrm{e}$, RAW) $\wedge \operatorname{GOAL}(\mathrm{e}$, DONE/BAKED $)]$
ii) $\lambda \mathrm{e}[\operatorname{BAKE}(\mathrm{m}, \mathrm{L}, \mathrm{e})]$

In (77.a), the goal of the baking is specified explicitly by an adverbial phrase, till half done. The source is left implicit, something that we find with other movement expressions as well (e.g. Mary walked to the capitol (in an hour). If the context does not say anything to the contrary, then the starting point is the origin of the scale, RAW. In (77.b), the goal of the whipping is specified by a resultative adjective, stiff. Resultative adjectives in general specify the goal of a movement event. (77.c) is ambiguous. In the version that allows for in an hour the lobster was done, but not in the version that allows for for an hour. This suggests the analyses in (i) and (ii). In (i), we may see the verb bake as doing semantically double duty: It describes the nature of the process and the state reached on the scale (we can denote this state regularly with the participle form of the verb, here baked). In (ii), bake just describes the process of moving the lobster higher on the dimension of baked-ness, and not necessarily to the natural end state.

### 4.7. Instantaneous movements

Among the quality changes that can be described as movements in certain dimensions there are those that appear to be instantaneous, Vendler's achievements. For example, Mary arrived in London describes an instantaneous change of Mary's position from not being in London to being in London, or perhaps the final part of this change. We can express this as saying that it refers to events that are minimal final events of movements of Mary to London:

$$
\begin{align*}
\text { Mary arrived in London. } \lambda \mathrm{e} \exists \mathrm{e}^{\prime} \in \mathrm{U}_{\mathrm{E}} \exists \mathrm{x} \in & \mathrm{U}_{\mathrm{H}}\left[\operatorname{MOVE}\left(\mathrm{M}, \mathrm{x}, \mathrm{e}^{\prime}\right) \wedge \operatorname{GOAL}\left(\mathrm{x}, \mathrm{~L}, \mathrm{e}^{\prime}\right) \wedge \mathrm{FIN}\left(\mathrm{e}, \mathrm{e}^{\prime}\right)\right.  \tag{78}\\
& \left.\wedge \forall \mathrm{e}^{\prime \prime}\left[\mathrm{FIN}\left(\mathrm{e}^{\prime \prime}, \mathrm{e}^{\prime}\right) \rightarrow \mathrm{e} \leq \mathrm{e}^{\prime \prime}\right]\right]
\end{align*}
$$

This representation makes plausible why neither interval adverbials nor measure adverbials go well with achievements. Achievements are quantized, which explains why they cannot be combined with for an hour. But then we should expect that they can be combined with in an hour. The problem here is that the length of minimal final event is not a fact about reality, but rather a matter of the granularity of the conceptual representation of reality. Depending on the granularity level, we can see the final event of Mary's arriving in London as aything between the event that spans the descent of her airplane in Heathrow and her leaving the gate of the airport (which typically takes well over an hour) and her stepping through the gate of the airport (which perhaps takes only a second). A interval adverbial would then not so much indicate the size of a real arrival event, but rather the finegrainedness of our conceptual representation, which is quite irrelevant for communicative purposes.
Movements in other dimensions can be seen as punctual as well. For example, transaction verbs like give or buy are often seen as punctual. This is so because in most cases we construct the legal implications in such a way that these events are atomic. If Mary buys a book from a bookseller, then the book changes its ownership exactly at the moment when Mary hands over the money. But we can conceive of such events in a non-punctual way as well; think of the buying of a house, which can be rather protracted.

