Semantic Values in Higher-Order Semantics

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Abstract: Recently, some philosophers have argued that we should take quantification of any (finite) order to be a legitimate and irreducible, sui generis kind of quantification. In particular, they hold that a semantic theory for higher-order quantification must itself be couched in higher-order terms. Øystein Linnebo has criticized such views on the grounds that they are committed to general claims about the semantic values of expressions that are by their own lights inexpressible. I show that Linnebo's objection rests on the assumption of a notion of semantic value or contribution which both applies to expressions of any order, and picks out, for each expression, an extra-linguistic correlate of that expression. I go on to argue that higher-orderists can plausibly reject this assumption, by means of a hierarchy of notions they can use to describe the extra-lingustic correlates of expressions of different orders.

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1 *Introduction*

Starting with George Boolos's seminal work (esp. 1985), various authors have argued for the view that higher-order languages are not only intelligible and legitimate devices which are not paraphrasable in first-order terms, but that an adequate semantics for such languages can and should itself be couched in higher-order terms. In particular, in such a semantics, the truth-conditions of object language second-order quantifications are specified by means of a second-order quantification in the meta-language. Call this view *higher-orderism.* In his 'Sets, Properties, and Unrestricted Quantification' (2006), Øystein Linnebo argues against higher-orderism by trying to show that its proponents face a peculiar kind of expressive limitation: they are committed to certain substantive semantic principles which, by their own lights, cannot be expressed. In this paper, I present a way for higher-orderists to resist this conclusion.

My argument proceeds in two main steps. First, I show that Linnebo's objection rests on the assumption that there is a notion of the semantic contribution of an expression which has the following two properties: it applies to expressions of *any* syntactic category, and for every such expression, the notion picks out a (typically) *extra-linguistic correlate* of that expression which determines the latter's effect on the truth-conditions of sentences in which it occurs. Second, I make a case that for all Linnebo has said, a tenable version of higher-orderism can be developed according to which there is no such notion. Roughly, the central claim of the view that emerges is that although for every expression, one can speak of something like its "extra-linguistic correlate", there is no unified, cross-categorically applicable notion that serves this purpose. Rather, there is a *hierarchy* of such notions, mirroring the hierarchy of syntactic categories in the object language. The temptation to "collapse" these into a single notion is explained as an instance of our more general unreflective but paradox-prone habit in natural language to nominalize expressions of non-nominal categories.

The structure of the paper is as follows: Sect. 2 explains what higher-orderism is and sketches the dialectical context of Linnebo's objection against the view. Sect. 3 examines that objection in detail and shows that it relies on the assumption of a unified notion of extra-linguistic semantic contributions. Sect. 4 argues that although this claim has some initial plausibility, higher-orderists can sensibly reject it, and they can offer a plausible explanation why, although false, the claim seems intuitively appealing. It presents the hierarchy of notions of extra-linguistic correlates that the higher-orderist can appeal to and indicates why it is tempting to "collapse" that hierarchy into a single first-order notion. Sect. 5 concludes.

2 *Background*

The first-order quantifiers '$\exists x$' and '$\forall x$' bind variables that stand in the syntactic position of singular terms. A *second*-order quantifier, in my use of the term, is a quantifier that binds variables which stand in the syntactic position of sentence-forming operators which take singular terms as arguments, i.e. ordinary *first*-order predicates.[1] A *third*-order quantifier, by analogy, is a quantifier that binds variables which stand in the syntactic position of sentence-forming operators which (in at least one of their argument places) take first-order predicates as arguments, i.e. *second*-order predicates. The characterization extends in the obvious way to n'th-order quantification, for arbitrary (finite) n. The view I call 'higher-orderism' consists of essentially three theses concerning this syntactic hierarchy of quantifiers: firstly, that quantification of any order is a legitimate, (in principle) intelligible linguistic device; secondly, that n'th-order quantifications cannot be adequately *paraphrased* by quantifications of an order lower than n; thirdly, that adequate *semantic* clauses for n'th-order quantifications must *employ* n'th-order quantifiers of the meta-language in much the same way in which the standard semantic clauses for first-order quantification employ first-order quantifiers of the meta-language.

We shall only consider the higher-order analogues to '$\exists x$' and '$\forall x$'. Their second-order analogues, for instance, permit the formulation of sentences such as

(1) $\exists X\ (Xa\ \&\ Xb)$.

(2) $\forall X\ (Xa \vee \neg Xa)$.

We can approximate their intended meanings by the natural language sentences 'some property is had by both a and b (namely, the property of being tall)' and 'every property is such that a has it or a does not have it'. According to higher-orderism, however, these readings cannot be strictly adequate, for they interpret the second-order quantifiers by means of *first-order* quantification over properties, as witnessed by the fact that the 'namely'-rider demands a singular noun phrase ('the property of being tall') as its complement.

[1] For simplicity, I ignore operators which form expressions other than sentences, such as singular term-forming functors on singular terms.

