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Go(Φ)d is Number: Plotting the Divided Line & the Problem of the Irrational

By Sandra Kroeker[*]

Plato believed that behind everything in the universe lie mathematical principles. Plato was inspired by Pythagoras (571 BCE), who developed a school of mathematics at Crotona that studied sacred geometry as a form of religion. The school's motto was "God is number," or "All is Number". Pythagoras believed that numbers represented God in pattern, symmetry, and infinity. When one of its students, Hippasus told the world the secret of the existence of irrational numbers, Greek geometry was born and Pythagoras' idea of divinity in numbers died because how could God not be perfect and symmetrical? In Plato's Republic he discusses something called The Divided Line, which is a map, of sorts, for reaching what he calls the highest Good, which is the ultimate truth where one realizes the true state of the universe and can see the world for what it really is. Many mathematicians have attempted to plot Plato's Divided Line only to come across a litany of problems and conundrums. Some have said that it the Divided Line cannot be plotted and is merely an allegory not meant to be plotted. This paper discusses some of the conundrums preventing the plotting of Plato's Divided Line (not an exhaustive list), including Whole 'vs' Separate, Equality 'vs' Ontological Dissimilarity, Linear 'vs' Non-linear, and Infinity 'vs' Finite. This paper also explores a new understanding of the Allegory of the Cave in light of 'the problem of the irrational.' In exploring the link between the Divided Line and the 'the problem of the irrational,' I was able to plot it. It was found that the Divided Line is not a line in the linear sense, but a spiral, the Golden Ratio! This paper is an example of a new category of scholarly inquiry I call "Math Theory" based on scholarly mathematical axioms in theory, rather than including actual maths. In my papers I use existing mathematical equations and place them in an encompassing theory, rather than finding new formulae to fit an existing theory.

Keywords: *Pythagoras, divided line, math theory, highest good, all is number*

Introduction

In this paper I plot Plato's Divided Line by exploring its connection to the problem of the irrational. The problem of the irrational is the existence of irrational numbers, which was highly controversial at the time of the Pythagorean school because the school's motto was "God is number" (Aczel 2000, p. 19) or "all is number" (Boyer 1991, p. 49). The school saw only whole numbers as representing God because numbers represented God in pattern, symmetry, and infinity (Aczel 2000), not irrational numbers that are random and chaotic, with no symmetry (Fossa 2005). Irrational numbers include numbers with decimals having no intelligible pattern (Aczel 2000, p. 18), like pi. Therefore, irrational numbers were

[*]PhD Candidate, Brock University, Canada.

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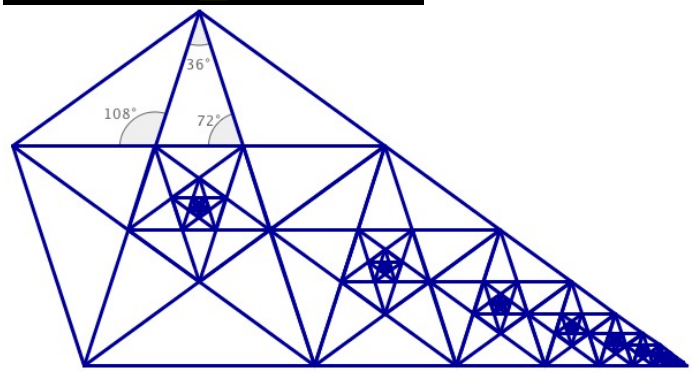
problematic, and their existence seemed to suggest that God was imperfect, so they kept the discovery of irrational numbers a secret (Aczel 2000). Hence, the "problem of the irrational" (Benjafield 2005, p. 6). Due to the controversial nature of irrational numbers, I postulate that this is why Plato did not reveal specifically, that the Divided Line is actually the Golden Ratio.

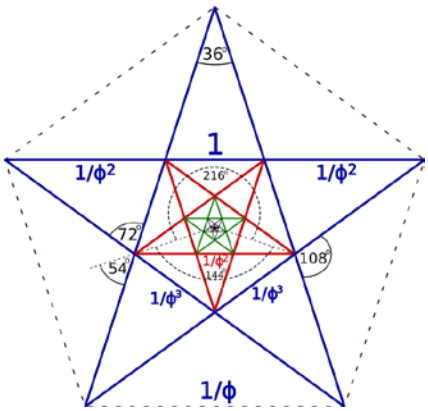
Many mathematicians have attempted to plot Plato's Divided Line only to come across a litany of problems and conundrums (Balashov 1994, Benjafield 2005). Some have said that it simply cannot be done. This paper discusses some of the conundrums involved in plotting Plato's Divided Line (not an exhaustive list) and explores its link to 'the problem of the irrational.' In overcoming some of these obstacles, I then show how and why the Divided Line has to be the Golden Ratio. Lastly, I will explore some reasons why this connection was or is not specifically stated by Plato, or anyone.

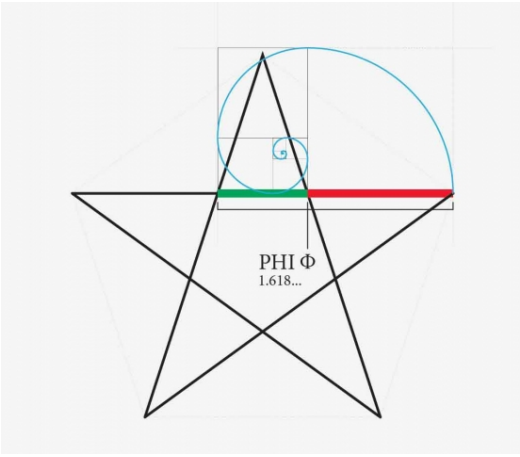
Plato believed that behind everything in the universe lie mathematical principles (Cornford 1965, Johnson and Reath 2007). Plato was inspired by Pythagoras (571 BCE), who developed a school of mathematics at Crotona that studied sacred geometry as a form of religion. "Both Pythagoras and Plato suggested that all citizens learn the properties of the first ten numbers as a form of moral instruction" (Schneider 1994, p. xxiii). The basic shapes that make up what are now called Platonic solids were revered so highly that it can be difficult to separate the math from the religion (Aczel 2000). In Plato's *Republic* he discusses something called The Divided Line, which is a map, if you will, for reaching what he calls the highest Good. The highest Good is the ultimate truth where one realizes the true state of the universe and can see the world for what it really is (Cornford 1965, Johnson and Reath 2007).