## 5. Conclusion

In this article I have developed a series of algebraic structures for parts, paths, times and events to formulate the essential properties of incremental relations between participants in a predication, the notion of derived extensive measure functions, and the formation of telic predicates. In particular, I developed a framework for the representation of spatial and other types of movement, which necessitated the assumption of an adjacency relation. I tried to assume just what seemed to be conceptually necessary. I could sketch a few applications of the apparatus developed. Some of the areas in which the idea of correspondences between the mereological relations between participants in predi-
cations have been applied that were not mentioned so far are the exploitation of partitive case for aspectual marking in Finnish (Krifka (1992)) and of prepositional objects in German (Filip (1989)), the use of perfective prefixes for the marking of definite and bounded NPs in Slavic (Krifka (1992), Filip (1997)), the description of iteration phenomena (Egg (1993)) and the semantic function of accusative adverbials in Korean (Wechsler \& Lee (1996)).
The examples considered in this article were limited in one important respect, as they worked under the assumption that incremental relations hold between an event argument and a particular participant of the event. This is not the case. First, the incremental relations need not hold with respect to an event argument. We have seen in § 4.1 with the marker se in Spanish that there is evidence for incremental relations between object parts and knowledge (Nishida (1994)). Jackendoff (1996) discusses cases like The road goes from New York to Boston, in which we have an incremental relation between the parts of the road and the parts of a path. Even when considering event predicates, there is some variation as to which participant stands in an incremental relation to the event argument, again discussed by Jackendoff (1996). In John crossed the river in an hour, the event is incremental with respect to some path across the river; in The army crossed the river in an hour, the event is more likely incremental with respect to the parts of the army. Hence we cannot fix the incremental argument once and for all in the lexical entry of a verb like cross. An important factor is that an incremental participant must have relevant parts in the first place; John doesn't have relevant parts, but an army has, when it comes to crossing a river. Often, a measure construction will indicate that a particular entity is seen as having parts. Hence we have contrasts as in Bill loaded three trucks with dirt versus Bill loaded the truck with three tons of dirt, in which the direct object vs. the withphrase refers to the incremental participant.
Finally, I would like to point out one possible line of investigation that remained unexplored in this paper. At several points I had to switch from a strictly incremental relation to a general incremental relation (e.g. in $\S 3.6$ to handle cases that allow for backups like read, and in $\S 4.3$ to handle "funny" movements). Alternatively, one could have defined structures that allow for objects like $\mathrm{p}_{1}+\mathrm{p}_{2}+\mathrm{p}_{2}+\mathrm{p}_{3}$ (three paragraphs of an article, where the middle paragraph occurs twice), or $\mathrm{a}+\mathrm{b}+\mathrm{c}+\mathrm{c}+\mathrm{b}+\mathrm{b}+\mathrm{c}+\mathrm{d}$ (the path of an Echternach movement). To do so, we would have to give up idempotency and commutativity for the sum operation ( $x \oplus x=x, x \oplus y=y \oplus x$ ), and assume entities like multisets and sequences. The mathematics of such structures is more complex, but they would have allowed us to stick with strictly incremental relations throughout.

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[^0]:    ${ }^{1}$ Aristotle, in Metaphysics, Book $\theta$ 6, distinguished between "actualities" like seeing something that are complete as soon as they started, and "movements" like learning something for which this is not the case.
    ${ }^{2}$ Other tradition makes use of the topological distinction between open and closed intervals; an atelic predicate is true at an open interval, whereas a telic predicate is true at a closed interval, cf. Bennett (1981) and Jackendoff (1991), or they make an ontological distinction between telic and atelic events, cf. Bach (1986), Piñon (1995). Also, I use "telicity" here in the sense of "boundedness" in Depraetere (1995), who reserves "telicity" for events with a natural or intended endpoint (as in Sheila deliberately swam for two hours) I feel justified in doing so, as the nature of the endpoint does not affect the points to be made here.

[^1]:    ${ }^{3}$ An alternative is to define $\mathrm{kg}(\mathrm{x} \oplus \mathrm{y})$ as $\mathrm{kg}(\mathrm{x})+\mathrm{kg}(\mathrm{y})$ - $\mathrm{kg}(\mathrm{z})$, where z is the greatest common part of x and y ; cf. Cartwright (1975).

[^2]:    ${ }^{4}$ See Clarke (1981) for a mereological approach that takes connection (= overlap or adjacency) as the basic relation and defines the part relation as $\mathrm{x} \leq \mathrm{y}$ iff all z connected with x are also connected with y . I do not follow this approach in order to stress that part structures are conceptually simpler than adjacency structures.

[^3]:    5 This is the conceptual basis for so-called "possessor raising constructions". Instead of saying The dog bit the leg of the policeman, we can say the dog bit the policeman, and perhaps specify the location by an oblique construction, e.g. in his leg or legwise.