There are two strategies that have been pursued in the literature in attempting to find better interpretations of second-order quantification in natural language. The first is Boolos's famous proposal of using *plural* nominal quantifiers, reading e.g. (1) roughly as follows: 'some things are such that *a* is one of them and *b* is one of them, too (namely, the tall things)' (cf. esp. Boolos 1984). As witnessed by the appropriate 'namely'-rider, the quantifier phrases used here govern the position of a *plural noun phrase*. The second strategy is to make use of *non-nominal* quantificational devices in English. The availability of such devices, and their apparent suitability for interpreting the formal second-order notation, was first emphasized by Arthur Prior (cf. his 1971, ch. 3) and recently developed in some detail by Agustín Rayo and Stephen Yablo 2001. On this proposal, we may read (1) and (2) as 'there is something such that *a* is that, and *b* is that, too (namely, tall)', 'everything is such that *a* is that or it is not the case that *a* is that', respectively. Here, the quantifiers are associated with *adjectival* rather than nominal position, as witnessed by the fact that the 'namely'-rider demands an adjectival expression as its complement.

For our purposes, it is not very important which of the readings is preferred, or whether perhaps neither can be considered fully satisfactory by higher-orderists' lights. For natural languages uncontentiously do not provide us with the resources required to translate quantifications of, say, the seventeenth order. So it is at any rate clear that in the general case, we have to come to understand the higher-order quantifiers without being able to rely on translation into independently understood vocabulary.[2] Since Linnebo does not object to this aspect of higher-orderism, I shall proceed on the assumption that in principle, we can in some way learn the meanings of quantifiers of arbitrarily high orders.

The plural and the non-nominal reading of second-order quantifiers plausibly nevertheless correspond to two different conceptions of the higher-orderists' hierarchy of quantifiers: one that it is natural to gesture at by talking of pluralities, pluralities of pluralities or super-pluralities, and so on, and one that is more naturally approximated by talk of properties, properties of properties, etc. The issues to be discussed below arise in the same way on either picture (cf. Linnebo 2006, 154); just to fix ideas, I shall be assuming the latter conception, since it seems to fit better with the intended predicational character of the formal quantifications (cf. Rayo and Yablo 2001, 78ff, Williamson 2003, 455ff).

One of the main motivations for accepting higher-orderism is that it appears to provide

[2] For some discussion of how far "up" we may be able to get using only natural language resources, see Rayo and Yablo 2001, sec. X, Linnebo and Nicolas 2008, the 'Appendix on Pairing' by Lewis, Burgess, and Hazen in Lewis 1990, as well as Hazen 1997.

a means of developing an adequate semantics for languages with unrestricted first-order quantifiers. Whether and how a semantics for such languages can be constructed is the main topic of Linnebo 2006, whose primary goals are to show that the higher-orderists' proposal faces severe difficulties, and to develop a better account. I shall here be concerned only with the extent to which Linnebo has achieved the first goal. In the remainder of this section, I sketch the problem that absolutely general first-order quantification poses for semantic theorizing, and how higher-orderism appears to provide a solution to that problem.

Let L_1 be an ordinary, monadic, extensional first-order language without function symbols whose quantifiers range over absolutely everything.[3] We assume that an adequate semantics for L_1 takes the shape of a specification of the truth-conditions of the sentences of L_1 relative to arbitrary ways of interpreting L_1's non-logical constants. The difficulty in developing a semantics for L_1 is that it appears to be impossible to specify a notion of an interpretation such that the following holds: If it is possible informally to interpret a predicate P of L_1 so that it applies to all and only Fs, then there is an interpretation under which it applies to all and only Fs.

On a popular approach, an interpretation of L_1 is a function mapping names and predicates of the object language to objects and sets of objects, respectively. Under such an interpretation i, a predicate P applies to all and only the members of $i(P)$. Yet it appears possible informally to interpret P so that it applies to all and only sets. Since there is no set of all sets, however, there is no interpretation of the envisaged kind under which P applies to all and only sets.

More generally, given any notion of an interpretation, it seems possible informally to interpret a predicate P as applying to something x just in case x is not an interpretation under which P applies to x.[4] But the hypothesis that there is an interpretation under which P applies to something x under just that condition logically implies a contradiction. For suppose that i_R is such an interpretation:

(3) For every object x, i_R is an interpretation under which P applies to x iff x is not an interpretation under which P applies to x.

Instantiating the universal quantification in (3) with i_R yields the inconsistent

[3] The assumption of monadicity, i.e. that every predicate of L_1 (other than '=') is one-place, is purely for ease of exposition.

[4] The argument to follow is Williamson's cf. 2003, 426.

(4) i_R is an interpretation under which P applies to i_R iff i_R is not an interpretation under which P applies to i_R.

Note that if the object language quantifiers were restricted to a domain D, the application conditions of predicates would need to be defined only for the members of D. The universal quantifier in (3) could thus be restricted to D, and instantiation of it with i_R would depend on the assumption that i_R is in D. The contradiction obtained on that assumption would thus simply be taken to show that the assumption is false. The problem arises because L_1's quantifiers are unrestricted.