What is fascinating about the problem of the irrational and Pythagoras' idea that God is number is that there is an irrational number hiding right inside his own formula. For example, "[w]hen the Pythagorean formula is applied to a triangle with two sides equal to one, the result is that the hypotenuse is given by the equation $c^2 = 1^2 + 1^2 = 2$, so that $c = \sqrt{2}$" (Aczel 2000, p. 18), which is an irrational number.

What is also curious about the school of Pythagoras and the connection to irrational numbers is that the school was represented by the symbol of the five-pointed star within a pentagon which is inset with another five-pointed star within a pentagon and so on (Fossa 2005, Wheeler 2005).



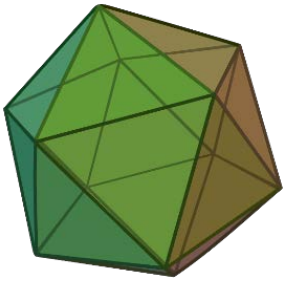




This symbol represents phi or the golden ratio, which is also an irrational number (Aczel 2000). The Golden Ratio, Spiral, Section, or Mean is represented by the equation: phi equals the square root of five plus one over two (Balashov 1994). This equals roughly 1.618. If the Pythagoreans wanted to keep the existence of irrational numbers a secret, why have one as their school's symbol? It interesting that an irrational number cannot be expressed by one *number* but can be expressed in one *symbol*. This symbol or pattern neatly sums up the ratio in one, elegant and simple design. But, first, a brief history of phi.

A Brief History of Phi

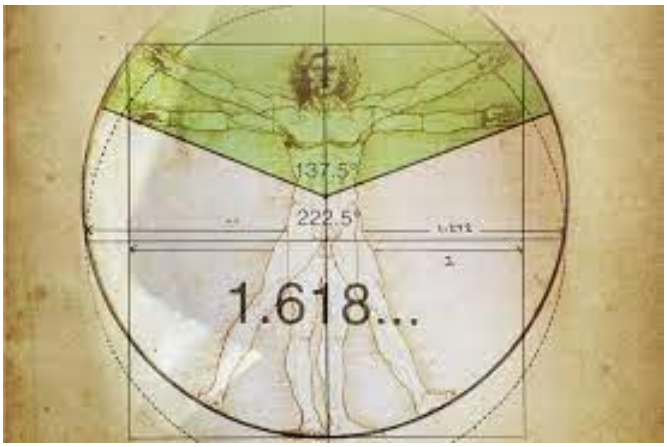
Some say that the Ancient Egyptians used phi and pi in the construction of the pyramids (Meisner 2012). This would date the sequence and its use to approximately 2575–2465 BC., when it was postulated that the pyramid of Khufu was under construction (Hemeda and Sonbol 2020). Others believe that Phidias (500 BC – 432 BC), used phi in the construction of the Parthenon (Fett 2006, Meisner 2012). Plato (circa 428 BC – 347 BC) is referenced next because of what he stated in the dialogue Timaeus (55C) about the Platonic solids or polyhedrons. It is said that Plato, like the Pythagoreans, believed it to be "key to the physics of the cosmos" (as cited in Meisner 2012, para. 4). Both the "Pythagoreans and Platonists were obsessive [about] models of harmony and proportions…but…of utmost importance" (Wheeler 2005, p. 3) was the 5th Platonic solid which "represented the kosmos" (Wheeler 2005). This fifth Platonic solid, called an icosahedron is also a representation of the irrational number phi (Fossa 2005, Wheeler 2005).



The Golden Spiral was also used by Euclid (365 BC – 300 BC) in Proposition 11 of Book II where he states" *To cut a given straight line so that the rectangle contained by the whole and one of the segments equals the square on the remaining segment*" (Porubský 2023, para. 4).

Fibonacci (circa 1170-1250 AD) is the most recognized for his sequence, which can be described as the equation Xn+2= Xn+1 + Xn (Grose 2023, para 1). The sequence looks like this: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, and so on infinitely and "each number is the sum of the two that precede it" (Ghose 2023, para. 1).