[^4]:    6 Jackendoff (1996) constructs predicates as relations between objects and times, e.g. EAT ( $\mathrm{x}, \mathrm{y}, \mathrm{t}$ ), where t is a time. I found that times are insufficient to formulate the properties of, say, eat two apples in such a way that it is guaranteed that this predicate is telic. The reason is that different objects can be subjected to the same type of action at the same time; it is possible that $\operatorname{EAT}(\mathrm{x}, \mathrm{y}, \mathrm{t})$, $\operatorname{EAT}\left(\mathrm{x}^{\prime}, \mathrm{y}^{\prime}, \mathrm{t}\right)$, and $\mathrm{x} \neq \mathrm{x}^{\prime}, \mathrm{y} \neq \mathrm{y}^{\prime}$, thus violating the important property of uniqueness of participants, (43).

[^5]:    ${ }^{7}$ Such one-to-one mappings seem to be cognitively very attractive. Some linguistic examples: Children understand sentences like Every kid has a ball as saying that every kid has a ball and every ball belongs to a kid (cf. Brooks \& Braine (1996)). Conditionals like If A then $B$, which say that every A-case is a B-case, are often strengthened to $A$ iff B. Also, a sentence like Five students read five articles, while allowing for a variety of interpretations, is preferrably understood as saying that each student read exactly one article.

[^6]:    ${ }^{8}$ There are actually various different ways. For example, concatenation may be defined for all non-overlapping events. If John sang from 3 p.m. to 5 p.m., and Mary sang from 4 p.m. to 6 p.m., then John and Mary sang for four hours according to this measure function (which may be the right one if, for example, John and Mary are paid for their singing individually by the hour). The resulting predicate is quantized. Or, concatenation may be defined for all events that do not overlap temporally (instead of all temporally adjacent events), in which case the resulting predicate is still telic. According to this definition, Mary walked for an hour if she walked from 9 to $9: 30$, and then from 11 to 11:30. There is actually some leeway in which measure phrases like for an hour can be understood. This is one of the reasons why I do not follow the quantificational analysis of measure adverbials in Dowty (1979) and Moltmann (1991).

[^7]:    ${ }^{9}$ The bare plural, apples, must be interpreted in the most general way, as applying to single apples and even parts of an apple. This is justified; witness the fact that the following dialogue is not contradictory: Did you eat apples today? -- Yes, but just a little piece.

[^8]:    ${ }^{10}$ Even without this interpretation of $a$ sequence of numbers, the sentence Mary wrote a sequence of numbers in ten seconds should be fine under the wide-scope reading of the object, a point argued for in § 3.8.

[^9]:    ${ }^{11}$ Moltmann (1991) discusses data that seem to show a narrow-scope interpretation of non-bare NPs, as in For several years John took a lot of pills. But all her examples seem to imply an iterative reinterpretation, in this case saying that there was an eventuality e lasting several years consisting of many events $e^{\prime}$, where at each of these events $\mathrm{e}^{\prime}$, John took a lot of pills.
    ${ }^{12}$ Such a dynamic view is proposed in Verkuyl (1993). But Verkuyl works with a discrete model and also assumes that the parts of the object subjected to an event are ordered by some precedence relation. This is perhaps adequate when it comes to movements along paths, but not for cases like eating apples; there is no specific intrinsic order of the apple parts in which they have to be eaten.

[^10]:    13 Alcatraz movements to commemorate the prison island off San Francisco, and Echternach movements after the medieval dancing procession of the town of Echternach in Luxembourg, still performed each Whit-Tuesday to honor St. Willibrord. Another instance of the latter type are the ceremonial marches of Ottoman janissaries, where the backward steps were supposed to express vigilance in military campaigns.

[^11]:    14 Actually, measure functions like km are originally measures for distances, not for paths. They can be generalized to paths: A path has the length of 2 km iff it has the same length as the shortest path between two points that have a distance of 2 km to each other.

    15 There are alternatives. For example, we can define a measure function $\mathrm{KM}^{\prime \prime}$ that just measures the path of a movement, irrespective of backups.