How does higher-orderism provide a way out of this difficulty? On the above approach, the assumption that it is possible informally to interpret a predicate in such and such a way is translated into a *first-order* existential quantification 'some interpretation i is such that ...'. Higher-orderism makes room for an alternative approach on which that assumption is translated into a *second-order* quantification.[5] Specifically, in place of the first-order variable 'i', we use a two-place second-order variable 'I'.[6] Whereas on the first-order approach, we define interpretation-relative application conditions for predicates by

(5) $\forall x\ (P \text{ applies}_i \text{ to } x \leftrightarrow x \in i(P))$

we now have

(6) $\forall x\ (P \text{ applies}_I \text{ to } x \leftrightarrow I(P,x))$

Accordingly, in order to capture the possibility of interpreting P as applying to all and only Fs, we no longer need it to be the case that

(7) $\exists i\ i(P) = \{x : Fx\}$

but only that the following second-order quantification be true:

(8) $\exists I\ \forall x\ (I(P,x) \leftrightarrow Fx)$

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5 There may be independent, informal reasons for regimenting talk of interpretations of predicates in terms of second-order vocabulary cf. Williamson, 2003, 453. – For discussion and development of the higher-order approach to the problem of absolute generality, see also Boolos 1985, Rayo and Uzquiano 1999, and Rayo 2006.

6 As is customary, I use uppercase letters for higher-order variables. For orders greater than 2, the order of the variable is indicated with a superscript. Note that I also use uppercase letters such as 'P' and 'S' as *first*-order *meta-lingustic* variables ranging over object language predicates and sentences, respectively.

On this approach, our counterpart to the singular term 'i_R' in (3) is a *predicate* 'I_R'. Since the universal quantifier 'for every object x' is first-order, it cannot be instantiated with that predicate, so the derivation of the inconsistent (4) is blocked.

Let us briefly consider how to interpret *names* on the higher-order approach. The simplest option is to restrict 'I' by the condition 'for every name n, $\exists x\ (I(n,x)\ \&\ \forall y\ (I(n,y) \rightarrow x = y))$' thus ensuring, roughly speaking, that every interpretation associates every name with a unique object. In what follows, I shall prefer the technically slightly less convenient approach of interpreting names by using a separate variable 'i' which forms a singular term when combined with one, thus emphasizing the difference between the way names and predicates are interpreted on the higher-order approach.

In order for the present proposal for a semantics for languages with absolute generality to work, we require the full strength of higher-orderism. Evidently, if a second-order quantification such as (8) could be adequately paraphrased by first-order quantifications over objects of some sort, we should immediately fall back into contradiction. So the use of higher-order quantification to regiment talk of interpretations helps only given higher-orderism's rejection of any such paraphrase. Similarly, unless it is denied that an adequate semantics can also be developed for the meta-language of L_1, the move to higher-order quantification helps only given higher-orderism's rejection of a first-order semantic account of second-order quantification. For on any such account, a second-order quantification such as (8) is true only if there is a suitable object (set) included in the range of the bound second-order variable.[7] Any such account would therefore render (8) equivalent to a first-order quantification like (7), once again reintroducing inconsistency.

Moreover, by an analogue of the argument from (3) to (4), one can show that, roughly speaking, an adequate semantics for a *second-order language* like our meta-language for L_1 must employ *third-order* quantification to regiment talk of interpretations.[8] A semantics for that meta-meta-language then must make use of fourth-order quantification, and so on.[9] Thus, unless we at some point deny even the in principle possibility of constructing an adequate semantics for a language we make use of in semantic theorizing,

[7] I set aside here (and throughout the paper) the possibility of a substitutional semantics for second-order quantification.

[8] The same conclusion can also be reached through a related but slightly simpler argument from cardinality considerations; see Linnebo and Rayo, 2012, Appendix B. Thanks here to Øystein Linnebo.

[9] Strictly speaking, the situation is slightly more complicated, but the crucial point remains: as long as we allow for the possibility of giving an adequate semantics for whatever language we use in constructing a semantic theory, we keep having to introduce additional ideology of higher and higher orders cf. Linnebo, 2006, 152f, n8. For a detailed examination of these issues, see Rayo 2006.

we are forced up through the entire hierarchy of (finite) orders of quantification. So the approach seems to also depend on the commitment of higher-orderism to the legitimacy and irreducibility of quantification of any finite order.[10]

3 Linnebo's Objection

Linnebo tries to show that higher-orderists – he calls them 'type-theorists' – suffer from a peculiar kind of expressive limitation: 'on their view, there are certain deep and interesting semantic insights that cannot properly be expressed' 2006, 154. His argument proceeds in three steps. In the first step, Linnebo introduces a notion of the *semantic value*, or *semantic contribution*, of an expression. The second step consists in a series of claims concerning the view higher-orderists must hold with respect to the nature of the semantic contributions made by various sorts of expressions. In the third step, Linnebo argues that in virtue of holding that view, higher-orderists are committed to certain claims that cannot, by their own lights, be properly expressed.

So as to have an idea of where we are going, it is helpful to have in mind what sort of claim might be an instance of this kind of elusive commitment of higher-orderism. Here are the examples Linnebo offers (ibid.):

Infinity. There are infinitely many different kinds of semantic value.