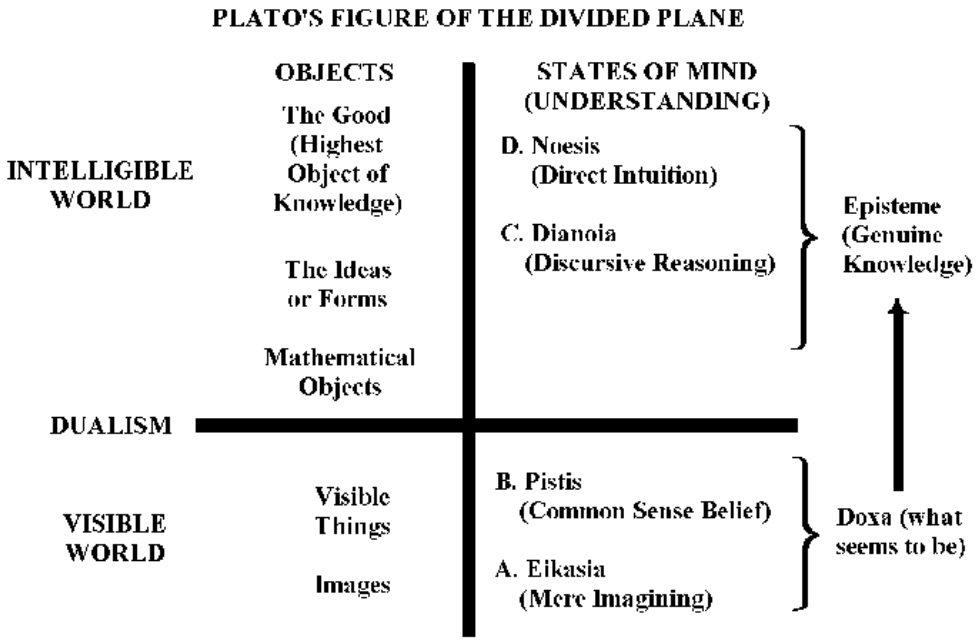
This brings us to DaVinci and his art, such as the Vitruvian man, which is an example of how these proportions work in humans.

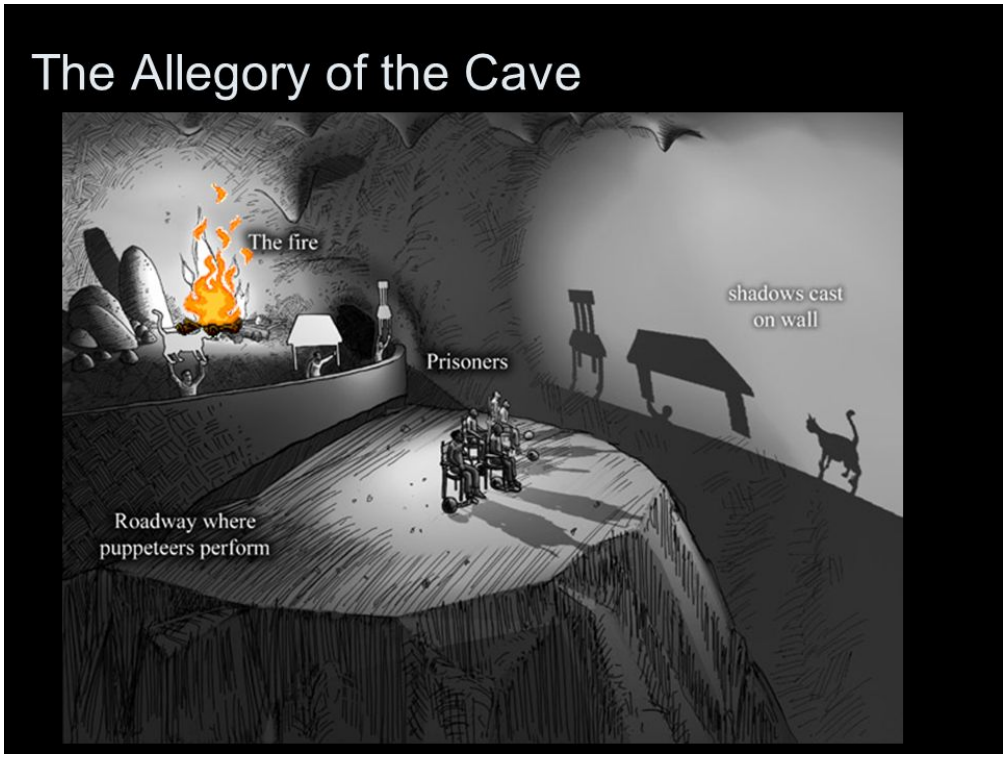


For example, the ratio of the width of the mouth to the width of the nose is the Golden Ratio, and the "total height of the body and the height from the toes to the navel" (Davis and Altevogt 1979, p. 341), is also the Golden Ratio or Section. This is where I will leave off with the history of phi. But, regarding the history of irrational numbers, when their existence was revealed to the world, Greek geometry was born and Pythagoras' idea of divinity in numbers died (Aczel 2000). Or did it? Plato was born in Greece about 100 years later. Plato was a "third generation Pythagorean" (Fossa 2005, p. 134).

Problems Plotting the Divided Line

As stated before, Plato believed that behind everything in the universe lie mathematical principles, these he refers to as 'Forms' (Aczel 2000, Cornford 1965, Johnson and Reath 2007, Sheldrake 1988). The belief that behind everything is a mathematical equation, however, remains strong today. Einstein's famous $E=mc^2$ is evidence to the truth in this statement. Some mathematicians have tried to tie all the different mathematical principles into one all encompassing principle called the Grand Unified Theory (Einstein 1956) or the Unified Field Theory (Hawking 1988). This journey is similar to those who try to construct Plato's Divided Line mathematically. This is not an exhaustive list, but many problems arise when it is attempted and most say it cannot be done (Balashov 1994, Personal communication C. Hayes October 2010). In order to properly explain this journey, I had to develop my own chart or graph comparing the Divided Line with the Allegory of the Cave. I believe the Divided Line to be the mathematical explanation of the Allegory of the cave.