Unique Existence. Every expression of every syntactic category has a semantic value which is unique, not just within a particular type, but across all types.

Compositionality. The semantic value of a complex expression is determined as a function of the semantic values of the expression's simpler constituents.

Let us grant that if higher-orderists are committed to these claims, but cannot, by their own lights, express them, then that result counts strongly against higher-orderism. So let us examine the antecedent of this conditional.

By way of explaining the intended notion of the semantic value of an expression, Linnebo writes:

[10] *If* we also allow for a *single* language containing quantifiers of *every* finite order, and if we allow that for that language, too, we can construct an adequate semantics, we have to go beyond the hierarchy of finite orders of quantification. Since the desire to deal with absolute generality by itself does not seem to force us to allow for such a language, I set this (considerable) issue aside for the purposes of this paper. For discussion, see Linnebo and Rayo, 2012.

> Each component of a sentence appears to make some definite contribution to the truth or falsity of the sentence. This contribution is its *semantic value*. It further appears that the truth or falsity of the sentence is determined as a function of the semantic values of its constituents. This is the *Principle of Compositionality*. Linnebo, 2006, 154

This gloss relies on an intuitive notion of an expression's (semantic) *contribution* to the truth or falsity of a sentence in which it occurs. That notion is assumed to have three important properties. First, it is assumed to have *general*, cross-categorical application, so that *every* component, of whatever syntactic category, of a given sentence makes a semantic contribution to the sentence's truth or falsity. Second, it is assumed to satisfy a *uniqueness* constraint. For each component of a given sentence, there is *exactly one* thing that is its semantic contribution to the truth-value[11] of the sentence. We may summarize these two assumptions in the following principle:[12]

(SC_1) Every (meaningful, unambiguous) expression makes a unique semantic contribution to the truth or falsity of sentences in which it occurs.

Finally, it is assumed that the truth-value of a given sentence is (functionally) determined by the semantic contributions of its constituents.

(SC_2) For each sentence, its truth or falsity is determined by the semantic contributions of its constituents.

The notion of an expression's *semantic value* is then identified with this intuitive notion of its semantic contribution, or perhaps some theoretical explication of it.

It is prima facie plausible that there is a notion of semantic contributions that satisfies (SC_1) and (SC_2), or at least some refinement of them. It should be noted, though, that the assumption that there is such a notion appears to have a dialectically somewhat peculiar status. For instance, (SC_2) seems very closely related to *Compositionality*, a claim that

[11] It is worth emphasizing that by speaking informally of a sentence's truth-value I do not mean to commit myself to the claim that the best semantic treatment of sentences assigns to them a special sort of object, namely a truth-value. Indeed, the most natural implementations of the kind of semantic views I shall suggest to the higher-orderist do not do so, but take a shape closer to the Davidson-Tarski tradition in which we simply specify truth-conditions for sentences without assigning to them objects as their truth-values.

[12] Unlike our above formulations, (SC_1) implies that expressions make the same contributions to all sentences in which they occur. At least for the kinds of formal language we are concerned with, this further assumption appears unproblematic.

according to Linnebo, higher-orderists would have to consider inexpressible.[13] If so, and if the same is true of (SC_2), then according to higher-orderism, the very sentences used in the attempt to *introduce* the notion of a semantic value would seem defective in that they could not possibly express what they are used to try to express. In that case, it seems that higher-orderists should have to consider Linnebo's argument not as a proof that they have inexpressible commitments, but as a *reductio* on the assumption that (SC_1)–(SC_2) succeed in characterizing a coherent notion. Of course, this would not mean that higher-orderists have nothing to worry about. For since it does appear initially plausible that the principles characterize a coherent notion, the result that they do not would also seem to count against higher-orderism. Our observation does hint, however, that strictly speaking, the expressibility of certain putative semantic commitments may not be exactly the right issue for higher-orderists to worry about. For now, we proceed on the assumption that (SC_1)–(SC_2) successfully characterize a coherent notion of semantic contributions.

We turn now to the second step in Linnebo's argument, in which he offers a number of claims about the semantic contributions of various sorts of expressions that he takes higher-orderism to be committed to. I quote the pertinent passage in full.

> We have seen that the type-theorists [deny] [...] that interpretations are objects. We have also seen that this forces the type-theorists up the hierarchy of higher and higher levels of quantification. This commits the type-theorists to a deep and interesting semantic view. On this view, proper names make a distinctive kind of semantic contribution to sentences in which they occur, namely the objects to which they refer. Likewise, monadic first-order predicates make a distinctive kind of semantic contribution: loosely speaking, a function from objects to truth-values, but, according to the type-theorist, properly represented only by means of second-order variables. And so it continues up through the types: for each natural number *n*, monadic *n*'th-order predicates make a distinctive kind of semantic contribution, properly represented only by means of (*n*+1)'th-order variables. Linnebo, 2006, 154.