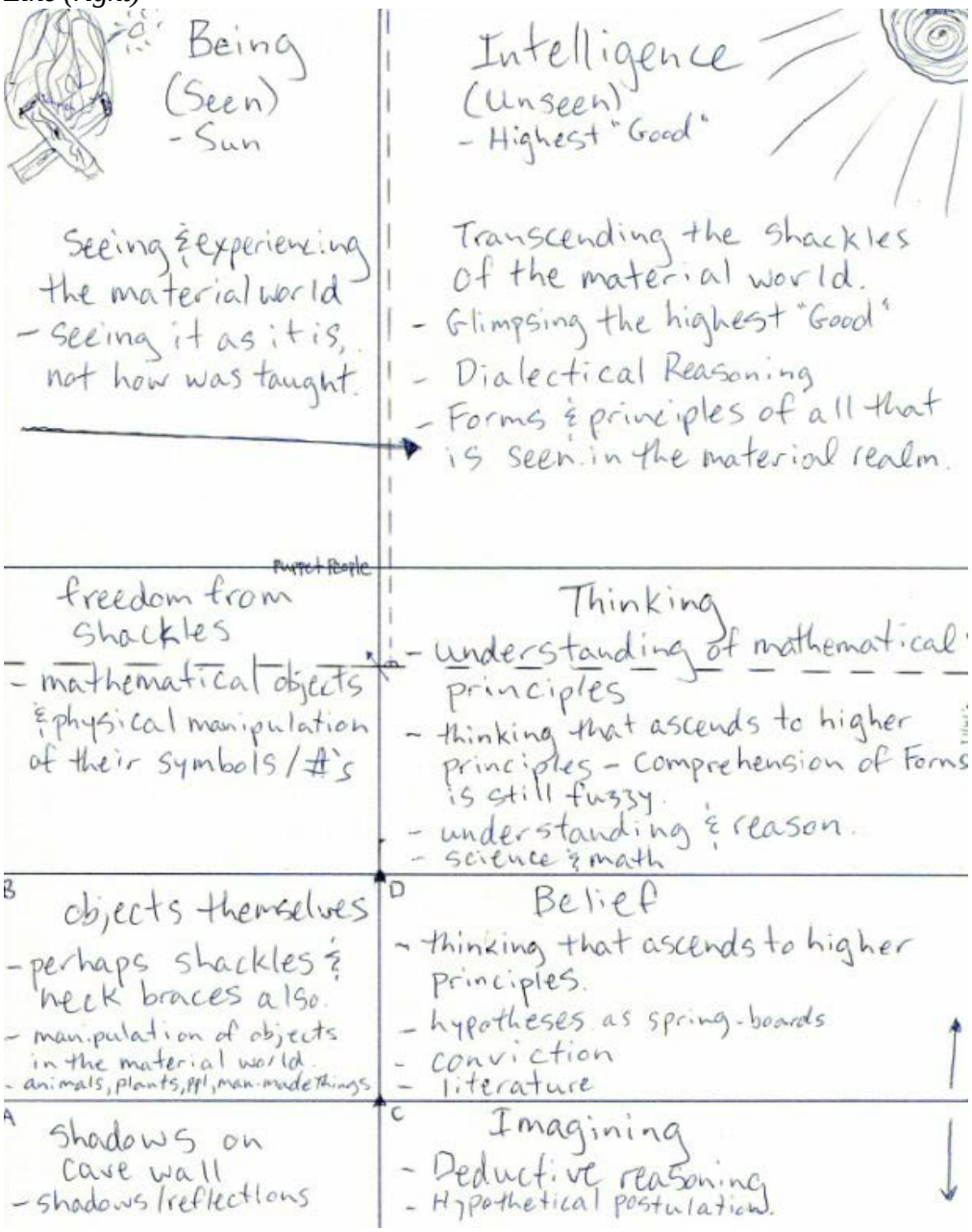
Above are diagrams found on the internet of both Plato's Divided Line and Allegory of the Cave. Below is my amalgamation. The left column is the Allegory of the Cave explanation and the right column, the Divided Line (see Figure 1).

Following Plato's Allegory of the Cave through the diagram in Figure 1, "A" represents shadows and reflections, like those seen on the cave wall; "B" represents the objects themselves that can be physically manipulated. This section also includes persons, animals, man-made things, plants etc. Moving from the material realm over to the intelligible realm, "C" represents deductive reasoning or hypothetical postulation. Assumptions are used in drawing conclusions at this stage (Johnson and Reath 2007). "D" represents studying literature and drawing conclusions from 'higher' principles. Here, hypotheses can be used as "springboards" to 'higher' understanding and reasoning which is the next level of clarity (Balashov 1994, p. 2). In the next level of clarity, science and mathematical principles are utilized in the material world's 'equivalent.' Mathematics and manipulation of its symbols and numbers is the physical representation for the higher equations which represent the Forms. These equations, however, are not completely understood or synthesized, just utilized (Johnson and Reath 2007). Level: "C" is the secular mathematics that is taught in grade school and high school that only manipulate the numbers without understanding them in their larger context. The assumptions are carried over to the university level.

The 'highest' level of the material realm is referred to as 'Being' and it represents seeing the world as it really is, and not how it is taught. Realizing this, one can move to the 'highest' Good or level of intelligence. This is where dialectical reasoning can be utilized and where the 'Forms' or guiding principles behind all physical objects can be understood or even synthesized. Now that the

Divided Line has been defined, I will move to resolving some issues or problems faced while trying to plot it.

Figure 1. *A Chart Comparison of Plato's Allegory of the Cave (left) & the Divided Line (right)*



Source: Kroeker 2009.