Let us first try to see exactly what the claims are that Linnebo thinks higher-orderists are committed to, and then ask whether they are indeed so committed. It is best to begin with the case of monadic first-order predicates. We may contrast two (partial[14]) readings, one

[13] The analogous observation applies to the uniqueness condition in (SC_1) and Linnebo's *Unique Existence*.

[14] My formulations lack the explicit implication that the contributions of monadic first-order predicates form a 'distinctive kind'. If they do, this will presumably follow from the description of what they are like.

fairly literal and one less so, of Linnebo's formulation of the higher-orderists' view of their semantic contributions and in particular of the description of some contributions as 'properly represented only by means of second-order variables':

(L.1) For every monadic first-order predicate *P*, there is something which is the semantic contribution of *P*. Loosely speaking, that thing is a function from objects to truth-values. Strictly speaking, it is properly represented only by a second-order variable.

(L.2) Loosely speaking, for every monadic first-order predicate *P*, there is a function from objects to truth-values which is the semantic contribution of *P*. Strictly speaking, only an analogue of this claim is true, in which the first-order quantification over functions from objects to truth-values is replaced by a monadic second-order quantification.

(L.1) is closer to Linnebo's actual formulation, so we shall consider it first. Setting aside the part which is supposed to be only loosely correct, an obvious semi-formal rendering of the claim reads as follows:

(9) $\forall P\ \exists x$ (x is the semantic contribution of P & x is properly represented only by a second-order variable).

(9), and thus (L.1), has a much simpler defect than giving rise to inexpressible commitments: it is clearly false. Surely, being a value of a given variable is sufficient for being properly represented by it. So every value of the first-order variable 'x' is properly represented by a first-order variable and therefore does not satisfy the open sentence 'x is properly represented only by a second-order variable'. However, there is no obvious reason why a higher-orderist should have to accept (9) or (L.1). So let us instead consider the second reading (L.2) of Linnebo's formulation.

Abbreviating 'function from objects to truth-values' by 'concept', we may regiment the claim that (L.2) presents as *loosely* correct as follows:

(10) For every monadic first-order predicate P: $\exists x$ (x is a concept & x is the semantic contribution of P).

What is strictly true, according to (L.2), is a paraphrase of (10), in which the first-order quantification over concepts is replaced by a monadic second-order quantification. If we simply substitute a second-order variable 'X' for the first-order 'x' in (10) and drop the restriction to concepts, we obtain

(11) For every monadic first-order predicate P: $\exists X\ X$ is the semantic contribution of P.

(11), however, is ill-formed, for instantiating the second-order quantification with, say, the predicate 'is red' yields the ungrammatical result 'is red is the semantic contribution of P'. To obtain a well-formed higher-order paraphrase of (10), we need a second-order predicate in place of 'is the semantic contribution of'. Thus, let 'SC' form a sentence from a monadic predicate and a singular term, so that

(P^1) For every monadic first-order predicate P: $\exists X$ SC(X, P).

is well-formed. Now if this sentence is to express an answer to the question what, on the sense of 'semantic contribution' delineated by (SC_1)–(SC_2), the semantic contributions of monadic first-order predicates are, then 'SC' must *express* that sense of 'semantic contribution'. So on our present proposal, Linnebo takes higher-orderism to be committed to two claims, namely to the claim that the intended notion of semantic contributions can be expressed by the second-order predicate 'SC', and to the claim expressed by (P^1) on that interpretation of 'SC'. Call the conjunction of these claims (P^1+).

I take it that this is what Linnebo has in mind. So let us now turn to the case of names. Linnebo claims that according to higher-orderism, the semantic contributions of names are the objects to which they refer. Since objects are properly represented only by *first-order* variables, given what we have just said, the appropriate analogue to (P^1) for names is

(N) For every name n: $\exists x$ sc(x, n).

Note that in order not to violate the rules of grammar, we have to take 'sc' in (N) to be a *first-order* predicate, taking names or name variables in both argument places. As before, if (N) is to express an answer to the question what, on the sense of 'semantic contribution' delineated by (SC_1)–(SC_2), the semantic contributions of names are, then 'sc' must *express* that sense of 'semantic contribution'. So again, the view ascribed to higher-orderism comprises two components: the claim that the intended notion of semantic contributions can be expressed by the first-order predicate 'sc', and the claim expressed by (N) on that interpretation of 'sc'. Call the conjunction of these claims (N+).

One may well suspect that if Linnebo is right so far, then already, things have gone badly wrong for higher-orderism. For one may well think that expressions which, like 'sc' and 'SC', belong to different syntactic categories, cannot possibly express one and the same notion. And although this is not beyond reasonable doubt, the strategy for higher-orderists that I want to explore in this paper is to deny the claims about names and predicates just presented. I shall therefore grant, for the sake of argument, that a version of higher-orderism that is committed to these claims is untenable, and provide only a brief sketch of how we might obtain that such a view carries commitments that are inexpressible by its own lights.[15] Thus, suppose we hold that 'sc' and 'SC' express a single notion of semantic contribution. We should then also hold that the infinity of analogues of these predicates for higher-order predicates express that notion as well. Given that we have this notion, it seems as though we should be able to make *general* claims about the entire range of semantic contributions, such as that it divides into infinitely many kinds – this is the claim *Infinity* mentioned above. Yet by higher-orderist lights, any given variable belongs to some order and therefore can range only over some semantic contributions: first-order ones over those suitable for names, second-order ones over those suitable for predicates, etc. If so, then although we have a general notion of semantic contributions, and there *seem* to be interesting general truths about the entirety of semantic contributions, no such truths can be expressed.