The problem of whole 'vs' separate parts. One problem includes, how can the line be a whole line at the same time having mutually exclusive sections in their respective 'boxes?' For example, the Golden Ratio works by the principle that "the ratio of its parts is equal to that between a part and the whole" (Balashov 1994, p. 294). Des Jardins (1976) states that "since the whole cannot exclude one

of its own parts, it cannot take part in any relation founded on mutual exclusion" (p. 494). This is an example of Russel's paradox. A simple example of this paradox can be found in a letter the Apostle Paul wrote to Titus: "All Cretans are liars, one of their own poets has said so" (as cited in Aczel 2000, p. 179). So, if this statement is true, then the poet is also lying, meaning the statement is entirely false. Paradoxes seem to not have a resolution, what causes paradoxes are dualistic or binaristic thinking. Binaries create paradoxes because life is not all or nothing. Categories in a binary are not mutually exclusive. The binaries or paradoxes that come to light when trying to plot the Divided Line are the reason why there is trouble plotting the Line. Since my previous research involves the breaking down of binaries, binaristic thinking, and resolving dualisms, perhaps there is a resolution after all to whole 'vs' parts. For example, I am a whole person made up of different parts and each of these parts have their own functions, but all work for the whole. The interpretation that the separate boxes in the Divided Line are mutually exclusive may not be correct. Plato does not seem to say this in the *Republic*. Perhaps the problem of plotting the Divided Line can be solved through resolving the dualisms that come about when trying.

The problem of equality 'vs' ontological dissimilarity. According to Plato's Divided Line, the subsections A and B relate to their equivalent sections in the intelligible realm (c and d) in a ratio that equals A + B (Balashov 1994). Therefore, A + B = C + D or A/B = C/D (Personal communication C. Hayes, October 2010). This relationship can also be interchanged showing that B = C (Balashov 1994). This implies equality of the sections, not only static equality, but unlimited or extended equality (Sayer 1983). But this is where the conundrum begins because Plato says that as one ascends, clarity increases. This implies ontological superiority as one climbs up the Line, not equality. Plato also states that the intelligible realm is superior to the material realm. Therefore, how can A, B, C and D all be equal if ontological superiority of the Highest Good is implied?

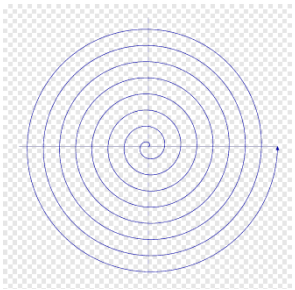
Another question related to this, that arises when trying to plot the Divided Line is: How is ontological superiority displayed? Does one ascend or climb up a line? Does that mean the line is vertical or is it horizontal? If it is true that 'as one ascends, clarity increases,' then it suggests that the line is vertical, rather than horizontal. When one normally thinks of a number line, (one from an English-speaking background), it is usually pictured as a horizontal line moving from left to right. Because ascension implies rising to a higher place, in my own diagram (Figure 1), I represent the Divided Line as a Rectangle, both vertically and horizontally, to resolve this issue.

The idea is that the line is to be divided into two unequal sections and then each section divided again using the same ratio (Balashov 1994). The two major, unequal sections represent the material realm of the seen, or "being" from the world of the unseen, or the "intelligible" (Johnson and Reath 2007, p. 54). It is important to note that the two major realms are ontologically unequal as the intelligible realm is considered superior to the visible/material realm. This makes sense because the physical world is subject to decay (as according to the second law of Thermodynamics), whereas the intelligible realm is not (first law of Thermodynamics). The ontological ranking is represented by the size of the 'box'

in Figure 1, the larger the area, the more ontologically superior that realm is. It is also divided in half horizontally; the lower half is illusion and the upper half, truth (see the broken line in Figure 1). Therefore, the line is not just horizontal or vertical, but both, making it possible that the Divided Line is not a line in the 'traditional' sense.

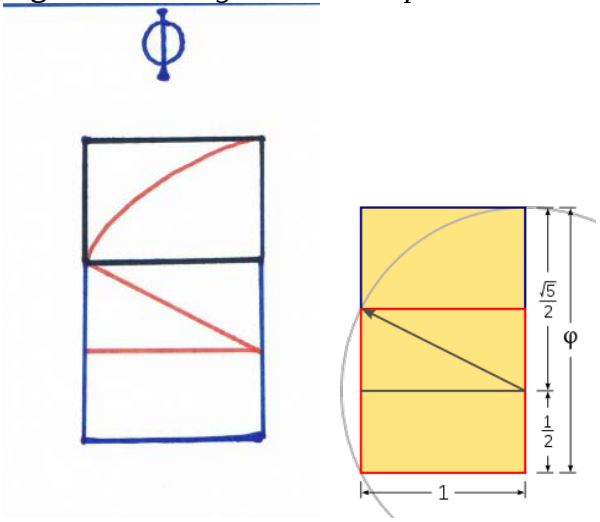
If, however, the idea is that the line is to be divided into two unequal sections and then each section divided again using the same ratio (Balashov 1994) and if phi works by the equal proportion of the smaller to the larger section as the larger to the entire line ["the smaller is to the larger as the larger is to the whole" (Benjafied 2005, p. 6)], then it would not be irrational to suggest that the Divided Line is not a line at all, but a spiral; the Golden Spiral. This makes sense because equality and ontologically dissimilar ideas can be represented here. For example, the ratio is an equal proportion, suggesting equality, but the ratios can be expressed as being smaller or larger examples of the proportion, implying ontological dissimilarity. By stating that the Line is not a linear line, but rather a pattern or a spiral, aids in solving the conundrum of the Divided Line because this way the proportions in the diagram can create a movement along the Line, as well as unfold on an equal plane, thus addressing the problem of ontology. All proportions represented would be inferior to the "Line" itself, which could be the location of the Highest Good. If the Highest Good is the line itself, this would make the Highest Good ontologically superior, as well as equal to all the proportions on the line, thus resolving the binary of equality and ontological dissimilarity.