What reasons are there for taking higher-orderism to be committed to (N+) and (P^1+)? In the passage quoted above, Linnebo seems to consider this an immediate consequence of the higher-orderists' proposal concerning the regimentation of talk of interpretations. On that proposal, we have the following principles concerning the interpretation of names and predicates:

(12) $\forall i\, \forall n\, \exists x\; x = i(n)$
(13) $\forall I\, \forall P\, \exists X\, \forall x\, (Xx \leftrightarrow I(P,x))$

The condition under which a given first-order predication $\ulcorner Pn \urcorner$ is true under an interpretation of the name and an interpretation of the predicate is then as follows:

(14) $\ulcorner Pn \urcorner$ is $\text{true}_{i,I} \leftrightarrow I(P, i(n))$.

The most plausible argument from these observations to the claim that higher-orderism is committed to (N+) and (P^1+), it seems to me, runs roughly as follows. *Firstly*, the singular term '$i(n)$' plays a certain role in the higher-orderist's treatment of names (12) such that that role is played in the treatment of first-order predicates (13) by the predicational

[15] It may be worth emphasizing that the concession I make here for the sake of argument is substantive, and that it may also be possible to develop a tenable version of higher-orderism that endorses (P^1+) and (N+) but blocks Linnebo's objection at a later stage. I make the concession here to bring into clearer view the premise in Linnebo's argument that I will argue the higher-orderist can reject.

expression '$I(P,\ldots)$'. At least in a rough approximation, this role can be described by saying that '$i(n)$' and '$I(P,\ldots)$' serve to specify the extra-linguistic correlates, or *denotations*, under the relevant interpretations, of the names or predicates in question. *Secondly*, any precise and strictly adequate reformulation of this rough, informal description will have to make use of a single, unambiguous expression to regiment the informal talk of extra-linguistic correlates or denotations. *Thirdly*, whatever notion is expressed by that putative expression will render true (SC_1)–(SC_2) and thus constitute the obvious candidate for the general notion of the semantic contribution of an expression.

The at any rate plausible third premise is actually redundant in that a straightforward variation on the above considerations regarding 'sc' and 'SC' will show the higher-orderist to be in trouble as soon as he accepts the first two premises. For the attempt to regiment talk of denotations by a single, unambiguous expression generates just the difficulties we saw to arise given the assumption that 'sc' and 'SC' express a common notion. It also seems very unpromising to deny that the somewhat vague first premise admits of a reading on which it is correct by the lights of higher-orderism. I therefore recommend that the higher-orderist reject the second premise. Since that premise, and more generally the claim that there is a general notion of denotation, has some intuitive appeal, a mere denial of that claim seems unsatisfactory. The higher-orderist's case would be significantly strengthened if he could give an explanation why the claim, although false, seems intuitively attractive. Ideally, he would show that we should *expect* the claim to seem plausible if his theory is correct.[16] My aim in what follows is to make a case that he can do so.

The next section shows how the higher-orderist can describe the common role played by '$i(n)$' and '$I(P,\ldots)$' in his treatments of names and predicates, respectively. I suggest that, in combination with the observation that in natural language, we habitually and unreflectively employ devices of nominalization to transform non-nominal expressions into nominal, first-order ones, this account provides at least a partial explanation of the intuitive appeal of the assumption of a general notion of denotation. That explanation can be supplemented by the point that it is intuitively plausible that there is a general notion of semantic contributions, playing roughly the role indicated by (SC_1)–(SC_2), and that the putative general notion of denotation would seem suited to play this role.

[16] This way of viewing the matter was suggested to me by Robbie Williams.

4 A Hierarchy of 'Denotation'-like Expressions

Where i is an interpretation of the names of some language L and n is such a a name, $i(n)$ is the denotation of n under i. We may write this formally as follows ('$=_1$' is the ordinary identity-predicate; the point of the subscripts will become clear shortly):

(15) $\forall i\, \forall n\ \mathrm{den}^i_1(n) =_1 i(n)$

Now it appears that the counterpart to '$i(n)$' in the higher-orderist's treatment of predicates is the expression '$I(P, \ldots)$'. First, to avoid the inconvenience of the gap-notation, let us think of second-order 'I' not as a two-place predicate but an operator that combines with a name to form a one-place predicate. We then write '$I(P)(x)$' instead of '$I(P,x)$' and '$I(P)$' instead of '$I(P, \ldots)$'. Roughly speaking, then, it appears that where I is an interpretation of the first-order predicates of L, and P is such a predicate, $I(P)$ is the denotation of P under I. Strictly speaking, however, according to the version of higher-orderism that I propose, this is incorrect. Specifically, it is incorrect as long as 'denotation' is taken here in the same meaning in which it is taken in connection with names. So how can we do justice to the impression that '$I(P)$' plays the same role in connection with predicates that '$i(n)$' plays in connection with names?