The problem of linear 'vs' non-linear. A major obstacle to seeing the superposition of both equality and ontological superiority is due to binaristic thinking. We tend to think of things in a linear fashion or hierarchy, when in reality, the relationship is neutral. If one is to reach the Highest Good, it seems as though you are to make progress towards something; thus, ascension is assumed. Referring to something as the "highest" Good, it is misleading because it makes one think it should be plotted linearly. Of course, calling it a "Line" also implies linearity. However, it is possible to ascend on a spiral. As one accrues knowledge, they move along the spiral, starting with 0, 1, 1, 2, 3 etc…If the spiral gets big enough, all sections or portions on the spiral-line are contained within and can be seen together on a grid, rather than separated linearly on a straight line, thus resolving the linear/non-linear dichotomy involved in the plotting of the Divided Line because it is both and they are not mutually exclusive categories.



The diagram or chart built from Plato's Divided Line, however, does not look like a line or spiral, but a rectangle. Euclid, however, shows "how to cut a line segment in this manner appears earlier in an equivalent form stated in terms of rectangles" (Porubsky 2011, para. 4). Therefore, the way the Golden Spiral, or in this case, the Fibonacci sequence starts off can be represented inside a rectangle (see Figure 2). This demonstrates how the Golden Spiral can start off as a linear line or a basic two-dimensional rectangle (also see the link to an animation displaying this at the bottom of Figure 2).

Figure 2. *Starting the Golden Spiral within a Rectangle Using Euclid's Formula*



* The black rectangle is the golden proportion to the blue rectangle.
* I may need help with finding permission to use the pictures inserted that were not drawn by me. Also see https://qph.cf2.quoracdn.net/main-qimg-93ccb33bb44fa2660bc8aaaccae98278.

If the Divided Line, however, is the Golden Ratio, this opens other problems. For example, this brings us back to the "problem of the irrational" because when using irrational numbers, they cannot be represented by one number (Benjafield 2005, p. 6). It is unclear whether rational or irrational numbers should be used in its construction (Balashov 1994, Benjafied 2005, Personal communication C. Hayes November 2010). Perhaps irrational numbers *are* to be used when constructing the Divided Line mathematically. But unfortunately, using an irrational number, like phi, will produce an answer that is always slightly off because it cannot be calculated or manipulated without rounding (Balashov 1994). If the Divided Line, however, is the Golden Ratio, this opens other problems. For example, using an irrational number, like phi, will produce an answer that is always slightly off because it cannot be calculated or manipulated without rounding (Balashov 1994). This might not be a problem, however, because Plato states that geometry or mathematics "are only *approximately* true of perceptible things" (University College London).This will be addressed at the end of the next section (see My response to the conundrum of infinity 'vs' finite p. 12).

The problem of infinity 'vs' the finite. Another issue with using phi to plot the Divided Line is the extended equality suggested by Sayer (1983) because this implies an infinite characteristic to the Divided Line. Dreher (1990), interprets

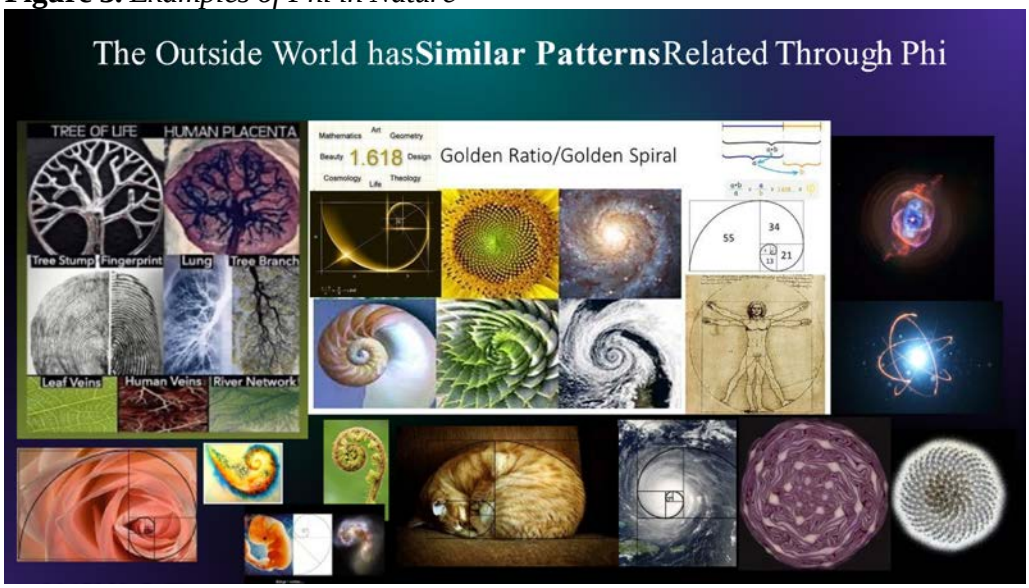
Plato's progression of knowledge as just this, as a never-ending attaining where "any cognitive success achieved by the mind intensifies the passion for further inquiry" (pp. 159–160). This is a stance I agree with, but perhaps this is not what Plato is saying in the *Republic* because he *has* an ending to the Divided Line, that of the realm of the Highest Good. This implies attainability of the highest knowledge or that the process of knowledge can be completed (Balashov 1994). If there is a limit to knowledge, then this means that the Line is not infinite and may not be represented by the phi sequence or the Golden Ratio. Plato himself states:

> Now in reasoning about all these things, a man might question whether he ought to affirm the existence of an infinite diversity of Universes or a limited number; and if he questioned aright he would conclude that the doctrine of an infinite diversity is that of a man unversed. (Plato in Timaeus, 55c)

Therefore, Plato does seem to say that the Divided Line is complete and not infinite. This throws a wrench in the Divided Line as Phi theory, but Plato only discusses "rational intuition (Noesis) and knowledge (Episteme)" in the *Republic* (Cornford 1965, p. 223). Perhaps this is key. When does intelligence give way to understanding (Katanóisi)? When does understanding give way to wisdom (Sophia)? Perhaps the Line *is* unfinished?

My response to the conundrum of infinity 'vs' the finite and the Divided Line is this: Even though an infinite, irrational number will produce an answer that is always slightly off because it cannot be calculated or manipulated without rounding (Balashov 1994), the concept of infinity *can* be represented by one finite symbol; ∞ for example. The irrational phi or Golden Spiral can also be neatly expressed as the pentagram (as stated earlier). Phi can also be found in all kinds of natural phenomena like weather, plants, and animals (see Figure 3). Plants and animals have a limit or a boundary to their 'bodies,' yet the Golden ratio can be seen in their construction and design. The problem of infinite/finite is here too because plants, animals, and weather patterns are not infinite, but eventually dissipate or die. However, then the next plant, animal or weather pattern comes along with the dimensions of phi…This pattern seems to go on until infinity. Here is the problem of binaries and binaristic thinking again because the universe is full of both finite and infinite characteristics, not just one or the other. Therefore, I argue that infinity and the finite are not mutually exclusive concepts, thus resolving the binary of infinity 'vs' the finite.

Figure 3. *Examples of Phi in Nature*



Mathematics itself is the perfect example of how infinity and the finite are interconnected and how they interact. Maths lie await in potentiality until a human mind manifests it into the finite material world. Mathematics, therefore, shows that not all things can be reduced to a physical explanation (take that Aristotle)! To bring the argument between infinity 'vs' finite to a close, Cantor proved conclusively, using infinite set theory, that "there are different orders of infinity. There is the order of infinity of the rational numbers, and there is another order of infinity that characterizes all the real numbers" (as cited in Aczel 2000, p. 116). He even postulated that one was more ontologically superior to another and even suggested that there might be another order of infinity between these (Aczel 2000). Therefore, infinity(ies) have boundaries. The human mind being another example, as well as the pentagram, icosahedron, ∞, Φ, π, √2, etc. Balashov (1994) may conclude that there is no "textual evidence" that the Divided Line is the Golden Section, making it a "no-go" (p. 294), but I say if we read between the lines, we can find the spiral. In conclusion, if we can resolve all the binaries that come about while plotting the line, then it is possible that the Divided Line could be the Golden Spiral.

Lack of Historical Connections between the Divided Line & the Golden Ratio

Balashov (1994) also concluded that there is a lack of historical evidence that Plato had any "acquaintance" with the Golden Spiral at the time he was writing the *Republic* (p. 294). Resolving this, will be the last inquiry of this paper. Going back to Plato's Solids or polygons and their mathematical relationships as the "key to the physics of the cosmos" (as cited in Meisner 2012, para. 4), why would Plato not reveal specifically, that the Divided Line is the Golden Spiral? Wheeler (2005) states that "The Divided Line symbolism of the Pythagoreans (of which Plato only parrots in the Republic) is missed by altogether most (if not all) 'Platonists' who

fail to see the root meaning to be gleaned from the unity and proportions of totality" (p. 4). Therefore, because Plato only repeated what the Pythagoreans said about the proportions and did not specify the connection to the Divided Line, the connection between them went right over the heads of the Platonists.

One obvious answer to why the search for the connection between Plato and phi has not been found is that the Golden Spiral, Section, Ratio, or Mean was not called this at the time Plato lived (Meisner 2012). It was not even referred to as phi until the 1800s, when Mark Barr used it to symbolize the Golden Ratio (Mann 2019). Therefore, the lack of historical connections could be that of semantics, or due to the fact that there was no name for it in Plato's time.

My answer, however, to why the connection between the Divided Line and the Golden Spiral was not obvious historically is because of politics. To explain this, I will need to go back to what Plato said in the Allegory of the Cave.

In the Allegory of the Cave section in his book the *Republic*, Plato states that:

> [W]e must conclude that education is not what it is said to be by some…the entire soul
> must be turned away from this changing world until its eye can bear to contemplate reality and that supreme splendour which we have called the Good…There is nothing wrong with… the power of vision, but it has been forced into the service of evil, so that
> the keener its sight, the more harm it works" (Cornford 1965, pp. 232–233).

Plato here is claiming that we are following or living a life that is based on a 'meaningless illusion;' influenced by those who are leading us astray from true reality. He believes we have not been educated properly regarding the truth (Johnson and Reath 2007). Plato seems to say that everything we are taught is based in falsehood and lies.

In the Allegory of the Cave, Plato discusses the puppet people. These are the ones holding the objects that get reflected onto the cave wall. The puppet people are the gatekeepers of knowledge. These gatekeepers do all they can to keep the truth hidden. There are a couple quotes from the Gnostic Gospels that say what Plato is trying to say regarding how we are taught. This is what Gnosticism would say about what we are taught in the illusion, and it helps explain the role of the 'puppet people.' The *Gospel of Philip* states:

> The rulers wanted to fool people, since they saw that people have a kinship
> with what is truly good. They took the names of the good and assigned them
> to what is not good; to fool people with names and link the names to what is not good…
> For, they wished to take free people and enslave them forever (Meyer 2005, p. 52).

There is a similar passage in the *Gospel of Thomas* 39 that states:

> The Pharisees and the scholars have taken the keys of knowledge and have hidden them. They have not entered, nor have they allowed those who want to enter to do so (Meyer 2005, p.14).