The strategy I shall pursue is as follows. First, we formulate strictly adequate counterparts to (15) for first- and higher-order predicates. We then show that the counterparts to 'den^i_1' in these principles, although different in meaning from it and from each other, behave in an important way *analogously* to it. We may then take this analogy to be what underlies the impression of a shared role. In addition, we may explain the intuitive appeal of the claim that there is a single notion of denotation on which '$i(n)$' and '$I(P)$' serve to specify the denotations of names and predicates, respectively, – i.e. the second premise of the argument from the end of the last section – as resulting in part from a natural but mistaken overestimation of the extent of the analogy.

We begin by formulating an analogue of (15) that has the right syntactic shape to express a correct principle concerning first-order predicates. Since we replace '$i(n)$' by '$I(P)$', we need to replace '$=_1$' by a second-order counterpart that conjoins first-order predicates rather than names. We shall write that expression '$=_2$', leaving open for now its exact interpretation. Accordingly, we need to replace the name-forming functor 'den^i_1' by a predicate-forming analogue, which we write 'den^I_2', obtaining as (15)'s counterpart for first-order predicates:

(16) $\forall I\, \forall P\ \text{den}_2^I(P) =_2 I(P)$

We can also say what analogues of (15) and (16) for higher-order predicates will look like. For the sake of generality, we indicate the form shared by all these principles with (15) and (16). Say that names are constants of order 0, and let instances of 'c^n' range over the n'th-order constants of L. Rewrite 'i' as 'i^1', 'I' as 'i^2', its third-order counterpart as 'i^3', etc. Then for any finite n, the instance of

(S-den$_n$) $\forall i^n\, \forall c^{n-1}\ \text{den}_n^{i^n}(c^{n-1}) =_n i^n(c^{n-1})$

obtained by replacing 'n' with the numeral for n is the counterpart for n'th-order constants to (15) and (16).[17] Again, we postpone the question of the interpretation of instances of '$=_n$' for $n > 1$.

It is worth being clear first of all about what we cannot claim by way of an analogy between the various 'denotation'-like functors. Just as we cannot hold them to be synonymous, we also cannot hold them to denote a common function. Indeed, on the present view, there is no sense of 'denote' on which they all denote something. Nor can we hold that there is a range of functions, each of them denoted, in some sense, by one of the functors, which fall under a common, unambiguous predicate we could take to characterize a kind of denotation-like function. More generally, what analogies our functors sustain must be expressible in meta-linguistic terms, without reference to the meanings of the functors. The most salient and interesting such analogy, I think, is that all these functors show what I call a quasi-*disquotational* behaviour with respect to expressions of their associated category. Let me explain what I mean by this.

It is often observed that the truth-predicate has a kind of disquotational function, in that appending 'is true' to the quotation-name of a given interpreted (object language) sentence S results in a meta-language sentence which is equivalent to (a meta-language translation of) S.[18] Something similar can be said about 'the denotation of': prefixing it to the quotation-name of a name n produces a name that is equivalent to n. And the phrase 'applies to' has the analogous feature with respect to first-order predicates: appending it to the quotation-name of a predicate P produces a predicate that is equivalent to P.

Can we specify a general sense for 'disquoter for expressions of category C' on which each of our three examples comes out a disquoter for expressions of the respective cate-

[17] Note that (S-den$_n$) is a somewhat peculiar schema in that, in contrast to more familiar schemata such as 'Fa' or '$p \rightarrow q$', its instances have different internal syntactic structures.

[18] I shall henceforth suppress the parenthetical qualifications. We may assume the meta-language to extend the object language so that the relevant translation is simply the identity-mapping.

gories? In a first step, we say that

(Dis) D is a disquoter for expressions of category $C \leftrightarrow_{df}$:

D combines with a name to form an expression of category C &

if n is a name of an expression e of category C,

then $\ulcorner D(n) \urcorner$ is equivalent to e.

In a second step, we need to see if we can make satisfactory sense of the notion of equivalence appealed to in (Dis). Since it is supposed to apply to names, predicates, and sentences alike, we cannot explain it in terms of a semantic relation such as co-denotation.

For names, the intended condition of equivalence is satisfied iff $\ulcorner D(n) = e \urcorner$ is true. For sentences it is satisfied iff $\ulcorner D(n) \leftrightarrow e \urcorner$ is true. For predicates, it is satisfied iff $\ulcorner \forall x\ (D(n)(x) \leftrightarrow e(x)) \urcorner$ is true. The expressions '$\leftrightarrow$', and '$\forall x\ (\ldots x \leftrightarrow \_\_x)$' are '='-like in that they satisfy analogues of the intersubstitutivity principle characteristic of '='.[19] That is, from $\ulcorner S \leftrightarrow T \urcorner$ and arbitrary Φ we may infer the result $\Phi(S/T)$ of replacing some or all occurrences of S in Φ by T, and from $\ulcorner \forall x\ (Px \leftrightarrow Qx) \urcorner$ and arbitrary Φ we may infer $\Phi(P/Q)$. Analogous constructions for higher-order predicates can be obtained by replacing the first-order quantifier with an appropriately higher-order one. We now define a general notion of an identity-like operator for expressions of category C as follows:

(Id) Id is an identity-like operator for expressions of category $C \leftrightarrow_{df}$:

Id combines with two expressions of category C to form a sentence &

if e_1 and e_2 are expressions of category C,

then from $\ulcorner Id(e_1, e_2) \urcorner$ and arbitrary Φ we may infer $\Phi(e_1/e_2)$.