This is what Plato is trying to say about education and how what we are taught is closely guarded. It is hard to accept that what we have come to know is based on misleading information and half-truths. So, let us explore this further regarding the 'problem of the irrational.'

When Hippasus of Metapontum, one of the students at the school of Pythagoras discovered irrational numbers in the pentagon (Fossa 2005), it was commonly understood back then, that "Hippasus was punished by the gods for having made public his terrible discovery" (von Fritz 1945, p. 260). Likewise, when Galileo discovered that the Earth went around the sun (heliocentrism), he was hounded by the Roman Catholic church for two decades (Wolf 2016). The reason Galileo was not sentenced to death was because he had powerful friends advocating for him (Wolf 2016). When Spinoza suggested the concept of pantheism, he was excommunicated because it went against Jewish Orthodoxy (Aczel 2000). These examples are only a drop in the bucket, but they are good examples of the gatekeeping of knowledge and the keeping of certain information from seeing the light.

In conclusion, Plato did not come out and say the Divided Line was actually the Golden Spiral because it did not have a name during his time, and he probably felt like he had to hide this information to avoid the 'puppet people' gatekeeping knowledge. Afterall, Socrates was sentenced to death by poisoning for corrupting the youth with his ideas. Plato witnessed this horror because wrote for Socrates. Plato most likely wanted to avoid persecution or death. Persecution and death are powerful motivators for secrecy or opaqueness. This is why the Golden Spiral has not been formally associated with the Divided Line.

Overall Conclusion

In the past, the Divided Line has not been plotted because of misunderstandings involved in binaristic thinking. The world divided into categories such as Whole 'vs' Separate, Equality 'vs' Ontological Dissimilarity, Linear 'vs' Non-linear, and Infinity 'vs' Finite is an illusion and problematic to furthering knowledge. The gatekeepers refereeing knowledge promote binaristic thinking and therefore keep learners shackled. If we could transcend the need to see the world in binaries, we can do amazing things, like Plot Plato's Divided Line (and solve the measurement problem in quantum physics, see Collapse Ontology: Implications of Quantum Physics on Research in the Social Sciences Kroeker 2019). Since the Divided Line as the Golden Spiral has not yet been falsified. It is quite probable that it is not a line in the linear sense, but rather a spiral.

References

Aczel AD (2000) *The Mystery of the Aleph: Mathematics, the Kabbalah, and the Search for Infinity*. Washington Square Press.

Balashov Y (1994) Should Plato's line be divided in the mean and extreme ratio? *Ancient Philosophy* 14: 283–295.

Benjafield JG (2005) *A history of psychology. Second edition*. Oxford University Press.

Boyer CB (1991) *A History of Mathematics.* Wiley.

Cornford F (1965) *The Republic of Plato*. Oxford University Press.

Davis AT, Altevogt R (1979) *Golden mean of the human body*. Available at: https://www.fq.math.ca/Scanned/17-4/davis-a.pdf .

Des Jardins G (1976) How to divide the divided line. *Review of Metaphysics* 29: 483–496.

Dreher JP (1990) The driving ratio in Plato's divided line. *Ancient Philosophy* 10: 159–172.

Einstein A (1956) *Out of my later years.* Random House.

Fossa JA (2005) On the pentagram as a Pythagorean emblem. *Revista Brasília de Historia da Mathematica* 6(12): 127–137.

Fett B (2006) An In-depth Investigation of the Divine Ratio. *The Mathematics Enthusiast* 3(2): 157–177.

Ghose T (2023, March 1) *What is the Fibonacci sequence?* LiveScience. Available at: https://www.livescience.com/37470-fibonacci-sequence.html.

Hawking S (1988) *A brief history of time: From the big bang to black holes*. Bantam Books.

Hemeda S, Sonbol A (2020) Sustainability problems of the Giza pyramids. *Heritage Science* 8(8): 1–28.

Johnson OA, Reath A (2007) *Ethics: Selections from classical and contemporary writers.* Edited by Thomson Wadsworth.

Mann A (2019, November 25) *Phi: The golden ratio*. LiveScience.

Meisner G (2012) *Golden Number.net.* https://www.goldennumber.net/golden-ratio-history/

Meyer M (2005) *The Gnostic Gospels of Jesus: The definitive collection of mystical gospels and secret books about Jesus of Nazareth*. HarperCollins Publishers Inc.

Plato (1925) *Plato in Twelve Volumes*, volume 9. Translated by WRM Lamb. Harvard University Press; William Heinemann Ltd. Vintage Books.

Porubský S (2023) *Golden Ratio.* Prague, Czech Republic: Interactive Information Portal for Algorithmic Mathematics, Institute of Computer Science of the Czech Academy of Sciences. Available at: https://www.cs.cas.cz/portal/AlgoMath/MathematicalAnalysis/MathematicalConstants/GoldenRatio.htm.

Sayre KM (1983) *Plato's late ontology: A riddle resolved*. Princeton University Press.

Schneider MS (1994) *A beginner's guide to the construction of the universe: The mathematical archetypes of nature, art, and science*. HarperCollins Publishers.

Sheldrake R (1988) *The Presence of the Past: Morphic Resonance and the Habits of Nature*.

von Fritz K (1945) The discovery of incommensurability by Hippasus of Metapontum. *Annals of Mathematics* 48(2): 242–264.

Wheeler G (2005) *Pythagoras, Plato and the golden ratio: The golden ratio and the pentagram in the philosophy of the Pythagoreans*. Dark Star Publications.

Wolf J (2016, December 22) *Newsroom UCLA.* Available at: https://newsroom.ucla.edu/releases/the-truth-about-galileo-and-his-conflict-with-the-catholic-church.