We may then take the intended general condition of equivalence between expressions e_1 and e_2 in (Dis) to be that filling the argument places of an identity-like operator for expressions of category C with e_1 and e_2 results in a true sentence.

Let us return to the instances of (S-den$_n$). We are now in a position to say how the higher-order versions of '=' we have made use of can be interpreted: we may simply regard them as abbreviations of the appropriate identity-like constructions just characterized.[20] Given this interpretation, the instances of 'den$_n^{i^n}$' turn out to show a quasi-disquotational behaviour. Since they carry a parameter for an interpretation, they of course

19 This of course holds only for extensional languages; for intensional languages, we should have to make use of intensional operators to construct '='-like expressions for non-names.

20 We may also read 'den$_2^{i^2}$' as 'applies$^{i^2}$ to'. Unsurprisingly, there do not seem to be any independently familiar expressions of English corresponding to the 'denotation'-functors for higher-order predicates.

do not satisfy the definition (Dis). Relative to a given interpretation i of names, however, when combined with a name of a name c, $\ulcorner \text{den}_1^i \urcorner$ forms a name which is equivalent to any expression that adequately translates c under i. Indeed, if we allow $\ulcorner i(c) \urcorner$ as a translation of c under i, this is immediate from the relevant instance of (S-den$_n$). Corresponding results are equally straightforwardly obtained for the other instances of '$\text{den}_n^{i^n}$'. The higher-orderist's many 'denotation'-predicates are therefore analogous in that they all behave quasi-disquotationally for expressions of the relevant categories. Based on this observation, we can now describe a role that is played by '$i(n)$' in the higher-orderist's treatment of names, and by '$I(P)$' in the treatment of first-order predicates, and so on: they are equivalent to – i.e. intersubstitutable with – quasi-disquotations of the relevant expressions.

This observation also enables us to give a plausible explanation of the intuitive appeal of the claim that there is a unified notion of denotation. For simplicity, we consider the de-parameterized 'denotation'-functors that disquote with respect to intended interpretations. Their shared disquotational character means that their inferential profiles are essentially the same, modulo the differences in grammatical category between the expressions they combine with. Thus, where 'a' is an (object language) name, we have 'den_1("a") $=_1$ a', from which we obtain the intersubstitutivity of 'den_1("a")' and 'a', and where 'F' is a first-order predicate, we have 'den_2("F") $=_2$ F', from which we obtain the intersubstitutivity of 'den_2("F")' and 'F'. It is therefore natural to suspect that these principles are merely superficially different manifestations of general laws of general notions of denotation and identity. Moreover, we could easily subsume the higher-order principles under the putative general first-order laws if we allowed ourselves free use of familiar devices for *nominalizing* non-nominal expressions, turning, say,

(17) den_2('is heavy') $=_2$ is heavy

into the first-order paraphrase

(18) den_1('is heavy') $=_1$ the property of being heavy

Since this kind of transformation is something that in natural language and in informal theorizing, we frequently and unreflectively carry out, it is to be expected that we should find it very plausible to consider as consequences of a theory claims that can be obtained from the theory essentially just by means of nominalization.[21]

[21] It is worth mentioning that the higher-orderist may have independent reasons to use this kind of argument to explain the intuitive appeal of claims he rejects. In particular, it seems intuitively plausible

5 Conclusion

Higher-orderism holds quantification of any (finite) order to be a legitimate, sui generis kind of quantification, an adequate semantics for which must itself employ quantification of at least the same order. Linnebo has argued that the view commits its proponents to the truth of certain general semantic clams that by their own lights are not expressible. In response, I have shown that his objection relies on the assumption that there is a general notion of the semantic contribution of an expression that picks out an extra-linguistic correlate of that expression. I have then tried to make a case that this assumption, and thus Linnebo's objection, can plausibly be resisted by higher-orderists. Specifically, I have suggested that to speak of the extra-linguistic correlates of expressions, higher-orderists may use a hierarchy of predicates of different orders, mirroring the hierarchy of syntactic categories in the object language. I conclude that although further investigation into the resulting view may be required before we can make a confident judgement about its adequacy, at the very least, the present proposal constitutes a promising start to a higher-orderist response to Linnebo's challenge.

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that there is a *general* notion of *existence* corresponding to the hierarchy of existential quantifiers of higher and higher orders. Here as above, it seems to me that the natural strategy for the higher-orderist is to claim that what underlies the intuition is the analogy in inferential profiles between the quantifiers combined with implicit nominalization.

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